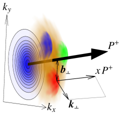
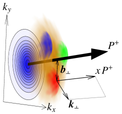


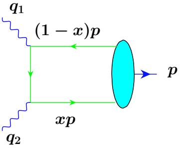
Transverse Momentum in Hard Exclusive Processes

A. Radyushkin

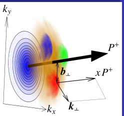
Old Dominion University
and
Jefferson Lab



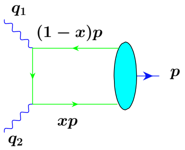
Photon-Pion Transition Form Factor



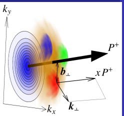
- $F_{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ relates two (in general, virtual) photons with the lightest hadron, the pion
- Plays special role among exclusive processes in QCD
- For real photons $F_{\gamma^*\gamma^*\pi^0}(0,0)$ determines rate of $\pi^0 \to \gamma\gamma$ decay, deeply related to axial anomaly
- For large photon virtualities, it has simplest structure analogous to that of form factors in deep inelastic scattering
- Comparing pQCD predictions with data gives information about shape of the pion DA $\varphi_\pi(x)$



Transition Form Factor in Perturbative QCD

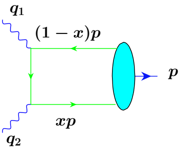


- Since only one hadron is involved, $\gamma^*\gamma^*\pi^0$ has simplest structure for pQCD analysis
- Nonperturbative information about pion is accumulated in pion DA $\varphi_\pi(x)$
- Short-distance amplitude for $\gamma^*\gamma^* \to \pi^0$ at leading order is given by single quark propagator
- Cleanest situation: both photon virtualities are large, but experiments are difficult due to very small cross section.



Pion Distribution Amplitude

- Pion DA $\varphi_\pi(x)$: momentum sharing for pion in valence $\bar{q}q$ configuration

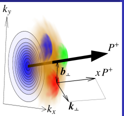


- Chernyak; A.R. 1977: function whose x^n moments

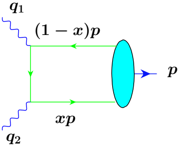
$$f_n = \int_0^1 x^n \, \varphi_\pi(x) \, dx$$

are given by reduced matrix elements of twist-2 local operators

$$i^{n+1} \langle 0 | \bar{d}(0) \gamma_5 \{ \gamma_\nu D_{\nu_1} \dots D_{\nu_n} \} u(0) | \pi^+, P \rangle \\ = \{ P_\nu P_{\nu_1} \dots P_{\nu_n} \} \; f_n$$



Pion DA in Light-Front Formalism



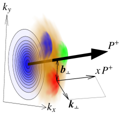
- Jackson, 1977; Lepage & Brodsky,1979: $k_\perp$-integral of light-front wave function $\Psi(x, k_\perp)$

$$\varphi_\pi(x,\mu) = \frac{\sqrt{6}}{(2\pi)^3} \int_{k_\perp^2 \leq \mu^2} \Psi(x, k_\perp)\, d^2 k_\perp$$

- zeroth moment of $\varphi_\pi(x)$: matrix element of the axial current

$$\int_0^1 \varphi_\pi(x)\, dx = f_\pi$$

pion decay constant $f_\pi \approx 130\,$MeV.



Shape and Evolution of Pion DA

- Integral under $\varphi_\pi(x)$ curve is fixed, but not its shape
- Shape of pion DA depends on renormalization scale μ: $\varphi_\pi(x) \to \varphi_\pi(x,\mu)$.
- Evolution equation for pion DA may be written in matrix form

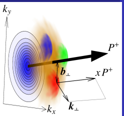
$$\mu \frac{d}{d\mu} f_n(\mu) = \sum_{k=0}^{n} Z_{nk} f_k(\mu)$$

A.R. 1977

- Or in kernel form

$$\mu \frac{d}{d\mu} \varphi_\pi(x,\mu) = \int_0^1 V(x,y)\, \varphi_\pi(y,\mu)\, dy$$

Lepage&Brodsky:1979



Solution of Evolution Equation

- Evolution kernel

$$V(x,y) = \frac{\alpha_s}{2\pi} C_F \left[\frac{x}{y}\,\theta(x<y) \left(1 + \frac{1}{x-y} \right) + \{x \leftrightarrow y\} \right]_+$$

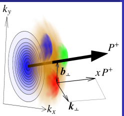
- The “+”-operation is defined by

$$[F(x,y)]_+ = F(x,y) - \delta(x-y) \int_0^1 F(z,y)\, dz$$

- Expansion over Gegenbauer polynomials

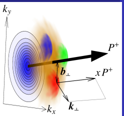
$$\varphi_\pi(x,\mu) = 6 f_\pi\, x(1-x) \left\{ 1 + \sum_{n=1}^{\infty} \frac{a_{2n} C_{2n}^{3/2}(2x-1)}{[\ln(\mu^2/\Lambda^2)]^{\gamma_{2n}/\beta_0}} \right\}$$

Efremov & A.R. 1978; Lepage & Brodsky,1979

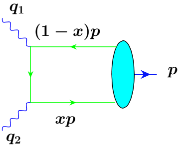


Shape of Pion DA at Low Scales

- What is shape of pion DA at low scales $\mu \lesssim 1$ GeV?
- Simplistic argument (A.R. 1980):
- For system of two equal-mass non-interacting particles, $\varphi(x) = f_\pi \delta(x - 1/2)$
- When interaction is on, width of $\varphi(x)$ increases
- It may be estimated as $\sim E_{\rm int}/m_q \sim \Lambda_{\rm QCD}/m_q$
- For heavy mesons (e.g., Υ), $\varphi(x)$ is narrow
- Taking $m_{u,d} \lesssim 10$ MeV gives very broad DA for pion
- Flat DA: $\varphi_\pi(x)$ close to f_π almost everywhere



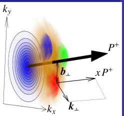
Different Large Photon Virtualities



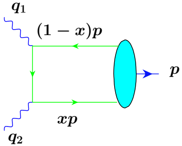
- Introduce asymmetry parameter $q_1^2 = -Q^2(1+\omega)/2$, $q_2^2 = -Q^2(1-\omega)/2$
- pQCD leading-order result

$$F^{\mathrm{pQCD}}_{\gamma^*\gamma^*\pi}(Q^2,\omega) = \frac{2\sqrt{2}}{3Q^2}\int_0^1 \frac{\varphi_\pi(x)}{1+\omega(2x-1)}\,dx \equiv \frac{\sqrt{2}f_\pi}{3\,Q^2}\,J(\omega)$$

- Invert integral transform to get pion DA $\varphi_\pi(x)$
- Experimentally feasible: one photon is real, $\omega = 1$



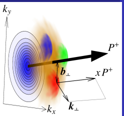
One Real and One Virtual Photon



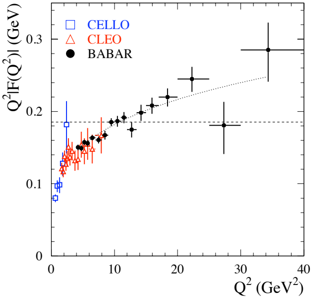
- $q_1^2 = -Q^2$, $q_2^2 = 0$
- Leading-order pQCD prediction

$$F^{\text{pQCD}}_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3Q^2} \int_0^1 \frac{\varphi_\pi(x)}{x}\, dx \equiv \frac{\sqrt{2} f_\pi}{3\, Q^2}\, J$$

- Information about pion DA is now accumulated in factor J
- $J = 2$ for infinitely narrow $\sim \delta(x - 1/2)$ DA
- $J = 3$ for asymptotic $\sim 6x(1-x)$ DA
- $J = 5$ for CZ $\sim 30x(1-x)(1-2x)^2$ DA
- Another measure of the width of pion DA
- $J = \infty$ for flat $\varphi_\pi(x) = f_\pi$ DA!



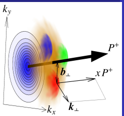
BaBar Data



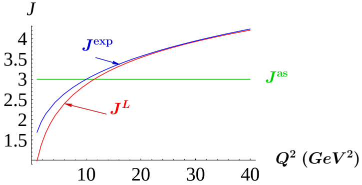
- Recent BaBar data may be fitted by

$$Q^2\, F_{\gamma^*\gamma\pi^0}(Q^2) \cong \sqrt{2} f_\pi \left(\frac{Q^2}{10\,\mathrm{GeV}^2}\right)^{0.25} \equiv \frac{\sqrt{2} f_\pi}{3}\, J^{\mathrm{exp}}(Q^2)$$

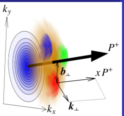
- $J^{\mathrm{exp}}(Q^2)$ does not flatten to some particular value!



Logarithmic Model



- $J^{\exp}(Q^2)$ is very close to logarithmic function
$$J^L(Q^2) = \ln\left(Q^2/M^2 + 1\right)$$
if one takes $M^2 = 0.6\,\text{GeV}^2$
- $J^L(Q^2)$ is obtained if $\varphi_\pi(x) = f_\pi$ and $xQ^2 \to xQ^2 + M^2$
$$J^L(Q^2) = Q^2 \int_0^1 \frac{dx}{xQ^2 + M^2}$$
- M is usually treated as average intrinsic transverse momentum
- $M = 0.77\,\text{GeV}$ is too large for such interpretation!



Tower of Higher Twists?

- Also: $1/xQ^2 \to 1/(xQ^2 + M^2)$ brings in a tower of $(-M^2/xQ^2)^n$ power corrections, higher twists
- Known (Musatov, A.R. 1997) : handbag diagram

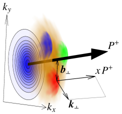
$$F(q,p) = \frac{1}{2\pi^2} \int e^{-iqz} \langle 0|\bar{\psi}(0)\gamma_5 \not{z}\psi(z)|p\rangle \frac{d^4z}{(z^2)^2} \, .$$

cannot generate infinite tower of power corrections

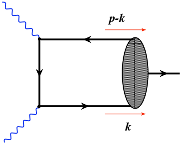
- Massless quark propagator is $\sim \not{z}/(z^2)^2$
- Matrix element of bilocal operator

$$\langle 0|\bar{\psi}(0)\gamma_5 \not{z}\psi(z)|p\rangle = \xi_2(zp)|_{z^2=0} + z^2\xi_4(zp)|_{z^2=0} + (z^2)^2\xi_6(zp)|_{z^2=0} + \ldots$$

- Twist-6 and higher cancel singularity of quark propagator $\to$ no tower of $(1/Q^2)^n$ terms!



"Sudakov" transverse momentum



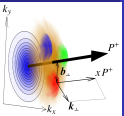
- Sudakov parametrization of integration momentum

$$k = xp + \eta q_1 + k_\perp$$

- Formally integrating over η by residue in *"p-k"* propagator:

$$F(Q^2) \sim \int_0^1 dx \int d^2 k_\perp \frac{\Psi(x, k_\perp)}{xQ^2 + k_\perp^2/(1-x)}$$

- But: $i)$ this formula generates infinite $(1/Q^2)^n$ tower
- And $ii)$ Ψ-functions depending on $k_\perp$ through $k_\perp^2/x(1-x)/\sigma$ give $k_\perp^2(x) \sim x(1-x)\sigma$ and $1/x$ singularity remains



Light-Front Formula and Gaussian Model

- Two-body ($i.e.$, $\bar{q}q$) contribution for $\gamma^*\gamma\pi^0$ form factor in light-front formalism (Lepage & Brodsky, 1980)

$$(\epsilon_\perp \times q_\perp) F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) \sim \int_0^1 dx \int \frac{(\epsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon} \Psi(x, k_\perp)\, d^2k_\perp$$

- Simplifies for wave functions of $\Psi(x, k_\perp) = \psi(x, k_\perp^2)$ type

$$F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{2\pi^2\sqrt{3}} \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ} \psi(x, k_\perp^2)\, k_\perp dk_\perp$$

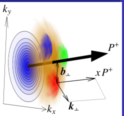
(Musatov & A.R. 1997)

- Gaussian ansatz for $k_\perp$-dependence (BHL 1984, JKR 1996)

$$\Psi^G(x, k_\perp) = \frac{4\pi^2}{x\bar{x}\sigma\sqrt{6}} \varphi_\pi(x) \exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right)$$

- Result for form factor

$$F^G_{\gamma^*\gamma\pi^0}(Q^2) = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right] dx$$

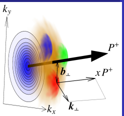


Features of Gaussian Model

- Result for form factor

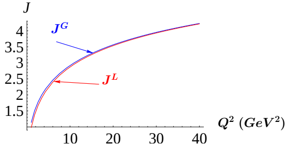
$$F^G_{\gamma^*\gamma\pi^0}(Q^2) = \frac{\sqrt{2}}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2}\left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right] dx \equiv \frac{\sqrt{2} f_\pi}{3}\, J^G(Q^2,\sigma)$$

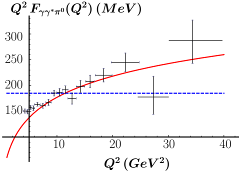
- Contains $1/xQ^2$ pQCD contribution and correction term making integral convergent for small x
- Extra term reflects $k_\perp$-dependence of pion wave function
- Extra term decreases faster than any power of $1/Q^2$ $\rightarrow$ not a higher twist $\rightarrow$ term invisible in OPE!
- For large Q^2 and flat DA:
  $J^G(Q^2,\sigma) = \ln\left(Q^2/(2\sigma)\right) + \gamma_E + \mathcal{O}(\sigma/Q^2)$
- In logarithmic model: $J^L(Q^2, M^2) = \ln\left(Q^2/M^2\right) + \mathcal{O}(M^2/Q^2)$
- Two models coincide up to $\mathcal{O}(1/Q^2)$ terms if $\sigma = M^2\, e^{\gamma_E}/2$
- Numerically: $\sigma = 0.53\,\text{GeV}^2$ for $M^2 = 0.6\ \text{GeV}^2$



Properties of Gaussian Model

- In fact, $J^L(Q^2, M^2 = 0.6\,\mathrm{GeV}^2)$ and $J^G(Q^2, \sigma = 0.53\,\mathrm{GeV}^2)$ practically coincide for $Q^2 > 1$ GeV^2





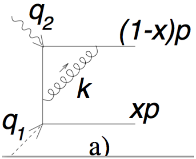
- Average transverse momentum for Gaussian model:

$$\langle k_\perp^2 \rangle = \frac{\sigma}{3} = (0.42\,\mathrm{GeV})^2$$

- $\sqrt{\langle k_\perp^2 \rangle}$ is close to folklore value of 300 MeV

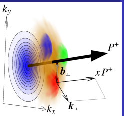
Perturbative source of transverse momentum: radiative corrections

- One-loop diagram in Sudakov decomposition:



- gluon momentum: $k = (\xi - x)p + \eta q + k_\perp$
  quark momentum: $(1-\xi)p + \eta q + k_\perp$, $d^4k \Rightarrow \; d^2k_\perp \, d\xi d\eta$
- After taking η-integral by residue

$$T_i(x, Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^1 d\xi \int M_i(x, Q^2; \xi, k_\perp) \frac{d^2k_\perp}{2\pi}$$



Virtual photon vertex correction

Exclusive $k_\perp$
Radyushkin
Transition FF
Definition
pQCD
Pion DA
Definition
Evolution
Shape
BaBar Data
Logarithmic model
Gaussian model
1 loop pQCD
Structure
Modified factorization
NONpQCD
Wave function
Born term
Summary

- Concentrate on $\xi > x$ part

$$M_a^{sing}(x, Q^2; \xi, k_\perp) = -\frac{1}{xQ^2} \frac{Q^2}{k_\perp^2 \left[\xi Q^2 + k_\perp^2/\bar{\xi}\right]} \left(\frac{\bar{\xi}}{\bar{x}}\right) \theta(\xi > x)$$

NB: singular for $k_\perp = 0$
Collinear divergence may be regulated by $k_\perp^2 \to k_\perp^2 + m^2$ producing evolution logarithm $\ln(Q^2/m^2)$. Using

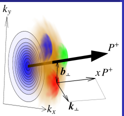
$$-\frac{\theta(\xi > x)}{\left[\xi Q^2 + k_\perp^2/\bar{\xi}\right] xQ^2} = \left(\frac{1}{\xi Q^2 + k_\perp^2/\bar{\xi}} - \frac{1}{xQ^2}\right) \frac{\theta(\xi > x)}{(\xi - x)Q^2 + k_\perp^2/\bar{\xi}}$$

and taking $k_\perp^2 = 0$ when it is added to $O(Q^2)$ terms gives

$$\frac{1}{\xi Q^2} \left[\left(\frac{\bar{\xi}}{\bar{x}}\right) \frac{\theta(\xi > x)}{\xi - x}\right]_+ \ln\left(\frac{Q^2}{m^2}\right) :$$

product of the "Born" term $1/\xi Q^2$ and V_a part of ERBL kernel with "+" prescription:

$$[F(\xi, x)]_+ = F(\xi, x) - \delta(\xi - x) \int_0^1 F(\zeta, x)\, d\zeta$$

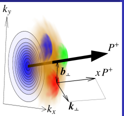


pQCD one-loop corrections

- NB: in impact parameter $b_\perp$ representation
  $[b_\perp$ is Fourier-conjugate to $k_\perp]$
  difference $[\ldots k_\perp \ldots] - [\ldots k_\perp = 0 \ldots]$ produces
  $\left(e^{ik_\perp b_\perp} - 1\right)$ factor vanishing at $b_\perp = 0$
- Similarly, it makes sense to isolate $x = \xi$ part,
  where longitudinal momentum does not change
  Factorization in $b_\perp$ space

$$M_a^{sing}(x, Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^1 d\xi \int B(\xi; bQ) \left[V_a(\xi, x) L(bm) \right.$$

$$\left. + \delta(\xi - x) S_a(x, bQ) + E_a(x, \xi; bQ) \right] \frac{d^2 b_\perp}{2\pi}$$



pQCD one-loop corrections

- Born term

$$B(\xi; bQ) = \frac{1}{2\pi}\int \frac{e^{-ik_\perp b_\perp}}{\xi Q^2 + k_\perp^2/\bar{\xi}}\, d^2k_\perp = \bar{\xi} K_0\left(bQ\sqrt{\xi\bar{\xi}}\right)$$

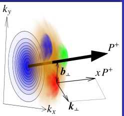
Evolution term

$$V_a(\xi, x) L(bm) \quad , \quad L(bm) = \frac{1}{2\pi}\int \frac{d^2k_\perp e^{ik_\perp b_\perp}}{k_\perp^2 + m^2} = K_0(bm)$$

"Sudakov" term

$$S_a(x; bQ) = \frac{1}{2\pi}\int d^2k_\perp \frac{e^{ik_\perp b_\perp} - 1}{k_\perp^2}\int_0^1 \left(\frac{\bar{\zeta}}{\bar{x}}\right) \frac{\theta(\zeta > x)d\zeta}{\zeta - x + k_\perp^2/\bar{\zeta}Q^2}$$

NB: singularity of evolution kernel at $\zeta = x$ is regularized here by $k_\perp^2/\bar{\zeta}Q^2$ rather than by $+$ prescription.



pQCD one-loop corrections (cont'd)

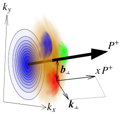
- Evolution-related term

$$E_a(x,\xi;bQ) = -\left[\frac{\bar{\xi}}{\bar{x}}\,\frac{\theta(\xi > x)}{\xi - x}\,K_0\left(bQ\sqrt{(\xi - x)\bar{\xi}}\right)\right]_+$$

- $b_\perp$ space (or modified) factorization at one loop level

$$F_{\gamma^*\gamma\pi^0}(Q^2) = \frac{4\pi}{3}\int_0^1 \varphi_\pi(x)\,dx\,\Bigg\{\frac{1}{xQ^2} + \frac{\alpha_s}{2\pi}\,C_F\int_0^1 d\xi\int\frac{d^2b_\perp}{2\pi}$$
$$\times B(\xi;bQ)\Big[V(\xi,x)\,L(bm) + E(\xi,x;bQ) + \delta(\xi - x)S(x,bQ)$$
$$+R(\xi,x;bQ)\Big]\Bigg\}$$

- Terms, nonsingular at $k_\perp = 0$, give $R(\xi,x;bQ)$



pQCD one-loop corrections (cont'd)

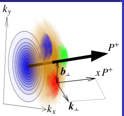
- Born term $B(\xi; bQ)$ and evolution terms $L(bm)$, $E_a(x, \xi; bQ)$ exponentially decrease at large b: $K_0\,(b\ldots) \sim \exp(-b\ldots)$
- Sudakov terms are doubly-logarithmic in b, e.g.,

$$S_a(x; bQ) \approx \frac{1}{2\pi} \int d^2k_\perp \frac{e^{ik_\perp b_\perp} - 1}{k_\perp^2} \ln\left(\frac{\bar{x}Q}{k_\perp}\right)$$

$$\approx \int_{1/b} \frac{dk_\perp}{k_\perp} \ln\left(\frac{k_\perp}{\bar{x}Q}\right) \approx -\frac{1}{2} \ln^2(\bar{x}Qb),$$

$\Rightarrow$ Resummation is needed

- NB: Derivation was done in perturbation theory
$\Rightarrow$ $d^2b_\perp$ integration gives the same result as standard pQCD factorization



Standard & modified factorization

- NB: Except $L(bm) = L(bQ * m/Q)$, $b_\perp$ everywhere appears through bQ. After $d^2 b_\perp$ integration

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{4\pi}{3}\int_0^1 dx\, \frac{\varphi_\pi(x)}{xQ^2}\left\{1 + C_F\frac{\alpha_s}{2\pi}\left[\left(\frac{3}{2} + \ln x\right)\ln\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\ln^2 x - \frac{x\ln x}{2(1-x)} - \frac{9}{2}\right]\right\}$$

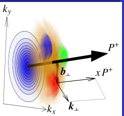
mass logarithm $\ln(Q^2/m^2)$ is accompanied by

$$\frac{1}{x}\left(\frac{3}{2} + \ln x\right) = \int_0^1 \frac{d\xi}{\xi} V(\xi, x)$$

forming standard evolution combination

$$\left[\delta(\xi - x) + \frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{m^2}\right)V(\xi, x)\right]\varphi_\pi(x)$$

suggesting the change $\varphi_\pi(x) \to \varphi_\pi(\xi, Q^2)$



Standard & modified factorization

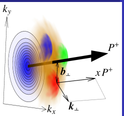
- In the impact parameter representation,

$$\varphi_\pi(\xi) \to \varphi_\pi(\xi) - \frac{\alpha_s}{2\pi} \ln(bm) \int_0^1 V(\xi, x)\, \varphi_\pi(x)\, dx$$

suggesting the change $\varphi_\pi(\xi) \to \varphi_\pi(\xi, 1/b^2)$
Symbolically:

$$\varphi(\xi, 1/b^2) = \exp\left[-\frac{\alpha_s}{2\pi} \ln(bm) V\right] (\xi, x) \otimes \varphi(x)$$

NB: $\varphi(\xi, 1/b^2)$ is usual ("collinear") pion DA, with $1/b^2$ serving as factorization scale



"Nonperturbative" transverse momentum

- Local duality relation for $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor

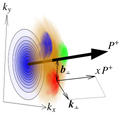
$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi f_\pi} \int_0^{s_0} \rho^{quark}(s, Q^2)\, ds$$

- Spectral density for triangle (anomaly) diagram

$$\rho^{quark}(s, Q^2) = 2 \int_0^1 \frac{x\bar{x}(xQ^2)^2}{[sx\bar{x} + xQ^2]^3}\, dx$$

- Substituting $s = k_\perp^2/(x\bar{x})$:

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{2}{\pi^2 f_\pi} \int_0^1 dx \int \frac{(xQ^2)^2}{(xQ^2 + k_\perp^2)^3}\, \theta(k_\perp^2 \le x\bar{x}s_0)\, d^2k_\perp$$



Effective wave function

- "Local duality" wave function for pion:

$$\Psi^{LD}(x, k_\perp) = \frac{2\sqrt{6}}{f_\pi}\,\theta(k_\perp^2 \le x\bar{x}s_0)$$

- $b_\perp$-space version

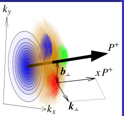
$$\widetilde{\Psi}^{LD}(x, b_\perp) = \frac{\sqrt{6}}{\pi f_\pi b_\perp}\sqrt{x\bar{x}s_0}\, J_1(b_\perp\sqrt{x\bar{x}s_0})$$

- Form factor In terms of effective LD wave function

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi^2\sqrt{6}}\int_0^1 dx \int \frac{(xQ^2)^2}{(xQ^2 + k_\perp^2)^3}\,\Psi^{LD}(x, k_\perp)\, d^2k_\perp$$

- In the impact parameter representation

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\sqrt{6}}\int_0^1 dx \int xQ^2b^2\, K_2\left(\sqrt{x}bQ\right)\widetilde{\Psi}^{LD}(x, b_\perp)\,\frac{d^2b_\perp}{2\pi}$$



Born term in LD formula

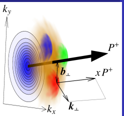
- $K_2\left(\sqrt{x}bQ\right)$: from Born term written in the b-space

$$\frac{1}{2\pi}\int e^{-ik_\perp b_\perp}\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\,d^2k_\perp = \frac{1}{4}xQ^2b^2K_2\left(\sqrt{x}bQ\right)$$

- Connection with pQCD Born term

$$\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3} = \frac{1}{xQ^2+k_\perp^2} - \frac{2k_\perp^2}{(xQ^2+k_\perp^2)^2} + \frac{k_\perp^4}{(xQ^2+k_\perp^2)^3}$$

- Differ only by $O(k_\perp^2)$ terms invisible in the analysis of effects induced by the $1/k_\perp^2$ singularity at small $k_\perp$.
- However, this difference is very essential when one extrapolates into the region of small Q^2.
- Local duality formula exactly reproduces pQCD asymptotics and also $F_{\gamma^*\gamma\pi^0}(0)$ value dictated by axial anomaly
- $\Psi^{LD}(x,k_\perp)$ is effective wave function describing all $\bar{q}G\ldots Gq$ Fock components of the usual light-front approach



Summary

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6. Summary