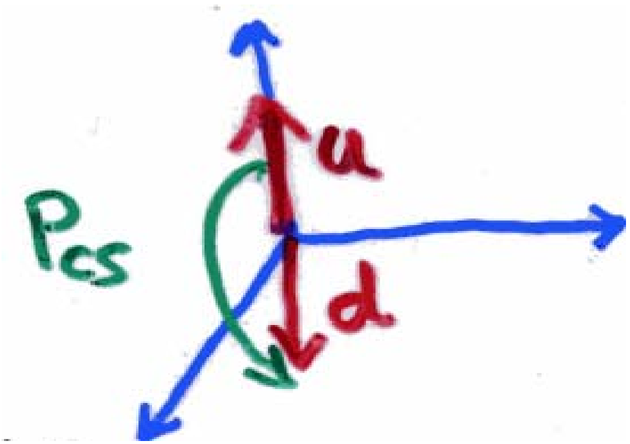
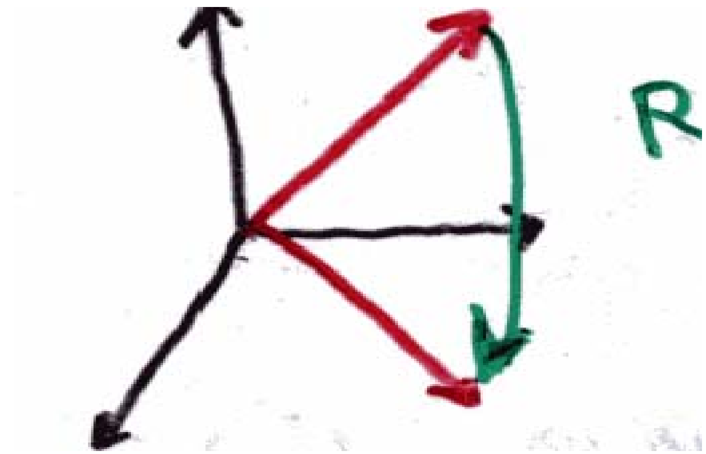
Charge Symmetry and Flavor Symmetry

G. A. Miller, UW

- Charge symmetry (u, d)
  - Hadronic masses
  - Relations for cross sections
- Flavor (Unitary) symmetry (u, d, s)
  - SU(3) relations for cross sections, magnetic moments
  - Effective field theory
  - Relations for cross sections, ΞΞ interactions-strange nuclei

Charge Symmetry: QCD if $m_d = m_u$, L invariant under u↔ d





SI invariant under $\begin{pmatrix} u' \\ d' \end{pmatrix} = U \begin{pmatrix} u \\ d \end{pmatrix}$

Isospin invariance $[H,T_i]=0$, charge independence of nuclear forces, CS does NOT imply CI

Example- CS holds, charge dependent strong force

- $m(\pi^+) > m(\pi^0)$, electromagnetic
- causes charge dependence of $^1S_0$ scattering lengths
- no isospin mixing

One π Exchange Potential is charge dependent, isospin conserving force

Example: isospin and cross sections, IsoBrk is not CSB

- CI $\rightarrow$ $\sigma(pd \rightarrow {}^3He\pi^0)/\sigma(pd \rightarrow {}^3H\pi^+)=1/2$
- NOT related by CS
- Isospin CG gives 1/2
- deviation caused by isospin mixing
- near η threshold- strong η N$\rightarrow\pi$ N contributes, not $\eta\rightarrow\pi$ mixing

Example –proton, neutron

proton (u,u,d) neutron (d,d,u)

m_p=938.3, m_n=939.6 MeV

If $m_p=m_n$, CS, $m_p<m_n$, CSB

CS pretty good, CSB accounts for $m_n>m_p$

$(m_p-m_n)_{Coul} \approx$ 0.8 MeV>0, also gluon hyperfine

m_d-m_u>0.8 +1.3 MeV

nucleon mass difference is CSB NOT IB

updated version of Miller Nefkens Slaus

Hadronic mass differences $d-u$

hadron	I, J^P	quarks	mass (MeV)	$d-u$ mass difference (MeV)
K^0	1/2,0^-	$d\bar{s}$	497.648±0.022	3.972±0.027
K^+		$u\bar{s}$	493.677±0.016	
K^{*0}	1/2,1^-	$d\bar{s}$	896.10 ±0.027	4.44 ±0.4
K^{*+}		$u\bar{s}$	891.66 ±0.026	
D^-	1/2,0^-	$d\bar{c}$	1869.4 ±0.05	4.78 ±0.10
$\bar{D}^0$		$u\bar{c}$	1864.6 ±0.5	
D^{*-}	1/2,0^-	$d\bar{c}$	2010. ±0.5	3.3±0.7
$\bar{D}^{*0}$		$u\bar{c}$	2006.7 ±0.5	
n	1/2,$1/2^+$	ddu	939.56536±0.00008	1.293317±0.000005
p		udu	938.27203±0.00008	
Σ^-	1,$1/2^+$	dds	1197.449±0.030	4.87±0.035
Σ^0		uds	1192.642±0.024	
Σ^0	1,$1/2^+$	dds	1192.642±0.030	3.27±0.07
Σ^+		uus	1189.37±0.07	
Σ^{*-}	1,$3/2^+$	dds	1387.2 ±0.5	3.5 ±0.5
Σ^{*0}		uds	1383.7 ±0.1	
Σ^{*0}	1,$3/2^+$	dds	1383.7 ±0.5	0.9 ±0.4
Σ^{*+}		uus	1382.8 ±0.4	
Ξ^-	1/2,$1/2^+$	dss	1321.31 ±0.13	6.48±0.24
Ξ^0		uss	1314.832±0.20	
$\Xi*^-$	1,$3/2^+$	dss	1535.0 ±0.6	3.2 ±0.7
Ξ^+		uss	1531.8 ±0.3	

- **All CSB arises from $m_d > m_u$ & electromagnetic effects–**

  Miller, Nefkens, Slaus, 1990
- **All CIB, that is not CSB, is dominated by fundamental electromagnetism**
- **CSB studies quark effects in hadronic and nuclear physics**

Importance of m_d-m_u

- >0, p, H stable, heavy nuclei exist
- too large, $m_n > m_p$ + Binding energy, bound n's decay, no heavy nuclei
- too large, Delta world: $m(\Delta^{++}) < m(p)$ also $m(\Sigma^{+}) < m(p)$, strange world
- <0 p decays, H not stable
- Existence of neutrons close enough to proton mass to be stable in nuclei a requirement for life to exist, Agrawal et al(98)

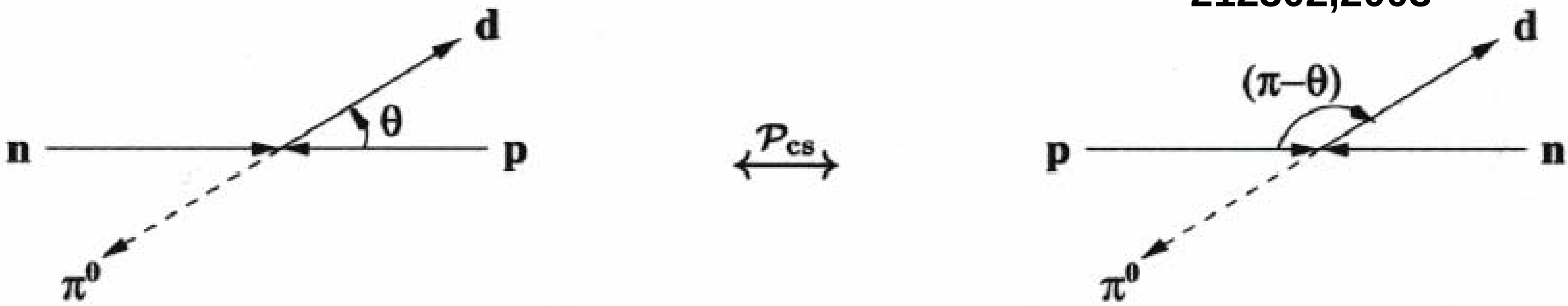
Importance of m_d-m_u

- >0, influences extraction of $\sin^2\theta_W$ ν -nucleus scattering, ½ of NUTEV anomaly
- ≈ 0, to extract strangeness form factors of nucleon from parity violating electron scattering
- hadronic vacuum polarization in g-2 of μ

$np \to d\pi^0$

Opper et al

Phys.Rev.Lett.91: 212302,2003



$$A_{fb}(\theta) \equiv \frac{\sigma(\theta)-\sigma(\pi-\theta)}{\sigma(\theta)+\sigma(\pi-\theta)} \text{ in cm}$$

$$A_{fb} \neq 0 \leftrightarrow \text{CSB}$$

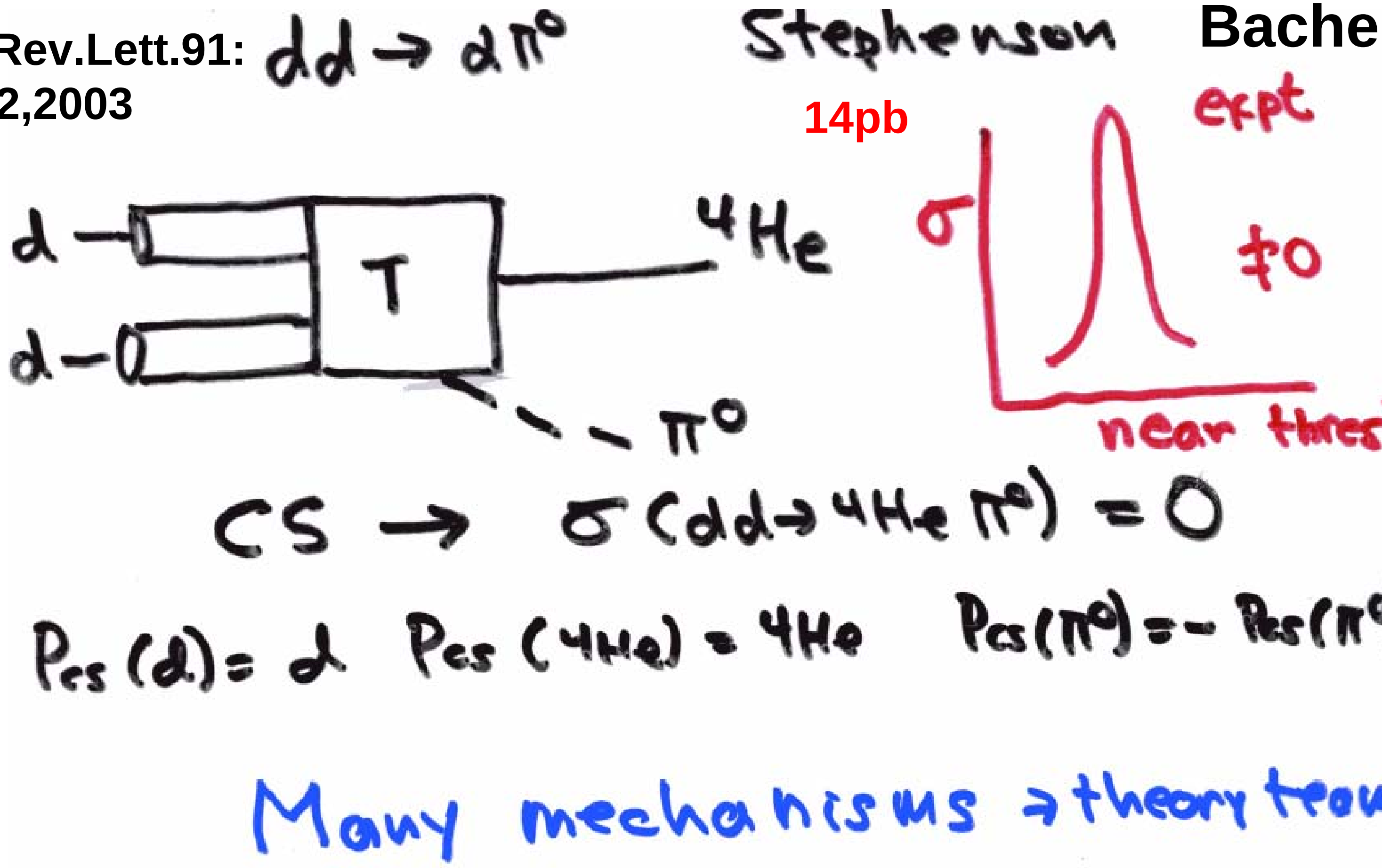
17.2 ± 8 (stat) ± 5.5 (sys)] 10^{-4},

Size is natural ≈ EFT

Phys.Rev.Lett.91: 142302,2003

Bacher

14pb



Gardestig, Nogga, Fonseca, van Kolck, Horowitz, Hanhart , Niskanen

plane wave, simple wave function, $\sigma = 23$ pb

GOAL of CSB in $np \rightarrow d\pi^0$, $dd \rightarrow {}^4He\pi^0$

- Use effective field theory (χ PT) to extract m_d-m_u at hadronic scale
- So far team of theorists has shown size of effects is natural

Flavor symmetry

- $m_u = m_d < m_s$ breaking is only in mass matrix
- Unitary symmetry SU(3)
- EFT –chiral Lagrangian

SU(3)

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \widehat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \widehat{U} = I - i \sum_{i=1}^{8} \lambda_i \theta_i$$

u d s

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Isospin

u d s

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \; \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

u d s

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

u d s

3 independent SU(2) subgroups, (λ_1,λ_2)(u,d) Isospin, (λ_4,λ_5) (u,s) Vspin,

(λ_6,λ_7) (d,s) Uspin-conserves charge, photon is Uspin scalar

Uspin raising operator

- **d$\rightarrow$ s** $\Lambda, \Sigma \rightarrow \Xi$
- **many states to be found**
- **will neutral Ξ ‘s always be lighter, than charged ?**
- **many questions – is there Ξ Roper –pionic effects suppressed ,maybe no Roper**
- **flavor exchange vs gluon exchange**
- **hybrids?**
- **di-quarks?**

SU(3) flavor symmetry Coleman-Glashow 1961

$$B^a_b = \begin{pmatrix} \Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix}, \quad Q = \text{ diag } e[2/3, -1/3, -1/3]$$

$$Q^k_p B^r_k - Q^r_k B^k_p = [Q, B]^r_p = \begin{pmatrix} 0 & \Sigma^+ & p \\ -\Sigma^- & 0 & 0 \\ -\Xi^- & 0 & 0 \end{pmatrix}$$

$$\bar{B} J^\mu_{\text{em}} B = \mu_1 q_\nu Tr(\bar{B}\sigma^{\mu\nu} BQ) + \mu_2 q_\nu Tr(\bar{B}\sigma^{\mu\nu} QB) + e_1 Tr(\bar{B}\gamma^\mu BQ) + e_2 Tr(\bar{B}\gamma^\mu QB)$$

2.46	$\mu(\Sigma^+) = \mu(p)$		2.79	
-.61	$\mu(\Lambda) = \frac{1}{2}\mu(n)$		-.96	
	$\mu(\Xi^0) = \mu(n)$			
-.65	$\mu(\Xi^-) = \mu(\Sigma^-)$	-1.16	$= -(\mu(p) + \mu(n))$	-.88
-.66	$\mu(\Sigma^0) = \frac{1}{2}\mu(n)$		-.96	
-1.61	$\mu(\Sigma^0 \to \lambda\gamma) = \frac{1}{2}\sqrt{3}\mu(n)$		-1.65	

Cloudy Bag Model need chiral loops +

Hope to predict electromagnetic interactions

U-spin conservation and strong reactions-Meshkov, Levinson, Lipkin

$$-\frac{1}{\sqrt{3}}\mathcal{M}(\pi^- p \to \pi^+\Delta^-) = \mathcal{M}(\pi^- p \to K^+\Sigma^{*-}) = \mathcal{M}(K^- p \to K^+\Xi^{*-}) = -\mathcal{M}(K^- p \to \pi^+\Sigma^{*-})$$

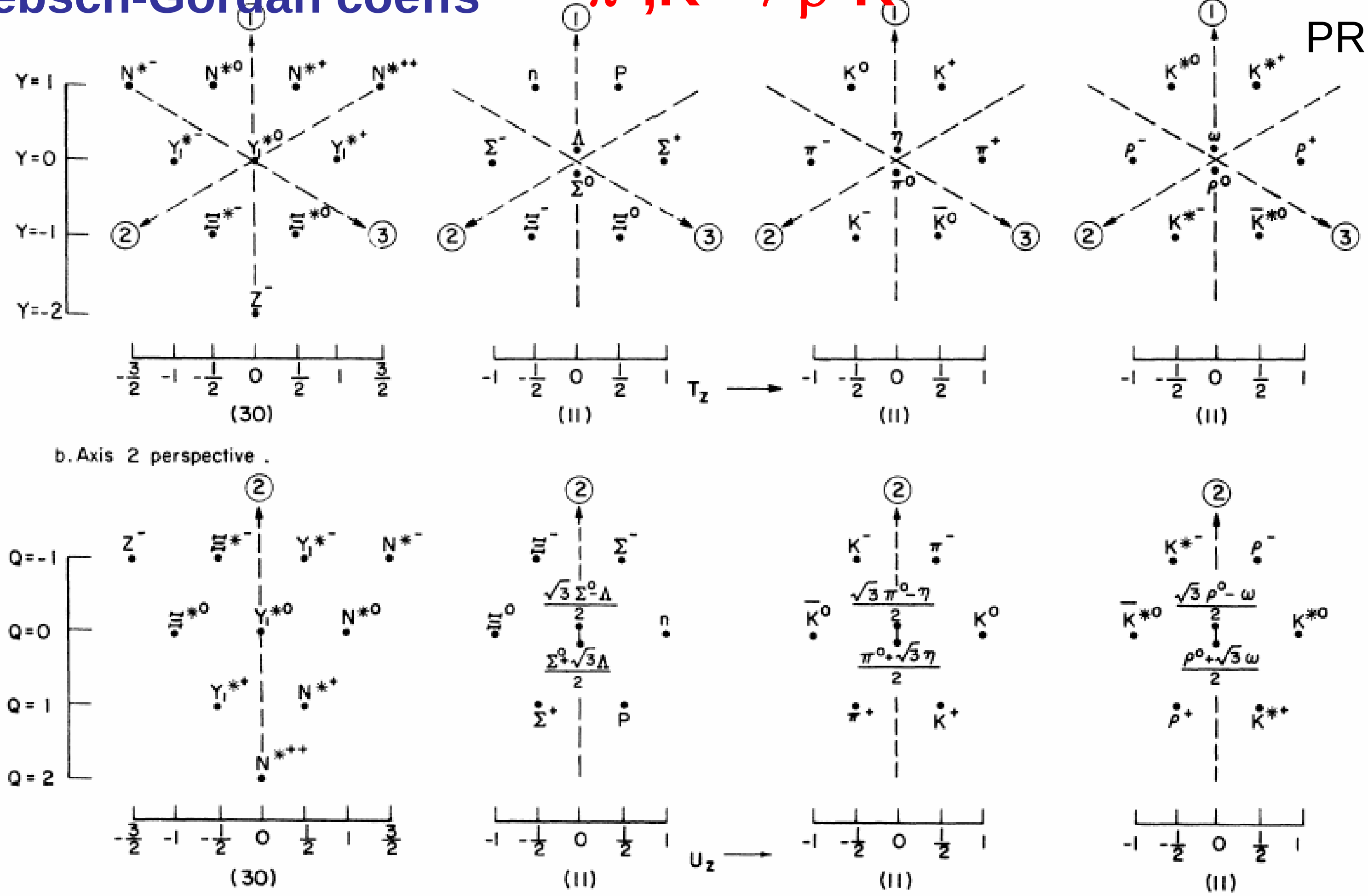
Clebsch-Gordan coeffs

$\pi^+, K^+ \to \rho^+ K^{*+}$

PRL10,361

Isospin

Uspin



New applications Nefkens' reactions

- $\pi^- p \to \eta\, n$
- initial $U, U_3 = 1,1$, final η (0,0) n(1,1)
- $K^- p \to \eta\, \Lambda$
- final Λ $U, U_3 = (1,0)$
- both reactions have U = 1
- amplitude for $K^- p$ to have U=1 is C-G coefficient $2^{-1/2}$

reaction matrix $|M(K^- p)|^2 = \frac{1}{2}\, |M(\pi^- p)|^2$

Nefkens $\sigma_{max}(\pi^- p) = 2.6 \pm 0.3$, $\sigma_{max}(K^- p) = 1.4 \pm .2$,

agrees

New applications Nefkens' reactions

- $\pi^- p \to N^*(1440) \to \pi^0\pi^0 n$
- $K^- p \to \Lambda(1600) \to \pi^0\pi^0\Lambda$
- $K^- p \to \Sigma(1660) \to \pi^0\pi^0\Sigma^0$
- intermediate states members of same octet, $(56,0^+_2)$ amplitudes related

- $\pi^- p \to N^*(1535) \to \pi^0\pi^0 n$
- $K^- p \to \Lambda(1670) \to \pi^0\pi^0\Lambda$
- $K^- p \to \Sigma(1620) \to \pi^0\pi^0\Sigma^0$
- intermediate state members of another octet,
- $(70,1^-)$ amplitudes related

Electromagnetic interactions SU(3)$\times$ SU(3) chiral PT

- Electromagnetic versions of Meshkov, Levinson, Lipkin relations –PRL7,81(63)
- photon is U spin scalar- selection rules
- **χ PT**
- **γ p $\rightarrow$ K^0 Σ^+ , K^+ Σ^0, K^+ Λ in three flavor heavy baryon chiral perturbation theory to one loop, Steininger and Meissner, 3 amplitudes related by CG**
- **reaction calculations for Ξ needed**

Unitary (flavor) symmetry for baryon-baryon interactions

- EFT calculation of Savage, Wise

$$\Pi = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{bmatrix}$$

$$B = \begin{bmatrix} \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{bmatrix}$$

$$\xi = \exp\left(\frac{i\Pi}{f}\right)$$

$$V_\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$$

$$A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$$

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

$$\mathcal{L}^{(1)} = \mathrm{Tr}B_j^\dagger i\partial_0 B_j + i\,\mathrm{Tr}B_j^\dagger[V_0, B_j] - D\,\mathrm{Tr}B_j^\dagger\vec{\sigma}_{jk}\{\vec{A}, B_k\} - F\,\mathrm{Tr}B_j^\dagger\vec{\sigma}_{jk}[\vec{A}, B_k]$$

Gives OBEP for exchange of Goldstone bosons +

$$\begin{aligned}\mathcal{L}^{(2)} = &-\frac{c_1}{f^2}\mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - \frac{c_2}{f^2}\mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i)\\ &-\frac{c_3}{f^2}\mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j) - \frac{c_4}{f^2}\mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i)\\ &-\frac{c_5}{f^2}\mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j)\\ &-\frac{c_6}{f^2}\mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i)\;.\end{aligned}$$

Lowest order potential

Ξ N, Ξ Ξ interactions

- **Evaluate Lagrangian**

$$\mathcal{L}^{(2)} \rightarrow \left(c_1 + c_5 + (c_2 + c_6)\frac{1}{2}\right)\left((\Xi^\dagger\Xi)^2 + (N^\dagger N)^2\right) + (c_2 + c_6)\frac{1}{2}\left(\Xi^\dagger\boldsymbol{\sigma}\Xi \cdot \Xi^\dagger\boldsymbol{\sigma}\Xi + N^\dagger\boldsymbol{\sigma}N \cdot N^\dagger\boldsymbol{\sigma}N\right)$$
$$+2(c_3 + c_4\frac{1}{2})\Xi^\dagger N^\dagger N\Xi + 2c_4\frac{1}{2}\left(\Xi^\dagger\boldsymbol{\sigma}N \cdot N^\dagger\boldsymbol{\sigma}\Xi\right)$$

$\Xi\Xi$ short range potential same as NN

$^1S_0$ channel – OPEP is small for NN

NN scattering length a =-17.3 fm

If $\Xi\Xi$ $^1S_0$ POTENTIAL same as for NN:

there will be a BOUND STATE dibaryon S=-4, decays

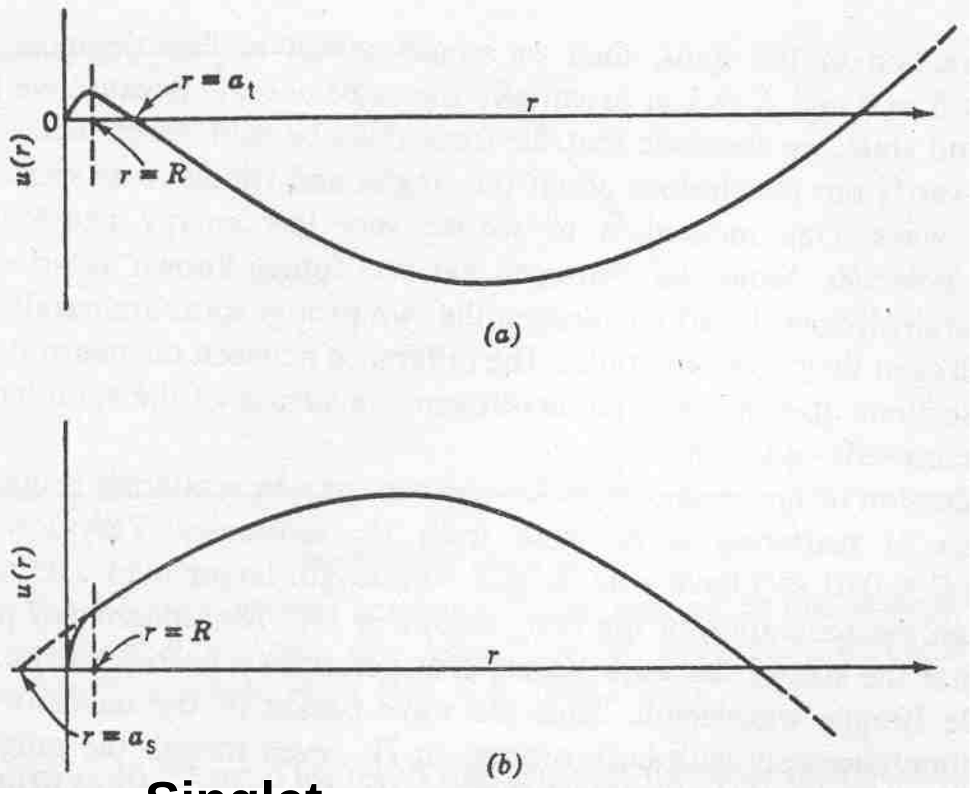
weakly Ξ $\Lambda\pi$

Wf for NNTriplet and Singlet Scattering

- (a) Wf for triplet np scattering, neutron energy 200 keV, well radius 2.1 fm. Positive scattering length
- (b) Wf for negative scattering length

Increase reduced mass bends wave into well causes singlet binding ≈ 10 MeV

- Triplet



Singlet

$\Xi\Xi$ binding many astrophysical consequences-quark stars, strangelets etc

- Witten, Bodmer → **neutron star**
- many searches at BNL, no findings
- but d,s,u ratios different with this mechanism

Finding ΞΞ bound states

- γ-D→(ΞΞ) 4K threshold photon energy 5 GeV (thanks to R Jones)
- KD →(ΞΞ) 3K ?
- RHIC (Huang) can detect decay products ΞΛ

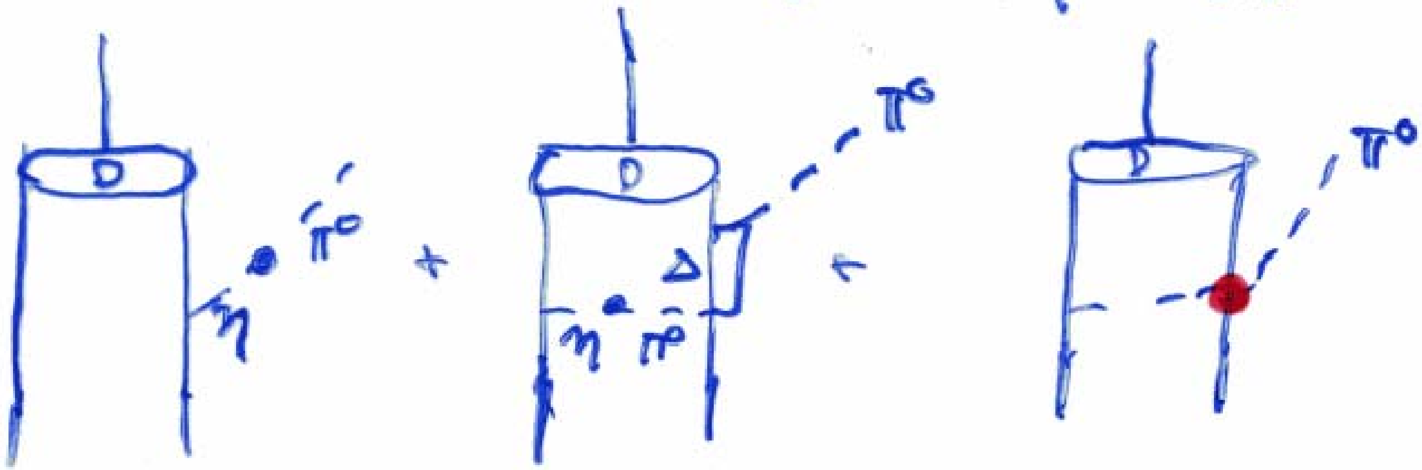
Summary of flavor symmetry

- many states should exist that have not been seen
- many doublet mass differences to study
- U spin conservation predicts ratios of cross sections

- chiral SU(3) EFT predicts $\Xi\Xi$ bound state

EXTRAS FOLLOW

CSB mechanisms for $np \to d\pi^0$



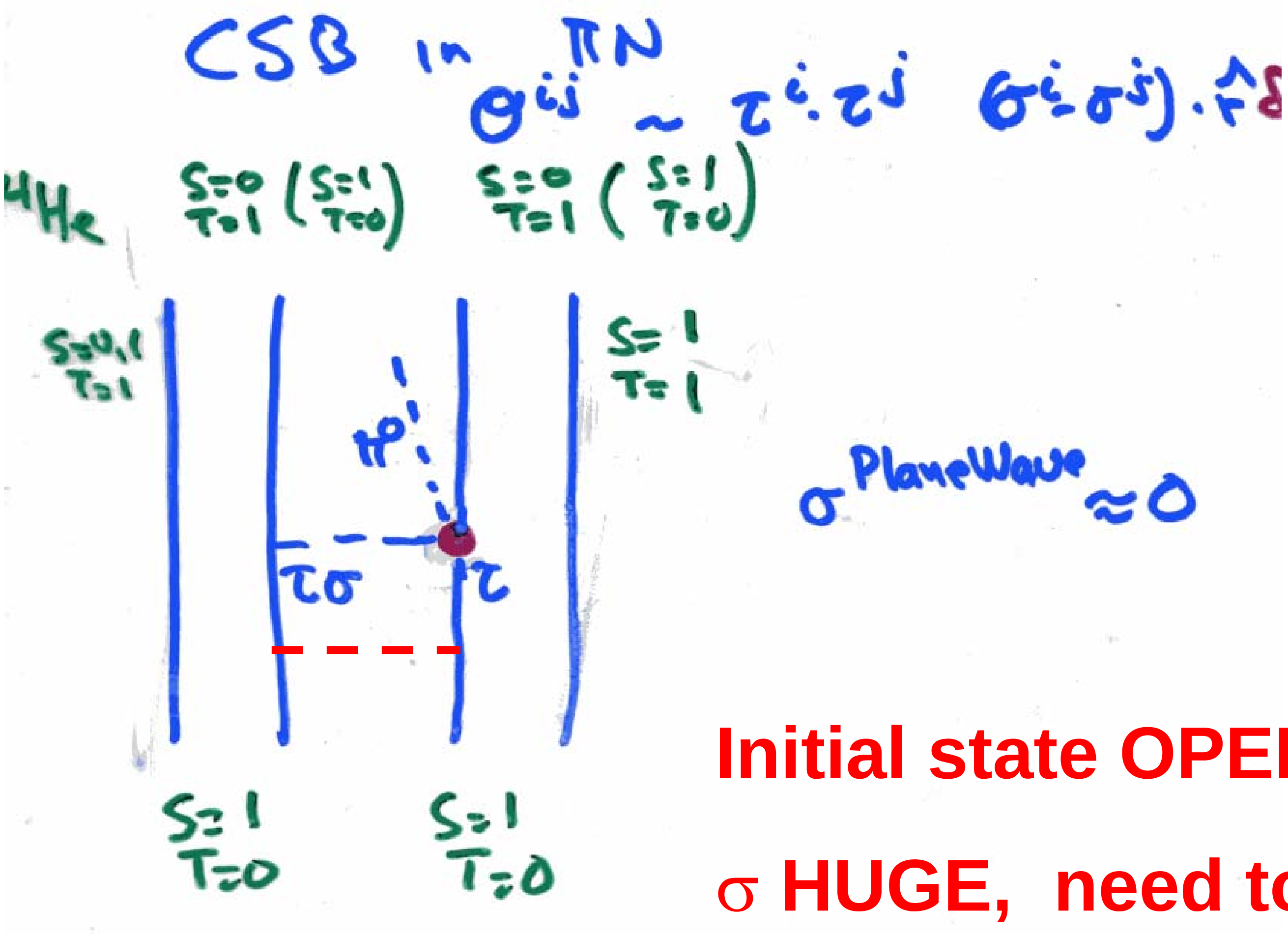
$$A_{fb} = -0.28\left[\frac{\langle \pi^0 | H | \eta \rangle}{-5900\,\text{MeV}^2} - 0.87\left(\delta m_N - \frac{\bar{\delta} m_N}{2}\right)\right]$$

van Kolck Niskanen Miller (2000)

−0.28, −0.87 from

old strong
int. calc.

Initial state interactions



Initial state OPEP, +

σ HUGE, need to cancel

Nogga, Fonseca

PHYSICAL REVIEW C 69, 044606 (2004)

Survey of charge symmetry breaking operators for $dd \rightarrow \alpha\pi^0$

A. Gårdestig* and C. J. Horowitz **et al**

The relative proportions of the pion-exchange $\left(\delta M - \frac{1}{2}\overline{\delta M}\right)$, photon-exchange, ρ-ω–mixing, and π-η–mixing (sum of one-body and HMEC) contributions to the matrix element are roughly $\pi : \gamma : \rho\text{-}\omega : \pi\text{-}\eta = 1{:}11{:}12{:}21$. Thus the for-

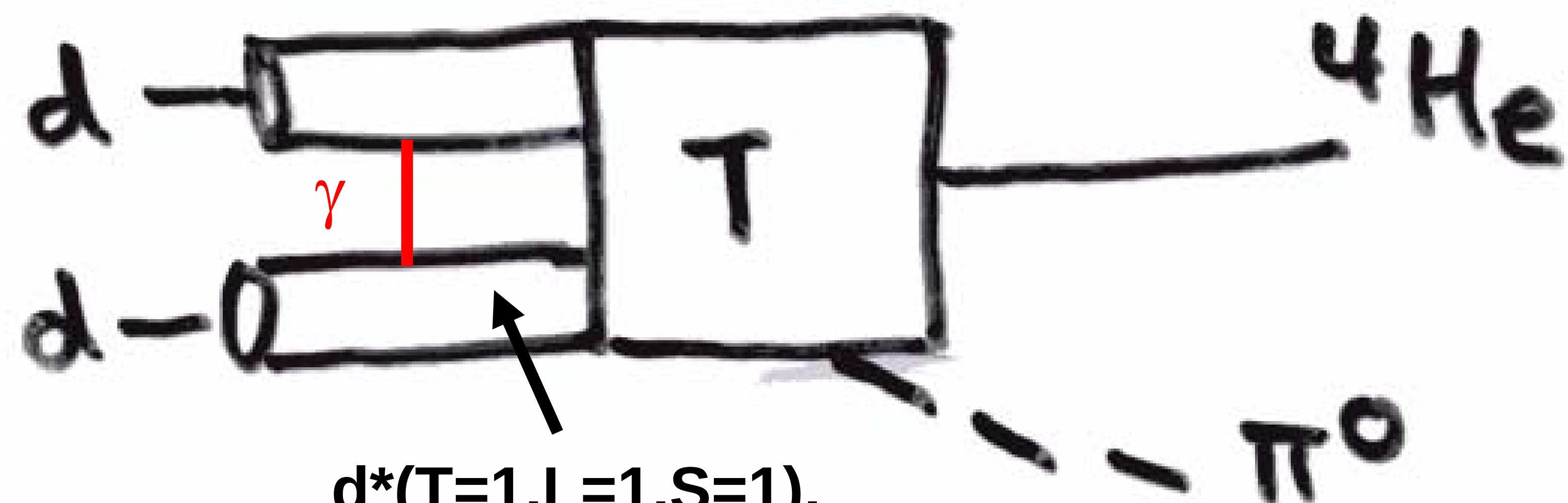
plane wave, Gaussian w.f.

photon in NNLO, LO not included

23 pb vs 14 pb

Initial state LO dipole γ exchange

$^3P_0$



d*(T=1,L=1,S=1), d*d OAM=0, π emission makes dd component of He, estimate using Gaussian wf

Theory requirements: $dd \to {}^4He\pi^0$

- **start with good strong, csb**

  np$\to$ dπ^0
- **good wave functions, initial state interactions-strong and electromagnetic**
- **great starts have been made**