Cascade baryons from overlap action

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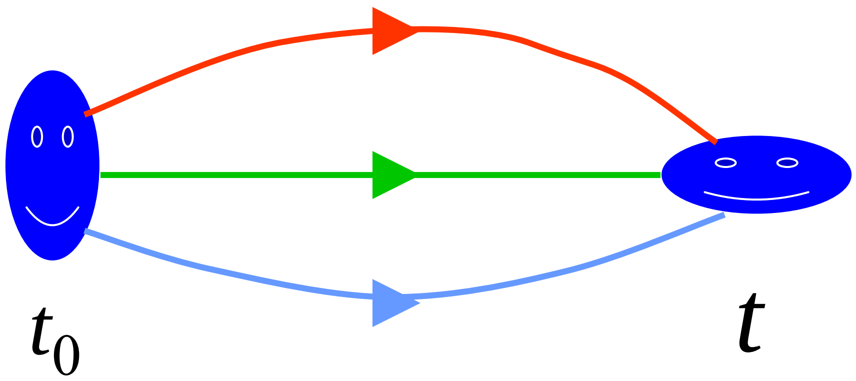
Collaboration : χQCD

Two Point Correlation Function



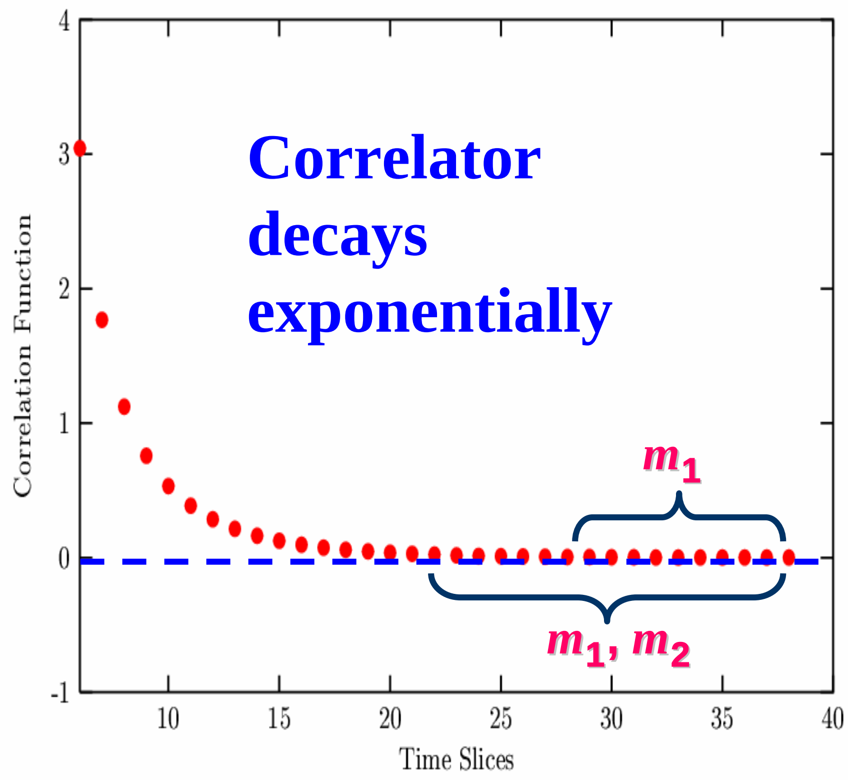
The two-point correlation function decays exponentially at large separation of time

$$G_{NN}^{\alpha\alpha}(t,t_0,\vec{p}) \equiv \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle T(\chi^\alpha(x)\overline{\chi}^\alpha(x_0))\rangle$$

$$\xrightarrow{t-t_0>>1} \langle 0|\chi^\alpha|N^\alpha(\vec{p})\rangle\langle N^\alpha(\vec{p})|\overline{\chi}^\alpha|0\rangle \frac{e^{-E_p(t-t_0)}}{2E_pV_3} \equiv \frac{E_p+m}{E_p}|\phi|^2 e^{-E_p(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(t) = \sum_{i=1}^{N} W_i\, e^{-m_i t} \underset{t\to\infty}{\approx} W_1\, e^{-m_1 t}$$



How to extract m_2 m_3… : excited states.
Non linear fitting.

Need help with statistics.

Sequential Bayesian Method.
….hep-lat/0405001

Alternative approach :
Solving generalized
cross correlator eigenvalue problem

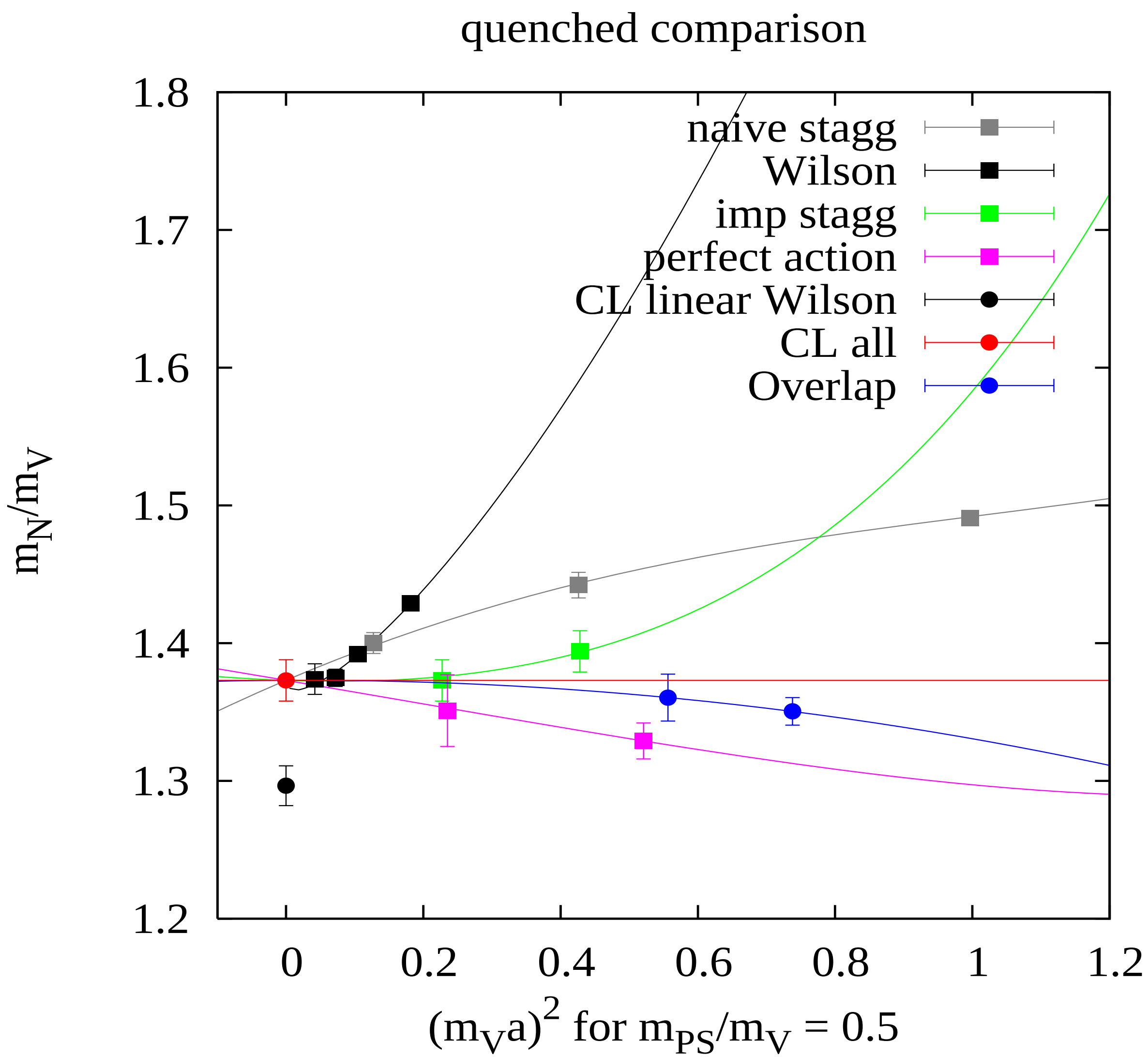
$$G(t)v = \lambda(t,t_0)G(t_0)v$$

Overlap Fermions

- Exact chiral symmetry at finite lattice spacing.
- No $O(a)$ errors.
- $O(a^2)$ and $O(m^2a^2)$ errors are small (found numerically).
- Multi-mass algorithm can be used.
- Numerically very intensive (~100 times of Wilson fermion to invert).

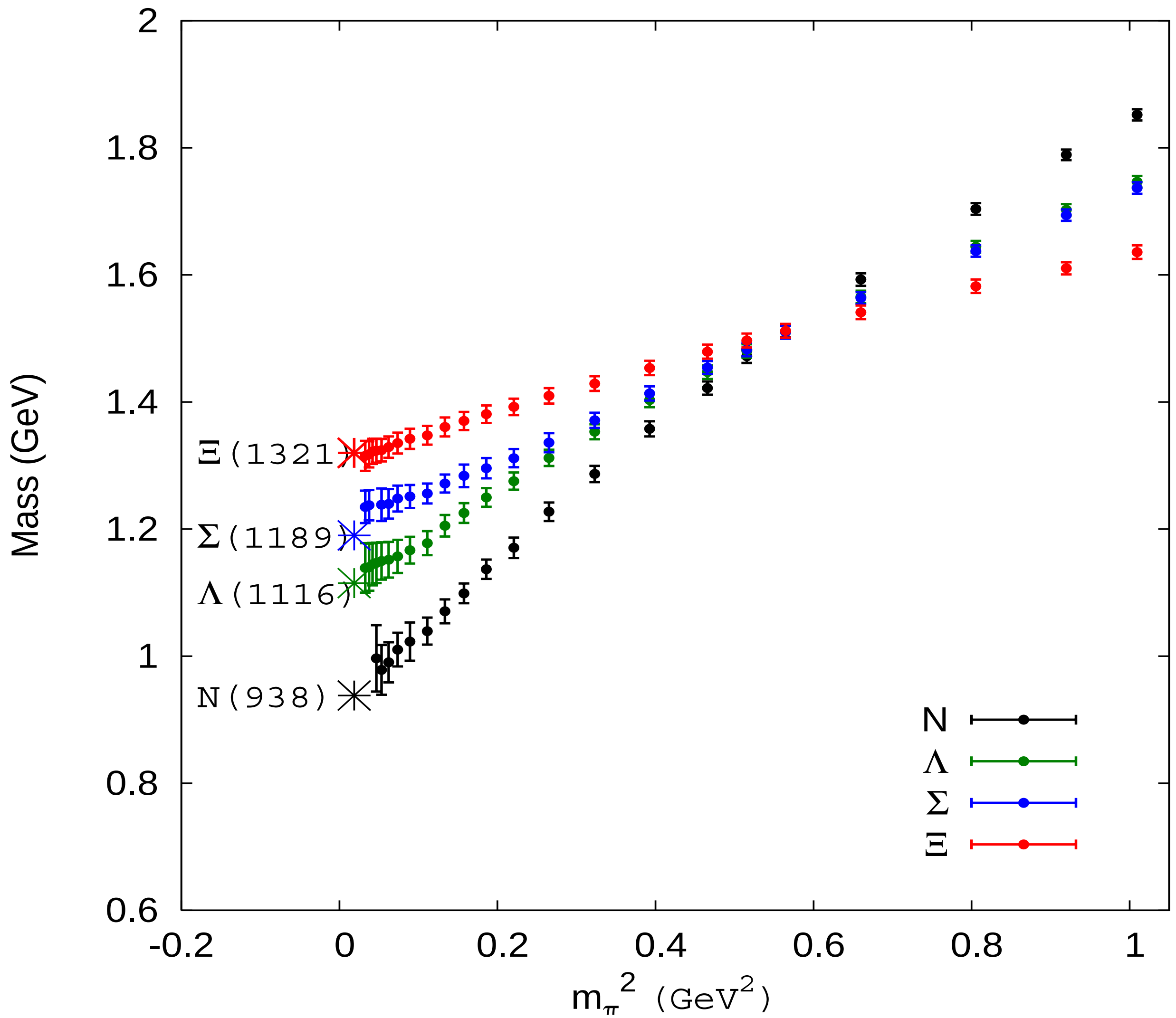
Parameters	16^3 x 28	20^3 X 32
Lattice spacing	0.2 fm	0.175 fm
Configurations	300	110

Scaling of overlap action



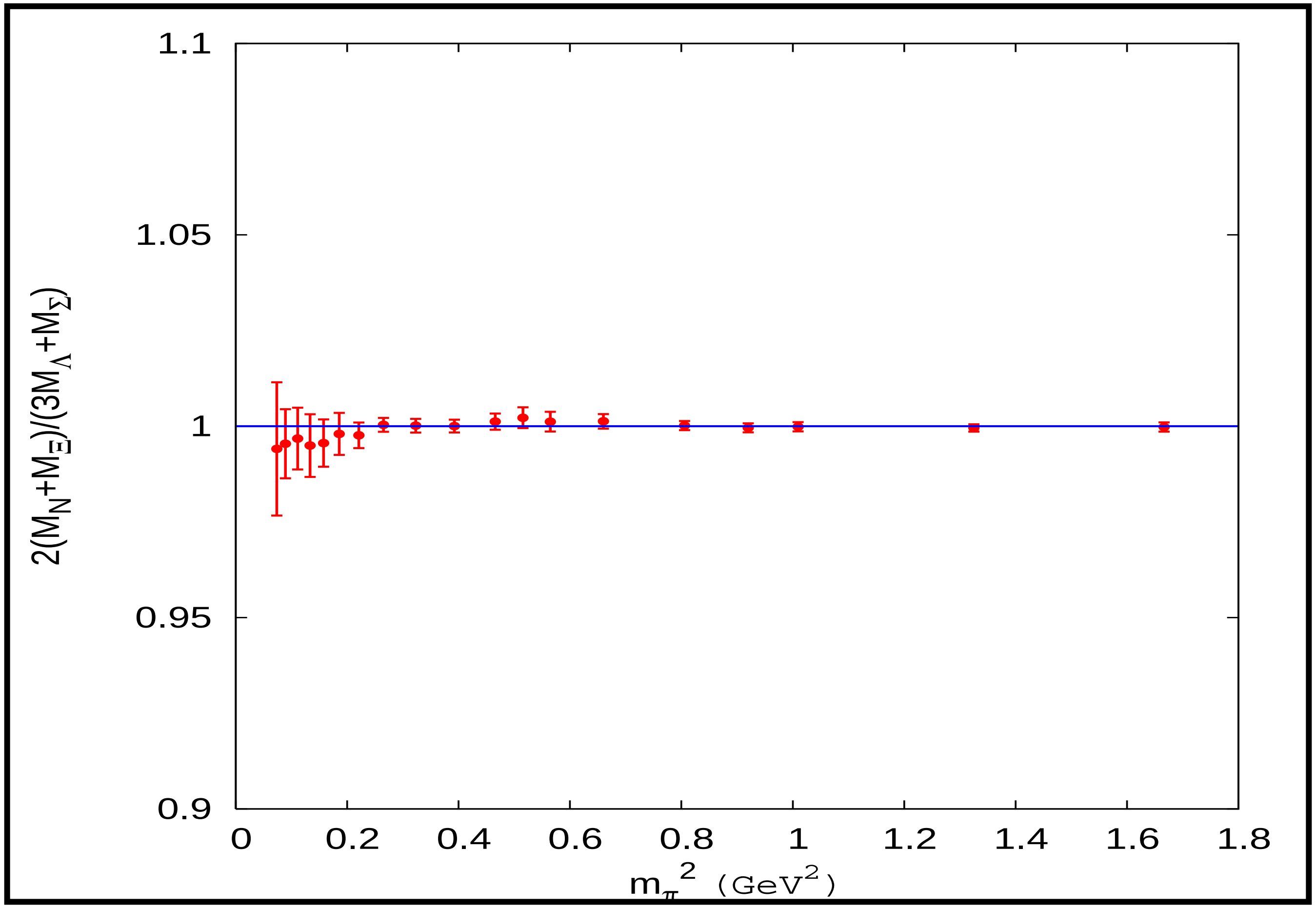
hep-lat/0510075

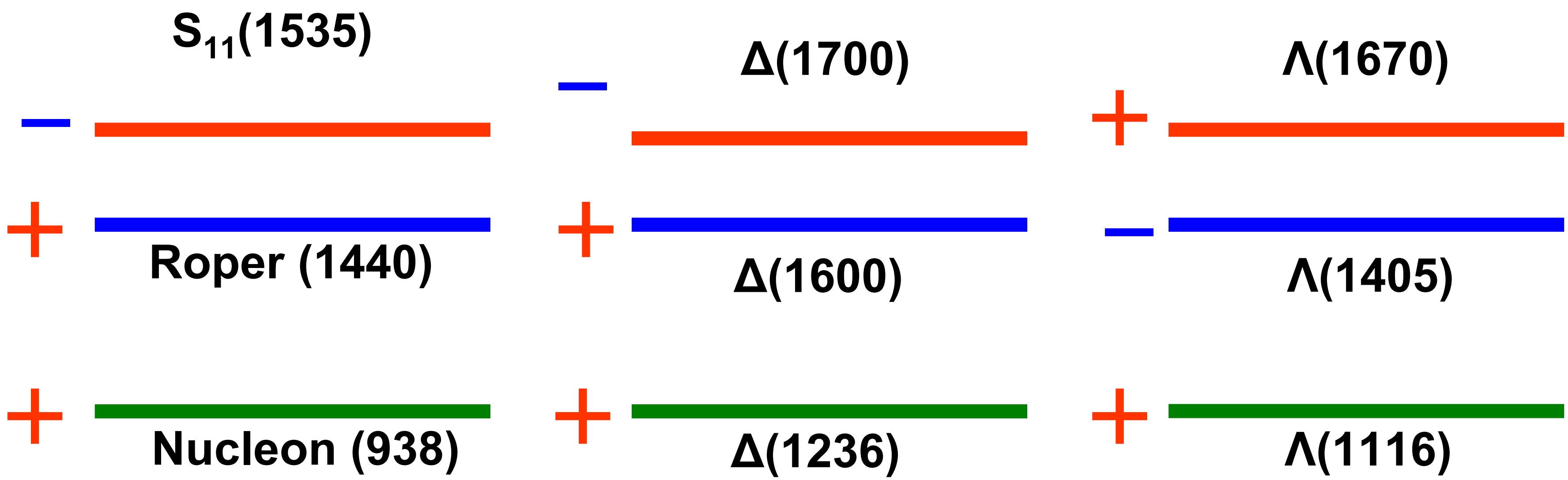
Ground state Octet Baryons



Gell-Mann Okubo Relation

$$2\left(M_N + M_\Xi\right) = 3M_\Lambda + M_\Sigma$$



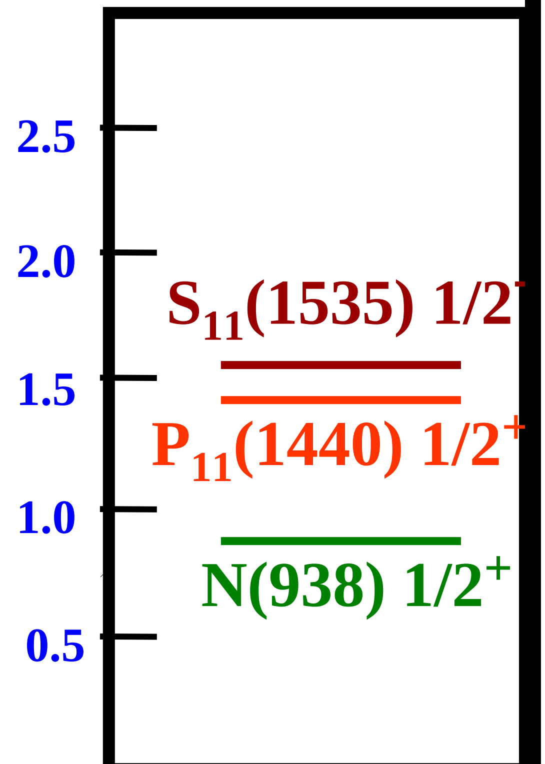


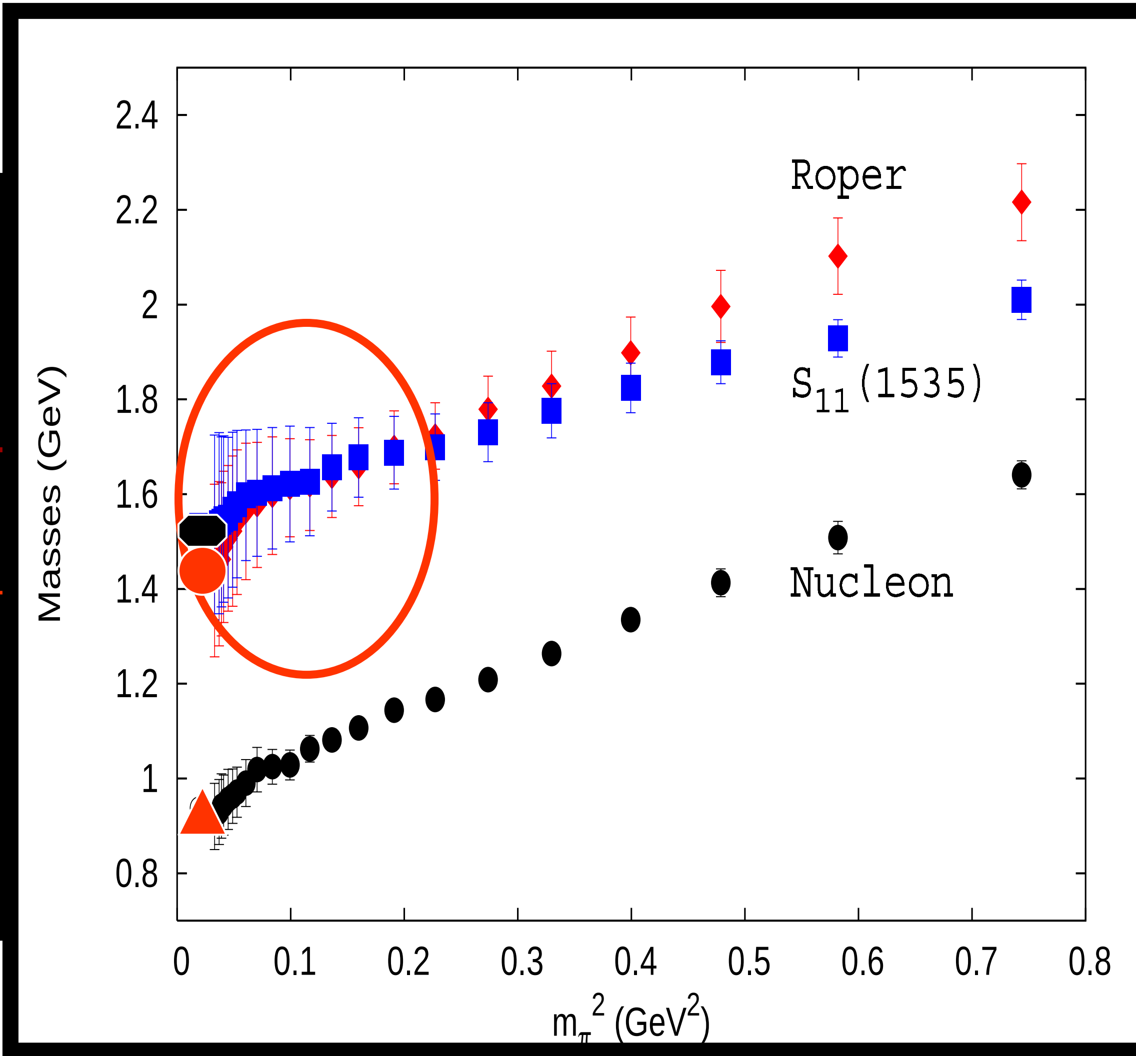
What is the ordering for Ξ spectrum?

Roper

Roper is seen on the lattice at the right mass with <u>three quark</u> interpolation field .....Mathur et.al. Phys. Lett. B605, 137 (2005)

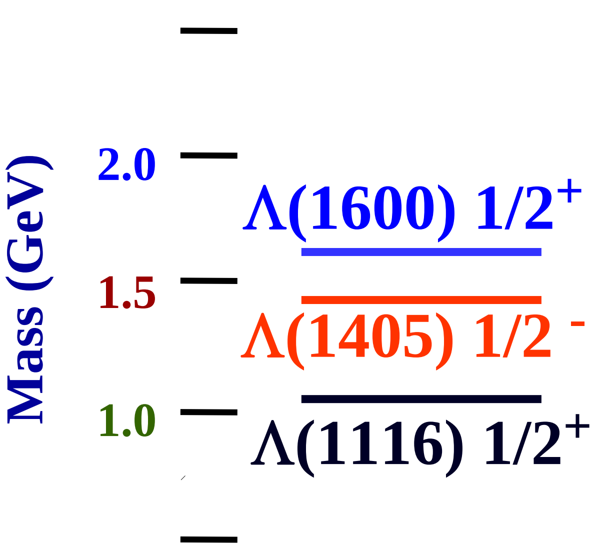
Cross over occurs in chiral domain

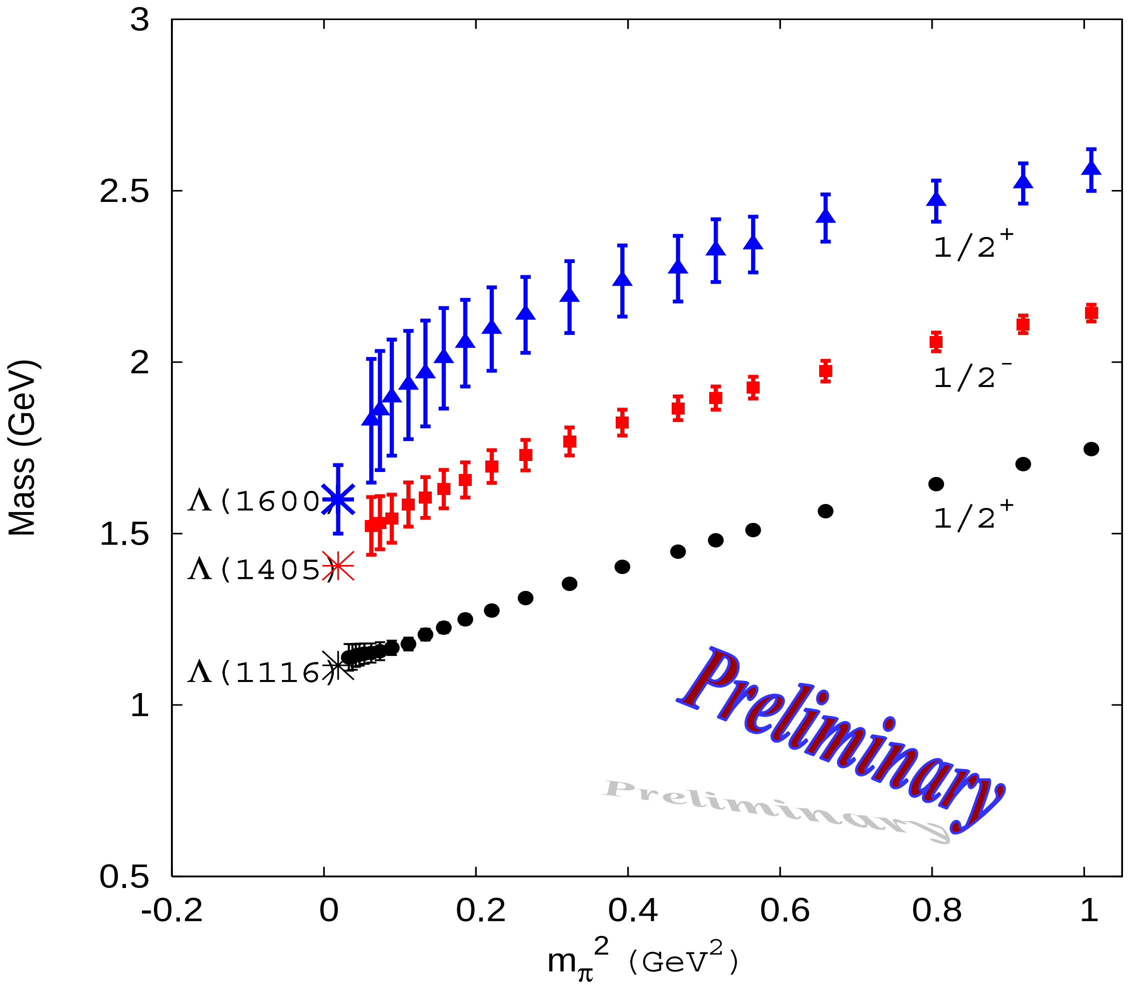


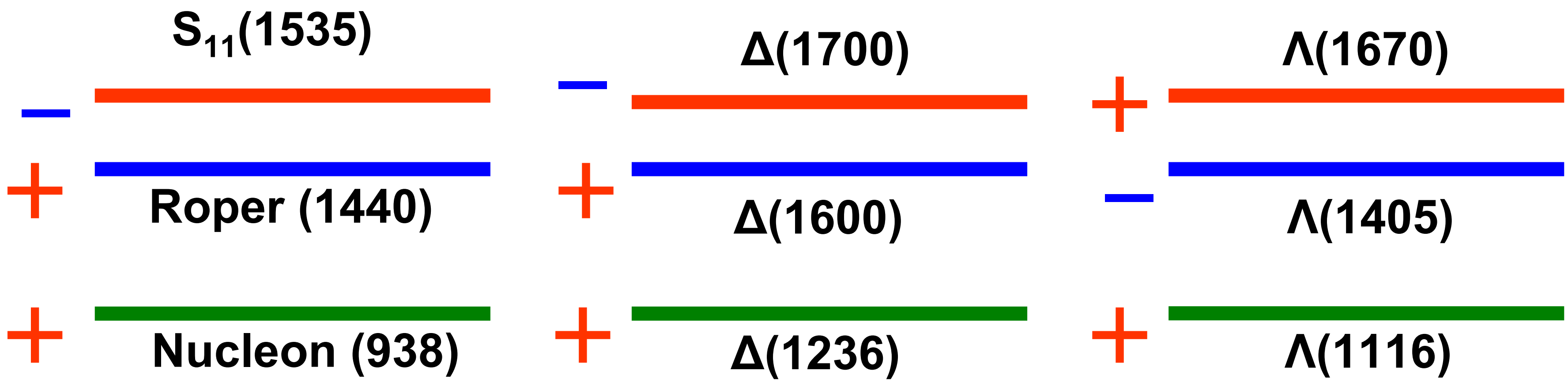


What about Hyperons?
The Λ(1405)?

***...different* story!!**







Hyperfine Interaction of quarks in Baryons

$$\lambda_c^1 \cdot \lambda_c^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Color-Spin Interaction

Excited positive > Negative ..Isgur

Our lattice results suggest this interaction dominates at higher quark mass (above 400 MeV pion mass)

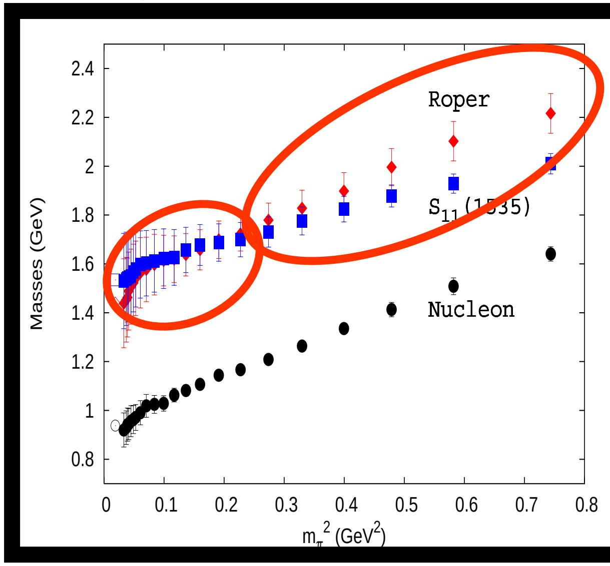
$$\lambda_F^1 \cdot \lambda_F^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Flavor-Spin interaction

Chiral symmetry plays major role

Negative > Excited positive

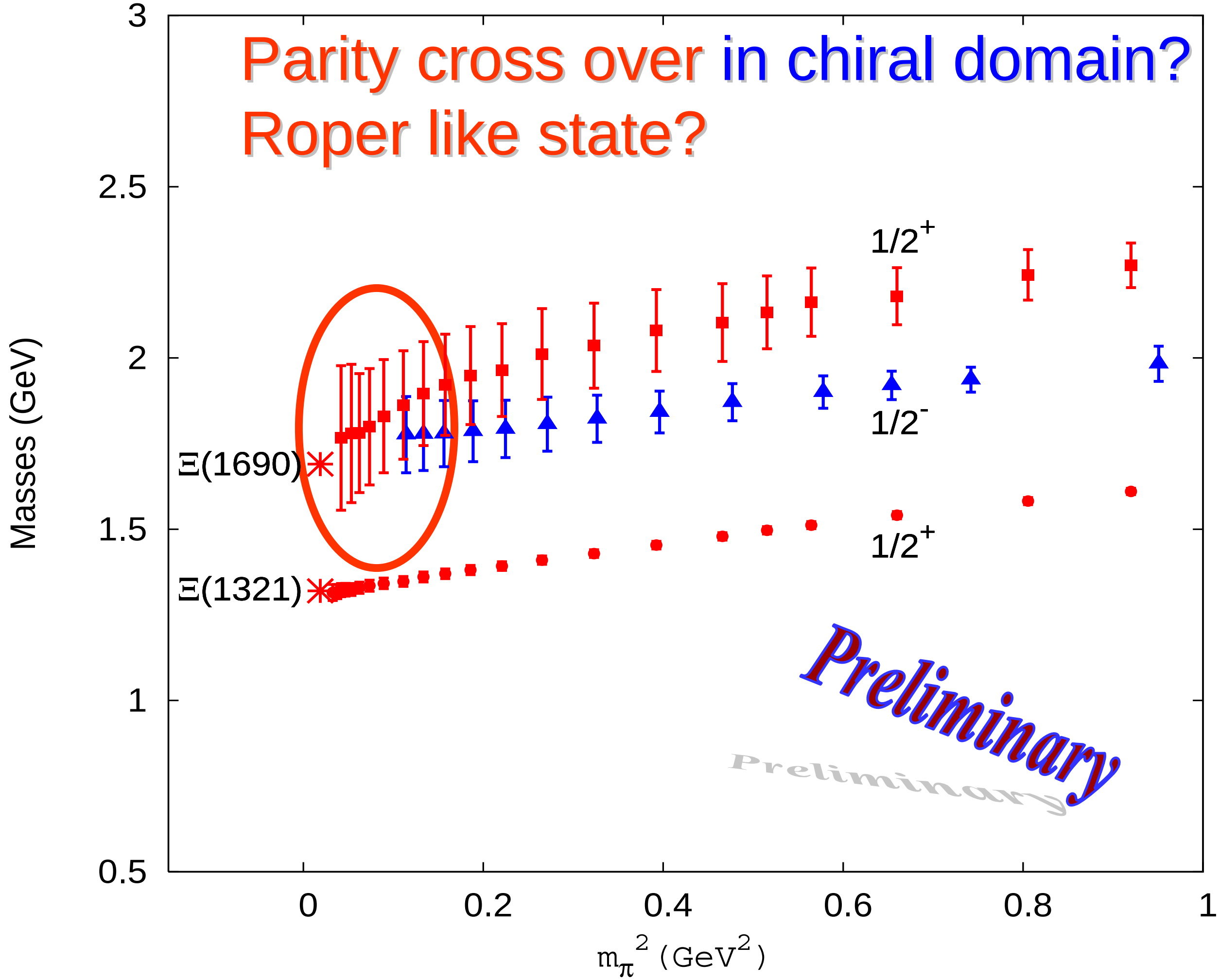
Glozman & Riska
Phys. Rep. 268,263 (1996)



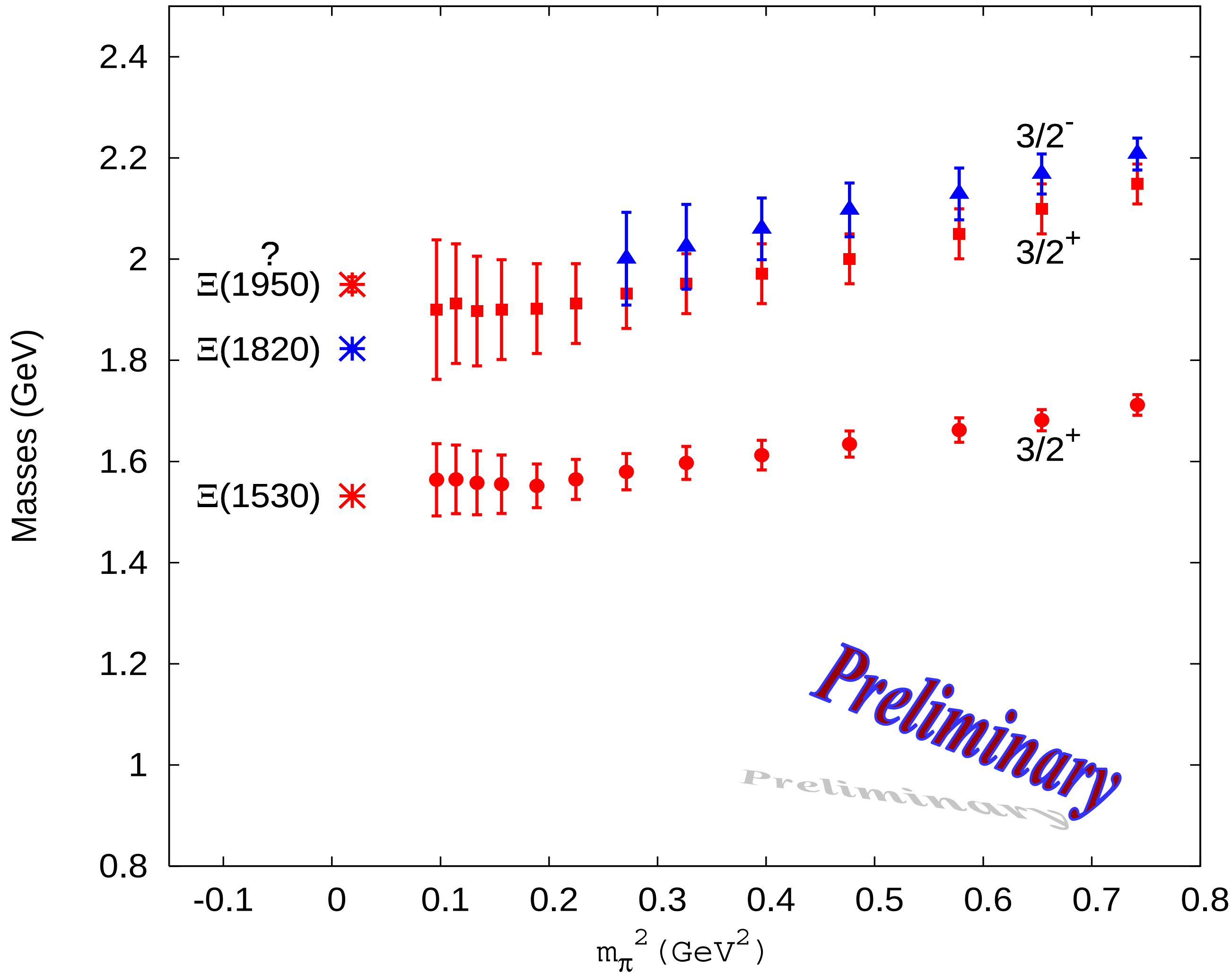
Our results suggest this interaction may dominate at lower quark mass (below 400-300 MeV pion mass)

...PRD59, 112001 (1999)

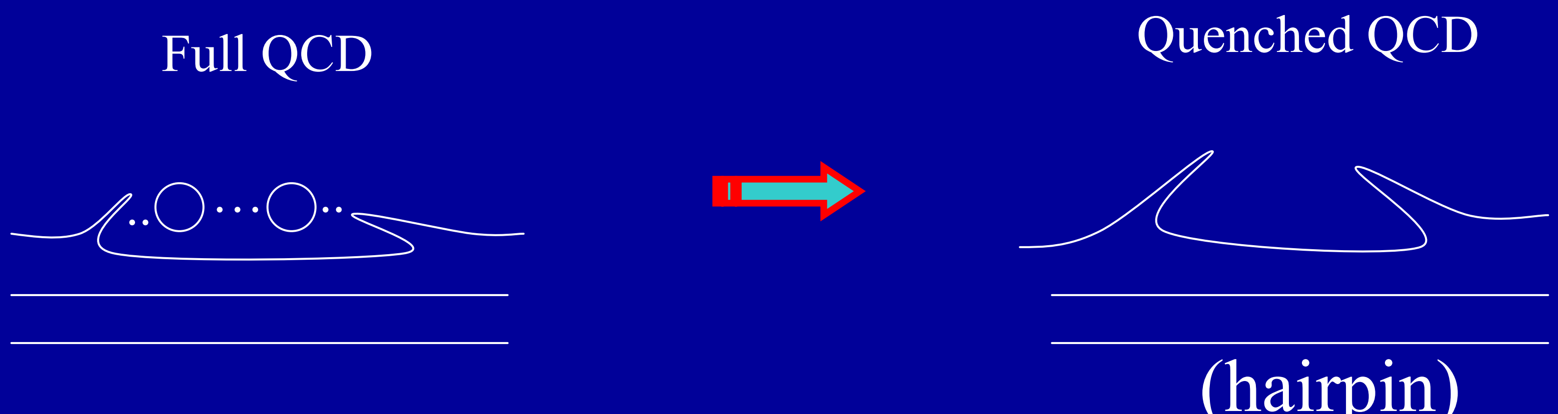
Octet Ξ Baryons



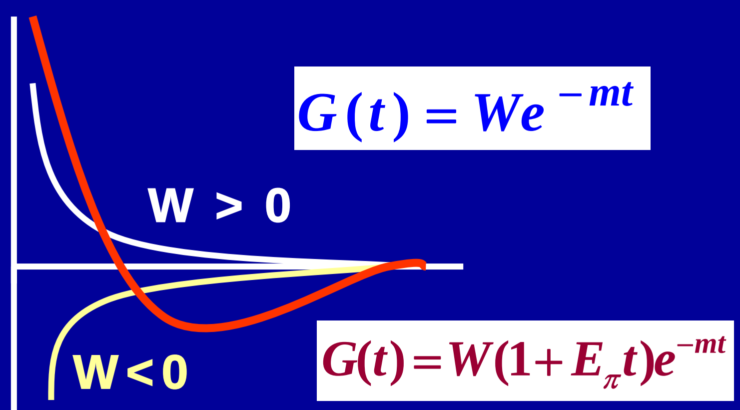
Decuplet Ξ Baryons



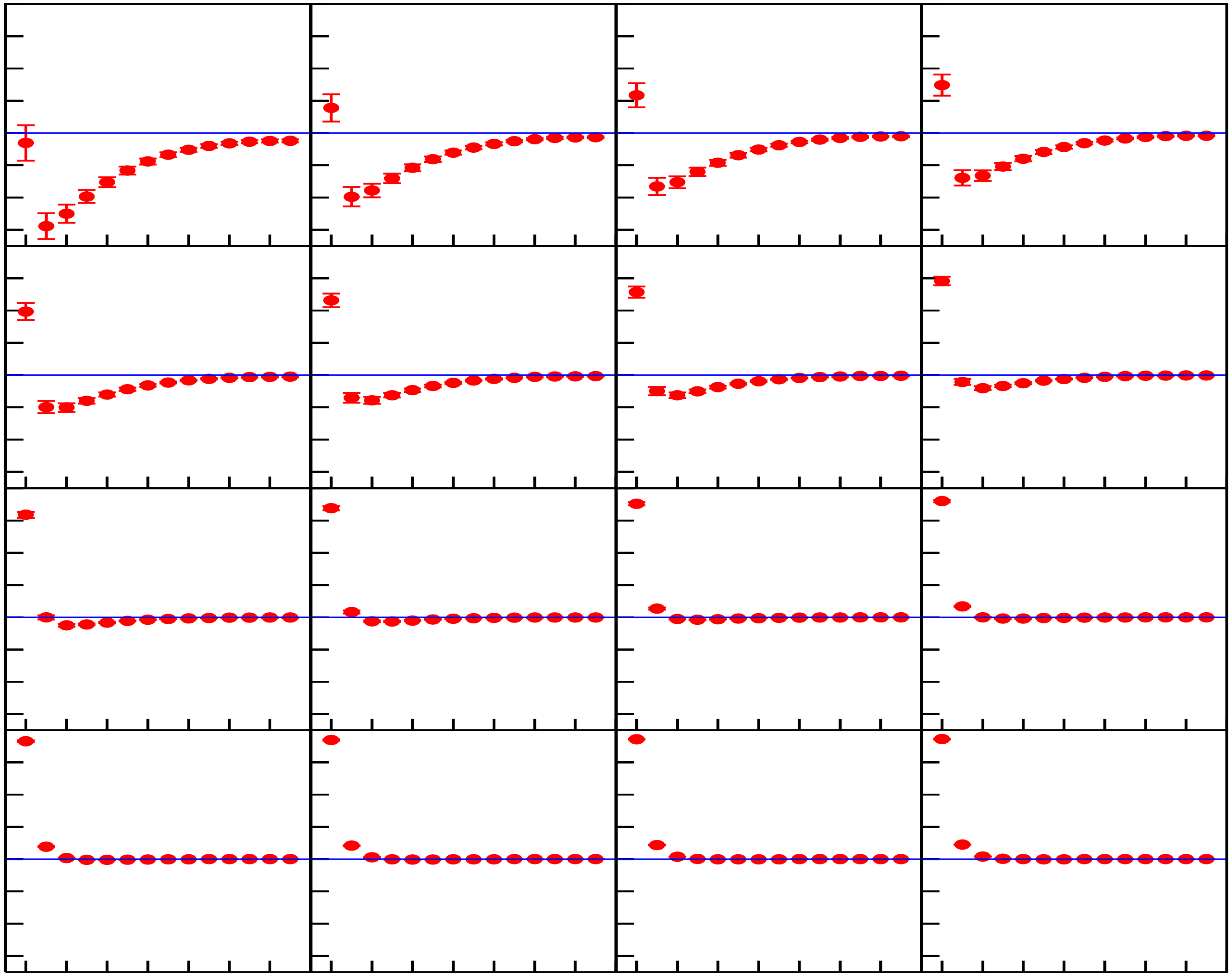
The η′ ghost in quenched QCD



- It becomes a light degree of freedom
  - with a mass degenerate with the pion mass.
- It is present in all hadron correlators G(t).
- It gives a negative contribution to G(t).
  - It is unphysical (thus the name ghost).



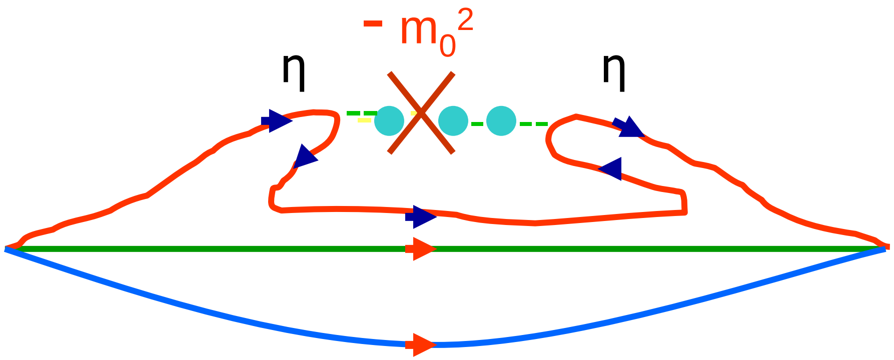
Is a_0 (1450) (0^{++}) a two quark state?

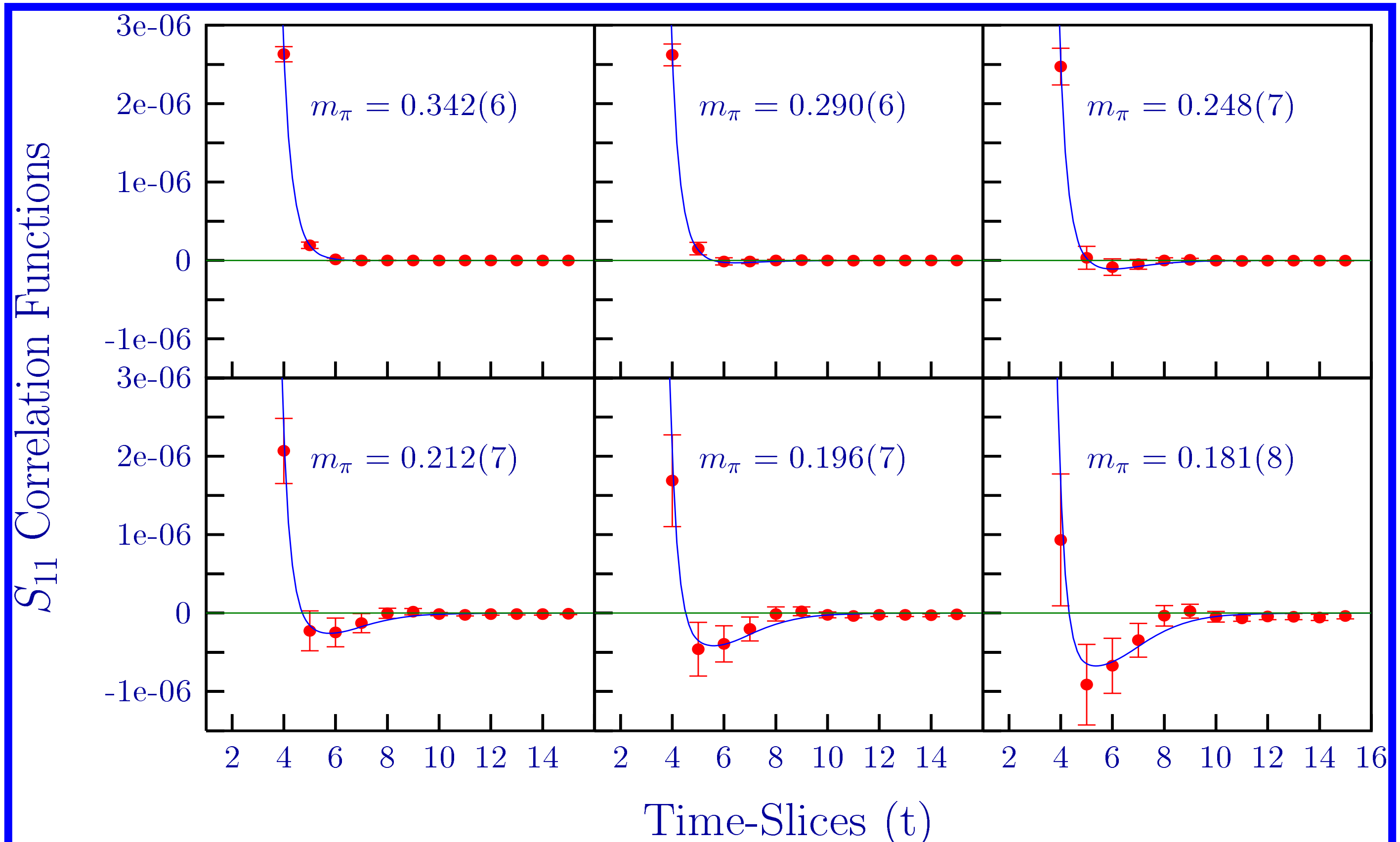


Correlation function for Scalar channel

Ground state : $\pi\eta'$ ghost state, Excited state : a_0

Evidence of η′N GHOST State in S_{11} (1535) Channel

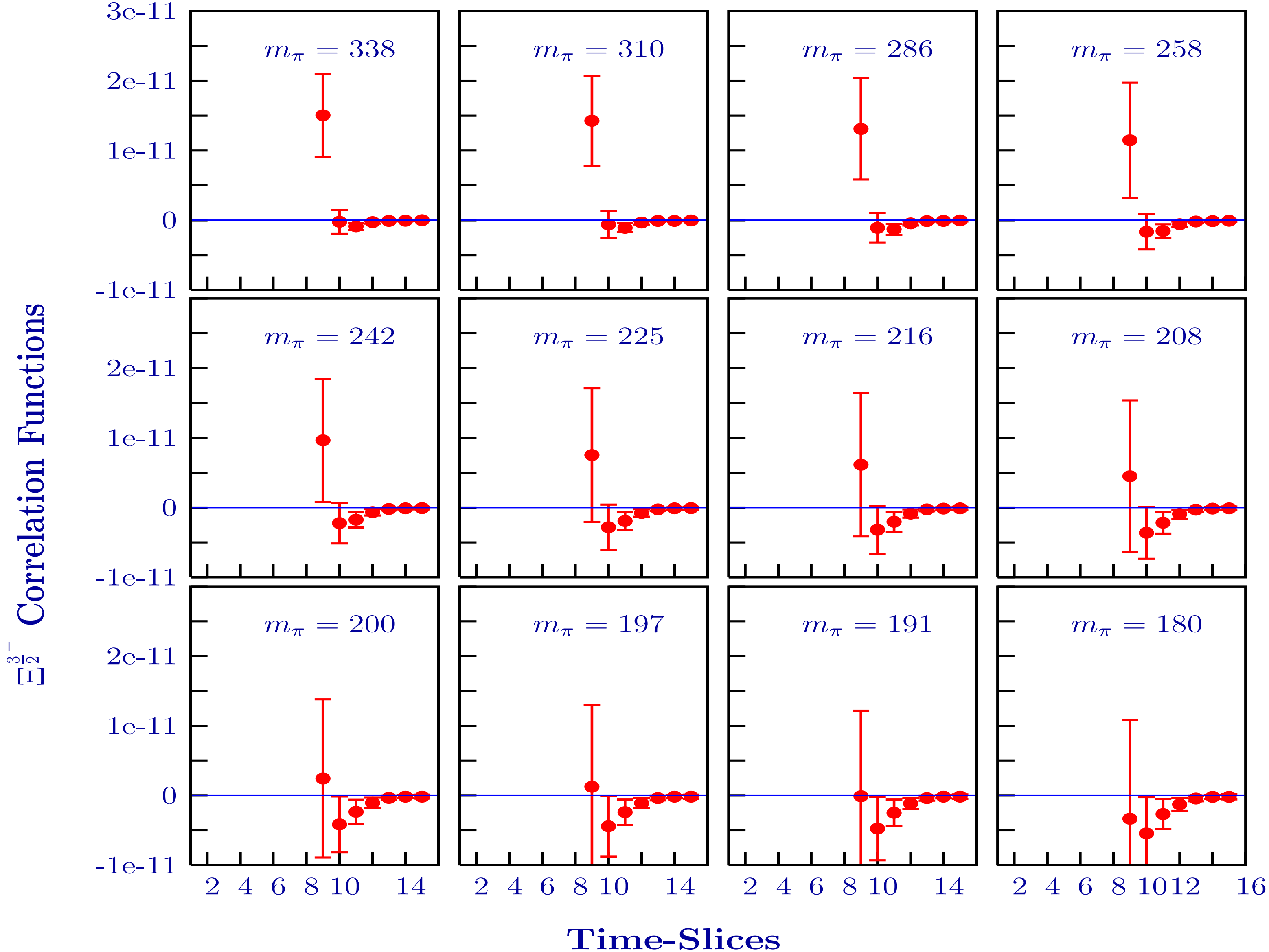




Ghost States in Ξ Spectrum

Particle	Ghost State	Presence
Ξ^*(?) $1/2^+$	P-wave $\Xi\eta'$	1st or 2nd excited state $\Xi(1/2^+)$, $\Xi\eta'$, Ξ^*..
Ξ^*(?) $1/2^-$	S-wave $\Xi\eta'$	Ground state $\Xi\eta'$, $\Xi(1/2^-)$. .
Ξ(1820) $3/2^-$	S-wave $\Xi\eta'$ (1530 +140 < 1820)	Ground state $\Xi\eta'$, $\Xi(3/2^-)$. .
Ξ^*(1950?) $3/2^+$	P-wave $\Xi\eta'$	1st or 2nd excited state $\Xi(3/2^+)$, $\Xi\eta'$, Ξ^*..

$\Xi(3/2^-)$ correlation function



Conclusions

- There are lattice signals for the $1/2^-$ ground and $1/2^+$ excited states.

- For octet Ξ states, our preliminary lattice results suggest a possible parity cross-over and the presence of a $1/2^+$ excited state similar to the nucleon channel.

  ($\Xi(1690) \rightarrow 1/2^+$?)

- Possibly there is an excited $3/2^+$ state around 1.9-2.0 GeV.

- One needs to consider the presence of unphysical ghost states while analyzing quenched lattice Ξ-spectrum in low quark mass region.

Interpolating Fields

$$\eta^{\Xi^+} = \varepsilon^{abc}\left[s^{aT} C\gamma_5 u^b\right] s^c$$

$$\eta_\mu^{\Xi^0} = \frac{1}{\sqrt{3}}\varepsilon^{abc}\left[2\left(s^{aT} C\gamma_\mu u^b\right)s^c + \left(s^{aT} C\gamma_\mu s^b\right)u^c\right]$$

$$P_{\mu\nu}^{3/2}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}\left(\gamma.p\gamma_\mu p_\nu + p_\mu\gamma_\nu\gamma.p\right)$$

$$P_{\mu\nu}^{1/2}(p) = g_{\mu\nu} - P_{\mu\nu}^{3/2}$$

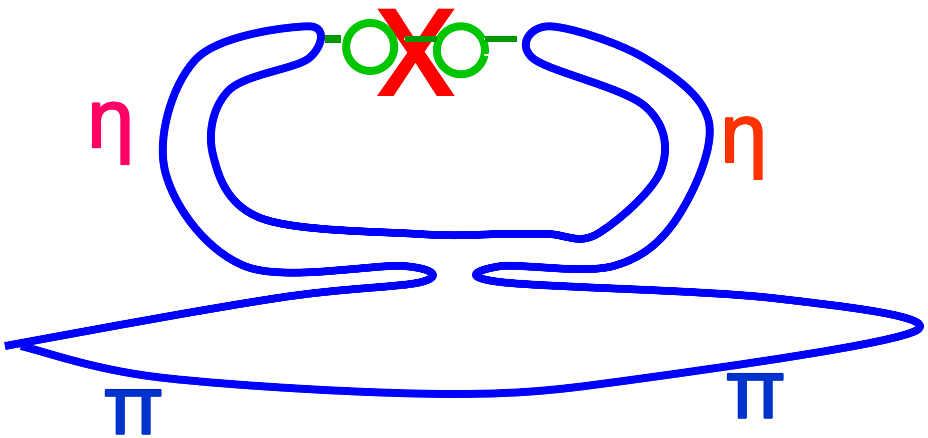
$$G_{ii}^{3/2(1/2)} = \sum_{\mu,\nu=1}^{4} G_{i\mu}\, g^{\mu\nu} P_{\nu i}^{3/2(1/2)}$$

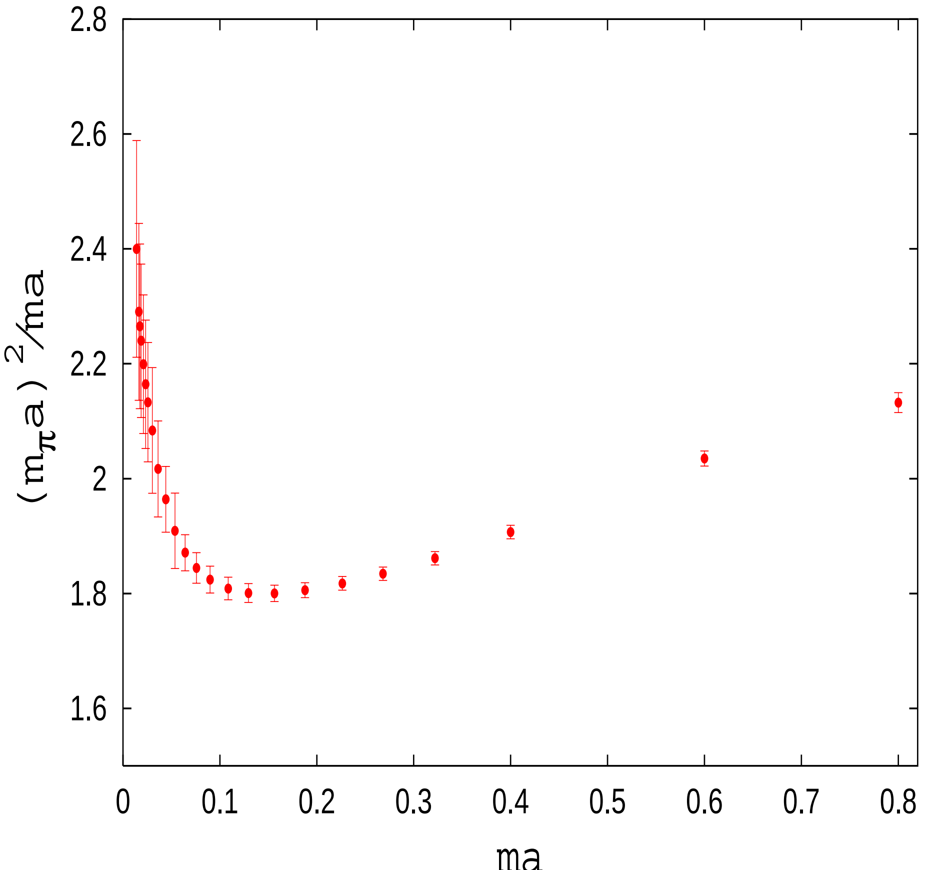
Quenched Artifacts

Chiral *log* in m_π^2

$$m_\pi^2 = Am\left\{1-\delta\left[\ln(Am/\Lambda_\chi^2)+1\right]\right\}+Bm^2$$

Quenched QCD





$\delta = 0.2 \pm 0.03$

Phys. Rev. D70, 034502 (2004)

Overlap Fermion

Overlap fermion (Neuberger, 1998)

$$D(m_0) = \left(\rho + \frac{m_0 a}{2}\right) + \left(\rho - \frac{m_0 a}{2}\right)\gamma_5 \varepsilon(H),$$

$$\varepsilon(H) = H \Big/ \sqrt{H^2}, \qquad H = \gamma_5 D_W,$$

$$-\rho = 1/2\kappa - 4 \quad \Rightarrow \quad \kappa_c < \kappa < 0.25, \quad \kappa = 0.19$$

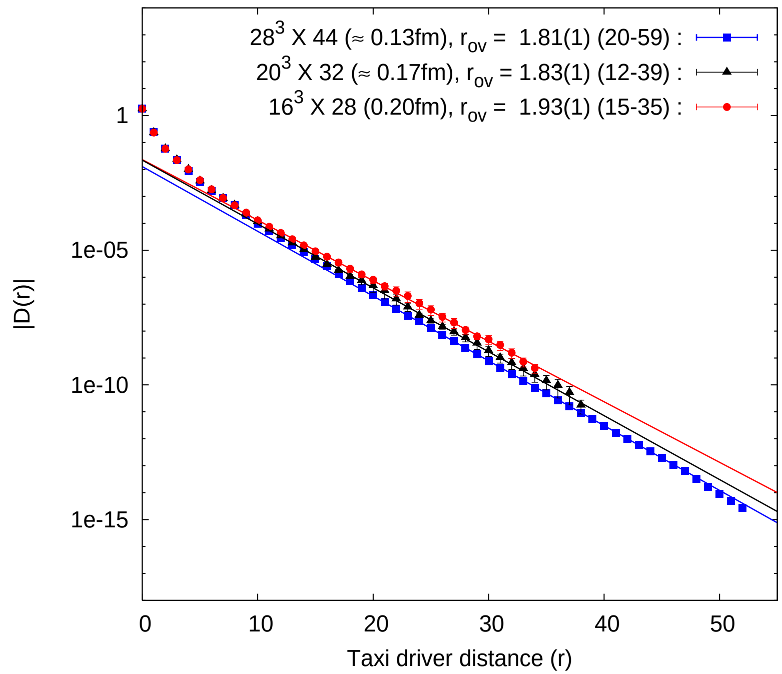
Ginsparg-Wilson relation

$$\{\gamma_5, D\} = D\gamma_5 D \underset{a \to 0}{\Rightarrow} 0, \; D = aD_a$$

Lattice chiral symmetry (Lüscher)

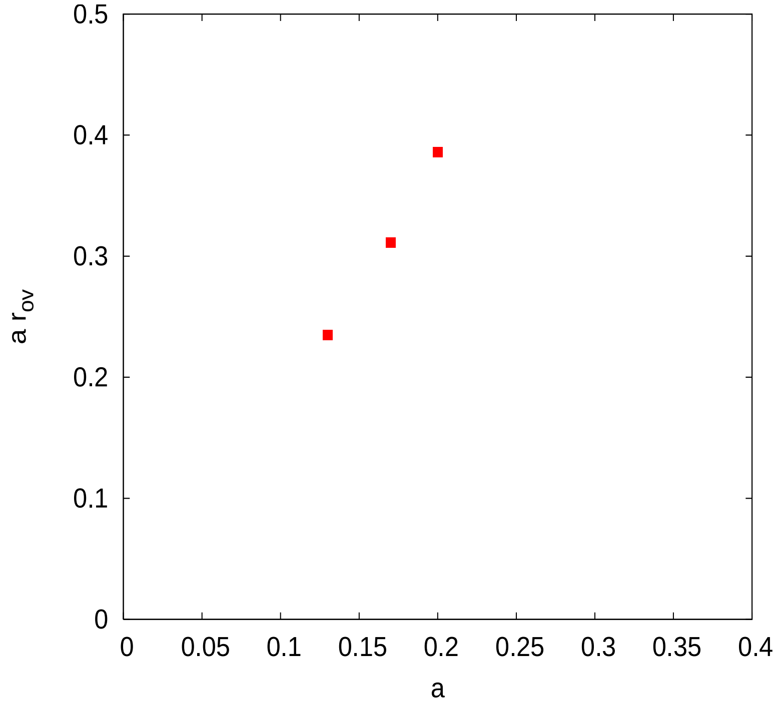
$$\delta\psi = T\gamma_5(1 - \tfrac{1}{2}D)\psi, \; \delta\overline{\psi} = \overline{\psi}(1 - \tfrac{1}{2}D)\gamma_5 T, \; \delta(\overline{\psi}D\psi) = 0$$

Locality of overlap action



Expection value of |D(r)|
as a function of the taxi-driver distance

$$r = \|x - y\|_1 = \sum_{\mu} |x_\mu - y_\mu|$$



Taxi-driver range in physical units as a function of lattice spacing

a (fm)	r_{TD}	r_{Eu}
0.20	1.93(1)	1.05(1)
0.17	1.83(1)	0.98(1)
0.13	1.81(1)	0.9(1)

hep-lat/0510075

Ghost States in Hadron Spectrum

Particle	Ghost State	Presence
a_0 (1450) 0^{++}	S-wave $\pi\ \eta'$ $2m_\pi < 1450$	Ground state $\pi\ \eta'$, a_0 . .
1^{-+} (1600?)	S-wave $a_1\ \eta'$ (1230 +140 < 1600)	Ground state $a_1\ \eta'$, 1^{-+}. .
N*(1440) $1/2^+$	P-wave $N\ \eta'$ $\sqrt{m_\pi^2+p_L^2}+\sqrt{m_N^2+p_L^2} < m_N^*$?	1st or 2nd excited state N, $N\ \eta'$, N* ...
N*(1535) $1/2^-$	S-wave $N\ \eta'$ (940 +140 < 1535)	Ground state $N\ \eta'$, N* . . .

Overlap Fermions

❖ Expected features:

- Numerically intensive (~100 times of Wilson fermion to invert)
  Zolotarev approximation of matrix sign function (< 10^{-9} accurancy)
- No $O(a)$ errors (no dimension 5 chirally symmetric action)
- No additive quark mass renormalization
- No exceptional configuration (eigenvalues on a circle)
- Well defined topology both globally and locally
- No mixing of operators in different chiral sectors

❖ Unexpected desirable features:

- ✓ $O(a^2)$ error are small (π and ρ masses on 3 lattices).
- ✓ $O(m^2a^2)$ errors are small (dispersion relation, renormalization constants). This justifies its application on both heavy and light quarks.
- ✓ Critical slowing down is gentle (m_π ~160 MeV).
- ✓ Topological charge density void of large ultra-violet fluctuation (overlap operator is exponentially local)