Baryons in 1/N_c

Iguazu Falls, Argentina

Outline

- Motivation for Ξ masses
- QCD in 1/N_c
- Baryons in 1/N_c
- Spin-flavor symmetry
- Ground state baryons
- Excited baryons: masses
- Predictions for Ξ masses
- Comments and conclusions

Cascade Physics Workshop

J L Goity
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Empirical status of Ξ masses and J^P

Ξ State	Mass [MeV]	Width [MeV]	J^P	PDG Status
$\Xi^0(1314)$	1314.83 ± 0.20	-	$\frac{1}{2}^+$	****
$\Xi^-(1320)$	1321.31 ± 0.13	-	$\frac{1}{2}^+$	****
$\Xi^0(1530)$	1531.80 ± 0.32	9.1 ± 0.5	$\frac{3}{2}^+$	****
$\Xi^-(1535)$	1535.0 ± 0.6	9.9 ± 1.8	$\frac{3}{2}^+$	****
$\Xi(1620)$	~ 1620	20-40	$?^?$	*
$\Xi(1690)$	1690 ± 10	< 30	$?^?$	***
$\Xi(1820)$	1823 ± 5	24^{+15}_{-10}	$\frac{3}{2}^-$	***
$\Xi(1950)$	1950 ± 15	60 ± 20	$?^?$	***
$\Xi(2030)$	2025 ± 5	20^{+15}_{-5}	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$	~ 2120	~ 25	$?^?$	*
$\Xi(2250)$	~ 2250	50 ± 30	$?^?$	**
$\Xi(2370)$	~ 2370	~ 80	$?^?$	**
$\Xi(2500)$	~ 2500	?	$?^?$	*

Ground states

GMO: 1320 vs 1331 MeV
EQS: 139:148:153 MeV

SU(3) symmetry requires

One Ξ I=1/2 per octet and per decuplet

Gell-Mann-Okubo mass relation in octet:

$$m_\Xi = \frac{m_\Sigma + 3m_\Lambda}{2} - m_N$$

Equal spacing mass relation in decuplet:

$$m_\Omega - m_\Xi = m_\Xi - m_\Sigma = m_\Sigma - m_\Delta$$

Ξs in O(3) x SU(6)

Excited Ξs in $O(3) \times SU(6)$ Multiplets											
$[\ell = 0, 56]^+$ Carlson & Carone			$[\ell = 1, 70]^-$ Schat, Scoccola & JLG			$[\ell = 2, 56]^+$ Schat, Scoccola & JLG			$[\ell = 4, 56]^+$ Matagne & Stancu		
State	$1/N_c$	Exp	State	$1/N_c$	Exp	State	$1/N_c$	Exp	State	$1/N_c$	Exp
$\Xi^8_{1/2}$	1825 ± 98	-	$\Xi^8_{1/2}$	1780 ± 20	-	$\Xi^8_{3/2}$	2081 ± 57	-	$\Xi^8_{7/2}$	2460 ± 166	-
$\Xi^{10}_{3/2}$	1955 ± 196	-	$\Xi^8_{3/2}$	1815 ± 20	1823 ± 5	$\Xi^8_{5/2}$	1997 ± 50	-	$\Xi^8_{9/2}$	2465 ± 165	-
			$\Xi^8_{1/2}$	1927 ± 20	-	$\Xi^{10}_{1/2}$	2237 ± 90	-	$\Xi^{10}_{5/2}$	2700 ± 266	-
			$\Xi^8_{3/2}$	1980 ± 20	-	$\Xi^{10}_{3/2}$	2216 ± 80	-	$\Xi^{10}_{7/2}$	2592 ± 203	-
			$\Xi^8_{5/2}$	1974 ± 20	-	$\Xi^{10}_{5/2}$	2181 ± 65	-	$\Xi^{10}_{9/2}$	2598 ± 250	-
			$\Xi^{10}_{1/2}$	1922 ± 20	-	$\Xi^{10}_{7/2}$	2131 ± 80	-	$\Xi^{10}_{11/2}$	2715 ± 260	-
			$\Xi^{10}_{3/2}$	1973 ± 20	-						

Other multiplets, e.g. $\ell = 0,\ 2\quad 70-\text{plets}$ too incomplete for a $1/N_c$ analysis.

QCD in $1/N_c$

The expansion parameters of QCD:

- Light quark masses: $m_u, \; m_d, \; m_s$

Approximate chiral $SU_L(3)xSU_R(3)$ chiral symmetry Goldstone bosons from SChSB.
Expansion in m_q. Non-analytic terms present from chiral loops with GBs.
Approximate SU(3) flavor symmetry.

- Heavy quark masses: $m_c, \; m_b$

Approximate Isgur-Wise spin-flavor symmetry expansion in $1/m_Q$.
Non-analytic terms present from QCD loops.

- $1/N_c$: ['tHooft (1974)]

Expansion analytic in $1/N_c$
Respects symmetries of QCD except $U_A(1)$
Gives rise to spin-flavor symmetry in baryons.

$N_c \to \infty$ limit gives consistent non-trivial theory if and only if $\alpha_s = \mathcal{O}(1/N_c)$

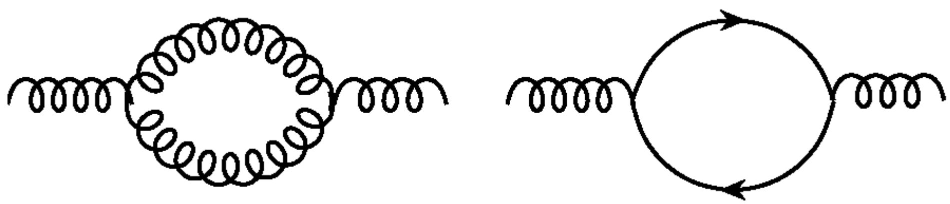
'tHooft-limit: $N_c \to \infty, N_f$ fixed.

Veneziano-limit: $N_c \to \infty, N_f/N_c$ fixed.

Consistency:

$$\beta(\alpha_s) = \frac{\alpha_s^2}{4\pi} \frac{2N_f - 11N_c}{3} + \mathcal{O}(N_c^2 \alpha_s^3)$$

$$\alpha_s(\mu) = \frac{12\pi}{(11N_c - 2N_f)\log \mu^2/\Lambda_{QCD}^2}$$

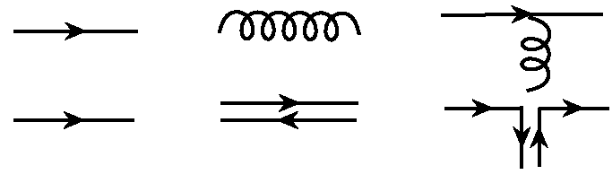


N_c^0 $\qquad$ N_f/N_c

In 'tHooft-limit quark loops are suppressed

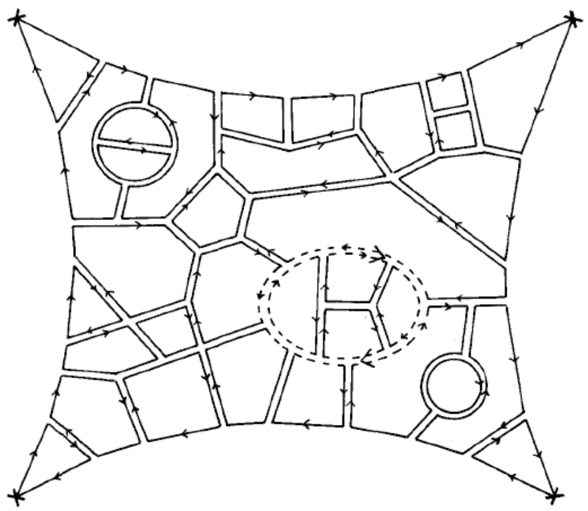
MESONS AND GLUEBALLS

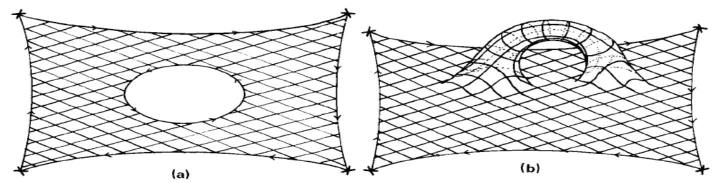
‘tHooft quark-line notation



etc

$1/N_c$ counting at level of Feynman diagrams





$1/N_c$ power: given by -Euler character of surface:

$$\nu = -2 + L + 2H$$

$1/N_c$ summary for mesons and glueballs

Mesons

$$M = \mathcal{O}(N_c^0)$$
$$\Gamma = \mathcal{O}(1/N_c)$$
$$F_M = \mathcal{O}(N_c^{\frac{1}{2}})$$
$$\sigma = \mathcal{O}(N_c^{-2})$$

Nonet symmetry

$$M_{\eta'}^2 = \mathcal{O}(1/N_c) \text{ in } \chi \text{ limit}$$
$$\langle \bar{q}q \rangle = \mathcal{O}(N_c)$$
$$\chi - \text{logs} = \mathcal{O}(1/N_c)$$

$qq\overline{qq}$ states are suppressed

Glueballs

$$M = \mathcal{O}(N_c^0)$$
$$\Gamma = \mathcal{O}(1/N_c^2)$$
$$F_M = \mathcal{O}(N_c)$$
$$0^- - \text{glueball} - \eta' \text{ mixing} = \mathcal{O}(N_c^{-\frac{1}{2}})$$
$$\langle GG \rangle = \mathcal{O}(N_c^2)$$

BARYONS IN LARGE N_c

Valence quark picture

Baryon radius=O(N_c^0)

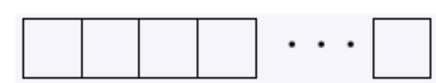
Baryon mass=O(N_c) → Hartree picture in large N_c (Witten)

For heavy enough quarks:

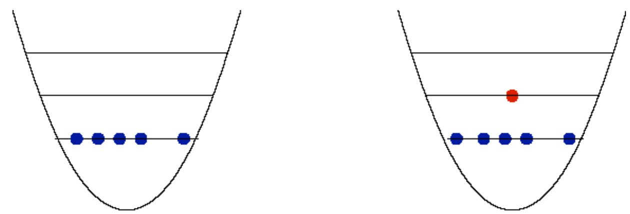
$$|\Psi> = \frac{1}{N_c!} \int \prod_{j=1}^{N_c} d^3x_j \Psi_{\xi_1 \cdots \xi_{N_c}}(x_1, \cdots, x_{N_c}) \epsilon_{\alpha_1 \cdots \alpha_{N_c}} |x_1, \xi_1, \alpha_1; \cdots ; x_{N_c}, \xi_{N_c}, \alpha_{N_c}>$$

Ground state baryons

$$\Psi^{GS}_{\xi_1 \cdots \xi_{N_c}}(x_1, \cdots, x_{N_c}) = \chi^S_{\xi_1 \cdots \xi_{N_c}} \prod_{i=1}^{N_c} \phi(x_i) \qquad \chi^S :$$



Excited baryons



$$\Psi^{S}_{\xi_1,\cdots,\xi_{N_c}}(x_1,\cdots,x_{N_c}) = \frac{1}{\sqrt{N_c}}\chi^{S}_{\xi_1,\cdots,\xi_{N_c}}\sum_{i=1}^{N_c}\phi(x_1)\cdots\phi'(x_i)\cdots\phi(x_{N_c})$$

$$\Psi^{MS}_{\xi_1,\cdots,\xi_{N_c}}(x_1,\cdots,x_{N_c}) = \frac{1}{\sqrt{N_c}(N_c-1)!}\sum_{\sigma}\chi^{MS}_{\xi_{\sigma_1},\cdots,\xi_{\sigma_{N_c}}}\phi(x_{\sigma_1})\cdots\phi(x_{\sigma_{N_c-1}})\phi'(x_{\sigma_{N_c}})$$

Matrix elements

1-body operators

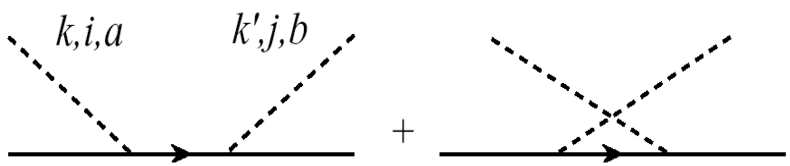
$$\begin{aligned} O_{1-body} &= q^\dagger\Gamma_1 q \\ \langle GS \mid O_{1-body} \mid GS\rangle &\sim N_c\,\langle\chi^S \mid \Gamma_1 \mid \chi^S\rangle\langle\phi^*\phi\rangle \\ \langle GS \mid O_{1-body} \mid ES\rangle &\sim \sqrt{N_c}\,\langle\chi^S \mid \Gamma_1 \mid \chi^{ES}\rangle\langle\phi^*\phi'\rangle \\ \langle\chi^S \mid \Gamma_1 \mid \chi^S\rangle &= \chi^{S*}_{\xi_1\cdots\xi_{N_c-1}\xi'}\Gamma_{1\,\xi'\xi}\chi^{S}_{\xi_1\cdots\xi_{N_c-1}\xi} \end{aligned}$$

2-body operators

$$\begin{aligned} O_{2-body} &= q^\dagger\otimes q^\dagger\Gamma_2 q\otimes q \\ \langle GS \mid O_{2-body} \mid GS\rangle &\sim \langle\chi^S \mid \Gamma_2 \mid \chi^S\rangle\langle\phi^*\phi^*\phi\phi\rangle \\ \langle GS \mid O_{2-body} \mid ES\rangle &\sim \frac{1}{\sqrt{N_c}}\langle\chi^S \mid \Gamma_2 \mid \chi^{ES}\rangle\langle\phi^*\phi^*\phi\phi'\rangle \\ \langle\chi' \mid \Gamma_2 \mid \chi\rangle &= \chi'^{*}_{\xi_1\cdots\xi_{N_c-2}\xi''\xi'''}\Gamma_{2\,\xi''\xi'''\xi\xi'}\chi_{\xi_1\cdots\xi_{N_c-2}\xi\xi'} \end{aligned}$$

SPIN-FLAVOR SYMMETRY

Pion-baryon coupling: $g_A \frac{N_c}{F_\pi} \partial_i \pi^a X^{ia} = \mathcal{O}(\sqrt{N_c})$



$$\propto \frac{N_c^2}{F_\pi^2} \left\{ \langle B' \mid X^{jb} X^{ia} \mid B \rangle \frac{i}{k_0} - \langle B' \mid X^{ia} X^{jb} \mid B \rangle \frac{i}{k_0'} \right\}$$

$$(\text{since } \; k_0 = k_0' + \mathcal{O}(1/N_c) :)$$

$$= \frac{N_c^2}{F_\pi^2} \frac{i}{k_0} \left(\langle B' \mid [X^{jb}, X^{ia}] \mid B \rangle + \mathcal{O}(1/N_c^2) \right)$$

Gervais-Sakita-Dashen-Manohar consistency $\langle B' \mid [X^{jb}, X^{ia}] \mid B \rangle = \mathcal{O}(1/N_c)$

$S^i, \quad T^a, \quad X^{ia}$ Generate SU(2N$_f$) contracted group X^{ia} semiclassical

In large N_c limit GS baryons fill a tower of states which are degenerate for states with spins of order 1.

Generators of SU(6): S^i, T^a, G^{ia} $\quad X^{ia} = 1/N_c\, G^{ia} + \cdots$

$$[S^i, T^a] = 0$$

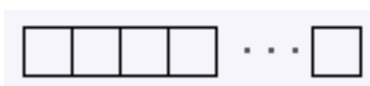
$$[S^i, S^j] = i\epsilon^{ijk} S^k \qquad [T^a, T^b] = i f^{abc} T^c$$

$$[S^i, G^{ja}] = i\epsilon^{ijk} G^{ka} \qquad [T^a, G^{ib}] = i f^{abc} G^{ic}$$

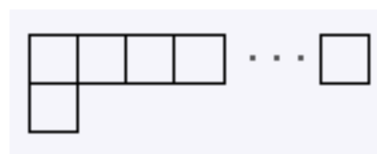
$$[G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2N_f}\delta^{ab}\epsilon^{ijk} S^k + \frac{i}{2}\epsilon^{ijk} f^{abc} G^{kc}$$

Contracted spin-flavor symmetry in large N_c limit

Spin-flavor basis of baryon states: multiplets of SU(6)



S

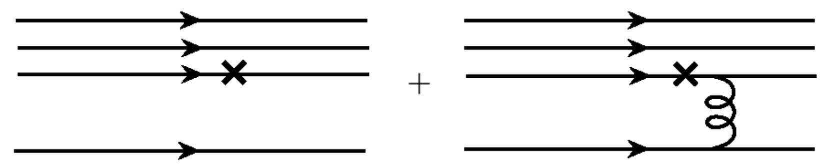


MS

etc.

OPERATOR ANALYSIS: GS Baryons

QCD operator $\mathcal{Q} = \bar{q}\,\Gamma\, q$



Effective operator at baryon level $\mathcal{Q} = \sum_{n,i} C_n^i Q_n^i$

Q_n^i n-body operator acting on baryon degrees of freedom

Use Wigner-Eckart: $\mathcal{Q}_n^i = W_n^i\, \mathcal{G}_1 \cdots \mathcal{G}_n$

$\mathcal{G}_j$: generator of $SU(6)$

$1/N_c$ power counting

$$\nu_{\mathrm{n-body}} = n - 1 - \kappa, \quad \kappa \geq 0 \ \text{ coherence index}$$

Examples

1-body

	1	S^i	T^a	G^{ia}
κ :	1	0	0	1

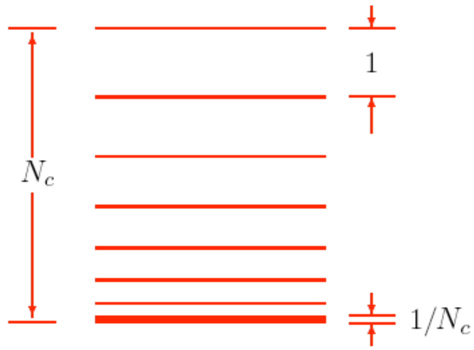
2-body

	$\frac{1}{N_c} S^i S^j$	$\frac{1}{N_c} S^i G^{ja}$	$\frac{1}{N_c} T^a T^b$	$\frac{1}{N_c} G^{ia} T^b$	$\frac{1}{N_c} G^{ia} G^{jb}$
κ :	0	1	0	1	2

RESULTS FOR GROUND STATE BARYONS

- Masses

$$M = c_0 N_c + c_2 \frac{S^2}{N_c} + \epsilon c_1 T^8$$



Parameter free mass relations

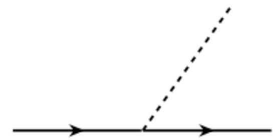
[Dashen, Jenkins & Manohar]

GM-O	$\Xi^8 - \Sigma^8 = \frac{1}{2}\left(-\Sigma^8 + 3\Lambda\right) - N$	128 MeV vs 141 MeV
EQS	$\Sigma^{10} - \Delta = \Xi^{10} - \Sigma^{10}$	153 MeV vs 145 MeV
EQS	$\Xi^{10} - \Sigma^{10} = \Omega^- - \Xi^{10}$	145 MeV vs 142 MeV
8-10	$\Sigma^{10} - \Sigma^8 = \Xi^{10} - \Xi^8$	212 MeV vs 195 MeV
8-10	$3\Lambda + \Sigma^8 - 2(N + \Xi^8) = -(\Omega - \Xi^{10} - \Sigma^{10} + \Delta)$	26 MeV vs 11 MeV

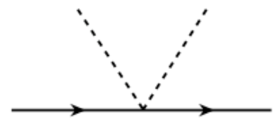
These relations are corrected at order $1/N_c^2$

NNLO mass analysis shows convergence of expansion [Jenkins & Lebed]

- Couplings to pseudoscalar mesons

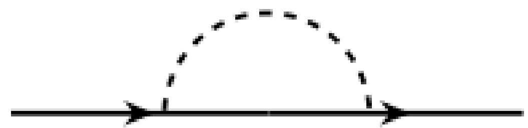


$\mathcal{O}(\sqrt{N_c})$



$\mathcal{O}(1/N_c)$

- Chiral loops in SU(3): baryon self-energy



$$\Sigma_{\chi-loop} \propto \frac{1}{F_\pi^2}\langle GS \mid G^{ia}G^{ia} \mid GS\rangle$$
$$= \frac{1}{F_\pi^2}\left(\frac{3}{8}N_c(N_c+6) - \frac{5}{12}\langle GS \mid S^i S^i \mid GS\rangle\right)$$

K and eta chiral loop effects on non-strange baryons:

$$\Sigma_{\chi-loop-K,\eta} \propto \frac{1}{F_\pi^2}\langle GS \mid \frac{3}{8} + \frac{1}{2}I^2 - \frac{1}{3}S^2 \mid GS\rangle$$

EXCITED BARYONS

[JLG; Pirjol & Yan; Carlson et al.; Schat, Scoccola and JLG]

Convenient basis of states: multiplets of O(3) x SU(6)

$$(N_c, 0)_{SU(6)} \rightarrow \oplus(S, (2S, \frac{N_c}{2} - S))$$

56-plet for N_c=3

$$(N_c - 2, 1)_{SU(6)} \rightarrow (S = 1/2, (1, \frac{N_c - 1}{2})) \oplus (S = 1/2, (0, \frac{N_c - 3}{2}))$$
$$\oplus \quad (S = 1/2, (3, \frac{N_c - 3}{2})) \oplus (S = 3/2, (1, \frac{N_c - 1}{2})) \oplus \cdots$$

70-plet for N_c=3

$$[0^+, 56], \quad [1^-, 70], \quad , [2^+, 56], \quad [0^+, 70], \quad \text{etc.}$$

Operator Analysis

$$\mathcal{Q} = \sum R \otimes \mathcal{G}$$

$R: \; O(3)$ tensor; $\qquad \mathcal{G}: \; SU(6)$ tensor

$$\mathcal{G} = N_c{}^{-n} \, \lambda \times \prod_{r=1}^{n} \Lambda_c^{(r)} \qquad \lambda + s^i, t^a, g^{ia} \quad \Lambda_c = S_c^i, T_c^a, G_c^{ia}$$

Masses of $[\ell = 1, \; 70]^-$ baryons

[Schat, Scoccola & JLG; SU(4): Carlson, Carone, JLG & Lebed]

Mass operator

$$M = \sum c_n O_n + \sum d_m B_m$$

$$O_n \; : \; \mathrm{SU(3) singlet}$$

$$B_m \; : \; \mathrm{SU(3) octet}$$

Analysis @ order $1/N_c$ and LO order in SU(3) breaking

Basis of mass operators and effective coefficients from fit to 70 masses

Operator	Coefficient [MeV]
$O_0 = N_c\ 1$	$c_0 =$ 449 ± 2
$O_1 = N_c\ t^a T^a_c - \frac{1}{2\sqrt{3}N_c} O_0$	$c_1 =$ -81 ± 36
$O_2 = l_h\ s_h$	$c_2 =$ 52 ± 15
$O_3 = \frac{3}{N_c}\ l^{(2)}_{hk}\ g_{ha}\ G^c_{ka}$	$c_3 =$ 116 ± 44
$O_4 = \frac{4}{N_c+1}\ l_h\ t_a\ G^c_{ha}$	$c_4 =$ 110 ± 16
$O_5 = \frac{1}{N_c}\ l_h\ S^c_h$	$c_5 =$ 74 ± 30
$O_6 = \frac{1}{N_c}\ S^c_h\ S^c_h$	$c_6 =$ 480 ± 15
$O_7 = \frac{1}{N_c}\ s_h\ S^c_h$	$c_7 =$ -159 ± 50
$O_8 = \frac{1}{N_c}\ l^{(2)}_{hk} s_h\ S^c_k$	$c_8 =$ 6 ± 110
$O_9 = \frac{1}{N_c^2}\ l_h\ g_{ka}\{S^c_k, G^c_{ha}\}$	$c_9 =$ 213 ± 153
$O_{10} = \frac{1}{N_c^2} t_a\{S^c_h, G^c_{ha}\}$	$c_{10} =$ -168 ± 56
$O_{11} = \frac{1}{N_c^2}\ l_h\ g_{ha}\{S^c_k, G^c_{ka}\}$	$c_{11} =$ -133 ± 130
$\bar{B}_1 = T^c_8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 =$ -194 ± 17
$\bar{B}_2 = \frac{1}{N_c}\ d_{8ab}\ g_{ha}\ G^c_{hb} + \frac{N_c^2-9}{16\sqrt{3}N_c^2(N_c-1)} O_0 + \frac{1}{4\sqrt{3}(N_c-1)} O_6 + \frac{1}{12\sqrt{3}} O_7$	$d_3 =$ -150 ± 301
$\bar{B}_3 = l_h\ g_{h8} - \frac{1}{2\sqrt{3}} O_2$	$d_4 =$ -82 ± 57

Results

State	Masses [MeV] Expt.	$1/N_c$	QM
$N_{1/2}$	1538 ± 18	1541	1490
$\Lambda_{1/2}$	1670 ± 10	1667	1650
$\Sigma_{1/2}$	(1620)	1637	1650
$\Xi_{1/2}$		1779	1780
$N_{3/2}$	1523 ± 8	1532	1535
$\Lambda_{3/2}$	1690 ± 5	1676	1690
$\Sigma_{3/2}$	1675 ± 10	1667	1675
$\Xi_{3/2}$	1823 ± 5	1815	1800
$N'_{1/2}$	1660 ± 20	1660	1655
$\Lambda'_{1/2}$	1785 ± 65	1806	1800
$\Sigma'_{1/2}$	1765 ± 35	1755	1750
$\Xi'_{1/2}$		1927	1900
$N'_{3/2}$	1700 ± 50	1699	1745
$\Lambda'_{3/2}$		1864	1880
$\Sigma'_{3/2}$		1769	1815
$\Xi'_{3/2}$		1980	1985
$N_{5/2}$	1678 ± 8	1671	1670
$\Lambda_{5/2}$	1820 ± 10	1836	1815
$\Sigma_{5/2}$	1775 ± 5	1784	1760
$\Xi_{5/2}$		1974	1930
$\Delta_{1/2}$	1645 ± 30	1645	1685
$\Sigma''_{1/2}$		1784	1810
$\Xi''_{1/2}$		1922	1930
$\Omega_{1/2}$		2061	2020
$\Delta_{3/2}$	1720 ± 50	1720	1685
$\Sigma''_{3/2}$		1847	1805
$\Xi''_{3/2}$		1973	1920
$\Omega_{3/2}$		2100	2020
$\Lambda''_{1/2}$	1407 ± 4	1407	1490
$\Lambda''_{3/2}$	1520 ± 1	1520	1490

Parameter free mass relations for 70-plet: GMO, EQS, and:

Relation	Test [MeV]
$\Lambda_{3/2} + \Lambda'_{3/2} - 27/14\Sigma_{5/2} = -9/2\Lambda_{5/2} - 18/7(\Sigma_{1/2} + \Sigma'_{1/2}) + 34/7(\Lambda_{1/2} + \Lambda'_{1/2})$	none
$-(\Sigma_{3/2} + \Sigma'_{3/2}) + 9/7(\Lambda_{1/2} + \Lambda'_{1/2}) = 3/2\Lambda_{5/2} - 9/14\Sigma_{5/2} - 1/7(\Sigma_{1/2} + \Sigma'_{1/2})$	none
$\Sigma''_{1/2} - 19/14\Sigma_{5/2} = -7/2\Lambda_{5/2} - 23/14(\Sigma_{1/2} + \Sigma'_{1/2}) + 45/14(\Lambda_{1/2} + \Lambda'_{1/2})$	none
$7/5\Sigma''_{3/2} - \Sigma_{5/2} = 14/5\Lambda_{5/2} - 11/10(\Sigma_{1/2} + \Sigma'_{1/2}) + 27/10(\Lambda_{1/2} + \Lambda'_{1/2})$	none

Remarks on 70-plet masses

- Hyperfine operator gives dominant spin-flavor breaking.
- Spin-orbit and other N_c^0 operators have small effects.
- Spin-orbit determined by singlet Λ splitting.
- O_4 necessary to resolve spin-orbit puzzle of QM.
- O_3 determines the mixing of the $8_{1/2}$ and $8_{3/2}$ states.
- GMO tested and satisfied in $8^{1/2}{}_{3/2}$.
- $\Xi^8{}_{5/2}$ and $\Xi'^8{}_{1/2}$ masses would test two more GMO.

$[\ell = 0,\ 56]^{+}$ Masses

Same mass relations as GS 56-plet

GM-O	$\Xi^8 = \frac{1}{2}\,(\Sigma^8 + 3\Lambda) - N$
EQS	$\Sigma^{10} - \Delta = \Xi^{10} - \Sigma^{10}$
EQS	$\Xi^{10} - \Sigma^{10} = \Omega^- - \Xi^{10}$
8-10	$\Sigma^{10} - \Sigma^8 = \Xi^{10} - \Xi^8$
8-10	$3\Lambda + \Sigma^8 - 2(N + \Xi^8) = -(\Omega - \Xi^{10} - \Sigma^{10} + \Delta)$

Results

State	PDG Mass [MeV]	$1/N_c$
N(1440)	1440-1470	
$\Lambda(1600)$	1560-1700	
$\Sigma(1660)$	1630-1690	
Ξ^8	–	1790 ± 110
$\Delta(1600)$	1550-1700	
$\Sigma(1840)$	~ 1840	1730 ± 125
Ξ^{10}		1860 ± 230
Ω^-	–	–

$[\ell = 2, 56]^+$ masses

[Schat, Scoccola & JLG]

	$1/N_c$ [MeV]	PDG Mass [MeV]
$N_{3/2}$	1674 ± 15	1700 ± 50
$\Lambda_{3/2}$	1876 ± 39	1880 ± 30
$\Sigma_{3/2}$	1881 ± 25	(1840)
$\Xi_{3/2}$	2081 ± 57	
$N_{5/2}$	1689 ± 14	1683 ± 8
$\Lambda_{5/2}$	1816 ± 33	1820 ± 5
$\Sigma_{5/2}$	1920 ± 24	1918 ± 18
$\Xi_{5/2}$	1997 ± 49	
$\Delta_{1/2}$	1897 ± 32	1895 ± 25
$\Sigma_{1/2}$	2068 ± 52	
$\Xi_{1/2}$	2237 ± 88	
$\Omega_{1/2}$	2408 ± 127	
$\Delta_{3/2}$	1906 ± 27	1935 ± 35
$\Sigma'_{3/2}$	2061 ± 44	(2080)
$\Xi'_{3/2}$	2216 ± 76	
$\Omega_{3/2}$	2373 ± 110	
$\Delta_{5/2}$	1921 ± 21	1895 ± 25
$\Sigma'_{5/2}$	2051 ± 37	(2070)
$\Xi'_{5/2}$	2181 ± 64	
$\Omega_{5/2}$	2313 ± 94	
$\Delta_{7/2}$	1942 ± 27	1950 ± 10
$\Sigma_{7/2}$	2036 ± 44	2033 ± 8
$\Xi_{7/2}$	2131 ± 76	
$\Omega_{7/2}$	2229 ± 110	

Basis of mass operators

Operator	Coefficient (MeV)
$O_1 = N_c\, \mathbf{1}$	$c_1 = 541 \pm 4$
$O_2 = \frac{1}{N_c} l_i\, S_i$	$c_2 = 18 \pm 16$
$O_3 = \frac{1}{N_c} S_i S_i$	$c_3 = 241 \pm 14$
$\bar{B}_1 = -\mathcal{S}$	$b_1 = 206 \pm 18$
$\bar{B}_2 = \frac{1}{N_c} l_i G_{i8} - \frac{1}{2\sqrt{3}} O_2$	$b_2 = 104 \pm 64$
$\bar{B}_3 = \frac{1}{N_c} S_i G_{i8} - \frac{1}{2\sqrt{3}} O_3$	$b_3 = 223 \pm 68$

Parameter free mass relations

	Test [MeV]
$\Delta_{5/2} - \Delta_{3/2} = N_{5/2} - N_{3/2}$	-40 ± 43 *vs* -17 ± 50
$5/7(\Delta_{7/2} - \Delta_{5/2}) = (N_{5/2} - N_{3/2})$	40 ± 20 *vs* -17 ± 50
$\Delta_{7/2} - \Delta_{1/2} = 3(N_{5/2} - N_{3/2})$	55 ± 27 *vs* 99 ± 151
$8/15(\Lambda_{3/2} - N_{3/2}) + 22/15(\Lambda_{5/2} - N_{5/2}) = \Sigma_{5/2} - \Lambda_{5/2} + 2(\Sigma_{7/2} - \Delta_{7/2})$	296 ± 59 *vs* 264 ± 31
$\Lambda_{5/2} - \Lambda_{3/2} + 3(\Sigma_{5/2} - \Sigma_{3/2}) = 4(N_{5/2} - N_{3/2})$	174 ± 110 *vs* -68 ± 200
$\Lambda_{5/2} - \Lambda_{3/2} + \Sigma_{5/2} - \Sigma_{3/2} = 2(\Sigma'_{5/2} - \Sigma'_{3/2})$	$(-80 \pm 45$ *vs* $-20 \pm 84)$
$7/12\, \Sigma'_{3/2} + 5/12\, \Sigma_{7/2} = \Sigma'_{5/2}$	2060 ± 17 *vs* 2070 ± 30
$4/5\, \Sigma_{1/2} + 1/5\, \Sigma_{7/2} = \Sigma'_{3/2}$	$--$
(GMO) $2(N + \Xi) = 3\,\Lambda + \Sigma$	$--$
(EQS) $\Sigma - \Delta = \Xi - \Sigma = \Omega - \Xi$	$--$

$1/N_c$ predictions for Ξ masses

Ξ State	Mass [MeV]	Width [MeV]	J^P	PDG Status
$\Xi^0(1314)$	1314.83 ± 0.20	-	$\frac{1}{2}^+$	****
$\Xi^-(1320)$	1321.31 ± 0.13	-	$\frac{1}{2}^+$	****
$\Xi^0(1530)$	1531.80 ± 0.32	9.1 ± 0.5	$\frac{3}{2}^+$	****
$\Xi^-(1535)$	1535.0 ± 0.6	9.9 ± 1.8	$\frac{3}{2}^+$	****
$\Xi(1620)$	~ 1620	20-40	$?^?$	*
$\Xi(1690)$	1690 ± 10	< 30	$?^?$	***
$\Xi(1820)$	1823 ± 5	24^{+15}_{-10}	$\frac{3}{2}^-$	***
$\Xi(1950)$	1950 ± 15	60 ± 20	$?^?$	***
$\Xi(2030)$	2025 ± 5	20^{+15}_{-5}	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$	~ 2120	~ 25	$?^?$	*
$\Xi(2250)$	~ 2250	50 ± 30	$?^?$	**
$\Xi(2370)$	~ 2370	~ 80	$?^?$	**
$\Xi(2500)$	~ 2500	?	$?^?$	*

Excited Ξs in $O(3) \times SU(6)$ Multiplets

$[\ell = 0, 56]^+$ Carlson & Carone			$[\ell = 1, 70]^-$ Schat, Scoccola & JLG			$[\ell = 2, 56]^+$ Schat, Scoccola & JLG			$[\ell = 4, 56]^+$ Matagne & Stancu		
State	$1/N_c$	Exp	State	$1/N_c$	Exp	State	$1/N_c$	Exp	State	$1/N_c$	Exp
$\Xi^8_{1/2}$	1825 ± 98	-	$\Xi^8_{1/2}$	1780 ± 20	-	$\Xi^8_{3/2}$	2081 ± 57	-	$\Xi^8_{7/2}$	2460 ± 166	-
$\Xi^{10}_{3/2}$	1955 ± 196	-	$\Xi^8_{3/2}$	1815 ± 20	1823 ± 5	$\Xi^8_{5/2}$	1997 ± 50	-	$\Xi^8_{9/2}$	2465 ± 165	-
			$\Xi'^8_{1/2}$	1927 ± 20	-	$\Xi^{10}_{1/2}$	2237 ± 90	-	$\Xi^{10}_{5/2}$	2700 ± 266	-
			$\Xi'^8_{3/2}$	1980 ± 20	-	$\Xi^{10}_{3/2}$	2216 ± 80	-	$\Xi^{10}_{7/2}$	2592 ± 203	-
			$\Xi^8_{5/2}$	1974 ± 20	-	$\Xi^{10}_{5/2}$	2181 ± 65	-	$\Xi^{10}_{9/2}$	2598 ± 250	-
			$\Xi^{10}_{1/2}$	1922 ± 20	-	$\Xi^{10}_{7/2}$	2131 ± 80	-	$\Xi^{10}_{11/2}$	2715 ± 260	-
			$\Xi^{10}_{3/2}$	1973 ± 20	-						

	Masses [MeV]	Spin-flavor content			
State	$1/N_c$	$^2 1$	$^2 8$	$^4 8$	$^2 10$
$\Xi_{1/2}$	1779		0.85	0.44	0.29
$\Xi_{3/2}$	1815		-0.98	0.03	-0.19
$\Xi'_{1/2}$	1927		-0.46	0.87	0.18
$\Xi'_{3/2}$	1980		-0.02	(-0.57)	(-0.82)
$\Xi_{5/2}$	1974			1.00	
$\Xi''_{1/2}$	1922		-0.14	-0.31	0.94
$\Xi''_{3/2}$	1973		-0.19	(-0.80)	(0.57)

70-plet:

- 7 masses in range 1750-2000 MeV
- Two pairs of same J states nearly degenerate
- Most significant predictions of $1/N_c$ analysis

L=2 56-plet:

- 6 masses in range 2000-2250 MeV
- Same J states separated by >140 MeV
- Larger errors in predictions than in 70-plet
- Parameter free mass relations rather well tested.

COMMENTS, CONCLUSIONS

Implications of new cascade data on theory:

- Test predictions and help further establish identification of states in multiplets. Important for further establishing $1/N_c$ approach to baryons.
- More information helps with problem of configuration mixing so far not addressed.
- It would be great to know partial decay widths in particular $\Xi^* \to \Xi\pi, \; \Xi^* \to \Lambda K$ to tighten the $1/N_c$ analysis of decays, in particuler 70-plet decays.

Predictions for cascades from $1/N_c$

- From pattern of O(3) x SU(6) multiplets broken primarily by hyperfine interactions with small spin-orbit effects. Results are not very different to those from quark model.
- Most significant predictions for cascade states in negative parity 70-plet and L=2 56-plet: 7 and 8 cascade states respectively. Only one established!.

70-plet

State	Masses [MeV] Expt.	Large N_c	QM	Spin-flavor content ^{21}	^{28}	^{48}	^{210}
$N_{1/2}$	1538 ± 18	1541	1490		0.82	0.57	
$\Lambda_{1/2}$	1670 ± 10	1667	1650	-0.21	0.90	0.37	
$\Sigma_{1/2}$	(1620)	1637	1650		0.52	0.81	0.27
$\Xi_{1/2}$		1779	1780		0.85	0.44	0.29
$N_{3/2}$	1523 ± 8	1532	1535		-0.99	0.10	
$\Lambda_{3/2}$	1690 ± 5	1676	1690	0.18	-0.98	0.09	
$\Sigma_{3/2}$	1675 ± 10	1667	1675		-0.98	-0.01	-0.19
$\Xi_{3/2}$	1823 ± 5	1815	1800		-0.98	0.03	-0.19
$N'_{1/2}$	1660 ± 20	1660	1655		-0.57	0.82	
$\Lambda'_{1/2}$	1785 ± 65	1806	1800	0.10	-0.38	0.92	
$\Sigma'_{1/2}$	1765 ± 35	1755	1750		-0.83	0.54	0.17
$\Xi'_{1/2}$		1927	1900		-0.46	0.87	0.18
$N'_{3/2}$	1700 ± 50	1699	1745		-0.10	-0.99	
$\Lambda'_{3/2}$		1864	1880	0.01	-0.09	-0.99	
$\Sigma'_{3/2}$		1769	1815		0.01	(-0.57)	(-0.82)
$\Xi'_{3/2}$		1980	1985		-0.02	(-0.57)	(-0.82)
$N_{5/2}$	1678 ± 8	1671	1670			1.00	
$\Lambda_{5/2}$	1820 ± 10	1836	1815			1.00	
$\Sigma_{5/2}$	1775 ± 5	1784	1760			1.00	
$\Xi_{5/2}$		1974	1930			1.00	
$\Delta_{1/2}$	1645 ± 30	1645	1685				1.00
$\Sigma''_{1/2}$		1784	1810		-0.14	-0.31	0.94
$\Xi''_{1/2}$		1922	1930		-0.14	-0.31	0.94
$\Omega_{1/2}$		2061	2020				1.00
$\Delta_{3/2}$	1720 ± 50	1720	1685				1.00
$\Sigma''_{3/2}$		1847	1805		-0.19	(-0.80)	(0.57)
$\Xi''_{3/2}$		1973	1920		-0.19	(-0.80)	(0.57)
$\Omega_{3/2}$		2100	2020				1.00
$\Lambda''_{1/2}$	1407 ± 4	1407	1490	0.97	0.23	0.04	
$\Lambda''_{3/2}$	1520 ± 1	1520	1490	0.98	0.18	-0.01	