Uses of Q^2 evolution in GPD phenomenology

Dieter Müller
LBNL

- ***GPD definitions and one trivial remark on Q^2 evolution***
- ***Uses of conformal symmetry***
- ***Modeling GPDs at the initial scale and their Q^2 evolution***
- ***Is evolution needed to describe present and future hard exclusive photon and meson electroproduction data?***

based on collaborations with

A. Belitsky (98-01)
K. Kumerički, K. Passek-Kumerički (05-...)
A. Schäfer, T. Lautenschlager, M. Meskauskas

Field theoretical GPD definition

GPDs are defined as matrix elements of
***renormalized** **light-ray** operators:*

$$F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa\, x\, n\cdot P} \langle P_2 | \mathcal{R}T : \phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n) : | P_1 \rangle,\ n^2 = 0$$

momentum fraction x , skewness $\eta = \frac{n\cdot\Delta}{n\cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\bar{\psi}_i \gamma_+ \psi_i \quad \Rightarrow \quad {}^i q^V = \bar{U}(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar{U}(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar{\psi}_i \gamma_+ \gamma_5 \psi_i \quad \Rightarrow \quad {}^i q^A = \bar{U}(P_2,S_2)\gamma_+ \gamma_5 U(P_1,S_1) \widetilde{H}_i + \bar{U}(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde{E}_i$$

shorthands:

chiral even GPDs: $F = \{H, E, \widetilde{H}, \widetilde{E}\}$ *& CFFs:* $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T = \{H_T, E_T, \widetilde{H}_T, \widetilde{E}_T\}$ $\mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \widetilde{\mathcal{H}}_T, \widetilde{\mathcal{E}}_T\}$

Q^2 evolution

"two-body" operators posses apart from self-energy insertions no singularities (in a generic scalar theory)

$$\mathcal{O}(x,y) = z_\phi T :\phi^{\mathrm{bar}}(y)\phi^{\mathrm{bar}}(x): \qquad (x-y)^2 \neq 0$$

usually a minimal subtraction (MS) scheme is used, e.g.

$$z_\phi = 1 + \frac{1}{\epsilon}\left(\frac{\alpha_s}{2\pi} z_\phi^{1,(0))} + O(\alpha_s^2)\right) + \frac{1}{\epsilon^2} O(\alpha_s^2) + \cdots$$

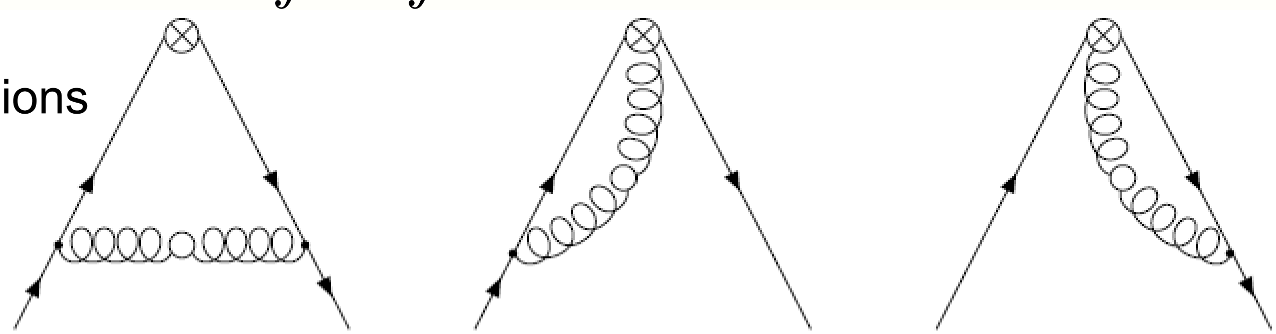
scale dependence is governed by anomalous dimensions

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g}\right]\mathcal{O}(x,y) = -2\gamma_\phi \mathcal{O}(x,y)\,, \quad \gamma_\phi = -\frac{1}{2} g \frac{\partial}{\partial g} z_\phi^{(1)}$$

leading twist operators on the light cone possess logarithmic singularities

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g}\right]\mathcal{O}(n,-n) = -\int d\kappa_1 \int d\kappa_2 \gamma(\kappa_1,\kappa_2)\,\mathcal{O}(\kappa_1 n, \kappa_2 n),\ n^2 = 0$$

LO anomalous dimensions are obtained from



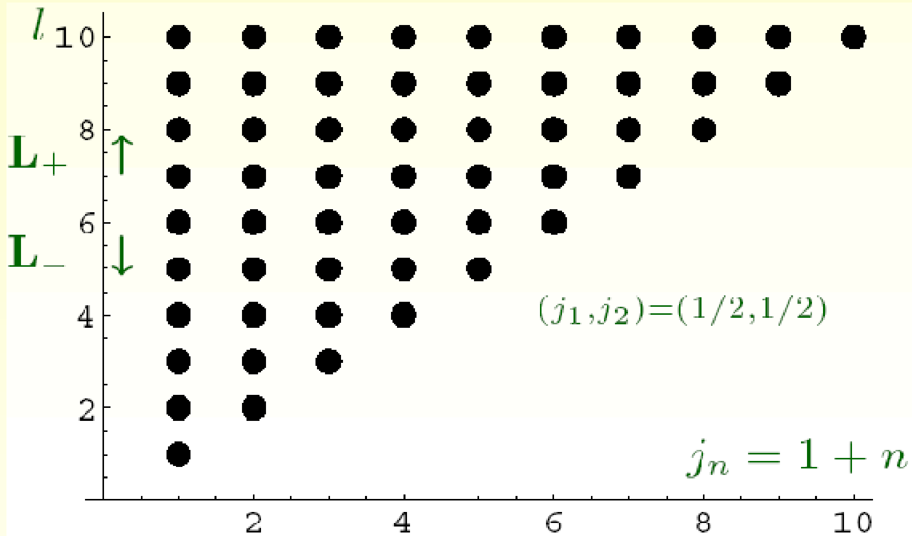
Conformal operator basis

irreducible representations :

$$\Phi_{j,l} = \partial_+^l \Phi_j(0)\,, \quad j = (d+s)/2$$

$$[j_1] \otimes [j_2] = \bigoplus_{n\geq 0} [j_n]\,, \quad j_n = j_1 + j_2 + n.$$

$$\mathcal{O}^{j,j}_{n,l} \propto \partial_+^{n+l} \left[\Phi_j C_n^{(2j-1)} \left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+} \right) \Phi_j \right]$$



✓ conformal symmetry is preserved at tree-level

► diagonal LO anomalous dimensions **[Ohrndorf 82, DM 91]**

! conformal symmetry is broken by the trace anomaly in *d=4-2ε* dimensions

► apart from *β*-proportional term it is also broken by the renormalization scheme

✓ conformal renormalization scheme exist so that the breaking appears only due to the β proportional trace anomaly in d=4 dimensions **[DM (97)]**

➢ anomalous dimensions and DVCS hard-scattering part @NLO **[Belitsky,DM (98)]**

➢ constructing all 12 twist-two NLO evolution kernels **[Belitsky, DM, Freund (00)]**
(two explicit calculated NLO kernerls **[Radyushkin et al (~85); Mikhailov, Vladimirov (09)]**)

conformal PW expansion of DAs

conformal symmetry in LO pQCD suggest Gegenbauer expansion

$$\phi_M(v,Q^2) = f_M \sum_{\substack{n=0 \\ \text{even}}}^{\infty} 6(1-v)v C_n^{3/2}(2v-1)\, E_n(Q,Q_0) a_n(Q_0^2)$$

(eigenfunction of the LO evolution operator)

- LO evolution equation is trivially solved

$$E_n(Q,Q_0) = \left(\frac{\ln(Q^2/\Lambda_{\mathrm{QCD}}^2)}{\ln(Q_0^2/\Lambda_{\mathrm{QCD}}^2)}\right)^{-\gamma_n^{(0)}/\beta_0} \qquad \gamma_n^{(0)} = \frac{4}{3}\left(4S_{n+1} - \frac{2}{(n+1)(n+2)} - 3\right)$$

- inverse moment enters in LO descriptions of form factors

$$\mathcal{I}_M(Q^2) = \frac{1}{3f_M}\int_0^1 du\, \frac{\phi_M(u,Q^2)}{u} = \sum_{\substack{n=0 \\ \text{even}}}^{\infty} E_n(Q,Q_0) a_n(Q_0^2)$$

Effective model for DAs

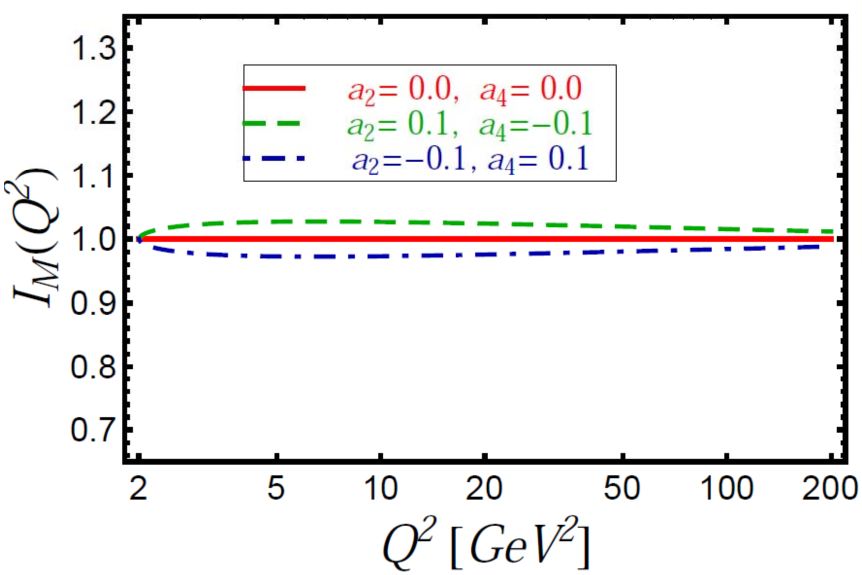
three conformal moments, two free parameters

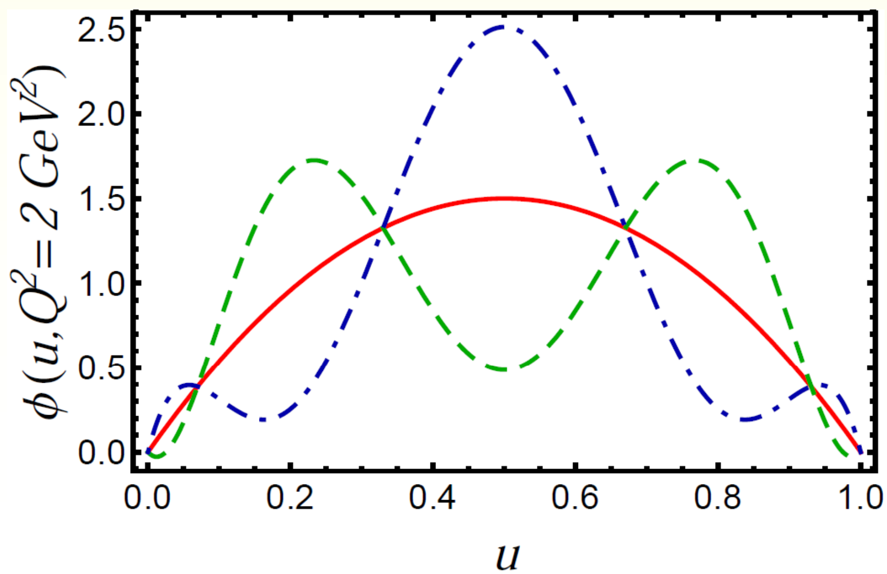
$$a_0 = 1 \text{ (fixed by normalization)}, \quad a_2, \quad a_4, \quad \text{for} \quad \mu^2 = Q_0^2$$

suppose we have a "measurement" , there is still freedom left

$$\mathcal{I}_M(Q^2 = Q_0^2) \; = \; 1 + a_2 + a_4 , \quad \text{fixed by data}$$
$$\mathcal{I}_M(Q^2 \to \infty) \; = \; 1 , \quad \text{asymptotic limit is slowly reached}$$

suppose $\mathcal{I}_M(Q^2 = Q_0^2) = 1$: Can one practically pin down such a model?





Conformal partial wave expansion of GPDs

- a GPD can be expanded with respect to conformal partial waves of the collinear conformal group SO(2,1) (similar to SO(3) expansion)
  - expansion in terms of discrete conformal spin j+2 for $\eta > 1, \ |x/\eta| \leq 1$

$$F(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) F_j(\eta,t)$$

$z = x/\eta$ ⟺ $j+2$

  - conformal moments (partial wave amplitudes) are polynomials:

$$F_j(x,\eta) = \frac{\Gamma(3/2)\Gamma(1+j)}{2^j\Gamma(3/2+j)} \int_{-1}^{1} dx\, \eta^{j+1} C_j^{3/2}\left(\frac{x}{\eta}\right) F(x,\eta,t)$$

  - conformal partial waves ensure the polynomiality condition:

$$p_j(x,\eta) = \frac{\Gamma(5/2+j)}{j!\Gamma(1/2)\Gamma(2+j)} \frac{d^j}{dx^j} \int_{-1}^{1} du (1-u^2)^{j+1} \delta(x-u\eta)$$

✓ ***crossing symmetry*** allows for a more convenient representation (technicality, e.g., Sommerfeld-Watson transform, numerous failures in the literature)

✓ partial waves evolve autonomously ⇨ trivial implementation of evolution

Summing up conformal PWs

- GPD support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

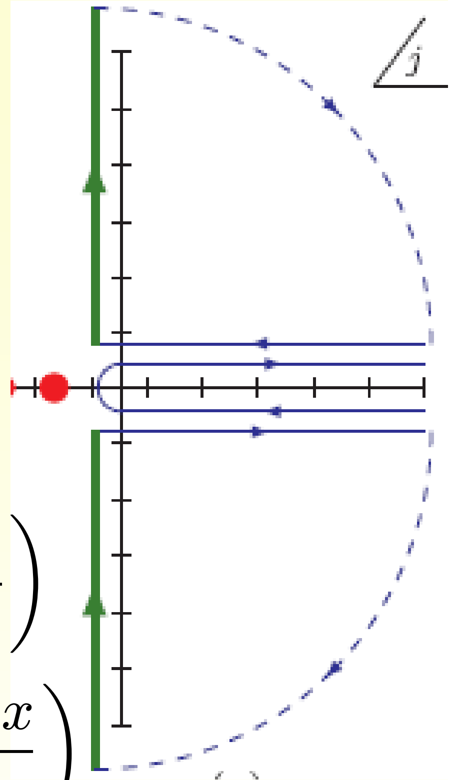
$$\mu \frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\; p_j(x,\eta) \propto \theta(|x|\le\eta)\frac{\eta^2-x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:
  - smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
  - mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
  - dual parameterization [M. Polyakov, A. Shuvaev (02)]
  - based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
  - ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Sommerfeld-Watson transform



✓ rewrite sum as an integral around the real axis:

$$F(x,\eta,\Delta^2) = \frac{1}{2i}\oint_{(0)}^{(\infty)} dj\, \frac{1}{\sin(\pi j)} p_j(x,\eta)\; F_j(\eta,\Delta^2)$$

✓ find appropriate analytic continuation of p_j and F_j (Carlson's theorem)

$$p_j(x,\eta) = \theta(\eta - |x|)\eta^{-j-1}\mathcal{P}_j\left(\frac{x}{\eta}\right) + \theta(x-\eta)\eta^{-j-1}\mathcal{Q}_j\left(\frac{x}{\eta}\right)$$

$$\mathcal{P}_j(x) \;=\; \frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(1/2)\Gamma(1+j)}(1+x)\;{}_2F_1\left(\begin{matrix}-j-1,\, j+2\\ 2\end{matrix}\,\middle|\,\frac{1+x}{2}\right)$$

$$\mathcal{Q}_j(x) \;=\; -\frac{\sin(\pi j)}{\pi}\; x^{-j-1}\;{}_2F_1\left(\begin{matrix}(j+1)/2,\,(j+2)/2\\ 5/2+j\end{matrix}\,\middle|\,\frac{1}{x^2}\right)$$

✓ change integration path so that singularities remain on the l.h.s.

$$F(x,\eta,\Delta^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\, \frac{1}{\sin(\pi j)} p_j(x,\eta)\, F_j(\eta,\Delta^2)$$

Advantages of the Mellin-Barnes integral

- ✓ another possibility to parameterize GPDs [similar to the dual parameterization] (basic properties are implemented, essential for flexible fitting routines)
- ✓ (LO) solution of the evolution equation is trivial implemented

$$F(x,\eta,\Delta^2,\mathcal{Q}^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\eta)}{\sin(\pi j)} \exp\left\{ -\frac{\gamma_j^{(0)}}{2} \int_{\mathcal{Q}_0^2}^{\mathcal{Q}^2} \frac{d\sigma}{\sigma} \frac{\alpha_s(\sigma)}{2\pi} \right\} F_j(\eta,\Delta^2,\mathcal{Q}_0^2)$$

- ✓ fast and robust numerical evaluation
- ✓ simple representation of amplitudes

$$\mathcal{F}(\xi,\Delta^2,\mathcal{Q}^2) = \int_{-1}^{1} dx \left[\frac{e^2}{\xi - x - i\epsilon} \mp \frac{e^2}{\xi + x - i\epsilon} \right] F(x,\xi,\Delta^2,\mathcal{Q}^2)$$

$$\mathcal{F} = \frac{e^2}{2i} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(3/2)\Gamma(3+j)} \left(i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right) F_j(\xi,\Delta^2,\mathcal{Q}^2)$$

- ✓ MS factorization conventions can be implemented at NLO
- ✓ CS factorization conventions enable us to explore NNLO corrections

What is ``dual'' GPD parameterization ?

- t-channel scattering angle and skewness parameter are related: $\cos\theta \approx -1/\eta$
- labeling the conformal moments by the t-channel ***angular momentum*** *J* (conjugated variable to θ or in some sense to η)

$$F_j(\eta,t) = \eta^{j+1} \sum_{J=J^{\min}}^{j+1} f_{j,J}(t)\, d_J(1/\eta)$$

partial wave amplitudes depending on j and J — *reduced Wigner rotation matrices*

[Polyakov (99) Ji, Lebed (00) Diehl (03), KMP-K (07)]

- primary `quantum numbers' are *j+2* and the difference *v= j+1-J*
- in ``dual'' parameterization *j+2* is replaced by conjugate momentum fraction *z*

$$F(x,\eta,t) = \sum_{\nu=0}^{\infty} \int_0^1 dz\, K_\nu(x,\eta|z) Q_\nu(z,t)$$

[Polyakov, Shuvaev (02)]

- GPD model building in terms of $f_{j,j+1-v}(t)$ or $Q_v(z,t)$ (one-to-one to DDs)

``dual'' parameterization ***[Guzey, Teckentrup (06)]*** effectively took *v=0* ***[Polyakov (07)]***

A flexible GPD model

- take three effective SO(3) partial waves

$$F_j(\eta,t) = \hat{d}_j(\eta) f_j^{j+1}(t) + \eta^2 \hat{d}_{j-2}(\eta) f_j^{j-1}(t) + \eta^4 \hat{d}_{j-4}(\eta) f_j^{j-3}(t), \quad j \geq 4$$

$$f_j^{j-k}(\eta,t) = s_k f_j^{j+1}(\eta,t)\,, \quad k = 2, 4, \cdots$$

- rewrite Mellin-Barnes integral

$$\mathcal{F} = \frac{1}{2i} \sum_{\substack{k=0 \\ \text{even}}}^{4} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \frac{2^{j+1+k}\Gamma(5/2+j+k)}{\Gamma(3/2)\Gamma(3+j+k)} \left(i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right)$$

$$\times s_k E_{j+k}(\mathcal{Q}^2) f_j^{j+1}(t) \hat{d}_j(\xi), \quad s_0 = 1$$

NOTE:

- first partial wave amplitude is fixed by PDFs (if they exist) and FFs
- “Regge poles” should be in the angular momentum *J*-plane (not in the *j*-plane)

$$H(x,x,t=0,\mathcal{Q}^2) \stackrel{x\to 0}{=} \sum_{\substack{k=0 \\ \text{even}}}^{4} s_k \frac{2^{\alpha+k}\Gamma(3/2+\alpha+k)}{\Gamma(3/2)\Gamma(2+\alpha+k)} q(x,\mathcal{Q}^2)$$

- a *J*-pole is associated with a series of spurious poles in the *j*-plane

Is the conformal ratio supported?

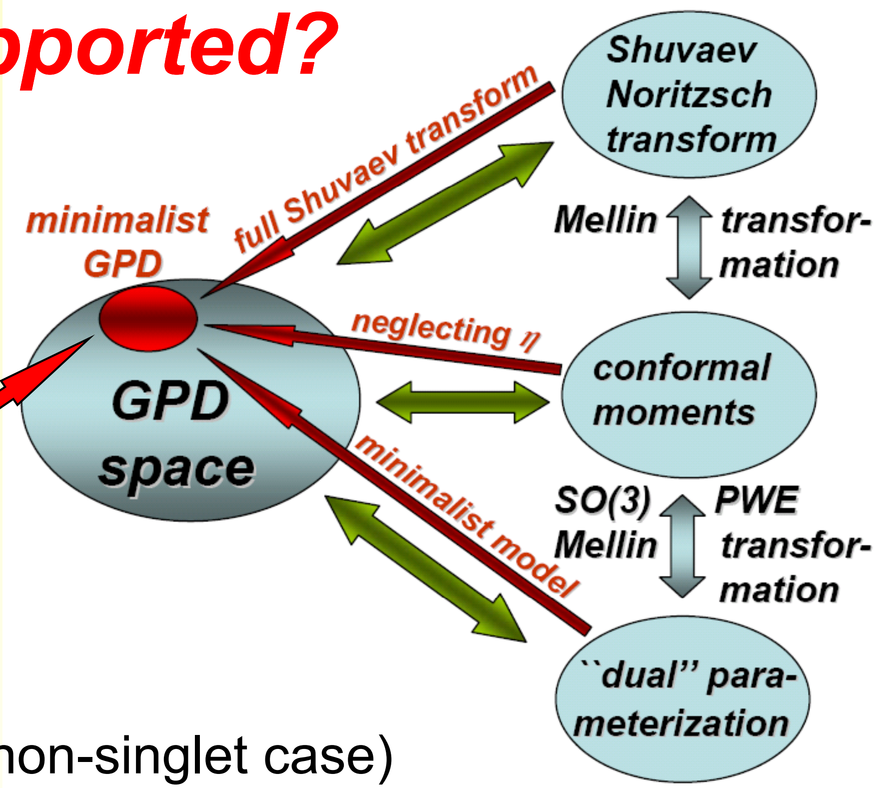
associating "Regge poles" with the *j*-plane yields ``erroneous small *x*-claim'' that GPDs are "tied" to PDFs:

$$r = \frac{H(x,x,t=0,Q^2)}{q(x,Q^2)}$$

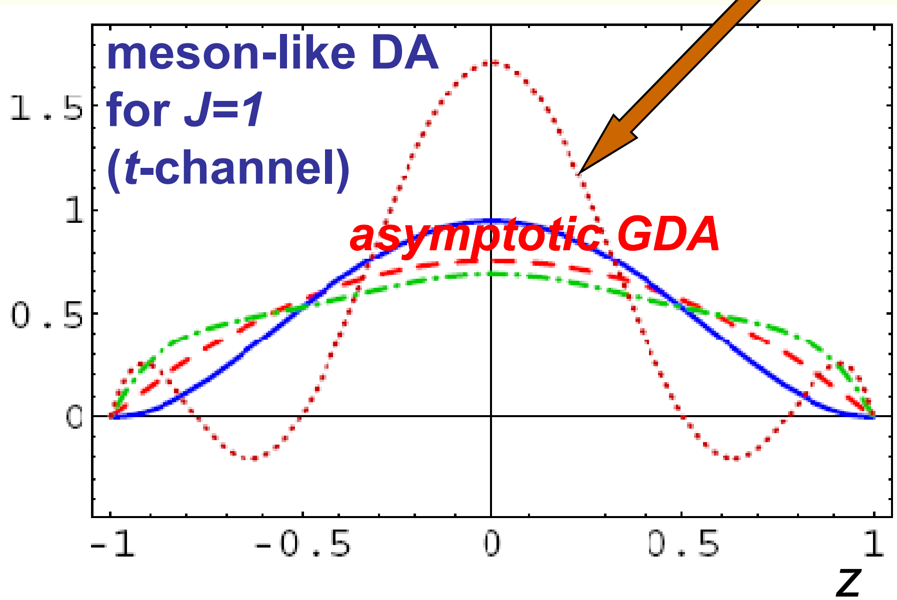
by the conformal (Shuvaev) ratio:

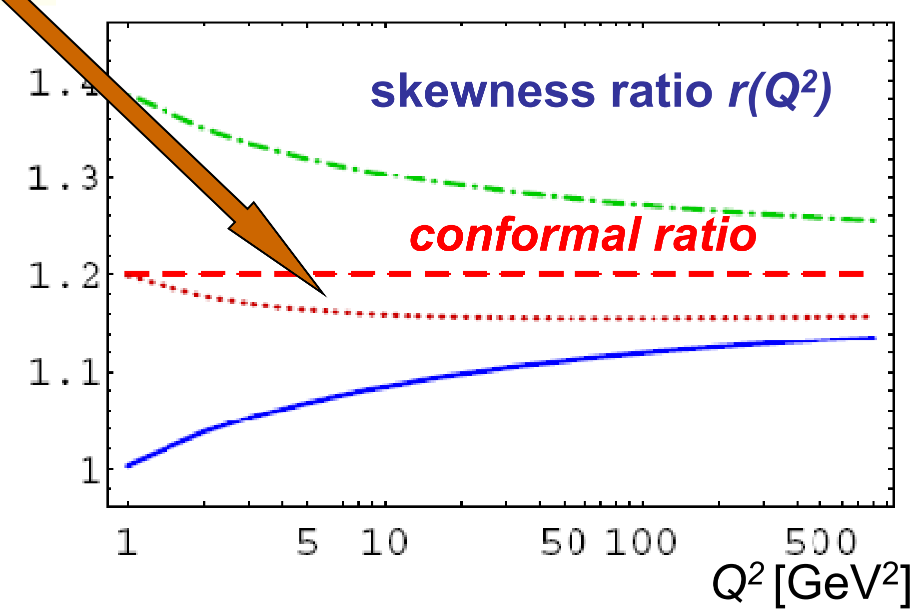
[Martin, Ryskin, Shuvaev et al.]

$$r_{\mathrm{con}} = \frac{2^{\alpha}\Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$$



counter example (non-singlet case)





Modeling & evolution in x-space

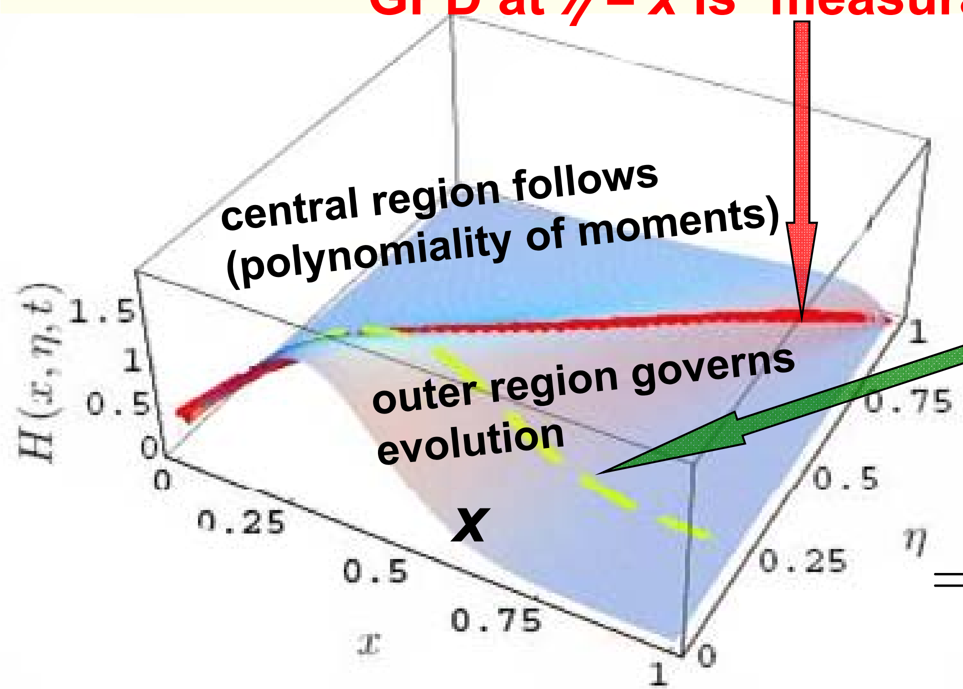
- "Dispersion relation" can be used at twist-two level:

$$\Re\mathrm{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right)\Im\mathrm{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

$$\frac{1}{\pi}\Im\mathrm{m}\mathcal{F}(\xi=x,t,Q^2) \stackrel{\mathrm{LO}}{=} F(x,x,t,Q^2) \mp F(-x,x,t,Q^2)$$

- outer region governs the evolution at the cross-over trajectory

$$\mu^2\frac{d}{d\mu^2}H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1,x/y,\alpha_s(\mu))H(y,x,\mu^2)$$

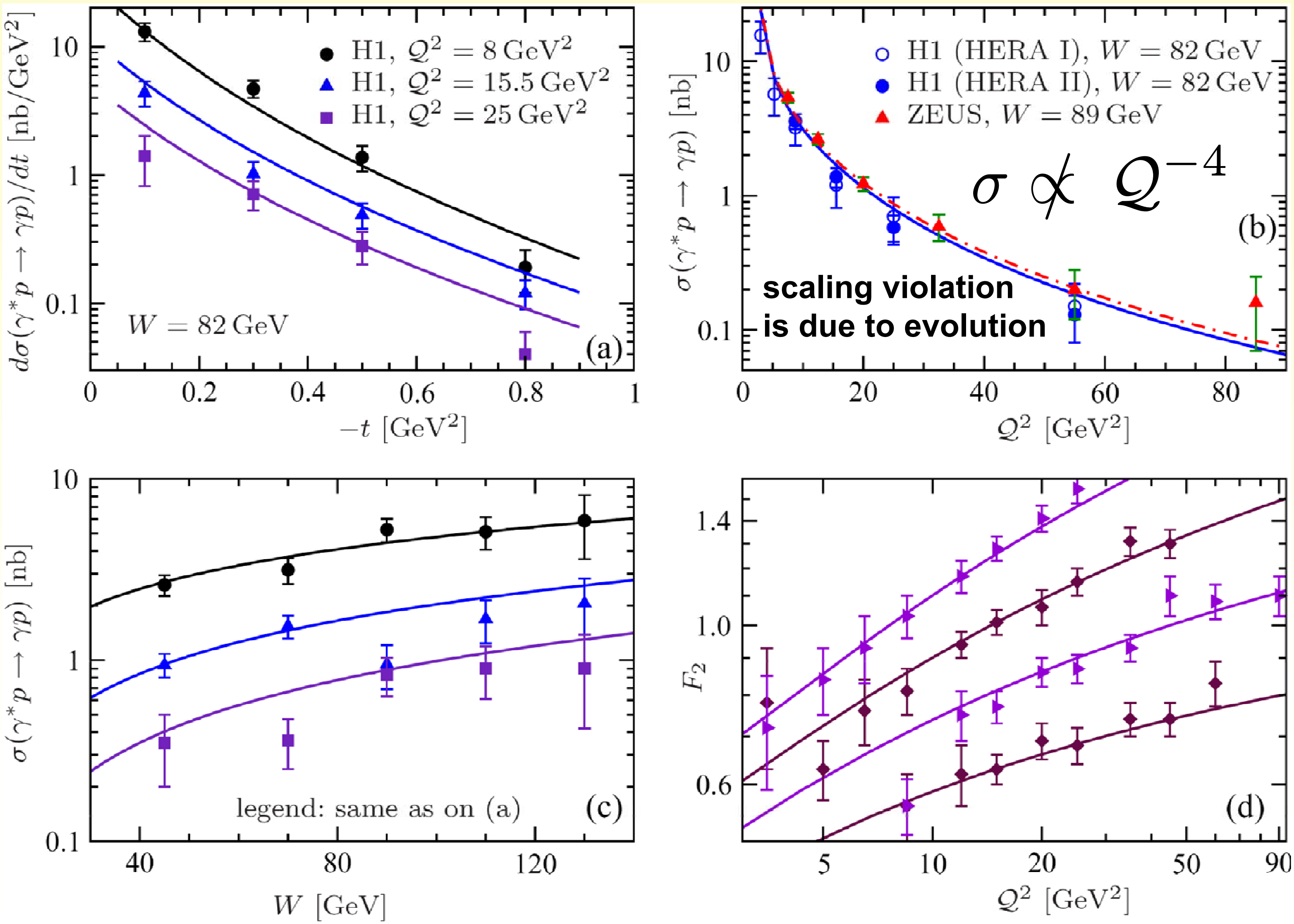


net contribution of outer + central region is governed by a sum rule:

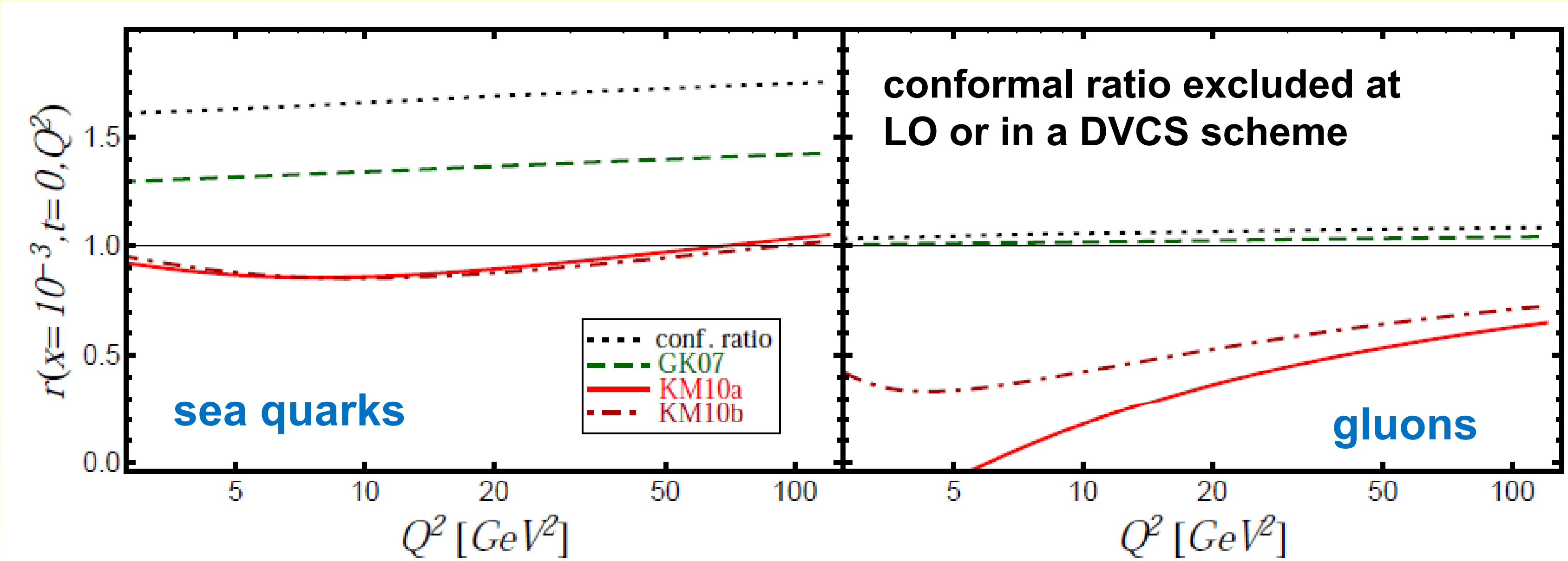
$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2-x^2}H^-(x,\eta,t)$$

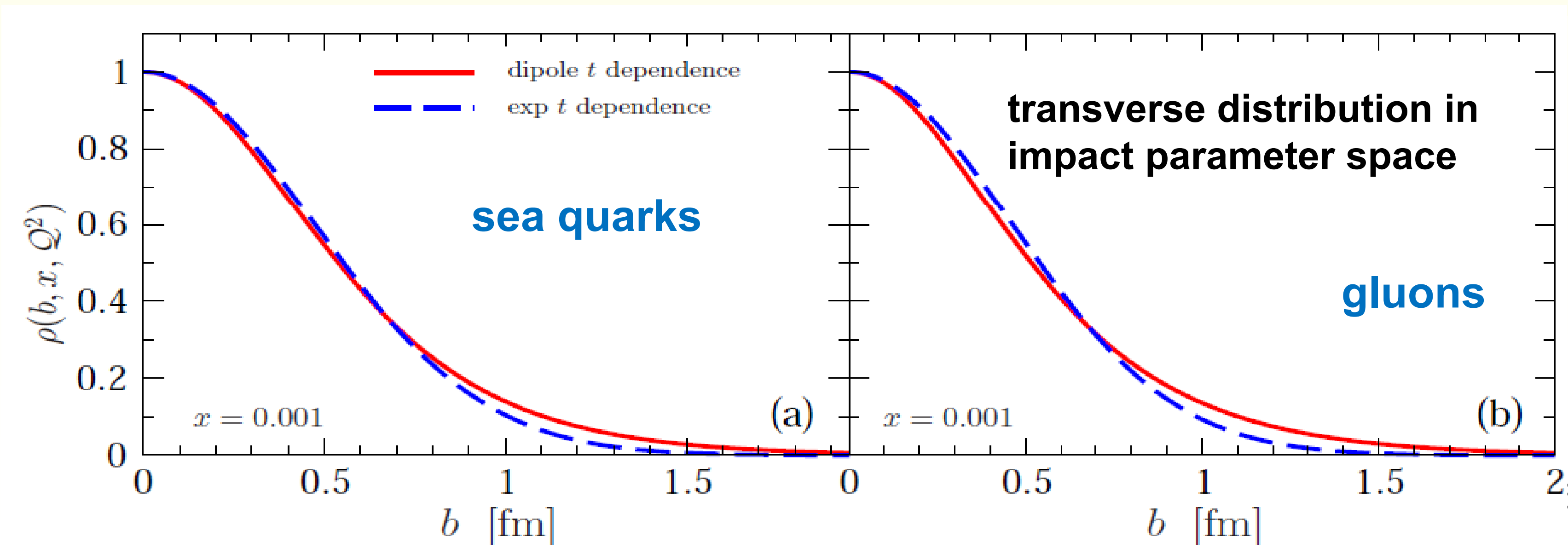
$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2-x^2}H^-(x,x,t) + C(t)$$

good DVCS fits to H1 and ZEUS data at LO, NLO, and NNLO with flexible GPD ansatz

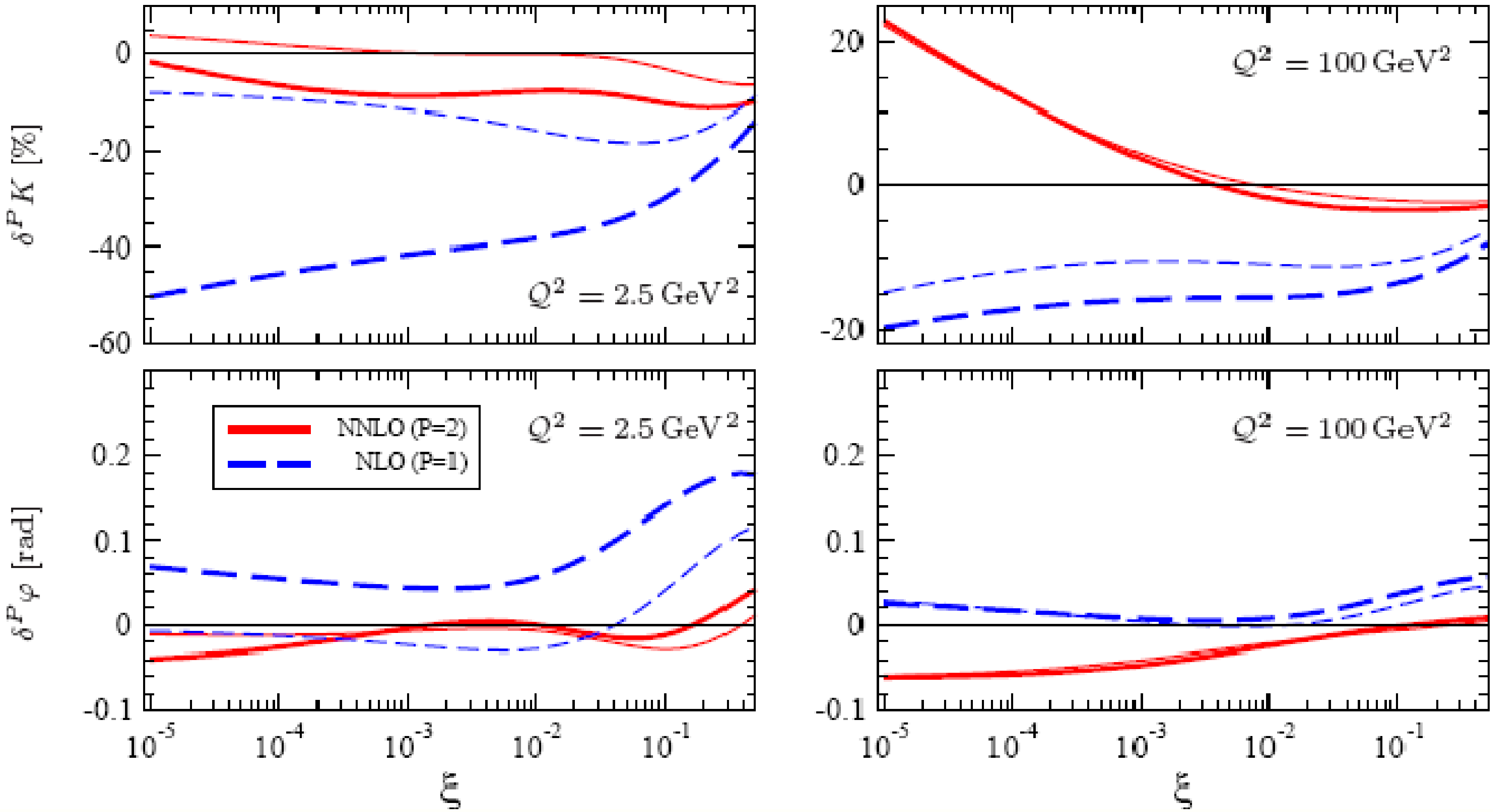


large Q^2 lever arm and the "pomeron" pole in the glonic sector allow to ask for gluon contributions in DVCS at small x





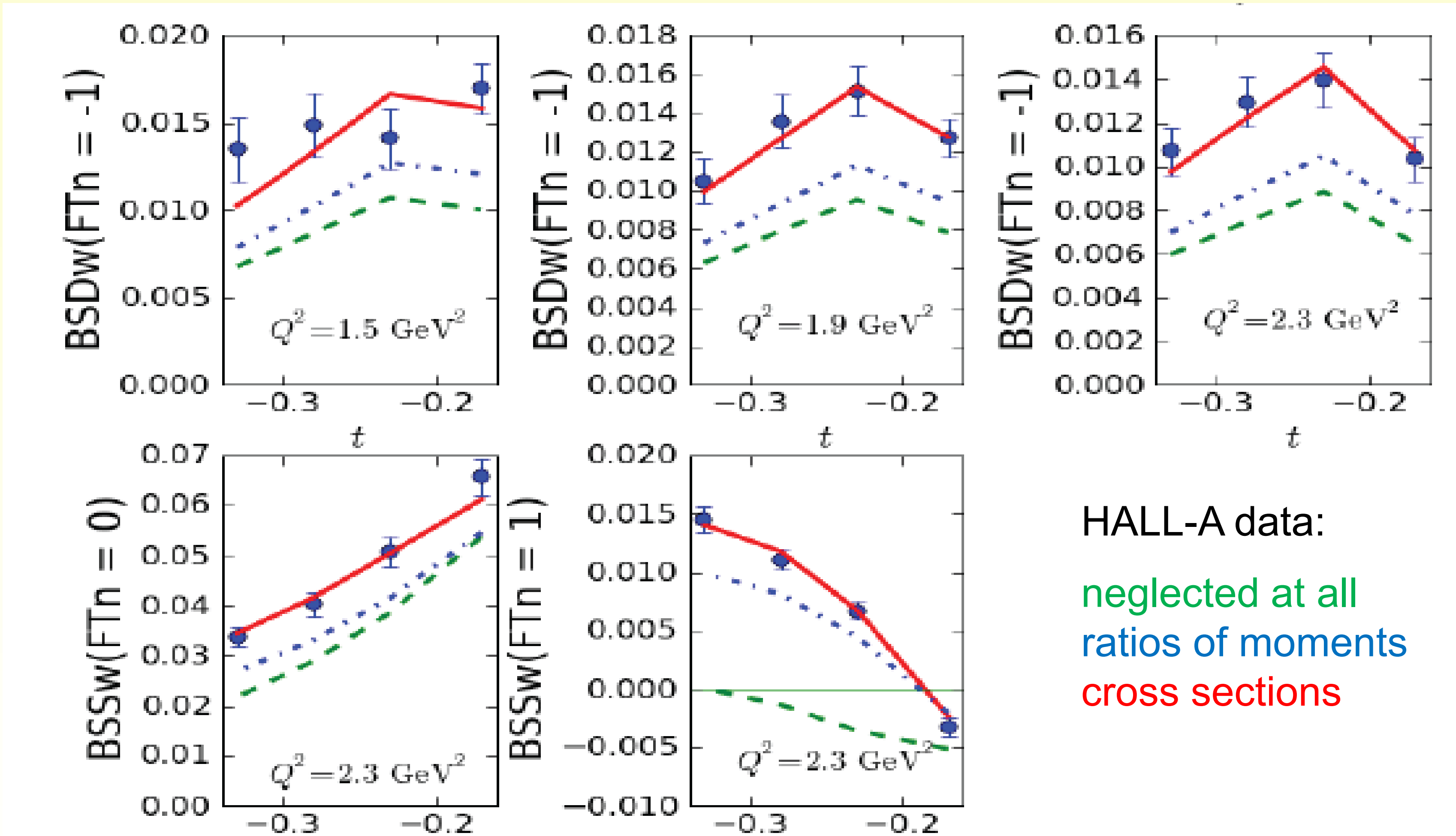
“pomeron pole” related NLO and NNLO corrections



- drastically reduction of perturbative corrections at NNLO for the hard part
- reduction of renormalization scale dependence
- but perturbative predictions for the evolution is unstable
- no improvement of factorization scale dependence

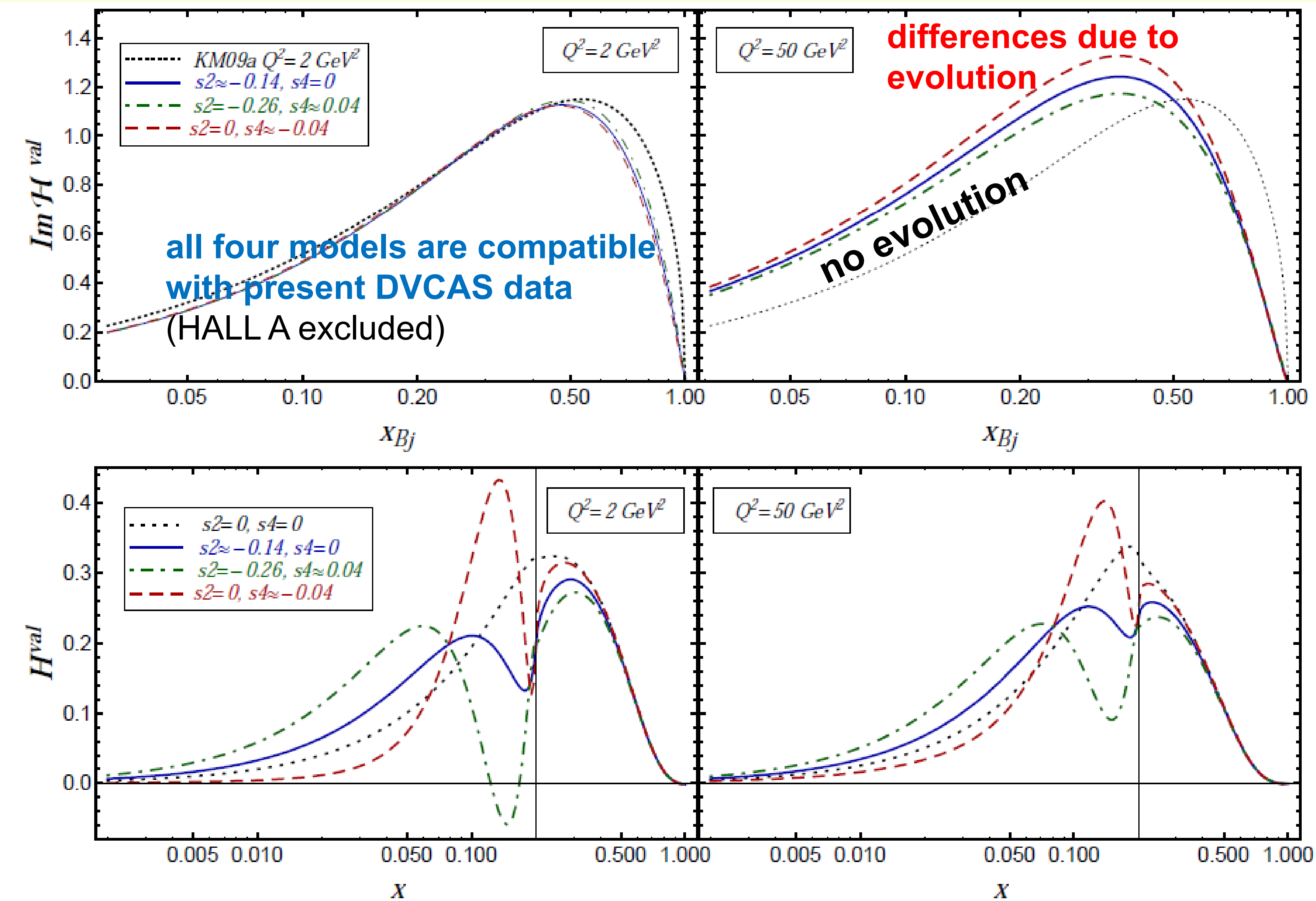
evolution is not needed to analyze fixed target DVCS data (HERMES, JLAB)

⇨ uses of "dispersion relation" approach (modeling accessible degrees of freedom)



- fits to HALL A harmonics are fine for unexpected large Ĥ or Ě contribution
- large Ĥ KM09 scenario is excluded from longitudinal TSA (HERMES, CLAS)

Can one use evolution to pin down valence GPDs in a future EIC measurement?



it will be a challenge to discriminate between models

Summary

- pQCD formalism for hard exclusive production is available at NLO
  - for DVCS even at NNLO in a specific subtraction scheme
  - pQCD@NLO will be needed for a global analysis of photon and meson data
- NLO evolution kernels where obtained from the understanding that conformal symmetry is broken by the normalization conditions
  - for *β=0* restoration of conformal symmetry is possible in any order
  - formally proved from conformal algebra and Ward identities
- evolution operator in the flavor singlet and parity even sector becomes unstable in the small *x*-region
  - fortunately, this is a universal feature
- a high luminosity machine with dedicated experiments is desired
  - to resolve the transverse degrees of freedom
  - within the discussed EIC it might be possible to employ evolution effects to explore GPDs apart from the cross-over line