Quark Distribution and Structure Function at Small-x

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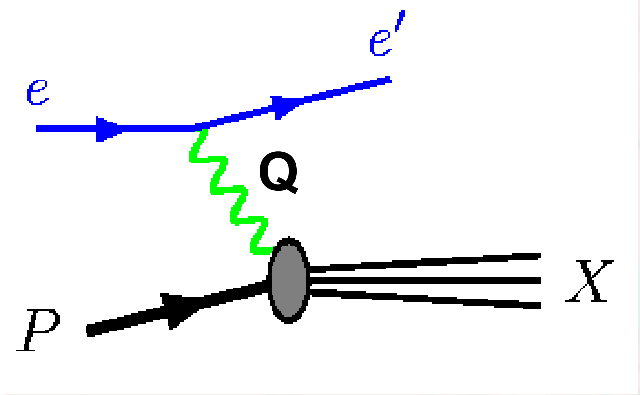
Outline

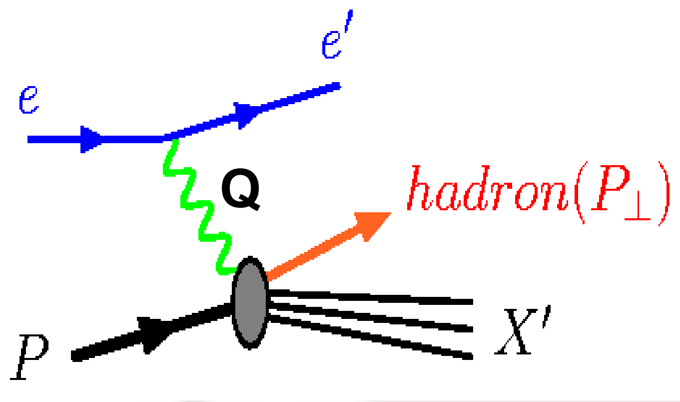
- Quark distribution at small-x
- Structure function at large Q is the leading-twist
- Geometric scalings in hard scattering processes

Inclusive and Semi-inclusive DIS



Inclusive DIS:

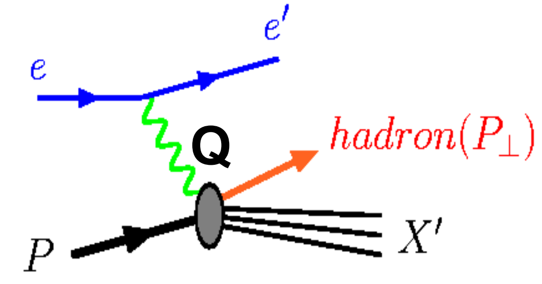
Partonic Distribution depending on the longitudinal momentum fraction



Semi-inclusive DIS:

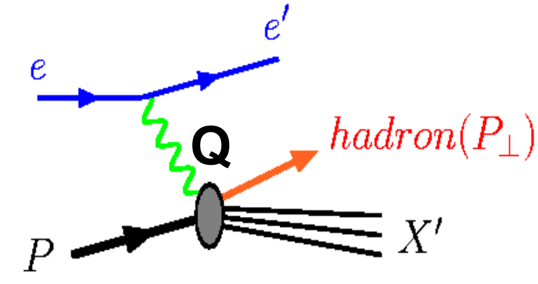
Probe additional information for parton transverse distribution in nucleon/nucle

Advantage of SIDIS



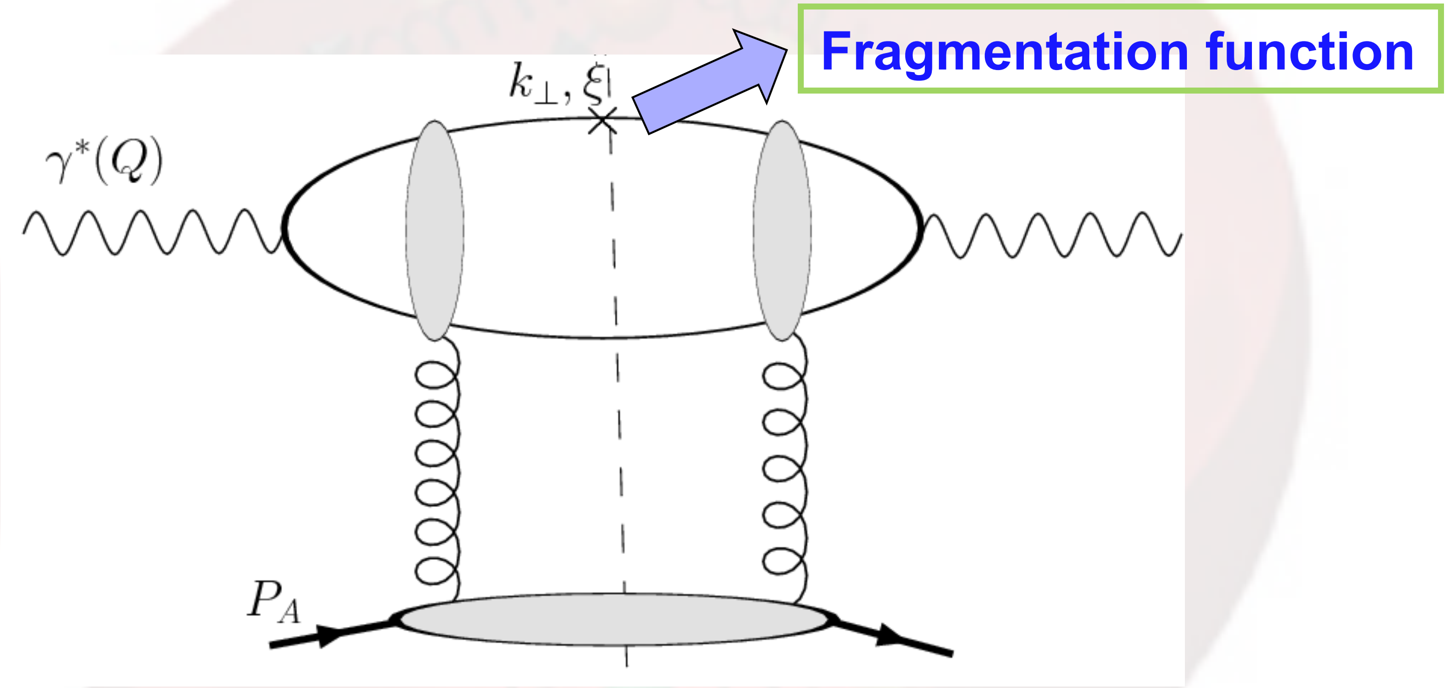
- Direct probe for the transverse momentum dependence of partons
  - Saturation effects explicitly show up in the transverse momentum distribution
- Factorization can be argued for large Q
- Can be related to the TMD factorization

SIDIS at small-x



- What are the relevant scales
  - Q, virtuality of the photon
  - Pt, transverse momentum of hadron
  - Qs, saturation scale
- We are interested in the region of Q>>Qs, Pt
  - TMD factorization makes sense

Dipole picture for DIS



$$\sigma_{\gamma^* H}(\tau, Q^2) = \int_0^1 dz \int d^2 r_\perp \, |\Psi(z, r_\perp; Q^2)|^2 \, \sigma_{\text{dipole}}(\tau, r_\perp)$$

$$\sigma_{\text{dipole}}(\tau, r_\perp) = 2 \int d^2 b_\perp \, \mathcal{N}_\tau(r_\perp, b_\perp)$$

SIDIS Differential Cross section

$$\frac{d\sigma(ep \to e'hX)}{d\mathcal{P}} = \frac{\alpha_{em}^2 N_c}{2\pi^3 x_B Q^2} \sum_f e_f^2 \int_{z_h} \frac{dz}{z} \frac{D(z)}{z^2}$$

$$\int d^2b d^2q_\perp F(q_\perp, x_B) \times \mathcal{H}(\hat{\xi}, k_\perp)$$

$$\mathcal{H}(\hat{\xi}, k_\perp) = \left(1 - y + \frac{y^2}{2}\right)(\hat{\xi}^2 + (1-\hat{\xi})^2) \left| \frac{k_\perp}{k_\perp^2 + \epsilon_f^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + \epsilon_f^2} \right|^2$$

$$+ (1-y) 4\hat{\xi}^2 (1-\hat{\xi})^2 Q^2 \left(\frac{1}{k_\perp^2 + \epsilon_f^2} - \frac{1}{(k_\perp - q_\perp)^2 + \epsilon_f^2} \right)^2 ,$$

$$F(q_\perp, x) = \int \frac{d^2r}{(2\pi)^2} e^{-iq_\perp \cdot r} (1 - T_{q\bar{q}}(r, x))$$

Unintegrated gluon dis.

Small kt limit: Q>>p$_T$

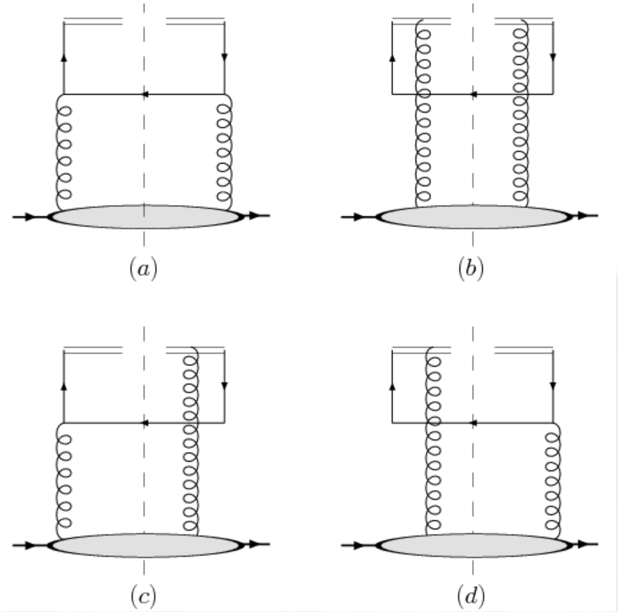
- Keep the leading power contribution, neglect all higher power corrections

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}|_{p_\perp\ll Q} = \frac{\alpha_{em}^2 N_c}{2\pi^3 Q^4}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\int\frac{d\xi}{x_B}$$

$$\times\int d^2b d^2q_\perp F(q_\perp,x_B)A(q_\perp,k_\perp),$$

$$A(q_\perp,k_\perp) = \left|\frac{k_\perp|k_\perp-q_\perp|}{(1-\xi)k_\perp^2+\xi(k_\perp-q_\perp)^2}-\frac{k_\perp-q_\perp}{|k_\perp-q_\perp|}\right|^2$$

TMD quark



McLerran-Venugopalan 98

$$q(x,k_\perp) = \frac{N_c}{8\pi^4}\int\frac{dx'}{x'^2}\int d^2b d^2q_\perp F(q_\perp,x')A(q_\perp,k_\perp)$$

- Reproduce the SIDIS cross section with the TMD quark distribution and the TMD factorization

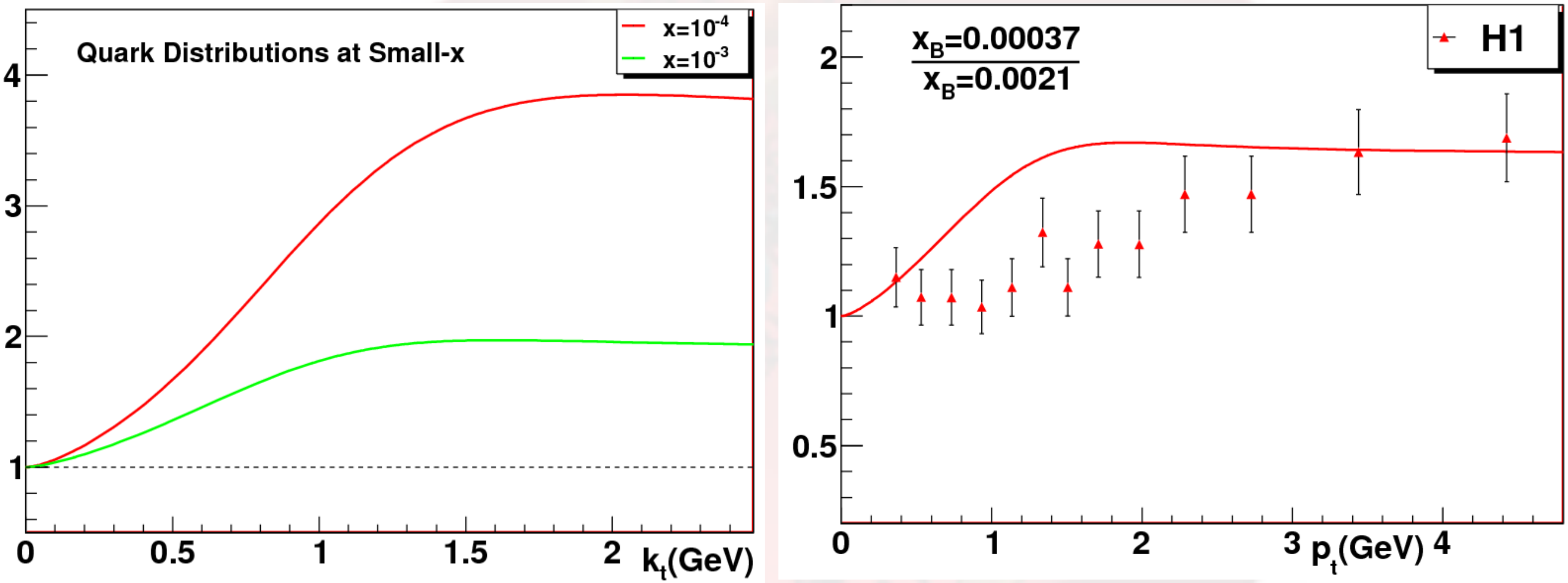
Interesting properties

$$\left.\frac{dx\tilde{q}^{\mathrm{DIS}}(x,q_\perp)}{d^2R_\perp}\right|_{q_\perp^2\gg Q_s^2} = \frac{N_c}{12\pi^4}\frac{Q_s^2}{q_\perp^2}$$

$$\left.\frac{dx\tilde{q}^{\mathrm{DIS}}(x,q_\perp)}{d^2R_\perp}\right|_{q_\perp^2\ll Q_s^2} = \frac{N_c}{4\pi^4}.$$

Mueller 99; McLerran-Venugopalan 99

Quark distribution at different x



Ratio relative to that at 10^{-2}

Comments

- We don't lose the sensitivity to the saturation physics even with Large Q
- We gain the direct probe for the transverse momentum dependence of partons
- Beyond the leading order?
- Additional dynamics involved
  - Soft gluon resummation

Integrated quark distribution

- Rewrite the quark distribution

$$x\tilde{q}^{\mathrm{DIS}}(x,q_\perp) = \frac{N_c}{4\pi^4}\int d^2R_\perp d^2k_\perp F(q_\perp - k_\perp, Q_s)$$
$$\times \int dy \left| \frac{\vec{q}_\perp}{q_\perp^2 + y} - \frac{\vec{k}_\perp}{k_\perp^2 + y} \right|^2$$

Mueller 1999
Xiao-Yuan, 2010

- Integrated quark distribution has Ultraviolet divergence

$$xq(x,\mu) = \frac{1}{\epsilon_{\mathrm{U.V.}}} + Q_s^2 \ln\frac{\mu^2}{Q_s^2} + \mathrm{finite} \quad \mathrm{terms}$$

GBW model

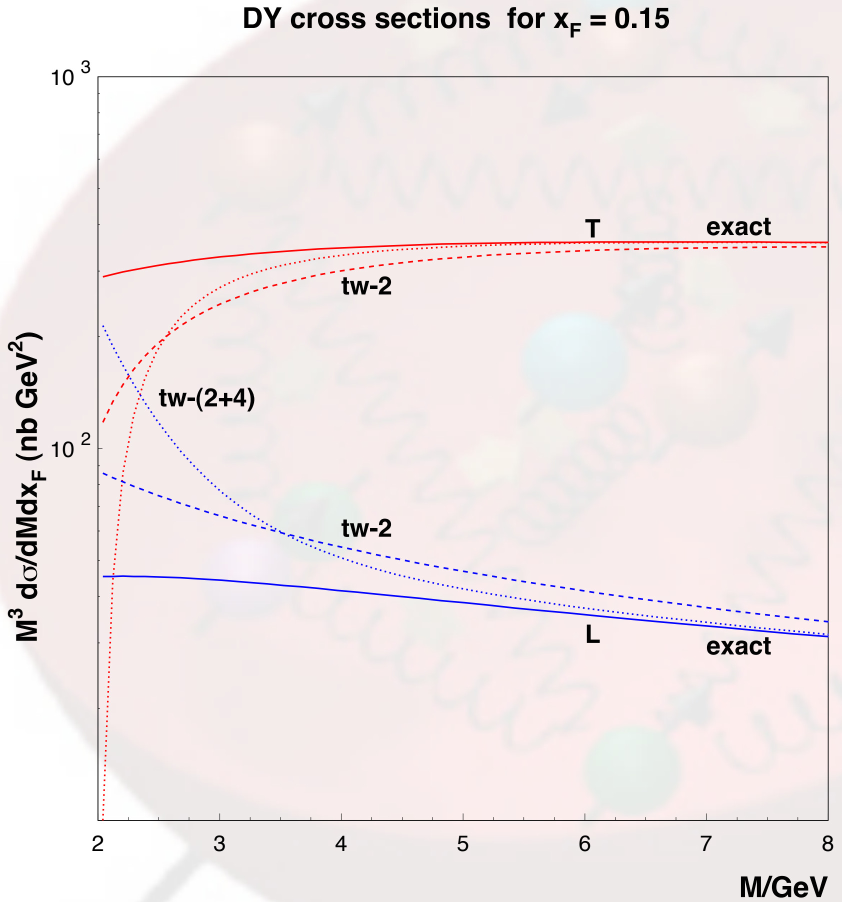
Comments

- Saturation physics (multiple interaction) are also included in the integrated parton distributions
- Reproduce the leading power (twist) expansion of the inclusive DIS structure function and Drell-Yan lepton pair production cross section
  - Bartels et al, 2009
  - Stasto et al, 2010

Prediction power

- Integrated parton distributions are universal
- NLO corrections are easy to compute
- We can use that to predict many other processes
  - EW processes at LHC
  - Higgs production in AA collisions

How good the approximation?



Golec-Biernat, Lewandowska, Stasto, **arXiv:1008.2652**

Back to the structure function

- Dipole (CGC) formalism

$$\sigma_{\gamma^* H}(\tau, Q^2) = \int_0^1 dz \int d^2 r_\perp \, |\Psi(z, r_\perp; Q^2)|^2 \, \sigma_{\text{dipole}}(\tau, r_\perp)$$

- Taking Q->0 limit will lead to infrared divergence

GBW model

$$\sigma(\gamma_T^* p)|_{Q^2 \to 0} \propto \frac{1}{\epsilon_{\text{I.R.}}} - \ln \frac{\mu^2}{Q_s^2} + \cdots$$

- Sensitive to the quark mass when Q=0
  - GBW 97, Log(mq^2)
- Associated with the real photon splitting to quark pair
  - Can be absorbed into the quark distribution in real photon (resolved photon)
- Small Q prediction is strongly model-dependent (wrong practice)

Geometric scaling in gluon distributions

- Kt-dependent gluon distributions in dijet correlation processes

$$\mathcal{F}_{qg}^{(1)} = xG^{(2)}(x, q_\perp), \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)}(q_1) \otimes F(q_2) \ ,$$

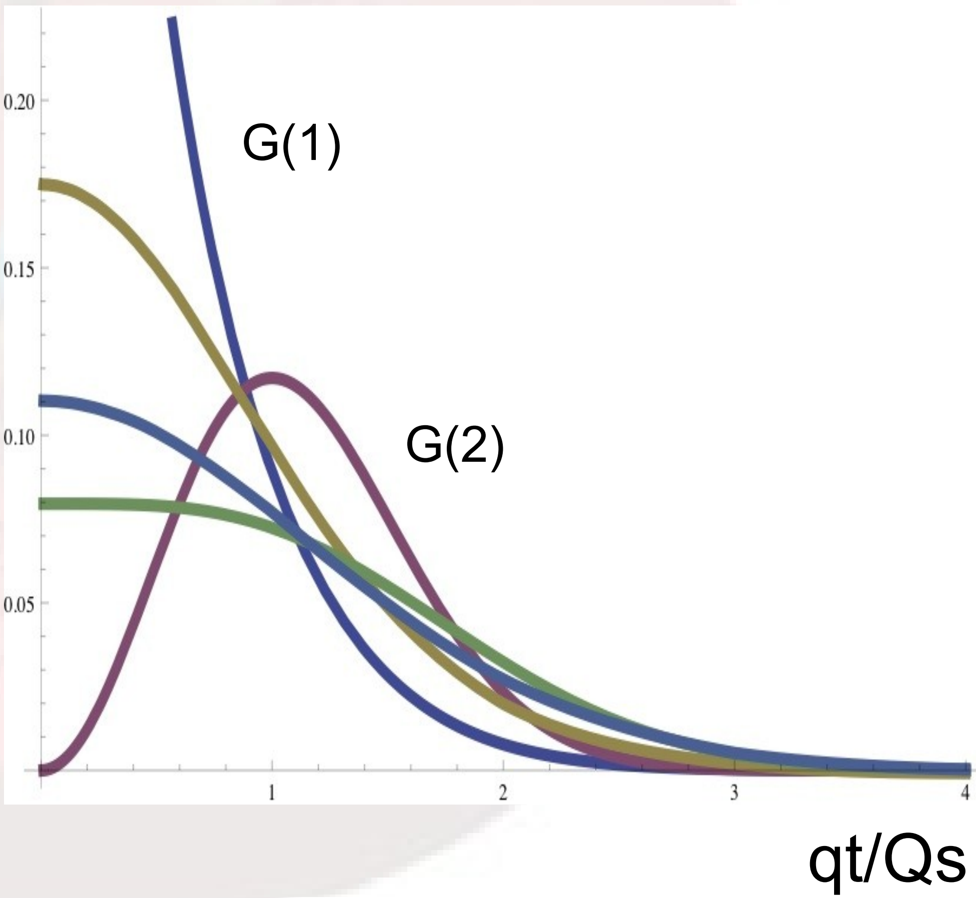
$$\mathcal{F}_{gg}^{(1)} = \int xG^{(2)}(q_1) \otimes F(q_2), \quad \mathcal{F}_{gg}^{(2)} = \int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)}(q_1) \otimes F(q_2)$$

$$\mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1) \otimes F(q_2) \otimes F(q_3) \ ,$$

Dominguez's talk

Different gluon distributions

- GBW model for the correlation functions
- Gluon distributions will only depend on qt/Qs
  - Geometry scaling



Modified factorization

- Dilute system on a dense target, in the large Nc limit,

$$
\frac{d\sigma^{(pA\to \text{Dijet}+X)}}{d\mathcal{P}.\mathcal{S}.}
= \sum_q x_1 q(x_1) \frac{\alpha_s^2}{\hat{s}^2} \left[\mathcal{F}_{qg}^{(1)} H_{qg\to qg}^{(1)} + \mathcal{F}_{qg}^{(2)} H_{qg\to qg}^{(2)}\right]
+ x_1 g(x_1) \frac{\alpha_s^2}{\hat{s}^2} \left[\mathcal{F}_{gg}^{(1)} \left(H_{gg\to q\bar{q}}^{(1)} + H_{gg\to gg}^{(1)}\right)
+ \mathcal{F}_{gg}^{(2)} \left(H_{gg\to q\bar{q}}^{(2)} + H_{gg\to gg}^{(2)}\right) + \mathcal{F}_{gg}^{(3)} H_{gg\to gg}^{(3)}\right] ,
$$

- Hard partonic cross section

$$H^{(1)}_{qg\to qg} = \frac{\hat{u}^2\left(\hat{s}^2+\hat{u}^2\right)}{-2\hat{s}\hat{u}\hat{t}^2}, \quad H^{(2)}_{qg\to qg} = \frac{\hat{s}^2\left(\hat{s}^2+\hat{u}^2\right)}{-2\hat{s}\hat{u}\hat{t}^2}$$

$$H^{(1)}_{gg\to q\bar{q}} = \frac{1}{4N_c}\frac{2\left(\hat{t}^2+\hat{u}^2\right)^2}{\hat{s}^2\hat{u}\hat{t}}, \quad H^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c}\frac{4\left(\hat{t}^2+\hat{u}^2\right)}{\hat{s}^2}$$

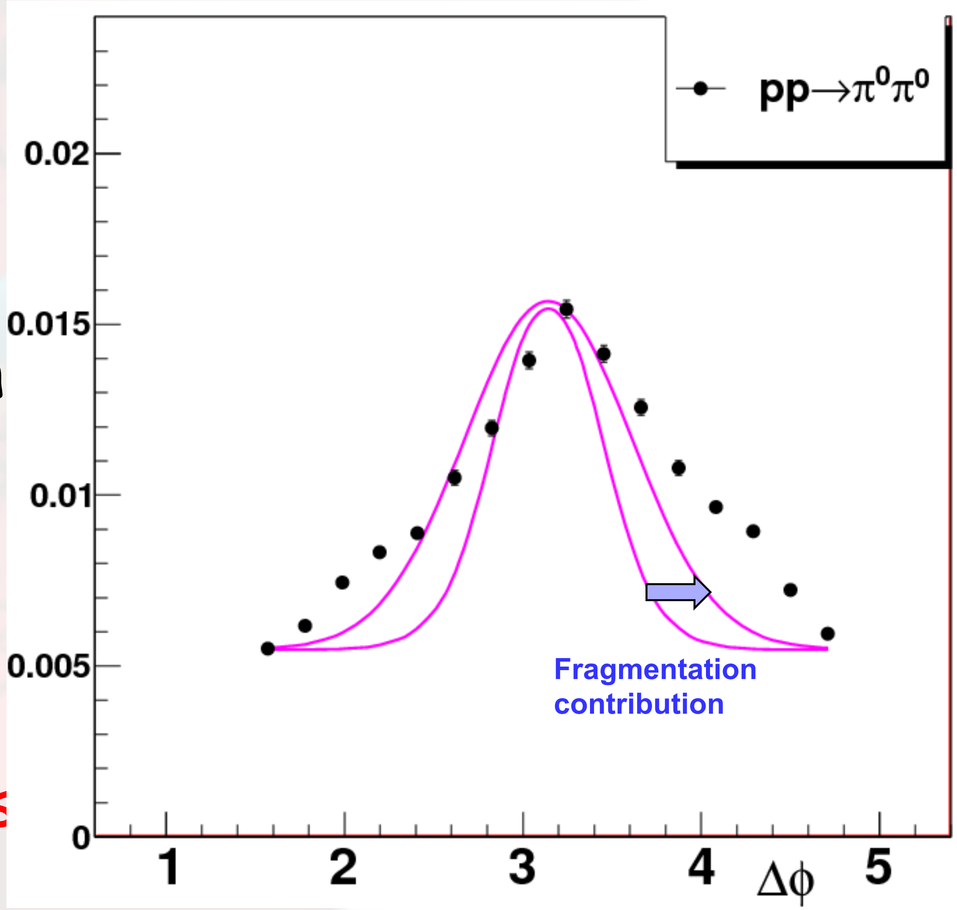
$$H^{(1)}_{gg\to gg} = \frac{2\left(\hat{t}^2+\hat{u}^2\right)\left(\hat{s}^2-\hat{t}\hat{u}\right)^2}{\hat{u}^2\hat{t}^2\hat{s}^2}, \quad H^{(2)}_{gg\to gg} = \frac{4\left(\hat{s}^2-\hat{t}\hat{u}\right)^2}{\hat{u}\hat{t}\hat{s}^2}$$

$$H^{(3)}_{gg\to gg} = \frac{2\left(\hat{s}^2-\hat{t}\hat{u}\right)^2}{\hat{u}^2\hat{t}^2},$$

  - Although the individual diagram depends on the gauge, the total contribution does not

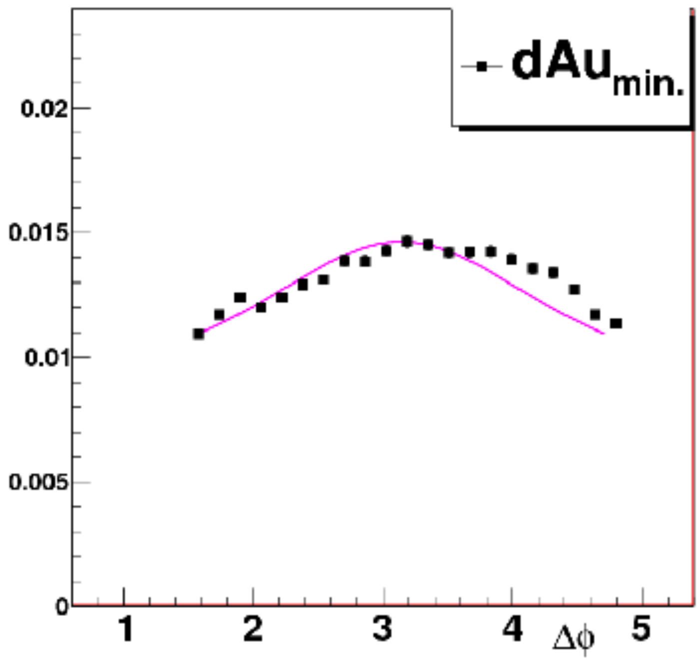
Compare to the STAR data

- $y_1 \sim y_2 \sim 3.1$
- GBW model for UGDs
- $Q_s^2 \sim (3.10^{-4}/x)^{0.28} GeV^2$
- Addition Fragmentation contribution
- Geometric scaling assumption to the UGDs

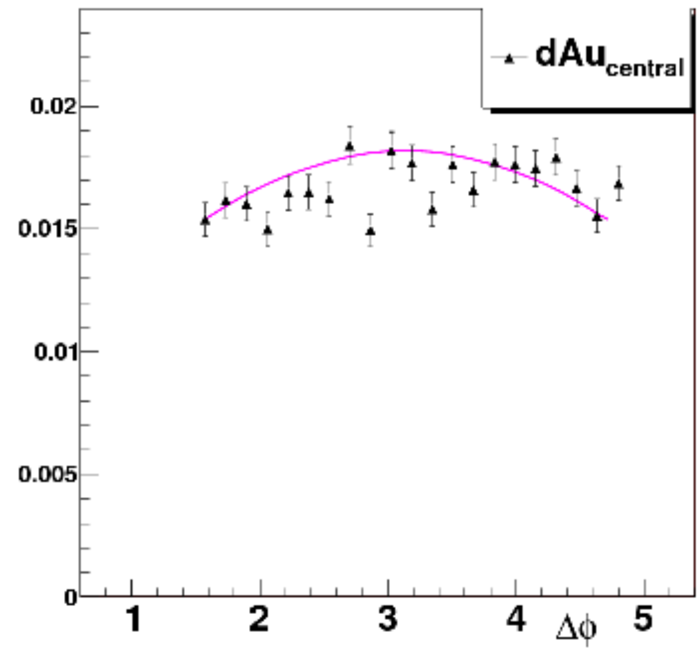


dAu collisions

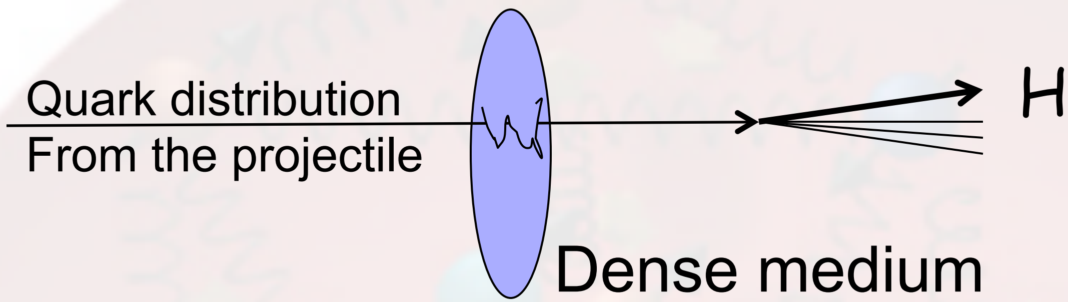
- $\eta_1 \sim \eta_2 \sim 3.2$
- $Q^2_{sA} \sim 0.8A^{(1/3)} Q_{sp}^{\ 2}$



- $\eta_1 \sim \eta_2 \sim 3.1$
- $Q^2_{sA} \sim 1.7\ 0.8A^{(1/3)} Q_{sp}^{\ 2}$



Scaling in Forward hadron production



$$\frac{d\sigma}{dyd^2P_{h\perp}} = \int \frac{dz}{z^2} x_1 q(x_1) D(z) x_2 G^{(2)}(x_2, q_\perp = P_{h\perp}/z)$$

Dumitru-Jalilian-Marian, 02
Dumitru-Hayashigaki-Jalilian-Marian, 06

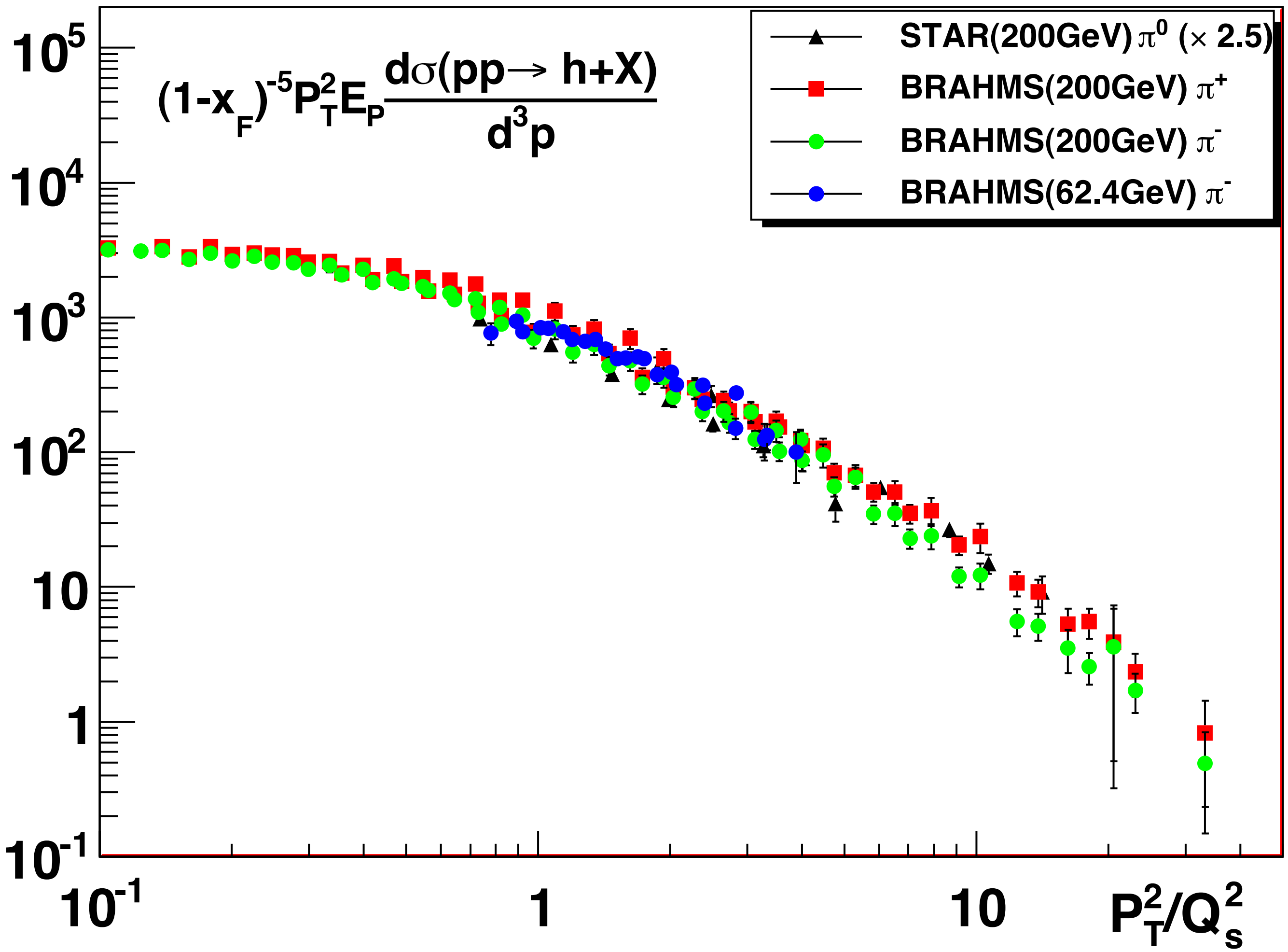
Simple power counting

- Forward region is dominated by the valence quark distribution $(1-x)^3$
- Similar power behavior for the fragmentation function, $(1-z)^{1\sim2}$, 1009.2481
- Pt-dependent-Geometric scaling,

$$P_{h\perp}^2 \frac{d\sigma}{dy d^2 P_{h\perp}} = (1-x_F)^5 \mathcal{F}\left(\frac{P_{h\perp}^2}{Q_s^2(x_2)}\right)$$

Similar study by
McLerran- Praszalowicz, 10

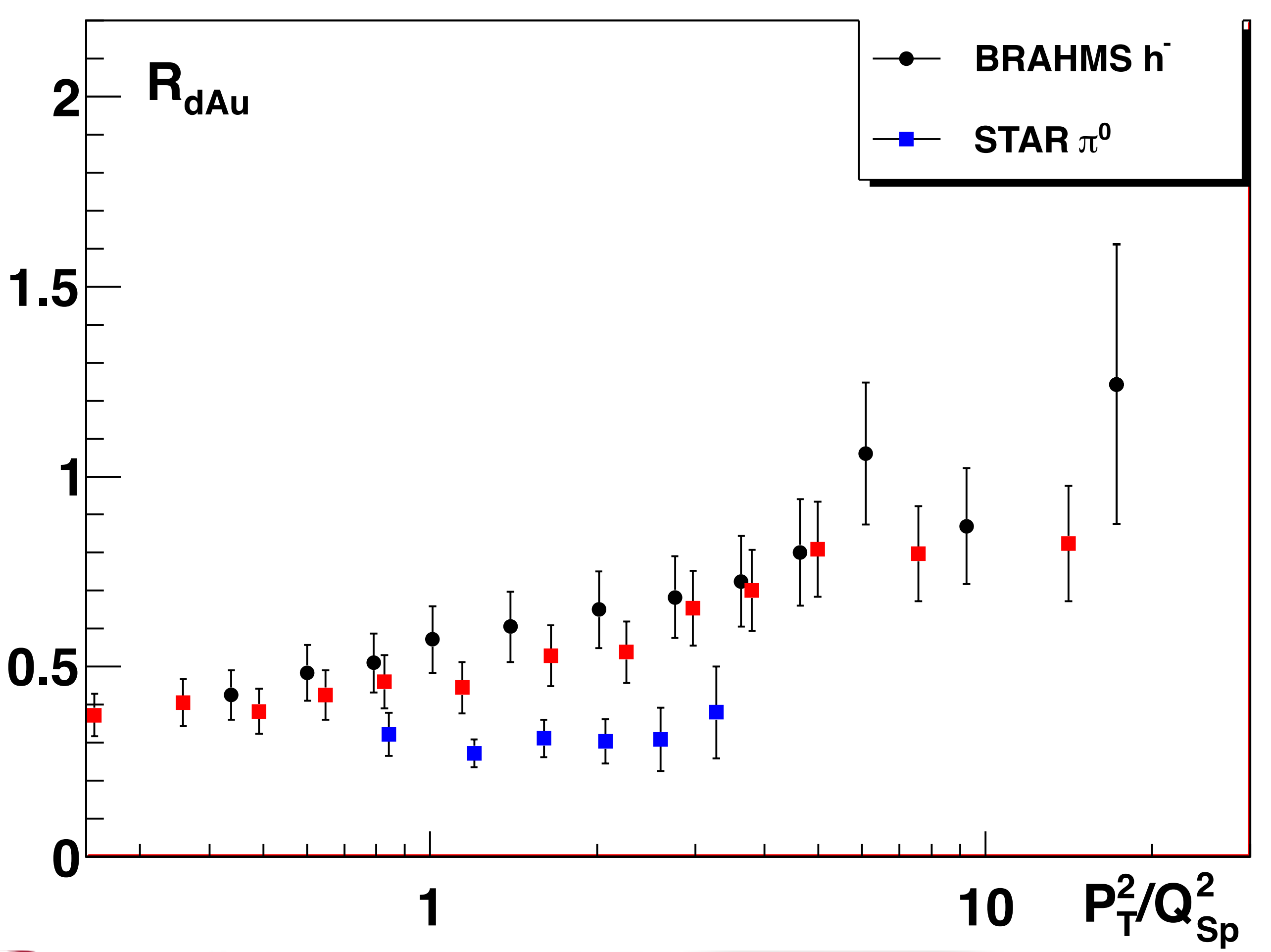
Scaling in pp collisions



BRAHMS:
2 rapidity bins

STAR:
3 rapidity bins

Geometric Scaling for R_{pA}?



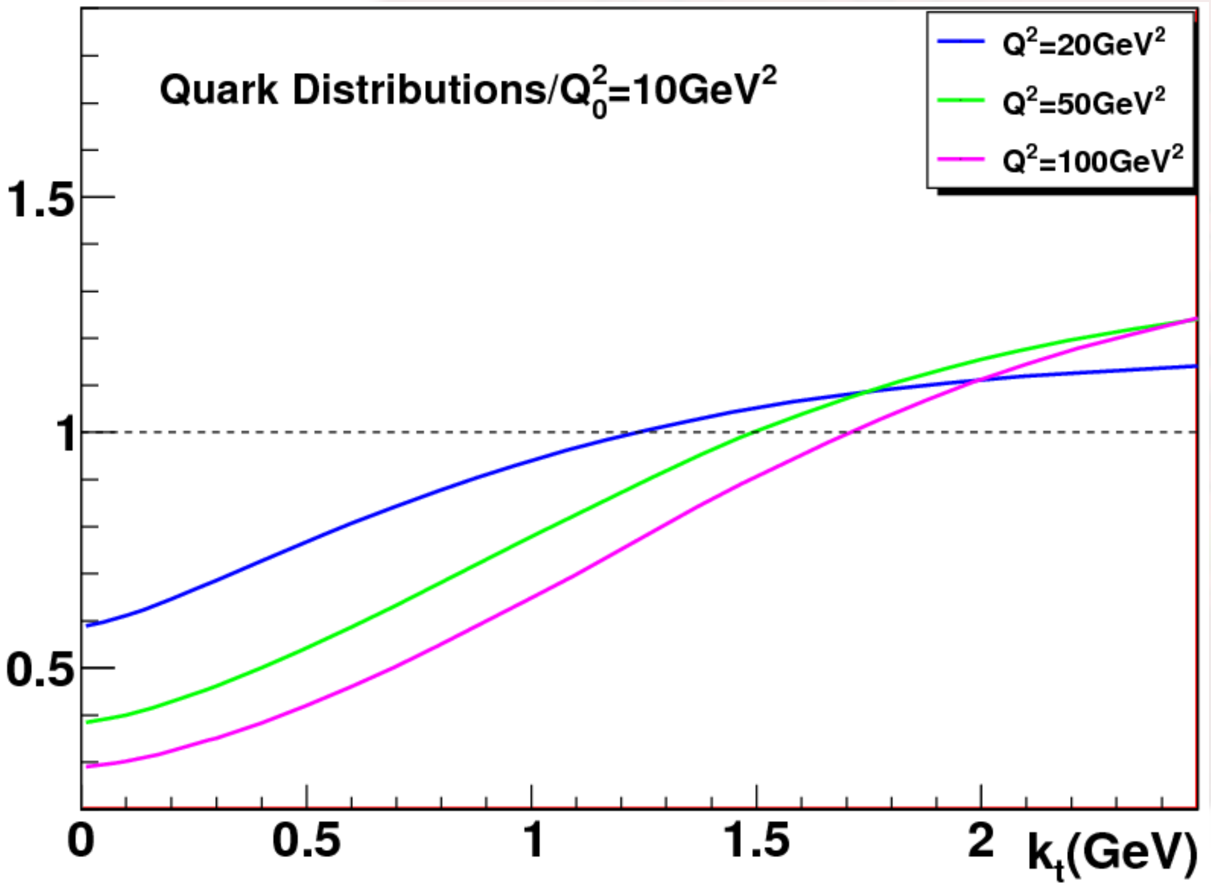
R ratio depends on the difference in the saturation scales

More data are needed to draw conclusion

Summary

- Both integrated and un-integrated quark distributions depend on the saturation scale, and can be used to probe the gluon saturation
- Geometric scaling of the un-integrated gluon distributions are used to predict the scaling of the shadowing of hadron and di-hadron production in pA collisions

Phenomenology: quark distributions ratios



Transverse Mometum Broading with Q

GBW model for dipole Cross section