ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

The full non-perturbative equation for the gluon effective mass

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Jefferson Lab
12|03|12-15|03|12

Workshop on Confinement Physics

Jefferson Lab

talk
Synopsis

PT-BFM gluon mass
- PT-BFM Schwinger-Dyson series
- *B*QQ vertex
- Dynamical gluon mass generation
- PT-BFM mass equation

Two-loop contributions
- Analytic Calculations
- Numerical analysis

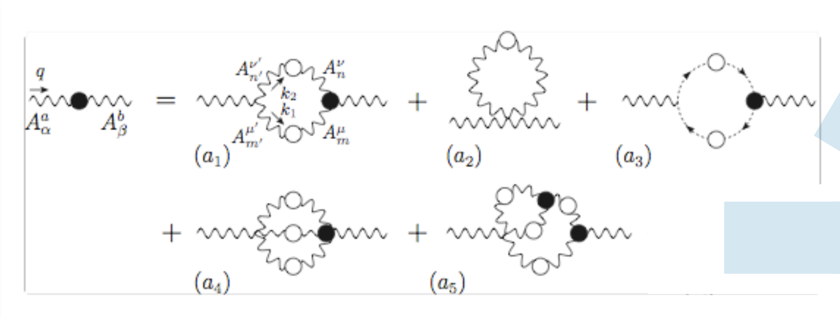
Conclusions

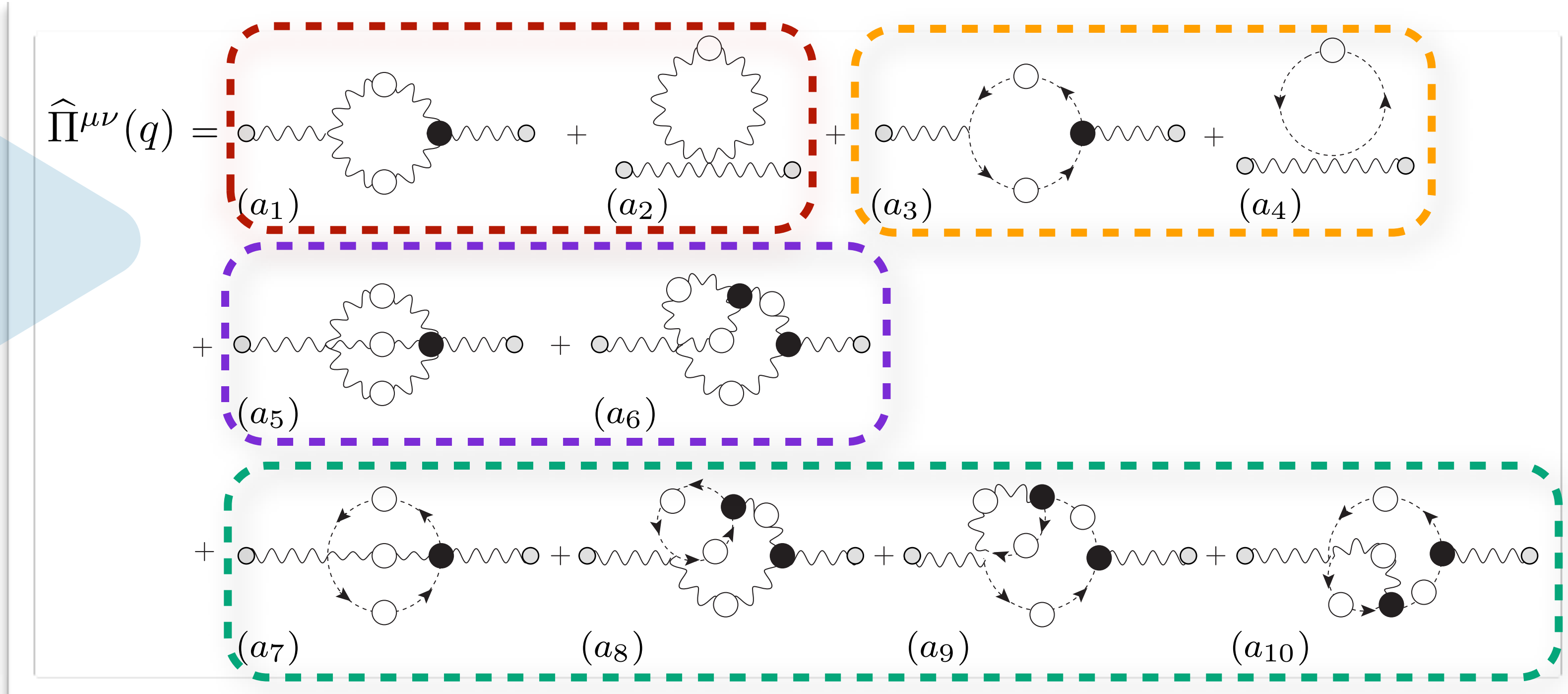
PT-BFM gluon mass

D. B. & J. Papavassiliou, Phys. Rev. D77, 061702(R) (2008)
D. B. & J. Papavassiliou, JHEP 11, 063 (2008)

PT-BFM resummed Schwinger-Dyson series

- Apply the pinch technique to the Schwinger-Dyson equation of the gluon propagator





  - graphs made out of **new vertices**, (inside **conventional props**)
  - new vertices corresponds to **BFM vertices**
    - **external gluons** dynamically converted into **background gluons**

- New Schwinger-Dyson equation has a **special structure**
  - **Subgroups** (one-/two-loop dressed gluon/ghost) are **individually transverse**

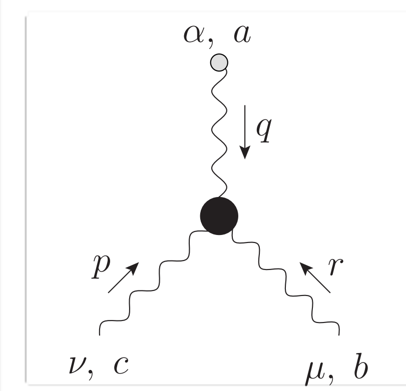
Problem

Not a genuine Schwinger-Dyson equation (**mixes pinch technique** and **conventional** propagators)

- Express the **Schwinger-Dyson eq** in terms of a background-quantum identity
  - $$\Delta^{-1}(q^2)[1+G(q^2)]^2 P_{\mu\nu}(q) = q^2 P_{\mu\nu}(q) \sum_{i=0}^{10} (a_i)_{\mu\nu}$$
    $$\widehat{\Delta}(q^2) = \left[1+G(q^2)\right]^{-2} \Delta(q^2)$$
  - In $4d$ the function G is directly related to the inverse of the ghost dressing function
    - $$F^{-1}(q^2) \approx 1 + G(q^2)$$

non-perturbative BQQ vertex

- Retaining the **full three gluon** vertex is fundamental for the **consistency** of the **whole scheme**



(differs from $\widetilde{\Gamma}$ by a tree level term singular in the Landau gauge)

- $$\widetilde{\rm I\Gamma}^{\alpha\mu\nu}(q,r,p) = \widetilde{\rm I\Gamma}^{\alpha\mu\nu}_{(\ell)}(q,r,p) + \widetilde{\rm I\Gamma}^{\alpha\mu\nu}_{(t)}(q,r,p)$$

- This ***BQQ*** vertex can be determined through a **gauge technique** procedure
  - Solving the WI/STIs yields 10 of 14 possible tensor structures $\widetilde{\rm I\Gamma}^{\alpha\mu\nu}_{(\ell)}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\ell_i^{\alpha\mu\nu}(q,r,p)$

$$q^\alpha \widetilde{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) = p^2 J(p^2) P_{\mu\nu}(p) - r^2 J(r^2) P_{\mu\nu}(r)$$

$$r^\mu \widetilde{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(r^2)\left[q^2 \widetilde{J}(q^2) P^\mu_\alpha(q) H_{\mu\nu}(q,r,p) - p^2 J(p^2) P^\mu_\nu(p) \widetilde{H}_{\mu\alpha}(p,r,q)\right]$$

$$p^\nu \widetilde{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(p^2)\left[r^2 J(r^2) P^\nu_\mu(r) \widetilde{H}_{\nu\alpha}(r,p,q) - q^2 \widetilde{J}(q^2) P^\nu_\alpha(q) H_{\nu\mu}(q,p,r)\right]$$

$$\Delta^{-1}(q^2) = q^2 J(q^2)$$

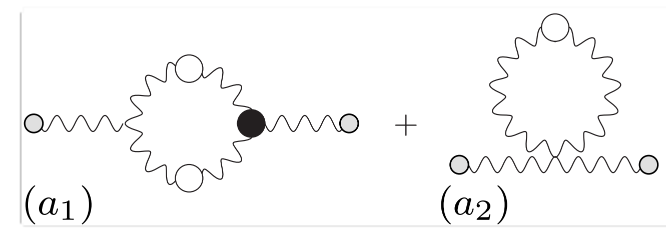
$$D(q^2) = \frac{F(q^2)}{q^2}$$

$$\widetilde{J}(q^2) = \left[1 + G(q^2)\right] J(q^2)$$

  - The system is over-constrained (9 equations for 7 independent form factors)
    - The required constraints are **provided** by the **BV identities** for the **auxiliary ghost Green's functions**
  - The 4 **totally transverse** form factors $\widetilde{\rm I\Gamma}^{\alpha\mu\nu}_{(t)}(q,r,p) = \sum_{i=1}^{4} Y_i(q,r,p)t_i^{\alpha\mu\nu}(q,r,p)$ are left **undetermined**
    - The Ys should however **vanish more rapidly** than the Xs **in the IR**

non-perturbative *BQQ* vertex

- Consider the one-loop dressed gluon diagrams



$$(a_1)_{\mu\nu} = \frac{1}{2}\, g^2 C_A \int_k \widetilde{\Gamma}^{(0)}_{\mu\alpha\beta} \Delta^{\alpha\rho}(k) \Delta^{\beta\sigma}(k+q) \widetilde{\Gamma}_{\nu\rho\sigma}$$

$$(a_2)_{\mu\nu} = g^2 C_A \left[g_{\mu\nu} \int_k \Delta^{\rho}_{\rho} + (1/\xi - 1) \int_k \Delta_{\mu\nu} \right]$$

- In the **absence** of a **dynamical mechanism**

$$\widehat{\Pi}(0) = 0$$

- The only combination of form factors surviving the $q \to 0$ limit is

$$X_4 + k \cdot (k+q) X_6 \to \frac{\Delta^{-1}(k+q) - \Delta^{-1}(k)}{(k+q)^2 - k^2}$$

- Then

$$\Pi(q) \propto \int_k k^2 \frac{\Delta(k+q) - \Delta(k)}{(k+q)^2 - k^2} + \frac{d}{2} \int_k \Delta(k) + \mathcal{O}(q)$$

$$\xrightarrow{q \to 0} \int_k k^2 \frac{\partial \Delta(k^2)}{\partial k^2} + \frac{d}{2} \int_k \Delta(k)$$

- The gluon remains massless due to the **seagull identity**

$$\int_k k^2 \frac{\partial \Delta(k^2)}{\partial k^2} + \frac{d}{2} \int_k \Delta(k) = 0$$

- The *BQQ* vertex is also such that there is **no residual** (seagull) **divergence** in the $q \to 0$ limit (**true only in the PT-BFM framework**)

BQQ vertex and
Schwinger mechanism

J. S. Schwinger, Phys. Rev. 125, 397 (1962)
J. S. Schwinger, Phys. Rev. 128, 2425 (1962)

Dyson resum

$$\Delta(q^2) = \frac{1}{q^2\left[1+\Pi(q^2)\right]}$$

Idea

If $\Pi(q^2)$ **has a pole at** $q^2 = 0$ the vector meson is **massive** even though it is massless in the absence of interactions

- Requires **massless**, **longitudinally coupled** Goldstone like **poles** $1/q^2$
- **Occur dynamically** (even in the **absence** of canonical **scalar fields**) as **composite excitations** in a **strongly coupled** gauge theory

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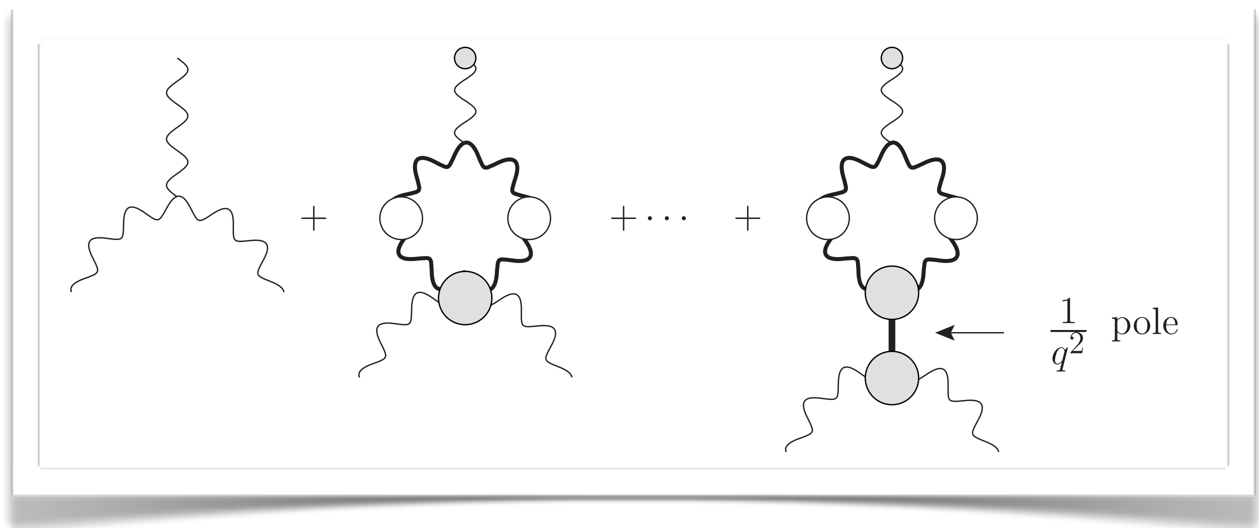
R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D8, 3338 (1973)
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

Dynamics enters through the **three-gluon vertex**

- **Longitudinally coupled massless poles**
  - **Not** a **kinematic singularity**, rather **bound states poles** non-perturbatively produced
  - **Do not appear** in the *S* matrix of the theory ("eaten-up" by the gluons to become massive)
- **Instrumental for ensuring that**

$$\Delta^{-1}(0) > 0$$



A. Aguilar, D. B. & J. Papavassiliou, Phys. Rev. D84, 085026 (2011)

DMG generation

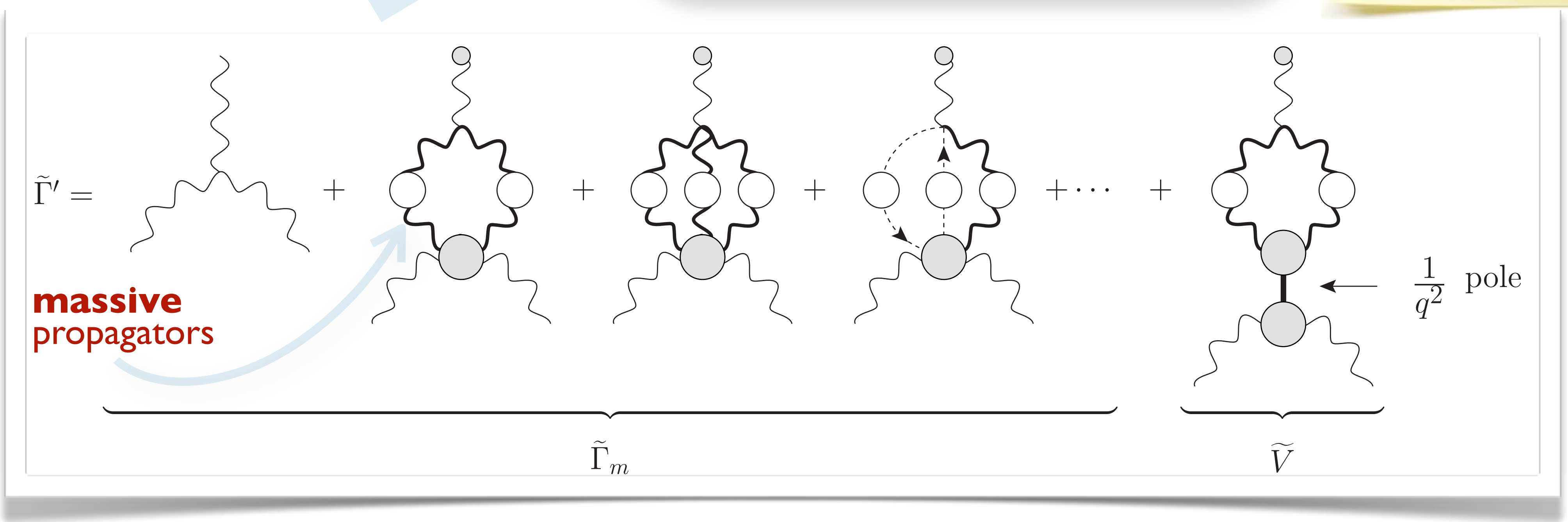
- How does dynamical gluon mass generation work in practice?
  - **Assumes** the formation of a **longitudinally coupled massless poles** that...
  - ...will **modify the vertex** of the theory...
  - ...which will lead to **massive type solutions** of the corresponding SDE
- Two levels
  - **Kinematical**

$$\Delta^{-1}(q^2) = q^2 J(q^2) \longrightarrow \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2)$$
$$J(q^2) \sim \ln q^2 \longrightarrow J_m(q^2) \sim \ln(q^2 + m^2)$$
$$q^2 J_m(q^2) \xrightarrow{q^2 \to 0} 0$$

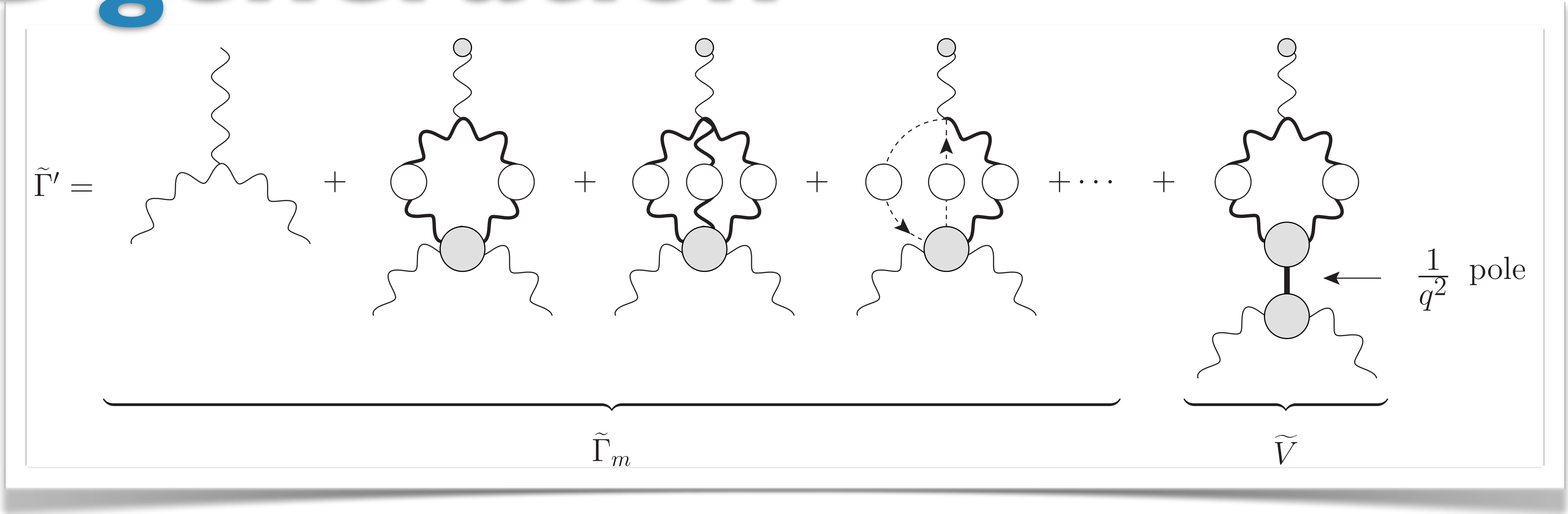
  - **Dynamical**

$$\widetilde{\Gamma} \longrightarrow \widetilde{\Gamma}' = \widetilde{\Gamma}_m + \widetilde{V}$$

V is **totally longitudinally coupled**



PT-BFM
DMG generation



- $\widetilde{\Gamma}_m$ satisfies the **same identities** as $\widetilde{\Gamma}$ with the replacement $J \longrightarrow J_m$

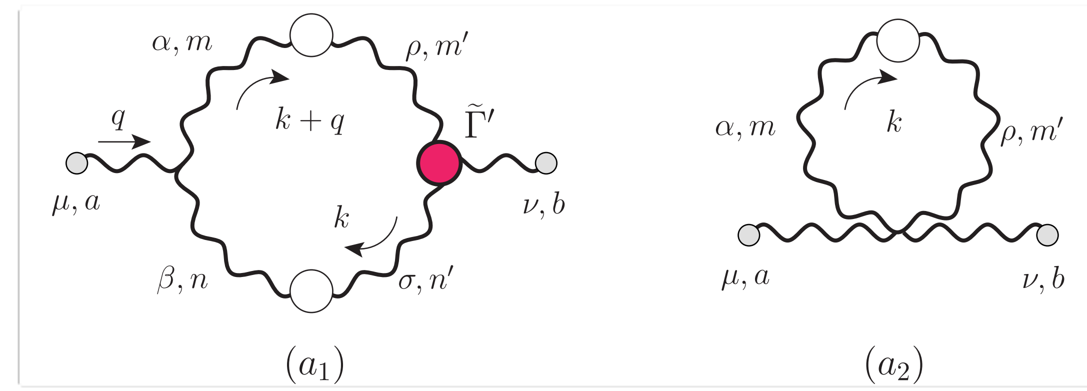
$$q_\alpha \widetilde{\Gamma}_m^{\alpha\mu\nu}(q,r,p) = p^2 J_m(p^2) P^{\mu\nu}(p) - r^2 J_m(r^2) P^{\mu\nu}(r)$$

- $\widetilde{\Gamma}'$ satisfies the **same identities** as $\widetilde{\Gamma}$ with the replacement $\Delta \longrightarrow \Delta_m$

  - $$q^\alpha \widetilde{\Gamma}'_{\alpha\mu\nu}(q,r,p) = p^2[J_m(p^2)P_{\mu\nu}(p) - m^2(p^2)] - [r^2 J_m(r^2) - m^2(r^2)]P_{\mu\nu}(r)$$

- The V and $\widetilde{V}$ vertices can be **explicitly determined** by **exploiting** the **total longitudinality** condition $PPPV = PPP\widetilde{V} = 0$ **and** the **STIs/WI** they satisfy
  - **Not needed** (in the Landau gauge) at the **one-loop** dressed level but **fundamental** at the **two-loop dressed** level

PT-BFM one-loop dressed mass equation



- Landau gauge mass equation (one-loop dressed)
  - **Dynamical equation** derived as what **survives** in the $q \to 0$ limit
  - **Seagull identity** can only happen in the $g_{\mu\nu}$ part
  - **Sufficient** to look at **what survives the limit in the longitudinal terms** (keeping in mind that the answer must be transverse)

$$m^2(q^2) = -\frac{3g^2 C_A}{1+G(q^2)}\frac{1}{q^2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} m^2(k^2)\Delta(k)\Delta((k+q)^2)\left[(k+q)^2 - k^2\right]$$

  - The $q \to 0$ limit is particularly interesting

m^2 **cannot be** a **monotonically decreasing** function

$$m^2(0) = -\frac{3}{2}g^2 C_A F(0)\int_k m^2(k^2)[k^2\Delta^2(k^2)]'$$

must reverse sign and display a sufficiently deep negative region at intermediate momenta

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- This mass equation is **different** from the one that has appeared in PRD **84**, 085026
  - Addresses a **very subtle issue** related to **taking the trace** and **completing the seagull identity** (resulting, rather ironically, in a breaking of transversality)
  - The $q \to 0$ **limit** of the equation is however the **same**

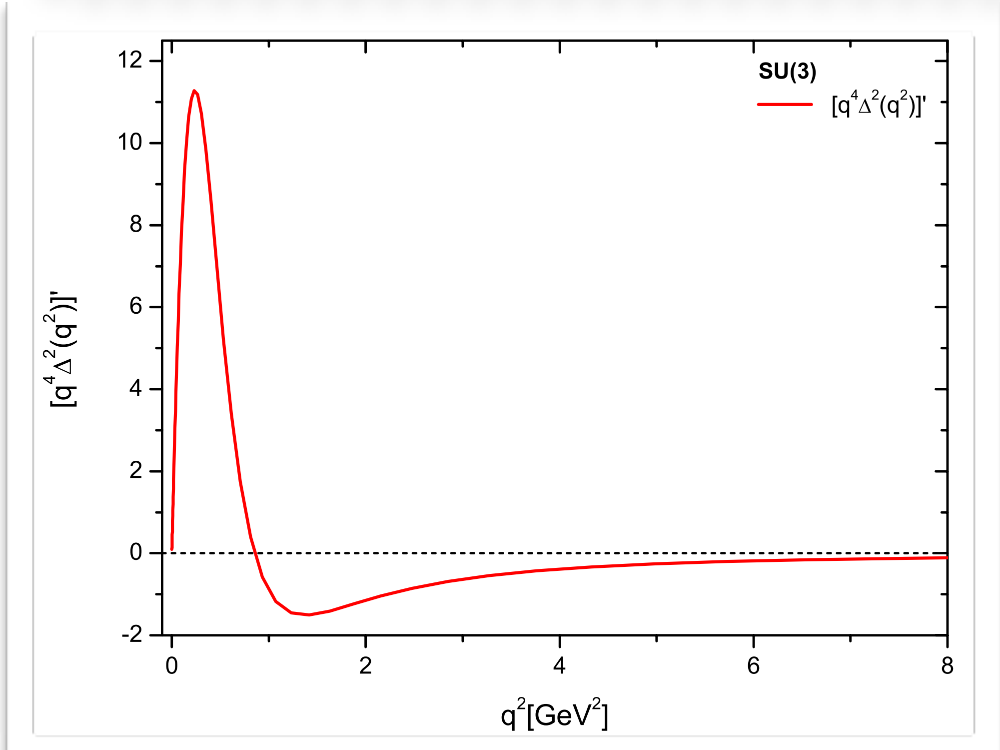
PT-BFM one-loop dressed mass equation

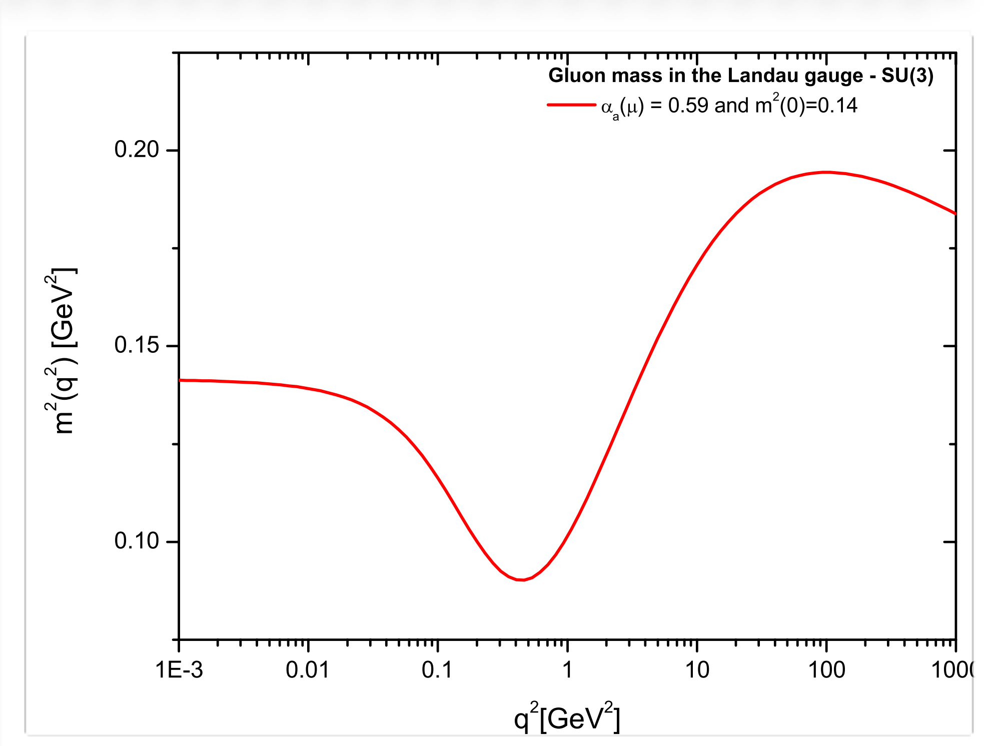
A. Aguilar, D. B. & J. Papavassiliou, Phys. Rev. D84, 085026 (2011)

- Within the standard angular approximation, the old equation yields

$$m^2(x) = m^2(0)\frac{F(x)}{F(0)} + \frac{\alpha_s C_A}{2\pi} F(x)\overline{R}(x)$$

$$\begin{aligned}\overline{R}(x) &= \frac{1}{2}\int_0^x \mathrm{d}y\, y\, m^2(y)\left(1-\frac{y}{x}\right)\Delta^2(y) + \Delta(x)\int_0^x \mathrm{d}y\, y\left(y-\frac{x}{4}\right)\frac{m^2(x)-m^2(y)}{x-y}\Delta(y)\\ &- m^2(x)\, x^2\Delta^2(x) + \frac{3}{4}\int_0^x \mathrm{d}y\, m^2(y)[y^2\Delta^2(y)]'\end{aligned}$$





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PT-BFM one-loop dressed mass equation

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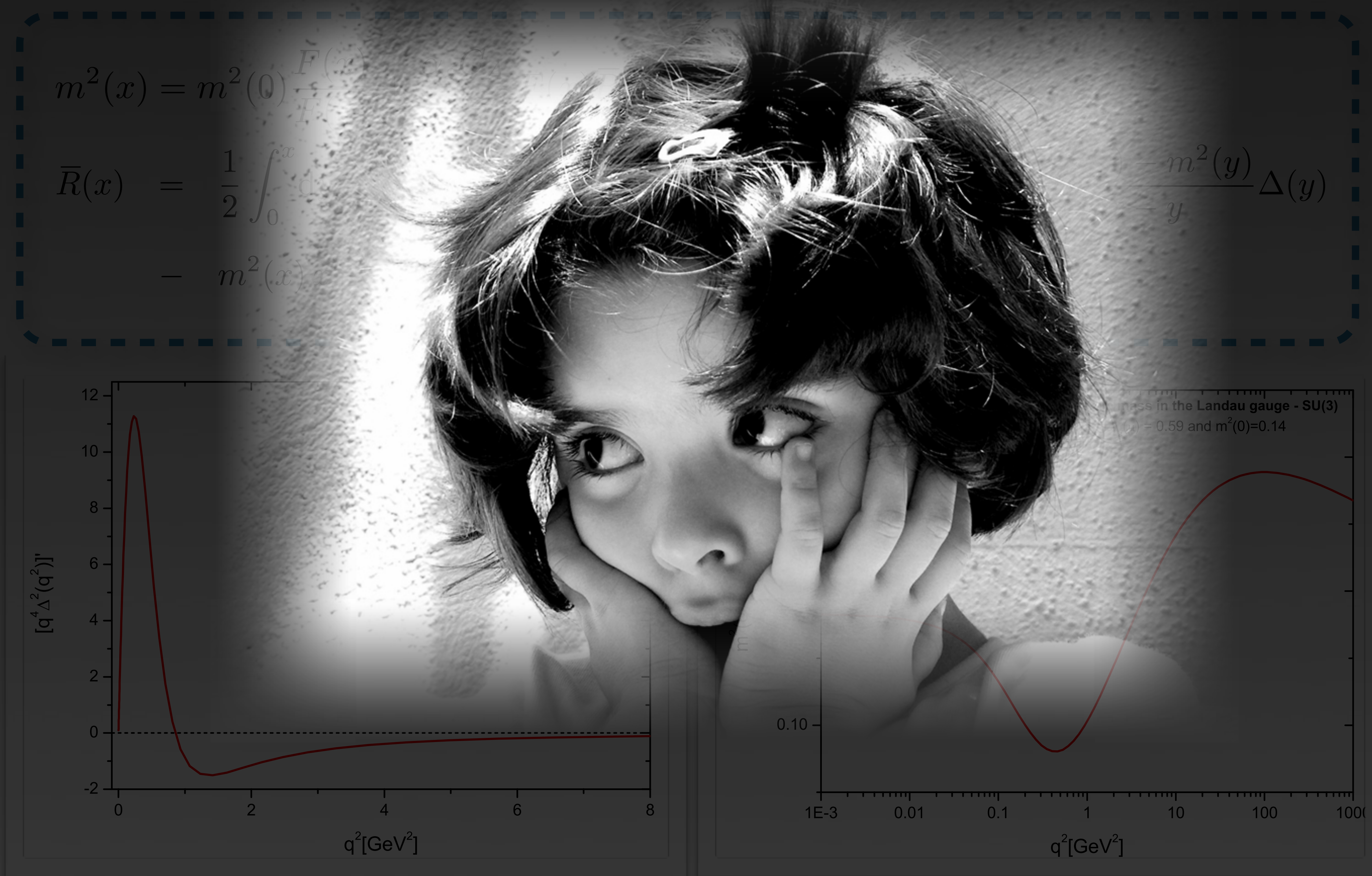
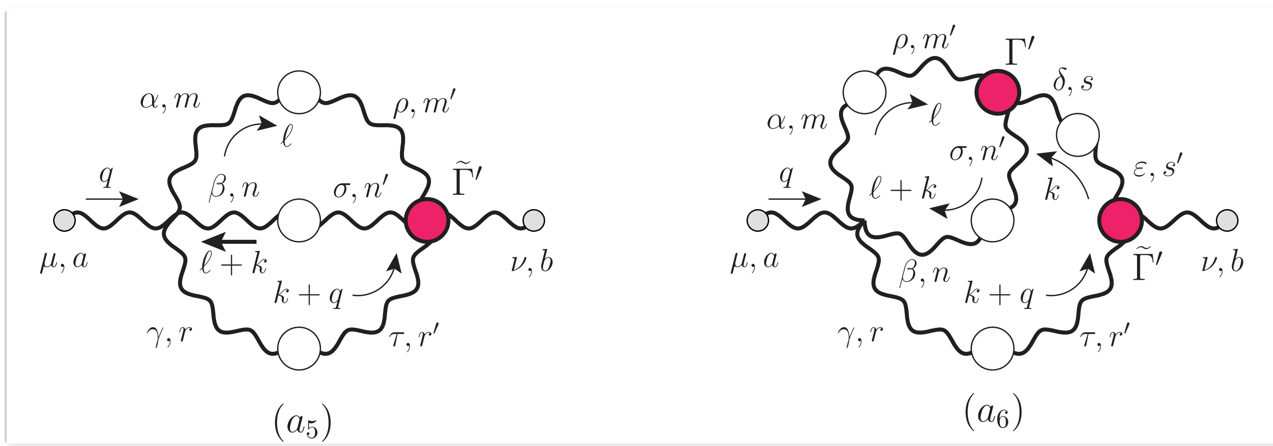


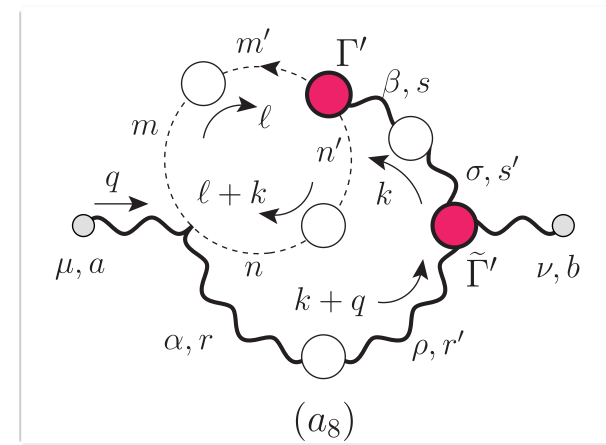
New equation, without approximations (using Chebishev polynomials)

no physical solution

Two-loop contributions

two-loop dressed diagrams





- We consider the **two-loop dressed** diagrams
  - If **ghosts** are **massless** these are the only contributions missing
- A **new ingredient** appears: $\widetilde{V}_4$ for the four-gluon vertex.
  - In principle **many new ghost Green's functions** appears due to the complicate STIs structure satisfied by the conventional four-gluon vertex
  - However in the **Landau gauge** we only need to know the contraction:

$$PPP\widetilde{V}_4 = \textit{linear combinations of } V_3$$

no additional ghost Green's function @ 2 loops

- It is therefore mandatory to explicitly determine the pole part of the three-gluon vertices $\widetilde{V}_3$ and V_3

trilinear

pole parts

- Can be determined by **solving** the **WI/STIs plus** the condition of **total longitudinality**

$$q^\alpha \widetilde{V}_{\alpha\mu\nu}(q,r,p) = m^2(r^2)P_{\mu\nu}(r) - m^2(p^2)P_{\mu\nu}(p)$$

$$r^\mu \widetilde{V}_{\alpha\mu\nu}(q,r,p) = F(r^2)\left[m^2(p^2)P_\nu^\rho(p)\widetilde{H}_{\rho\alpha}(p,r,q) - \widetilde{m}^2(q^2)P_\alpha^\rho(q)H_{\rho\nu}(q,r,p)\right]$$

$$p^\nu \widetilde{V}_{\alpha\mu\nu}(q,r,p) = F(p^2)\left[\widetilde{m}^2(q^2)P_\alpha^\rho(q)H_{\rho\mu}(q,p,r) - m^2(r^2)P_\mu^\rho(r)\widetilde{H}_{\rho\alpha}(r,p,q)\right]$$

$$P^{\alpha\beta}(q)P^{\mu\rho}(r)P^{\nu\sigma}(p)\widetilde{V}_{\beta\rho\sigma}(q,r,p) = 0$$

$$\begin{aligned}\widetilde{V}_{\alpha\mu\nu}(q,r,p) = {} & \frac{q_\alpha}{q^2}\left[m^2(r^2) - m^2(p^2)\right]P_\mu^\rho(r)P_{\rho\nu}(p) \\ & + D(r^2)\left[m^2(p^2)P_\nu^\rho(p)\widetilde{H}_{\rho\alpha}(p,r,q) - \widetilde{m}^2(q^2)P_\alpha^\rho(q)P_\nu^\sigma(p)H_{\rho\sigma}(q,r,p)\right]r_\mu \\ & + D(p^2)\left[\widetilde{m}^2(q^2)P_\alpha^\rho(q)H_{\rho\mu}(q,p,r) - m^2(r^2)P_\mu^\rho(r)\widetilde{H}_{\rho\alpha}(r,p,q)\right]p_\nu\end{aligned}$$

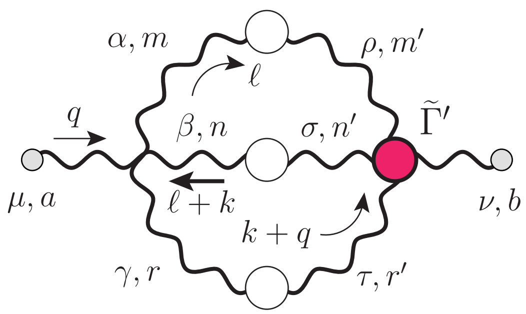
  - The same procedure yields V_3

- Luckily for $\widetilde{V}_4$ we only need

$$\begin{aligned}P^{\mu\beta}(r)P^{\nu\gamma}(p)P^{\rho\delta}(t)\widetilde{V}^{abcd}_{\alpha\beta\gamma\delta}(q,r,p,t) = {} & ig^2\frac{q_\alpha}{q^2}P^{\mu\beta}(r)P^{\nu\gamma}(p)P^{\rho\delta}(t)\left[f^{abx}f^{xcd}V_{\gamma\delta\beta}(p,t,q+r)\right. \\ & \left. + f^{acx}f^{xdb}V_{\delta\beta\gamma}(t,r,q+p) + f^{adx}f^{xbc}V_{\beta\gamma\delta}(r,p,q+t)\right]\end{aligned}$$

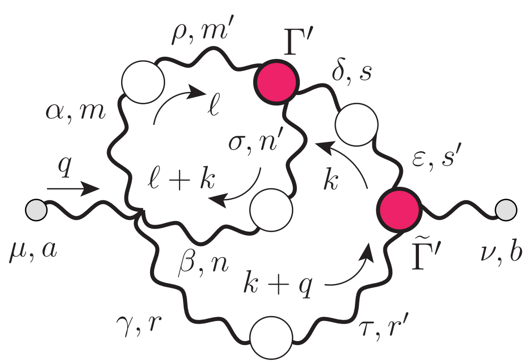
PT-BFM gluon

two-loop dressed diagrams

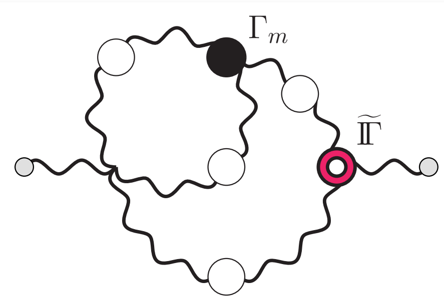


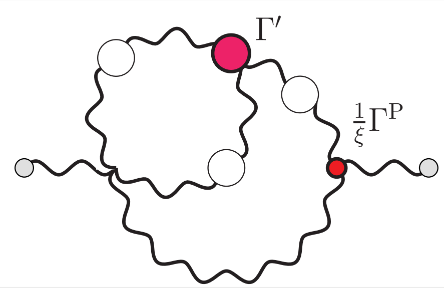
(a_5)

- The **pole part** of this diagram (surprisingly) **vanishes**
- The remaining term does not contribute to the mass equation



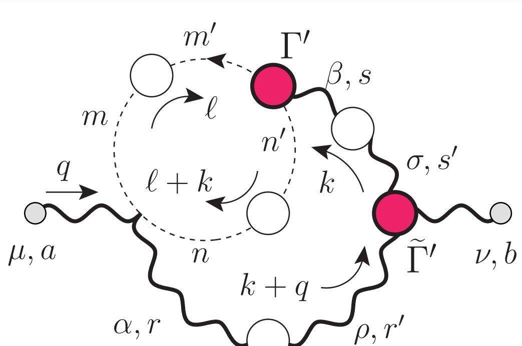
(a_6)





The pole part of this diagram is the only surviving piece

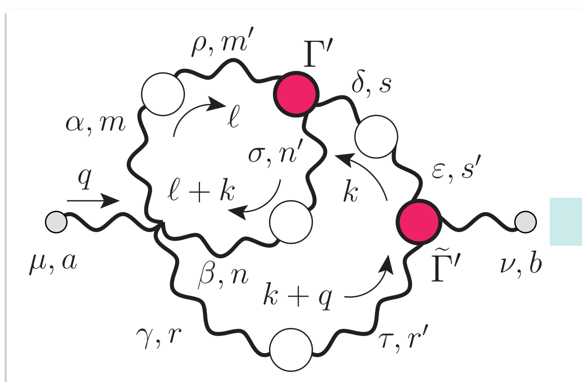
- The **pole part** of this diagram **vanishes**; the rest does not contribute



(a_8)

- The **pole part** of this diagram **vanishes**; the rest does not contribute

two-loop contribution to the **mass equation**



(a_6)

$$\frac{3}{2} i \int_k \frac{Y(k^2)}{q^2 k^2} \Delta(k)\Delta(k+q)(k \cdot q)[m^2(k) - m^2(k+q)]$$

- $$Y(k^2) = k^\alpha \int_\ell \Delta(\ell)\Delta(\ell+k) P_{\alpha\rho}(\ell) P_{\beta\sigma}(\ell+k) \mathrm{I}\Gamma^{\sigma\rho\beta}(-\ell-k, \ell, k)$$

- Add this to the (one-loop) mass equation to get (Euclidean space)

$$m^2(q^2) = -\frac{g^2 C_A}{1+G(q^2)} \frac{d-1}{q^2} \int_k m^2(k)\Delta(k)\Delta(k+q)\left[(k+q)^2 - k^2\right] - \frac{g^4 C_A^2}{1+G(q^2)} \frac{3}{2q^2} \int_k \frac{Y(k^2)}{k^2} (k \cdot q)\Delta(k)\Delta(k+q)\left[m^2(k+q) - m^2(k)\right]$$

- Take the $q \to 0$ limit, use the seagull identity and introduce spherical coordinates

$$m^2(0) = -\frac{3C_A}{8\pi} \alpha_s F(0) \int_0^\infty \mathrm{d}y\, m^2(y) \left\{ \left[1 - \frac{1}{2} g^2 C_A \frac{Y(y)}{y} \right] y^2 \Delta^2(y) \right\}'$$

two-loop contribution to the mass equation

- Calculate Y to lowest order in perturbation theory

$$Y(k^2) = k_\alpha \int_\ell \frac{1}{\ell^2(\ell+k)^2} P^{\alpha\rho}(\ell) P^{\beta\sigma}(\ell+k) \Gamma^{(0)}_{\sigma\rho\beta}(-\ell-k,\ell,k)$$
$$= \frac{1}{(4\pi)^2} k^2 \left[\frac{15}{4}\left(\frac{2}{\epsilon}\right) - \frac{15}{4}\left(\gamma_{\mathrm{E}} - \log 4\pi + \log\frac{k^2}{\mu^2}\right) + \frac{33}{12}\right]$$

- Renormalize subtractively @ μ

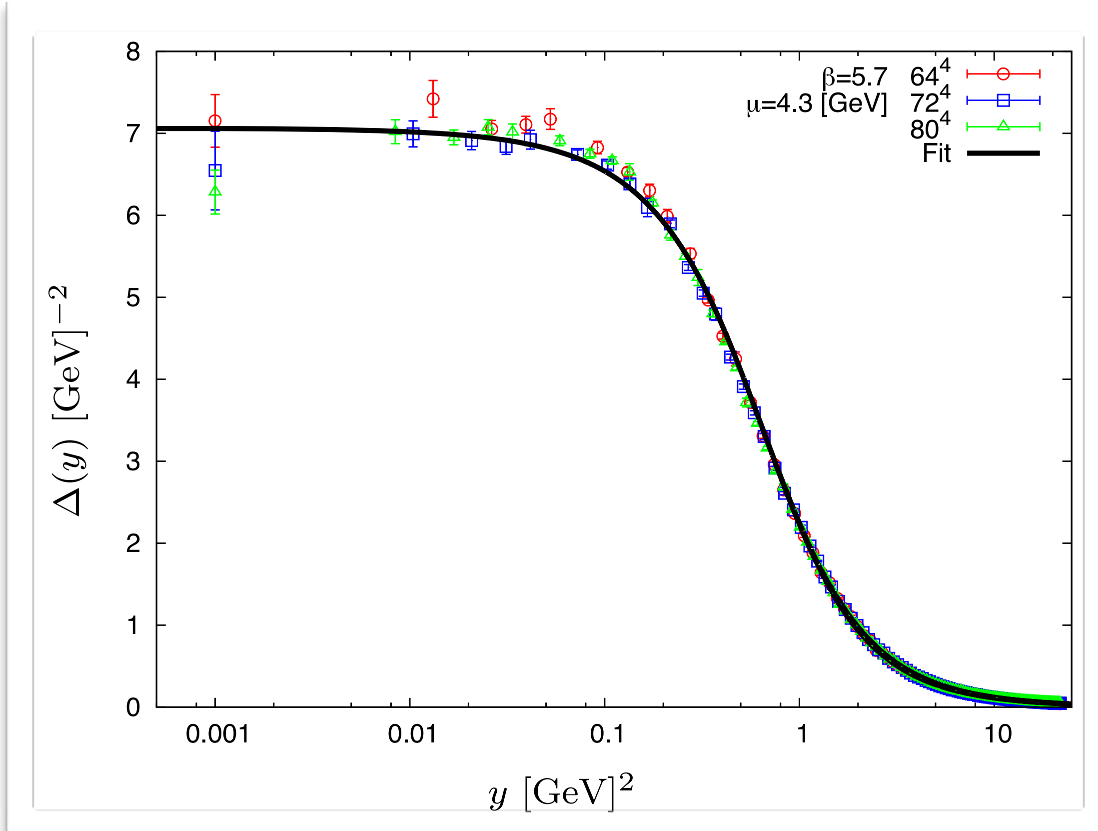
$$Y_{\mathrm{R}}(k^2) = -\frac{1}{(4\pi)^2}\frac{15}{4}k^2 \log\frac{k^2}{\mu^2}$$

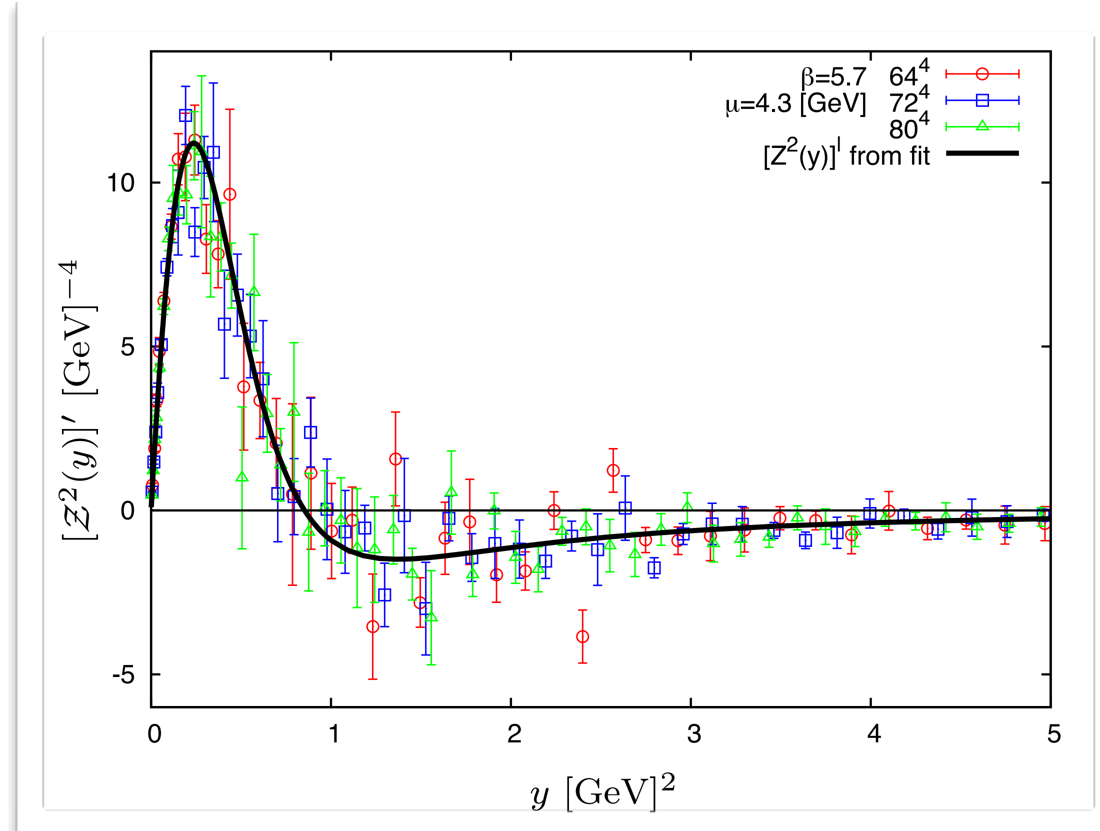
- Substitute to the mass equation to get the final equation

$$m^2(0) = -\frac{3}{8\pi}\alpha_s C_A F(0) \int_0^\infty \mathrm{d}y\, m^2(y) \underbrace{\left[\left(1 + \frac{15 C_A}{32\pi}\alpha_s \log\frac{y}{\mu^2}\right)\overbrace{\mathcal{Z}^2(y)}^{y^2\Delta^2(y)}\right]'}_{\mathcal{K}(y)}$$

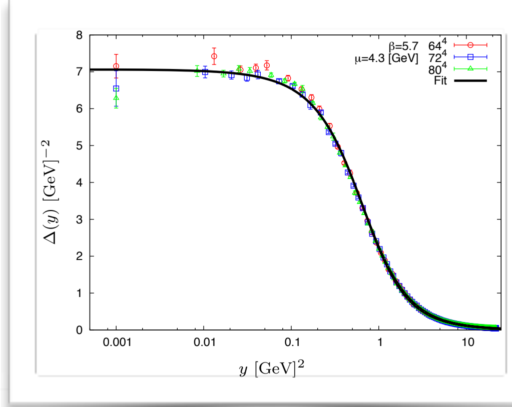
(quenched) numerical analysis

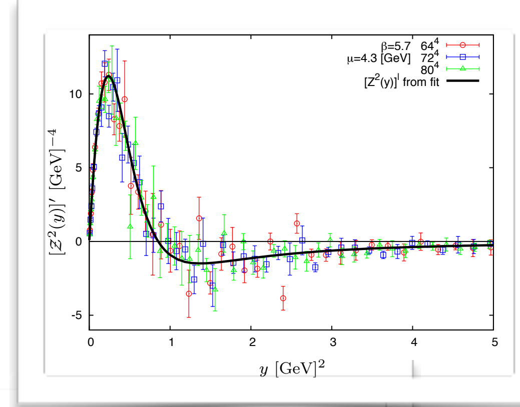
I. L. Bogolubsky *et al.*, Phys. Lett. B676, 69 (2009)

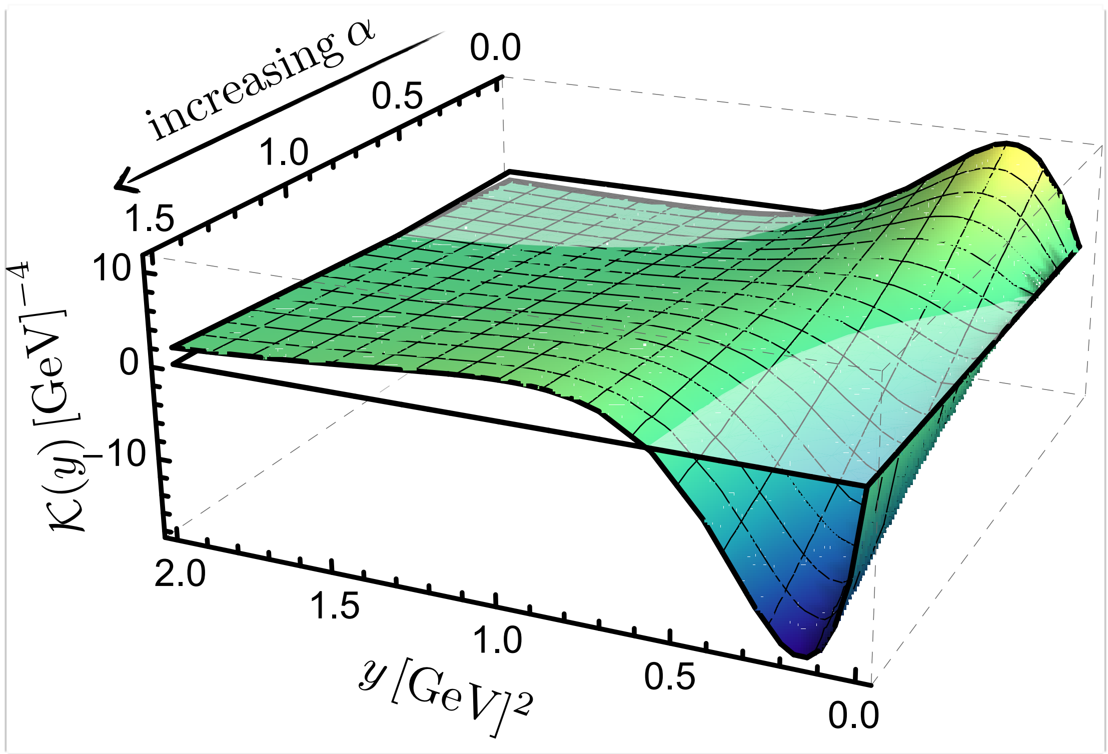


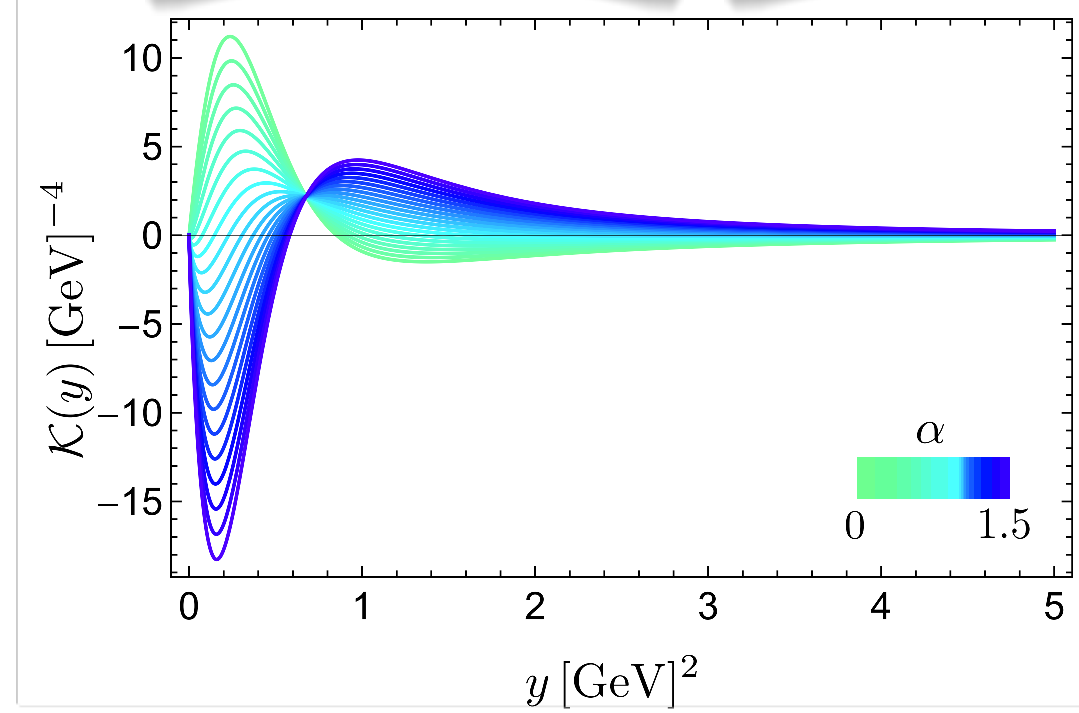


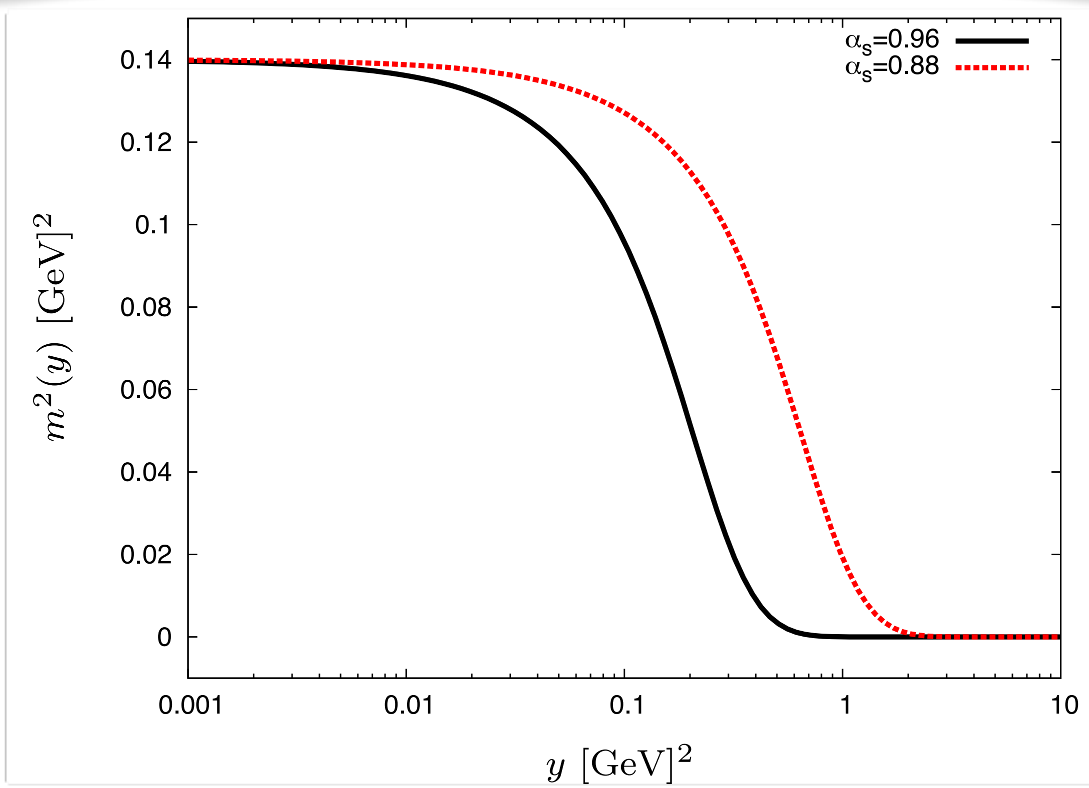
(quenched)
numerical analysis











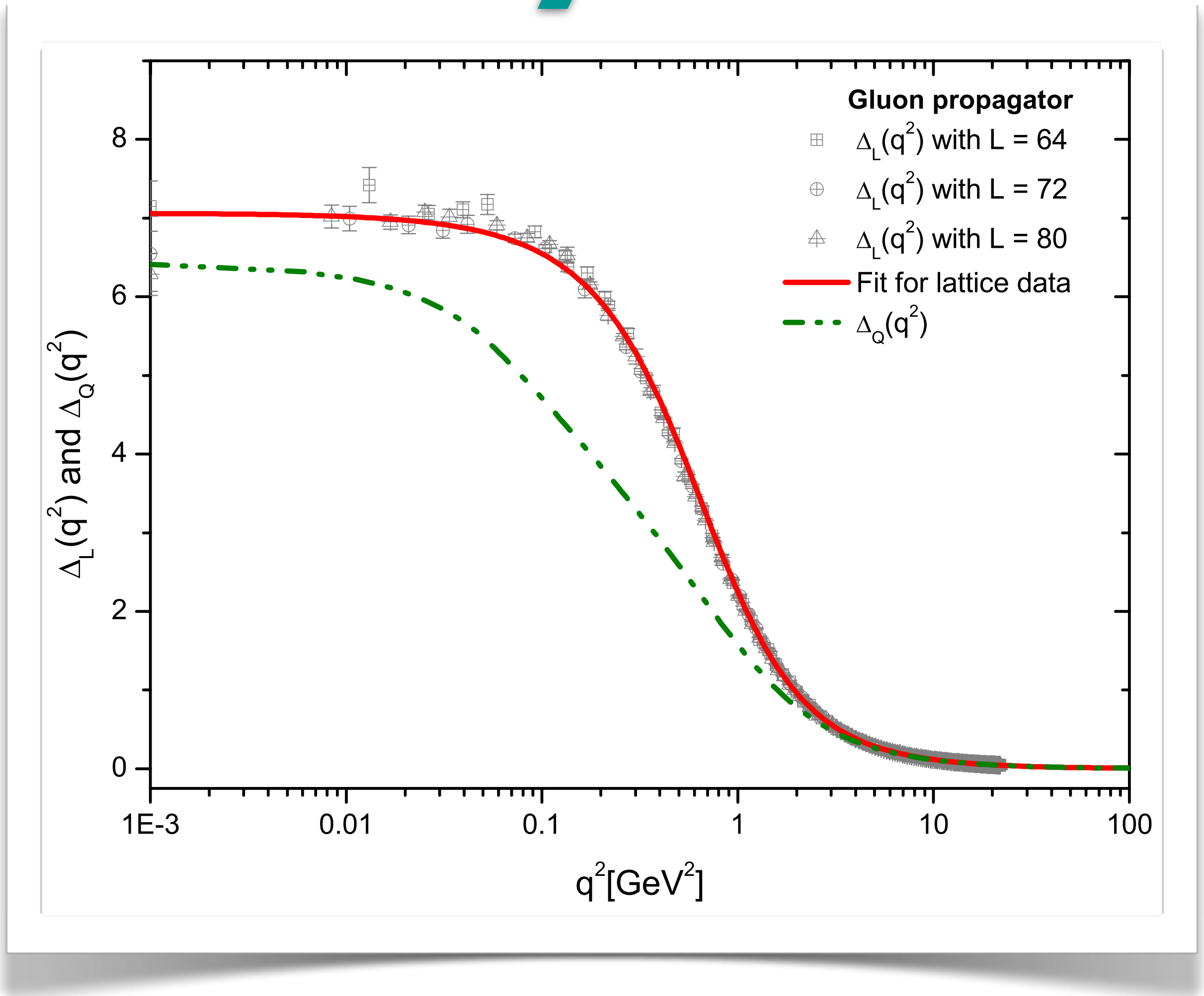
- **Solutions** of the **integral condition** for the quenched $SU(3)$ mass found in lattice simulations

$$m^2(0) \approx 0.14$$

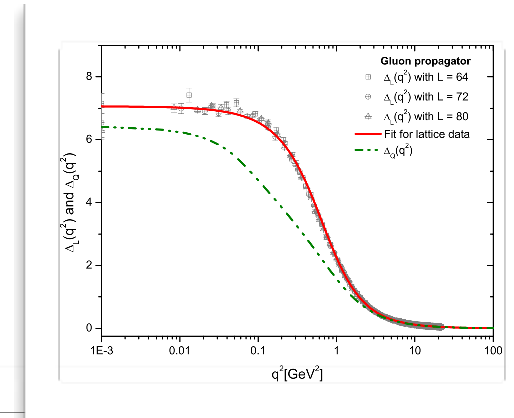
- Back to **monotonically decreasing masses**
- **Natural** notion of a **critical coupling**
  - QCD has to be "**strong enough**" **to dynamically generate a gluon mass**
  - For the unquenched lattice case we find

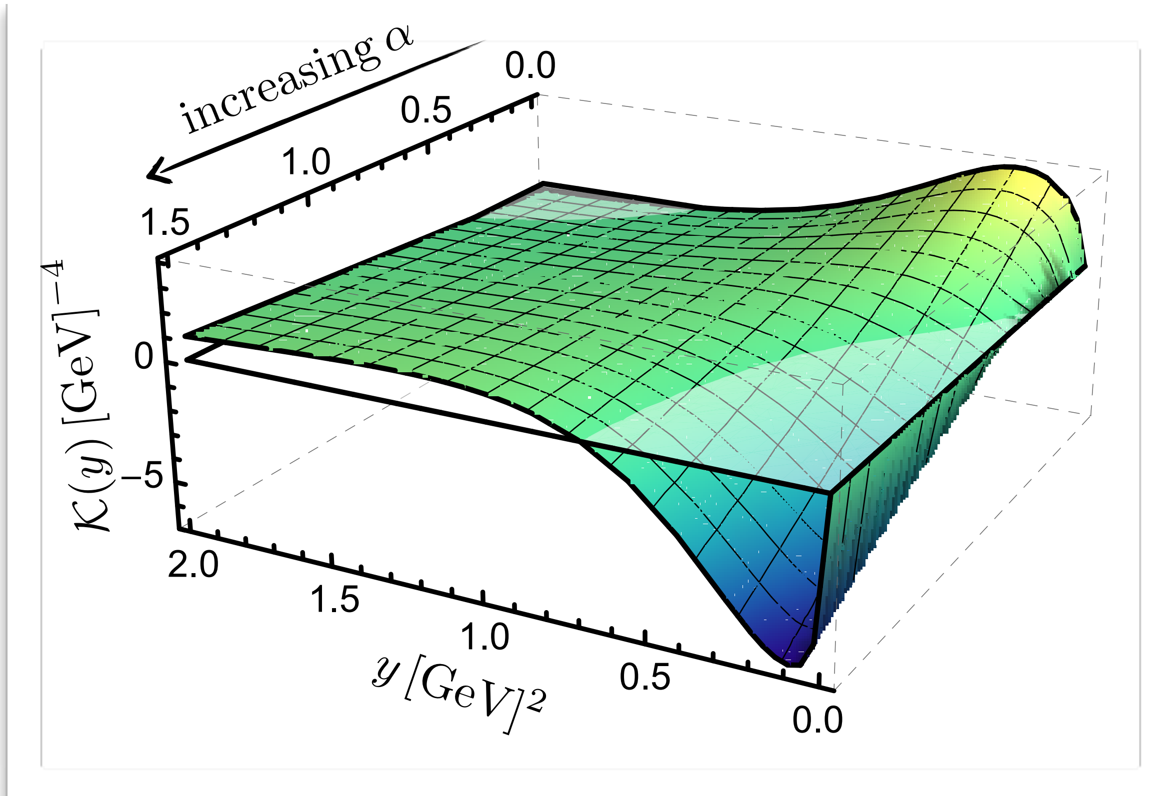
$$\overline{\alpha}_s \approx 0.83$$

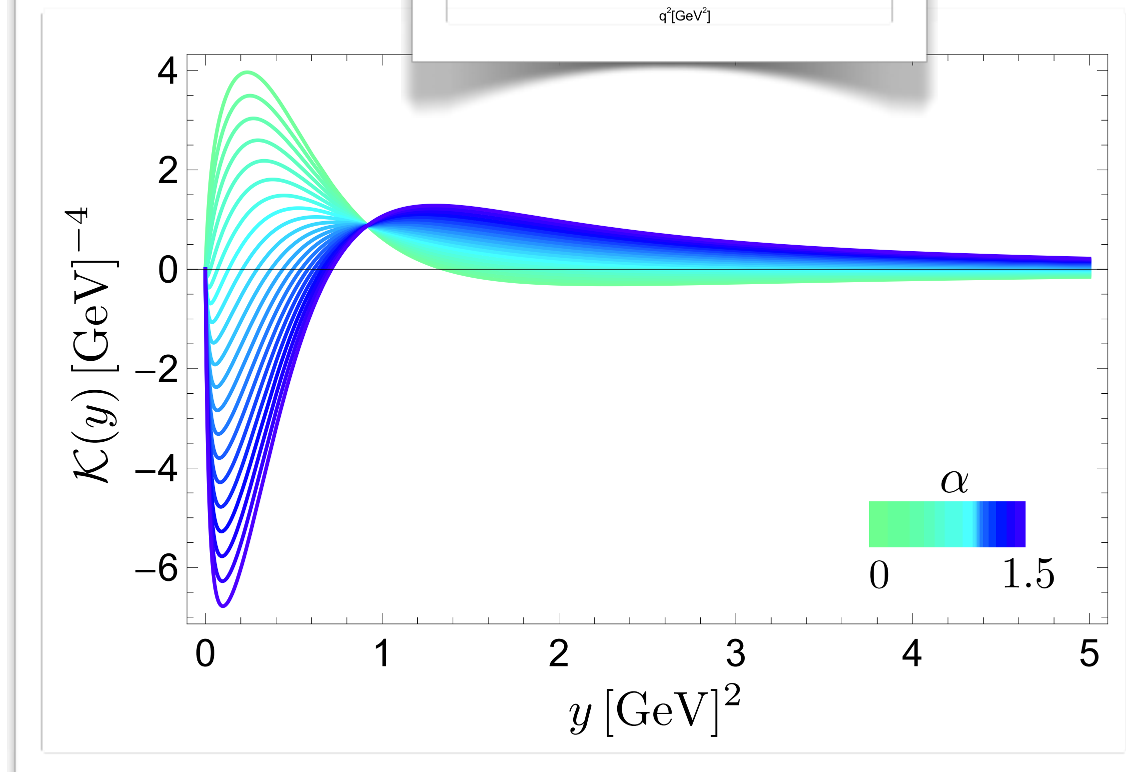
(unquenched)
numerical analysis

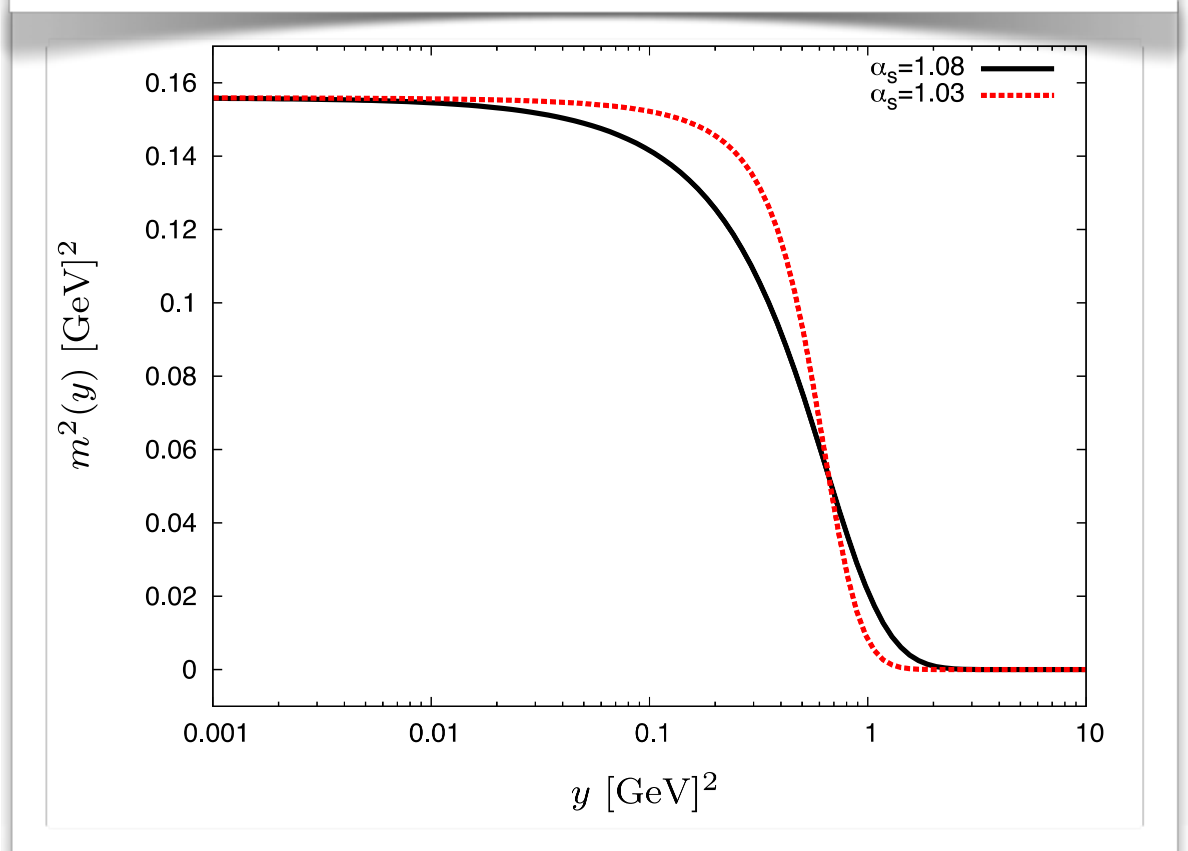


(unquenched)
numerical analysis





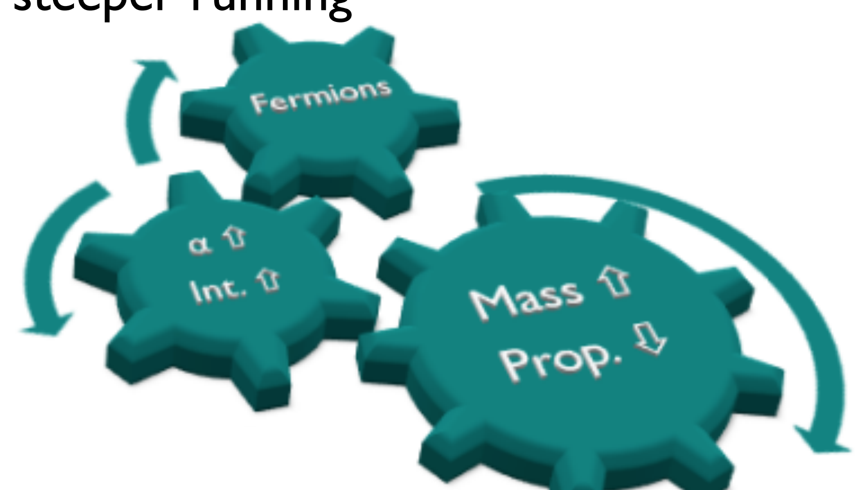




- **Solutions** of the **integral condition** for the unquenched $SU(3)$ mass found in *SDE studies* (n_f=2)

$$m^2(0) \approx 0.156$$

- Solutions requires a bigger (~20%) coupling and, possibly, a steeper running

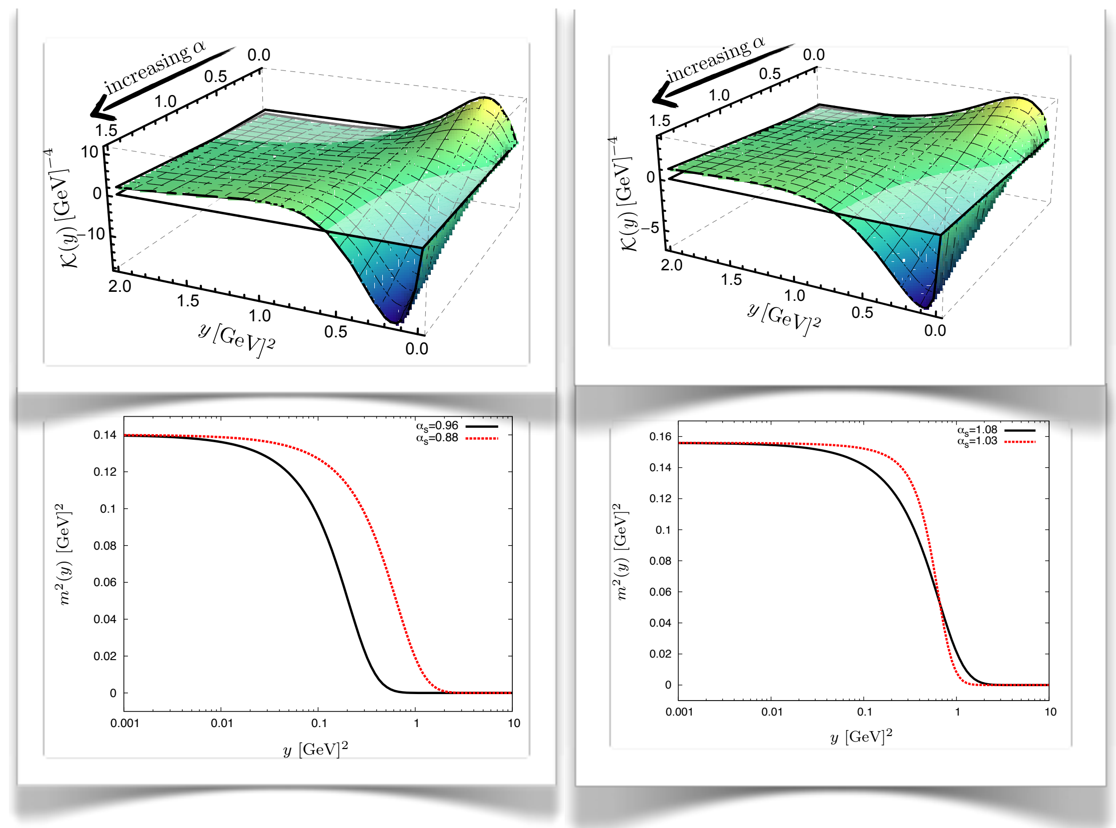


Conclusions & outlook

conclusions & outlook

- Adding **two-loop dressed diagrams** contribution to the mass equation is **necessary** in order to get **physical sensible solutions**

- Dramatic change wrt the one-loop dressed case
  - Above a certain value of the coupling the **kernel "flips"**
  - Solutions are **monotonically decreasing**
  - Notion of a **critical coupling**



- Beyond lowest order calculation for Y, using the full BQQ vertex

- Numerical study of the full equation

the end