Scale dependence of Twist-3 correlation functions

Jianwei Qiu
Brookhaven National Laboratory

Based on work with Z. Kang

QCD Evolution Workshop: from collinear to non collinear case
Thomas Jefferson National Accelerator Facility, April 8-9, 2011
Newport News, Virginia, USA

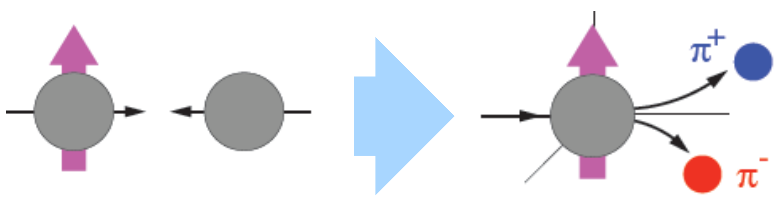
Outline of my talk

- Go beyond leading power collinear pQCD
- Single transverse spin asymmetry
- Twist-3 correlation functions, fragmentation functions
- Evolution equations and evolution kernels
- Global QCD analysis of SSAs
- Summary

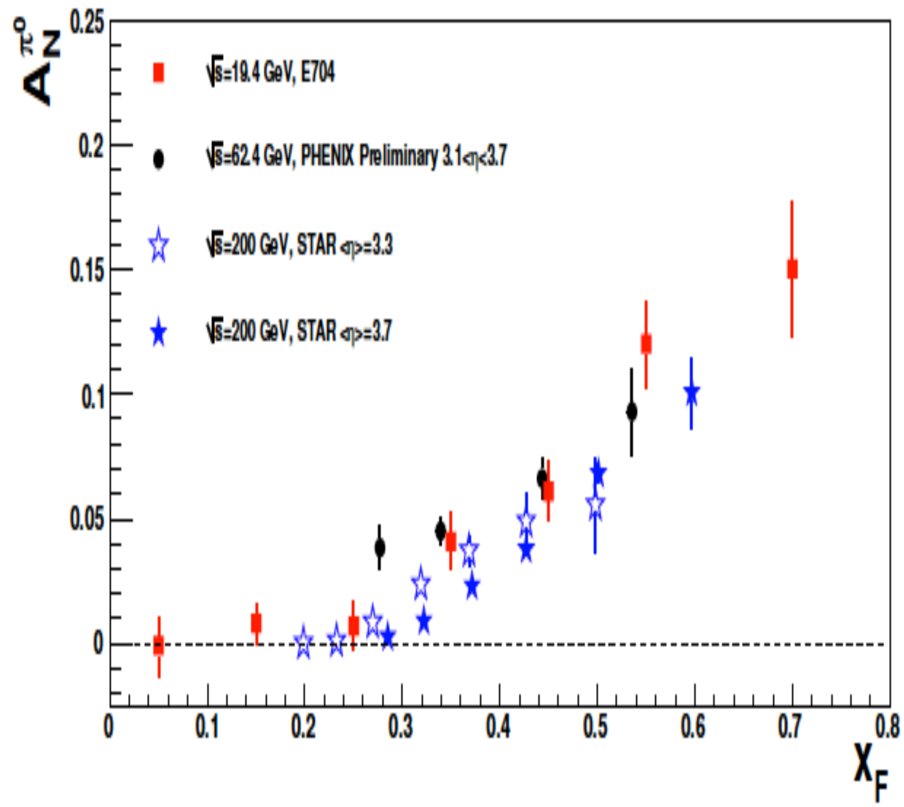
Transverse spin phenomena in QCD

- **Left-right asymmetry:**

$$A(p_A, s_\uparrow) + B(p_B) \to \pi(p) + X$$



$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$



- **Vanish without parton's transverse motion:**

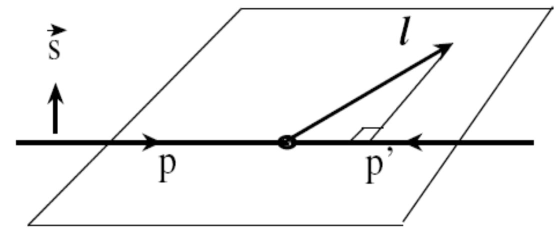
➡ **A direct probe for parton's transverse motion**

A direct probe of QCD quantum interference

Single transverse spin asymmetry

❑ **SSA corresponds to a naively T-odd triple product:**

$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \;\Rightarrow\; i\,\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$$

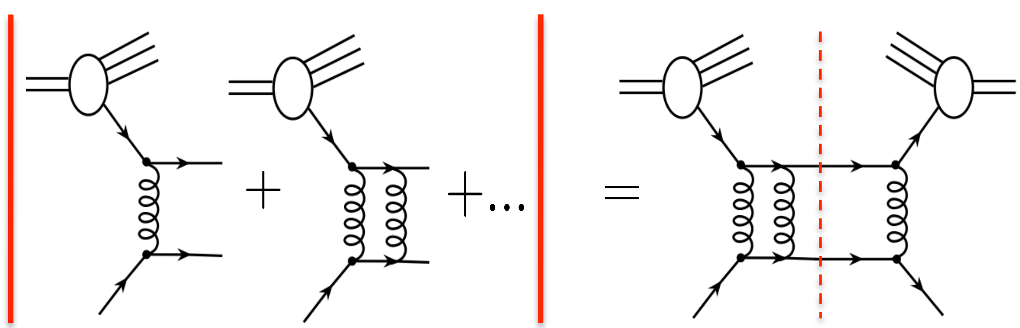


Novanish A_N requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

❑ **Leading power in QCD:**

Kane, Pumplin, Repko, PRL, 1978

$$\sigma_{AB}(p_T, \vec{s}) \propto$$



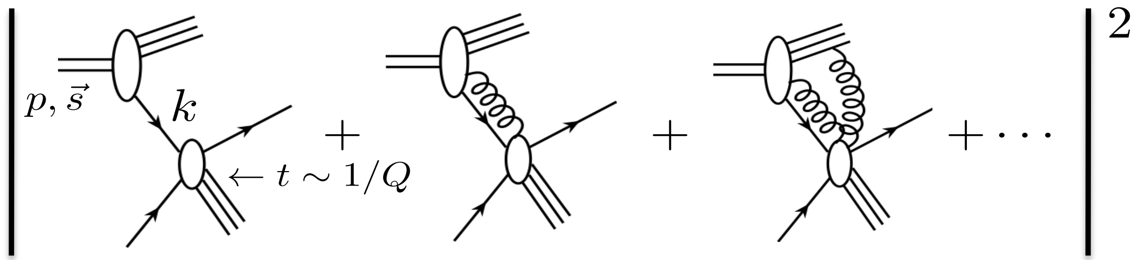
$$\propto \alpha_s \frac{m_q}{p_T}$$

A_N connects to parton's transverse motion!

Collinear factorization

- **Cross section with one large momentum transfer: Q >> Λ_{QCD}**

$\sigma(Q, \vec{s}) \propto$



$$= \sigma^{\rm LP}(Q, \vec{s}) + \frac{Q_s}{Q}\sigma^{\rm NLP}(Q, \vec{s}) + \ldots \qquad Q_s^2 \sim \langle k^2 \rangle, \langle k_T^2 \rangle$$

$$\approx H^{\rm LP} \otimes f_2 \otimes f_2 + \frac{Q_s}{Q} H^{\rm NLP} \otimes f_2 \otimes f_3 + \ldots$$ **pQCD factorization**

- **Single transverse spin asymmetry:**

$$A_N \propto \sigma(Q, S_\perp) - \sigma(Q, -S_\perp)$$

$$\propto H(Q) \left[\langle p, S_\perp | \mathcal{O}(\psi, A^\mu) | p, S_\perp \rangle - \langle p, -S_\perp | \mathcal{O}(\psi, A^\mu) | p, -S_\perp \rangle \right]$$

- **Parity and time-reversal invariance:**

$$\langle p, \vec{s} | \mathcal{O}(\psi, A^\mu) | p, \vec{s} \rangle = \langle p, -\vec{s} | \mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi, A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1} | p, -\vec{s} \rangle$$

Not all operators contribute to SSA!

Inclusive DIS

- **Inclusive DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$
- **Leptonic tensor is symmetric:** $L^{\mu\nu} = L^{\nu\mu}$
- **Hadronic tensor:** $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$
- **The difference of two cross sections:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right]$$

- **P and T invariance:**

$$\langle P, \vec{s}_\perp |\, j^\dagger_\mu(0) j_\nu(y)\, | P, \vec{s}_\perp \rangle = \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0) j_\mu(y)\, | P, -\vec{s}_\perp \rangle$$

$$\Rightarrow \qquad W_{\mu\nu}(\vec{s}_\perp) = W_{\nu\mu}(-\vec{s}_\perp) \qquad \Longleftrightarrow \qquad A_N = 0$$

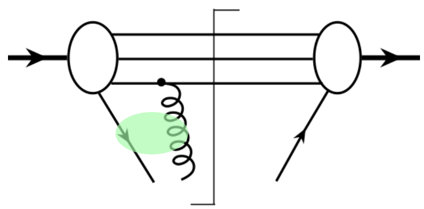
Single hadron inclusive

- **One large scale:** $A(p_A, S_\perp) + B(p_B) \to h(p) + X$ **with $p_T >> \Lambda_{QCD}$**

$$A_N \propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp)$$
$$\propto T^{(3)}(x, x, S_\perp) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x, S_\perp) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \ldots$$

Leading power contribution to cross section cancels!

- **Twist-3 three-parton correlation functions:** Qiu, Sterman, 1991, …

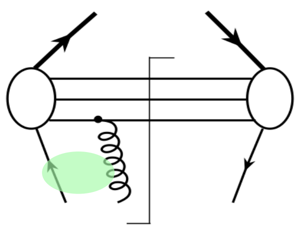
$T^{(3)}(x, x, S_\perp) \propto$ 

Moment of Sivers function?

Single jet inclusive

- **Twist-3 three-parton fragmentation functions:**

Kang, Yuan, Zhou, 2010

$D^{(3)}(z, z) \propto$ 

Moment of Collins function?

No probability interpretation!

Twist-3 correlation functions

Kang, Qiu, PRD, 2009

❑ Twist-2 parton distributions:

✧ **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

✧ **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Evolution equations and kernels

❑ Evolution equation is a consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

❑ Evolution kernel is process independent:

✧ Calculate directly from the variation of process independent twist-3 distributions

Kang, Qiu, 2009
Yuan, Zhou, 2009

✧ Extract from the scale dependence of the NLO hard part of any physical process

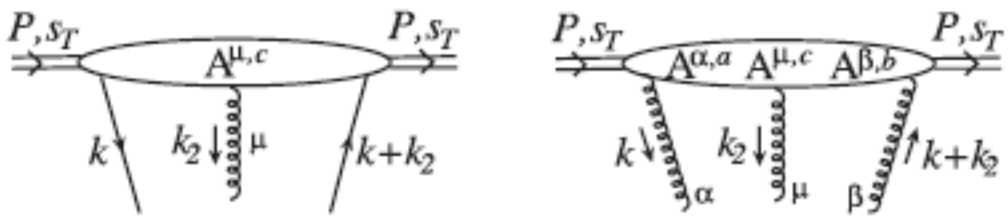
Vogelsang, Yuan, 2009

✧ Renormalization of the twist-3 operators

Braun et al, 2009

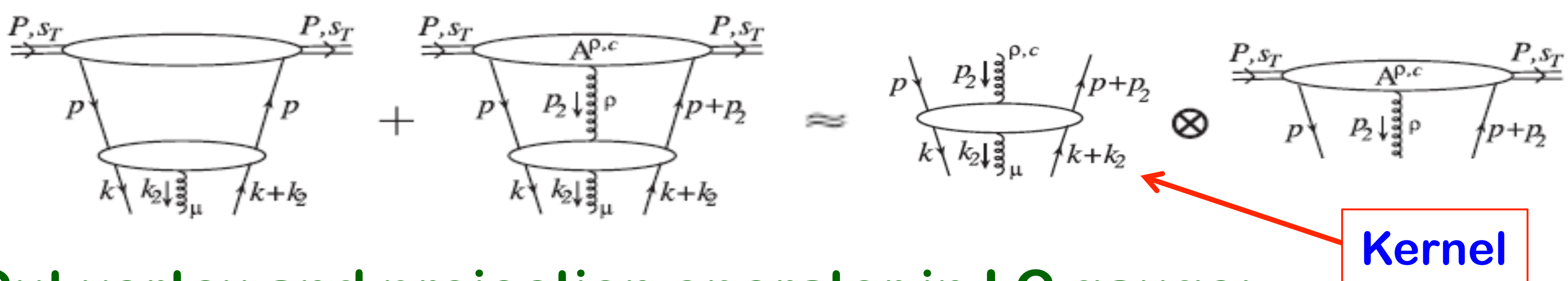
The Feynman diagram representation

- **Feynman diagram representation of twist-3 distributions:**



Different twist-3 distributions ⇔ diagrams with different cut vertices

- **Collinear factorization of twist-3 distributions:**



- **Cut vertex and projection operator in LC gauge:**

$$\mathcal{V}^{\rm LC}_{q,F} = \frac{\gamma^+}{2P^+}\delta\!\left(x - \frac{k^+}{P^+}\right)x_2\delta\!\left(x_2 - \frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}})[-g_{\sigma\mu}]\mathcal{C}_q$$

$$\mathcal{P}^{\rm (LC)}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{\mathcal{C}}_q$$

Variation of twist-3 correlation functions

❑ **Closed set of evolution equations (spin-dependent):**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(i)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{qg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{q\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)].$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(j)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{gg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(j)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{g\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_{q} \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{gq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{g\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)],$$

Plus two more equations for:

$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T)$ **and** $\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T)$

Evolution equations

- **Distributions relevant to SSA:**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x+x_2, x, \mu_F, s_T)\right].$$

- **Important symmetry property:**

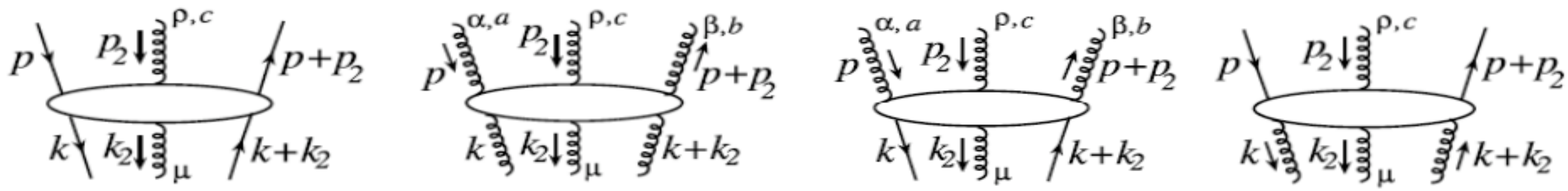
$$T_{\Delta q,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = 0,$$

$$T^{(f,d)}_{\Delta G,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \left(\frac{1}{x}\right) \mathcal{T}^{(f,d)}_{\Delta G}(x, x+x_2, \mu_F) = 0.$$

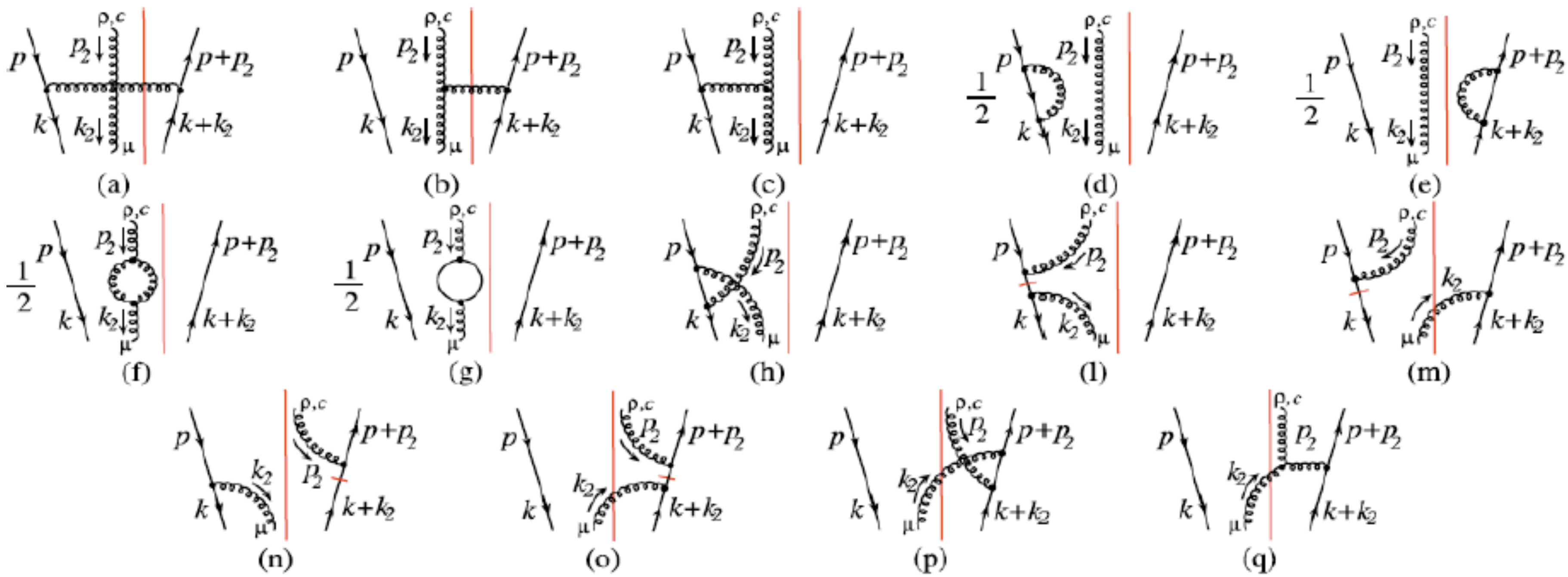
These two correlation functions do not give the gluonic pole contribution directly

Evolution kernels

❑ **Feynman diagrams:**



❑ **LO for flavor non-singlet channel:**



Leading order evolution equations - I

❑ **Quark:**

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F)$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F)-T_{q,F}(\xi,\xi,\mu_F)\right]+z\,T_{q,F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right]$$
$$+P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)+T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\Bigg\}$$

❑ **Antiquark:**

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F)$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]+z\,T_{\bar{q},F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta\bar{q},F}(x,\xi,\mu_F)\right]$$
$$+P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)-T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\Bigg\}$$

✧ **All kernels are infrared safe**

✧ **Diagonal contribution is the same as that of DGLAP**

✧ **Quark and antiquark evolve differently – caused by tri-gluon**

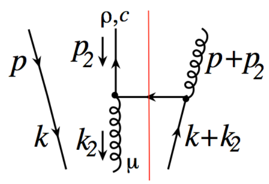
Leading order evolution equations - I

❑ **Quark:**

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{ P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F) - T_{q,F}(\xi,\xi,\mu_F)\right] + z\,T_{q,F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) + T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\}$$

❑ **Antiquark:**

Missing a term



Braun et al, 2009

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{ P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F) - T_{\bar{q},F}(\xi,\xi,\mu_F)\right] + z\,T_{\bar{q},F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) - T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\}$$

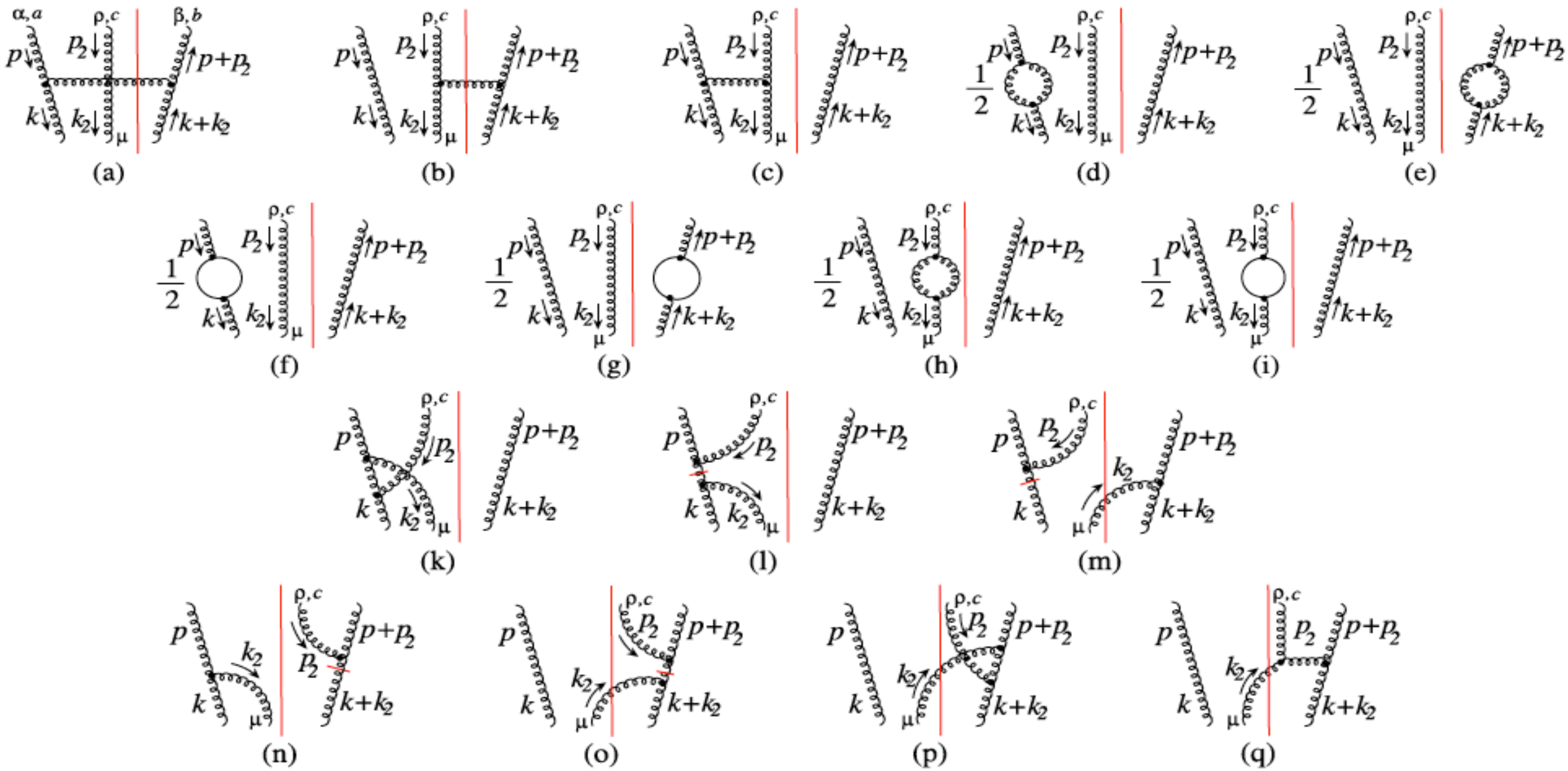
- ✧ **All kernels are infrared safe**
- ✧ **Diagonal contribution is the same as that of DGLAP**
- ✧ **Quark and antiquark evolve differently – caused by tri-gluon**

Three-gluon correlation and evolution

- **Two possible color contributions:**

$$d^{abc}, \;\; if^{abc} \quad \Longrightarrow \quad T_{G,F}^{(d)}(x_1, x_2), \quad T_{G,F}^{(f)}(x_1, x_2)$$

- **LO diagrams:**



Leading order evolution equations - II

❑ **Gluons:**

$$\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{P_{gg}(z)\,T^{(d)}_{G,F}(\xi,\xi,\mu_F)$$
$$+\frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right.$$
$$\left.+2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right]$$
$$+P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\sum_q\left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Bigg\}$$

Similar expression for $T^{(f)}_{G,F}(x,x,\mu_F)$

- ✧ **Kernels are also infrared safe**
- ✧ **diagonal contribution is the same as that of DGLAP**
- ✧ **Two tri-gluon distributions evolve slightly different**
- ✧ $T^{(d)}_{G,F}$ **has no connection to TMD distribution**
- ✧ **Evolution can generate** $T^{(d)}_{G,F}$ **as long as** $\sum_q [T_{q,F}+T_{\bar{q},F}] \neq 0$

Leading order evolution equations - III

- Evolution equations for diagonal correlation functions are not closed!

- "Model" for the off-diagonal correlation functions:

For the symmetric correlation functions:

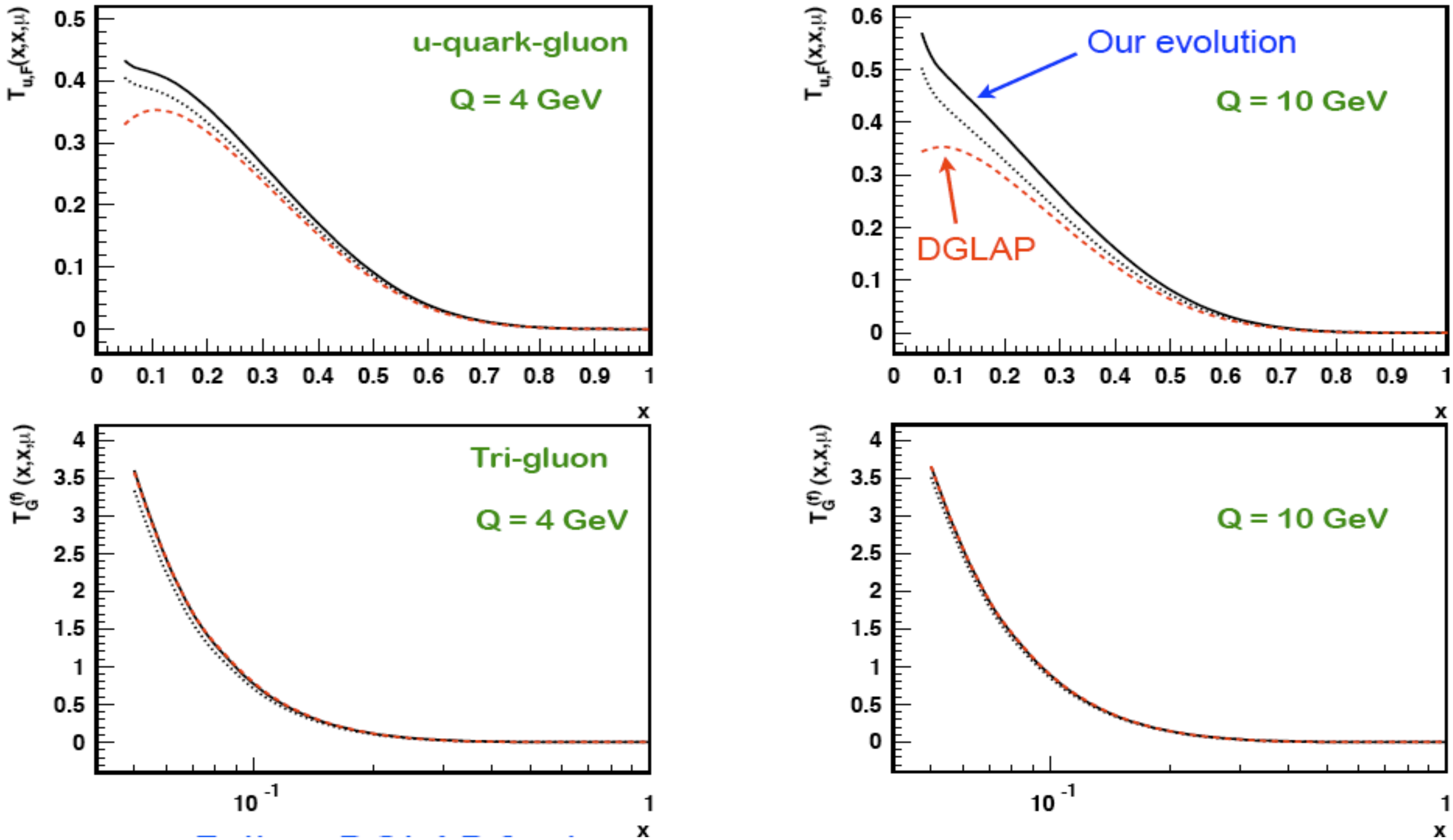
$$T_{q,F}(x_1, x_2, \mu_F) = \frac{1}{2}[T_{q,F}(x_1, x_1, \mu_F) + T_{q,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$$\mathcal{T}^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \frac{1}{2}[\mathcal{T}^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \mathcal{T}^{(f,d)}_{G,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

⇒

$$T^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \frac{1}{2}\left[T^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \frac{x_2}{x_1}T^{(f,d)}_{G,F}(x_2, x_2, \mu_F)\right]e^{-[(x_1-x_2)^2/2\sigma^2]}.$$

Scale dependence



- ✧ **Follow DGLAP at large x**
- ✧ **Large deviation at low x (stronger correlation)**

QCD global analysis of SSAs

- **Factorization for physical observables:**

$$A(p_A, S_\perp) + B(p_B) \to h(p) + X$$

$$A(p_A, S_\perp) + B(p_B) \to \text{jet}(p) + X$$

$$A(p_A, S_\perp) + B(p_B) \to \gamma(p) + X$$

$$\ldots$$

- **Urgently needed – NLO hard parts:**

Beyond LO!

$$A_N \propto \Delta\sigma(Q, S_\perp) \propto T_f^{(3)}(x,x) \otimes \hat{H}_f \otimes \ldots$$

Only NLO calculation – SSA for p_T weighted Drell-Yan Vogelsang, Yuan, 2009

- **A completely new domain to test QCD!**

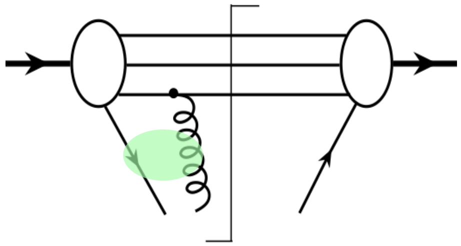
From paton's transverse motion to direct QCD quantum interference

"Interpretation" of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$$T^{(3)}(x,x,S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

❑ "Expectation value" of QCD operators:

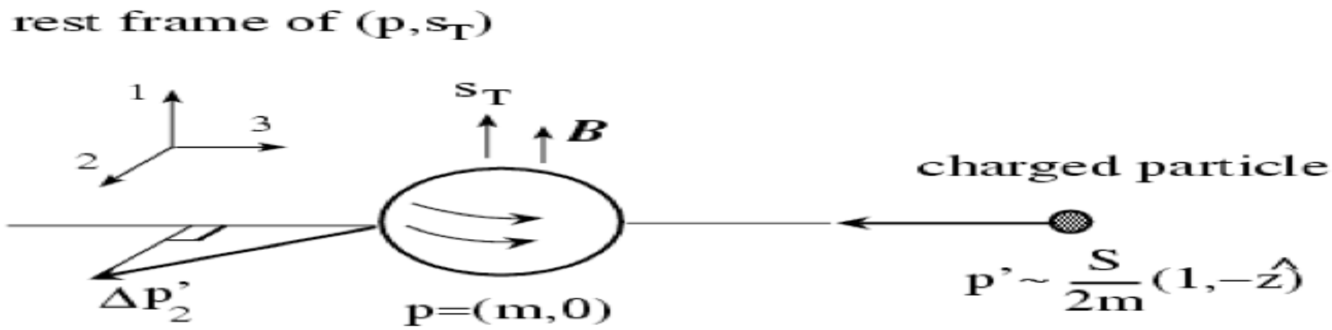
$$\langle P,s|\overline{\psi}(0)\gamma^+\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

$$\langle P,s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P,s\rangle \;\longrightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[i\,g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+$$

- The total change: $\Delta p_2' = e \int dy^- \epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y^-)$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Collinear vs TMD factorization

- Cover two different kinematic regions:

  Collinear: $Q_1 \dots Q_n \gg \Lambda_{QCD}$

  TMD: $Q_1 \gg Q_2 > \Lambda_{QCD}$

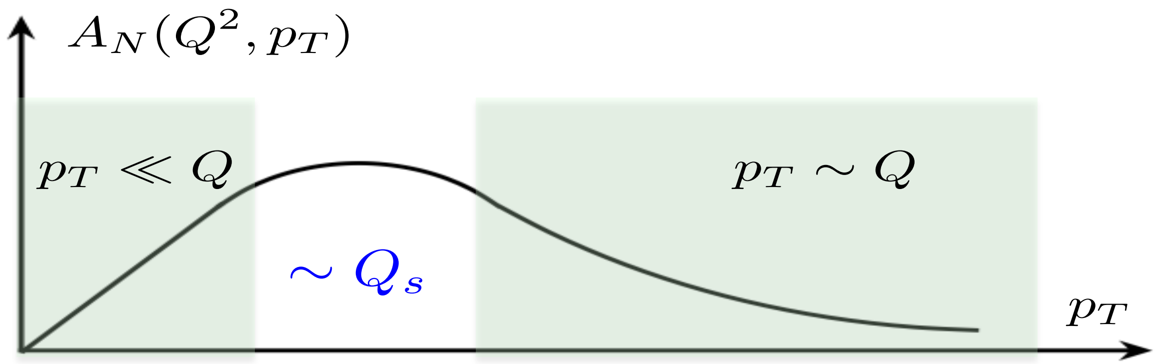
  Twist-3 correlation functions: Integrated effect of parton k_T

$$\frac{1}{M_p}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x,k_\perp) + \mathrm{UVCT}(\mu^2) = T_F(x,x,\mu^2)$$

  TMDs: direct information on parton k_T

  – more interesting if we can measure them

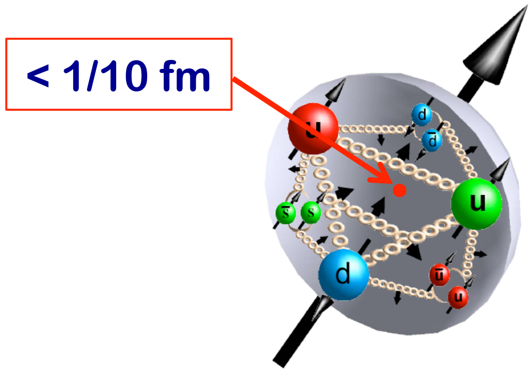
- Consisitent in the overlap (perturbative) region:



Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Summary

- QCD factorization/calculation have been very successful in interpreting HEP scattering data



- What about the hadron structure?

Not much!

- Experiments with a polarized hadron beam opened up new ways to test QCD and to study hadron structure

Parton's transverse motion and hadron's transverse structure

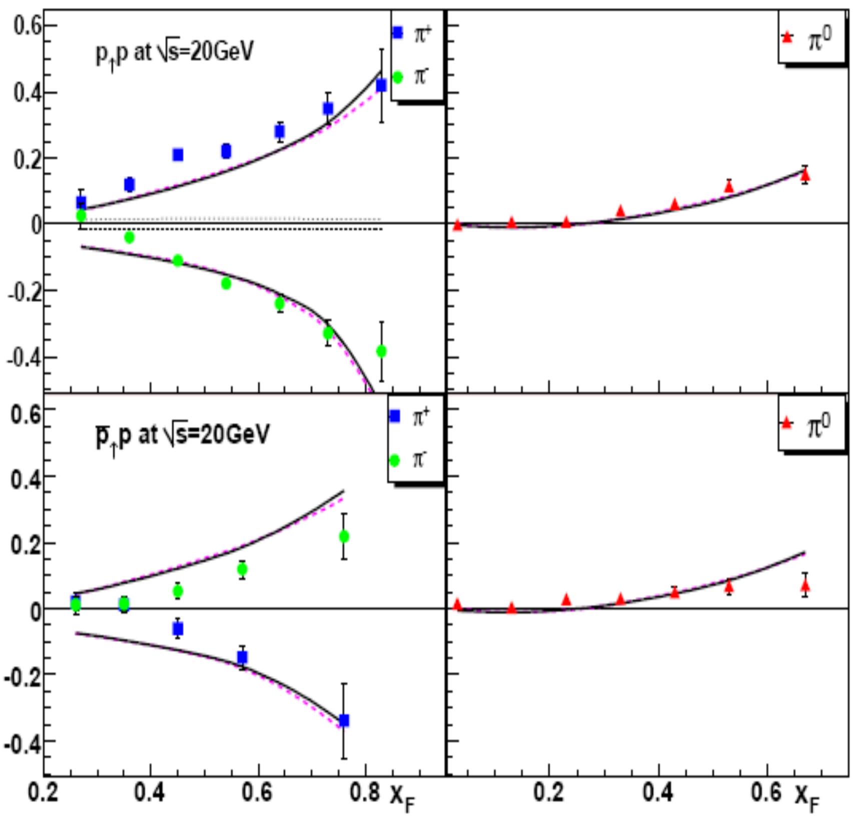
- Collinear and TMD factorization give complementary descriptions of QCD dynamics

Thank you!

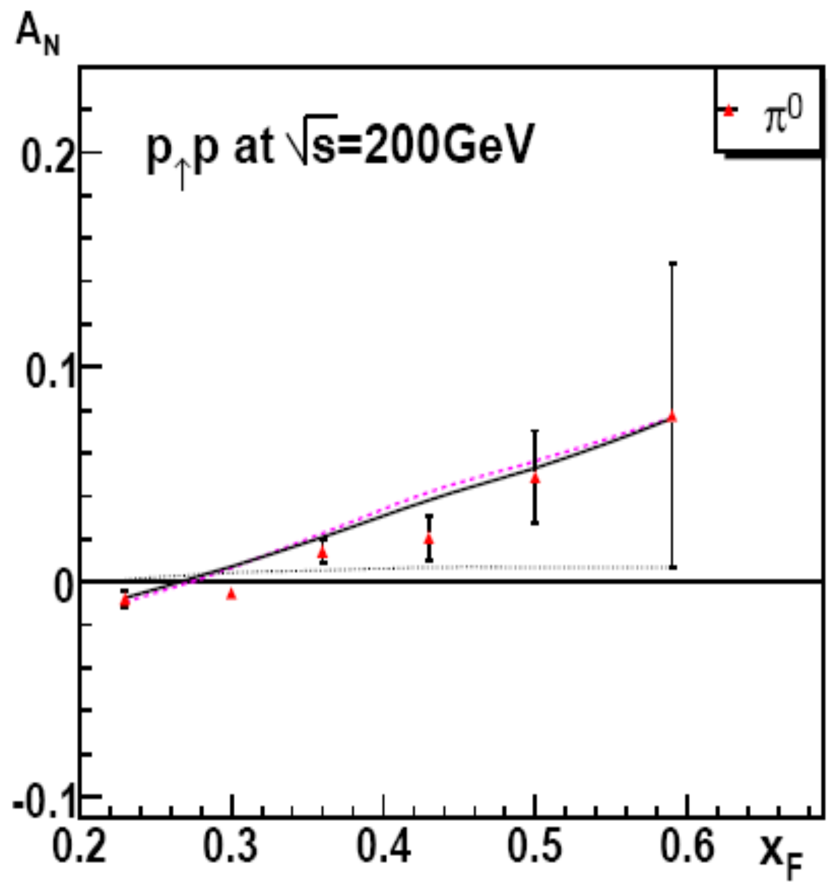
Backup slices

SSA from quark-gluon correlation

(FermiLab E704)



(RHIC STAR)

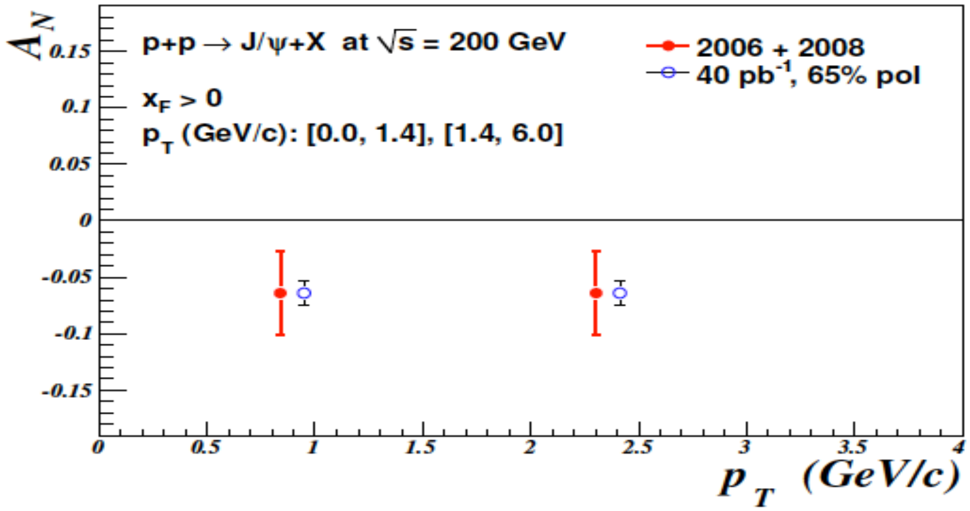


Kouvaris,Qiu,Vogelsang,Yuan, 2006

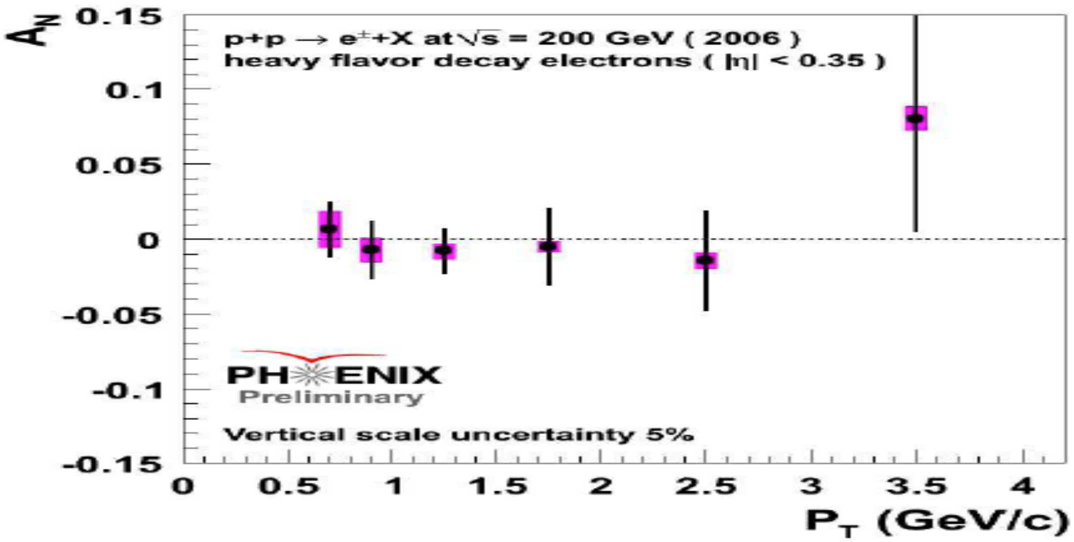
Nonvanish twist-3 function ➡ Nonvanish transverse motion

First hint of tri-gluon correlation

❑ PHENIX data on J/psi:



❑ PHENIX data on open charm:



Collinear factorization:

✧ tri-gluon correlation – direct quantum interference

Challenges:

✧ J/psi production mechanism

✧ Initial- vs final-state effect

✧ Connection to Gluon Sivers function

Collins, Qiu, Vogelsang, Yuan, Rogers, Mulder, …