Transverse Momentum Distributions from Effective Field Theory

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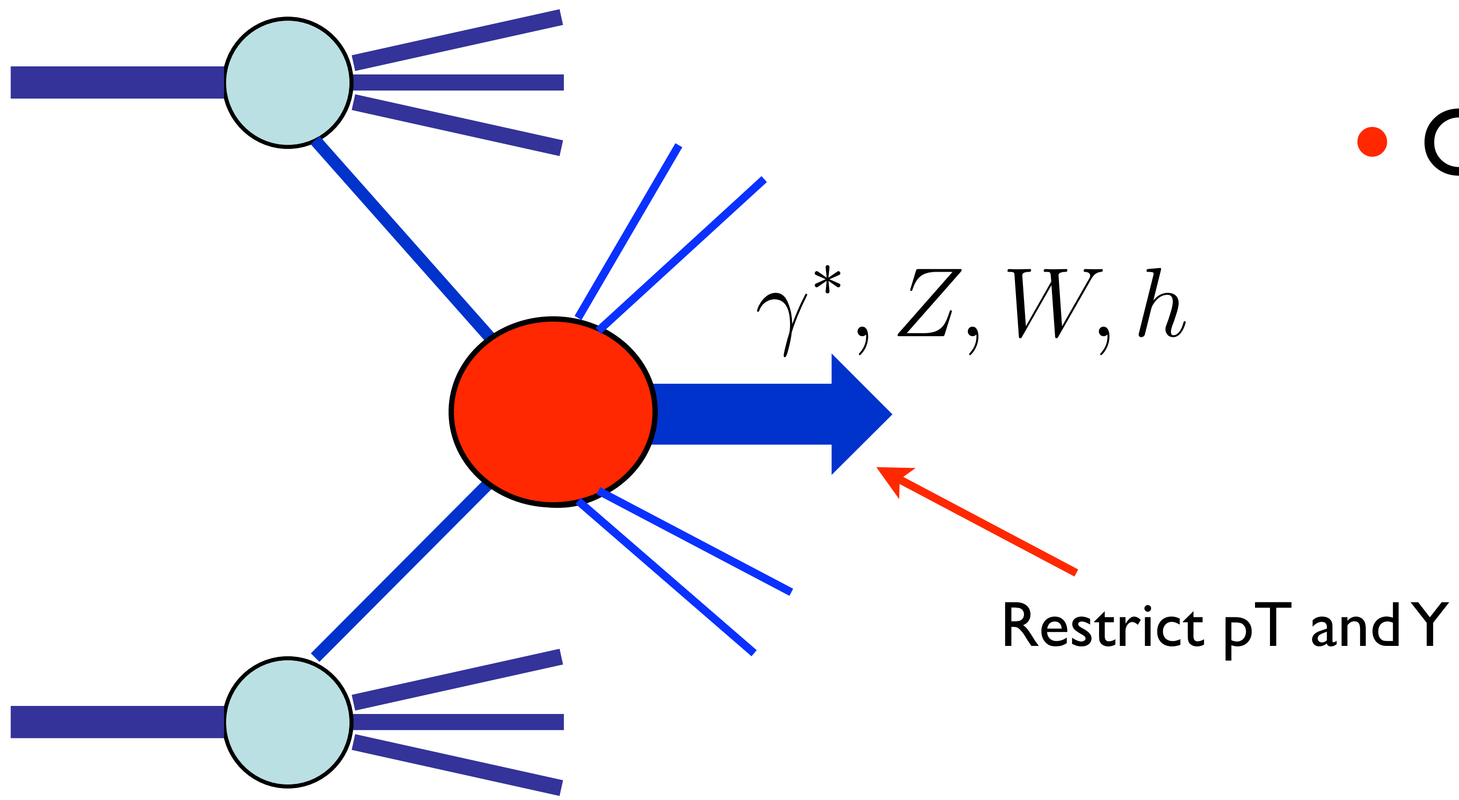
arXiv:1011.0757

QCD Evolution Workshop: from Collinear to Non-collinear Case
JLAB, April 9th, 2011

Outline

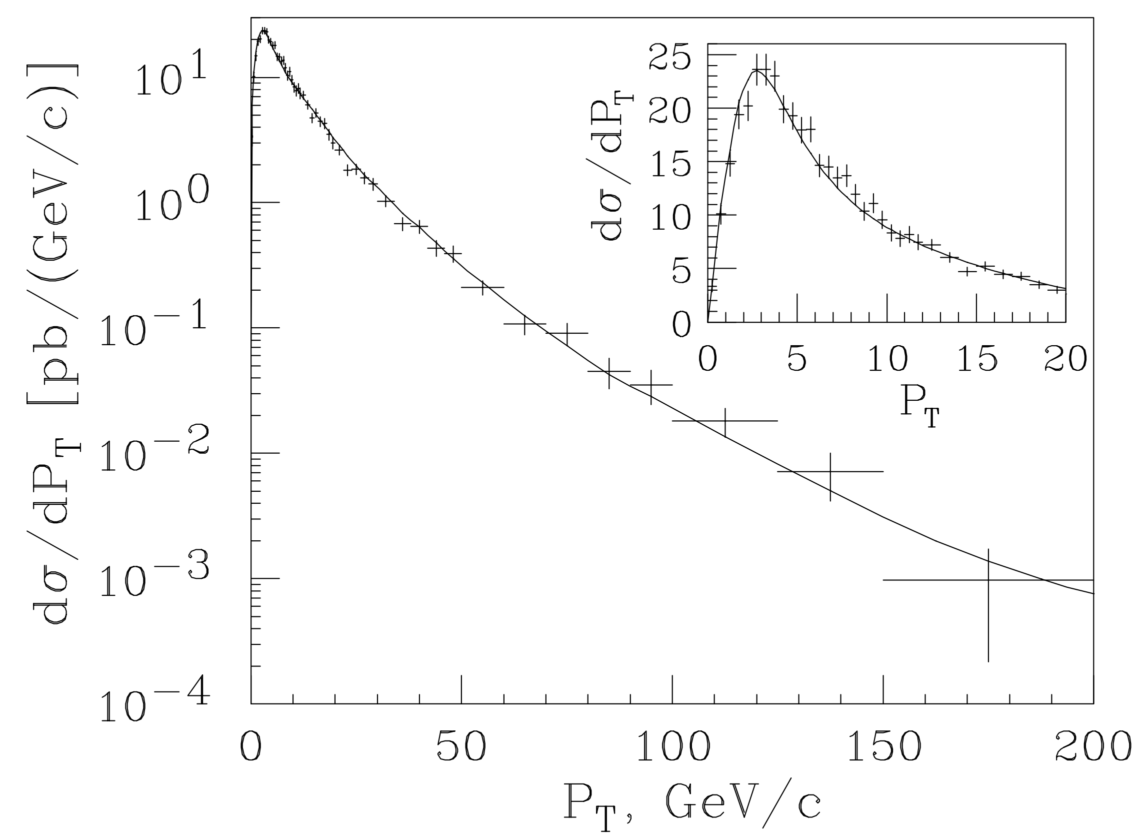
- Introduction
- Effective field theory Approach
- Numerical Results and Comparison with Data
- Non-perturbative transverse momentum region
- Conclusions

Transverse Momentum Spectrum



- Observable of interest

$$\frac{d^2\sigma}{dp_T^2 dY}$$

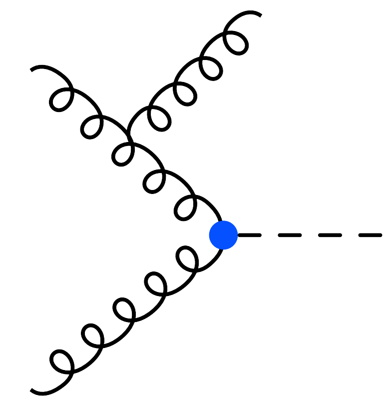


CDF Data
for Z-production

Motivations

- Higgs Boson searches
- W-mass measurement
- Tests of pQCD
- Transverse nucleon structure

Low pT Region



- The schematic perturbative series for the pT distribution for $pp \longrightarrow h + X$

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_2\alpha_S^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

Large Logarithms spoil perturbative convergence

- Resummation has been studied in great detail in the Collins-Soper-Sterman formalism.

(Davies, Stirling; Arnold, Kauffman; Berger, Qiu; Ellis, Veseli, Ross, Webber; Brock, Ladinsky Landry, Nadolsky; Yuan; Fai, Zhang; Catani, Emilio, Trentadue; Hinchliffe, Novae; Florian, Grazzini, Cherdnikov, Stefanis; Belitsky, Ji,....)

- Resummation has also been studied recently using the EFT approach. (Idilbi, Ji, Juan; Gao, Li, Liu; SM, Petriello; Becher, Neubert)

Low pT Region

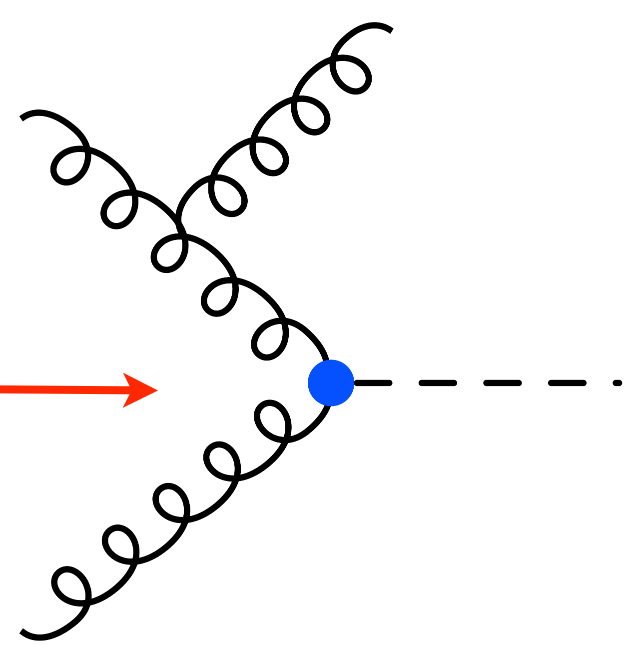
$$A(P_A) + B(P_B) \to C(Q) + X, \quad C = \gamma^*, W^\pm, Z, h$$

- The transverse momentum distribution in the CSS formalism is schematically given by:

$$\frac{d\sigma_{AB\to CX}}{dQ^2\, dy\, dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\, dy\, dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\, dy\, dQ_T^2}$$

Most singular contribution

Soft or collinear pT emission



Low pT Region

Focus of this talk

$$\frac{d\sigma_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{resum})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} + \frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

- Singular as at least Q_T^{-2} as $Q_T \to 0$
- Important in region of small Q_T.
- Treated with resummation.

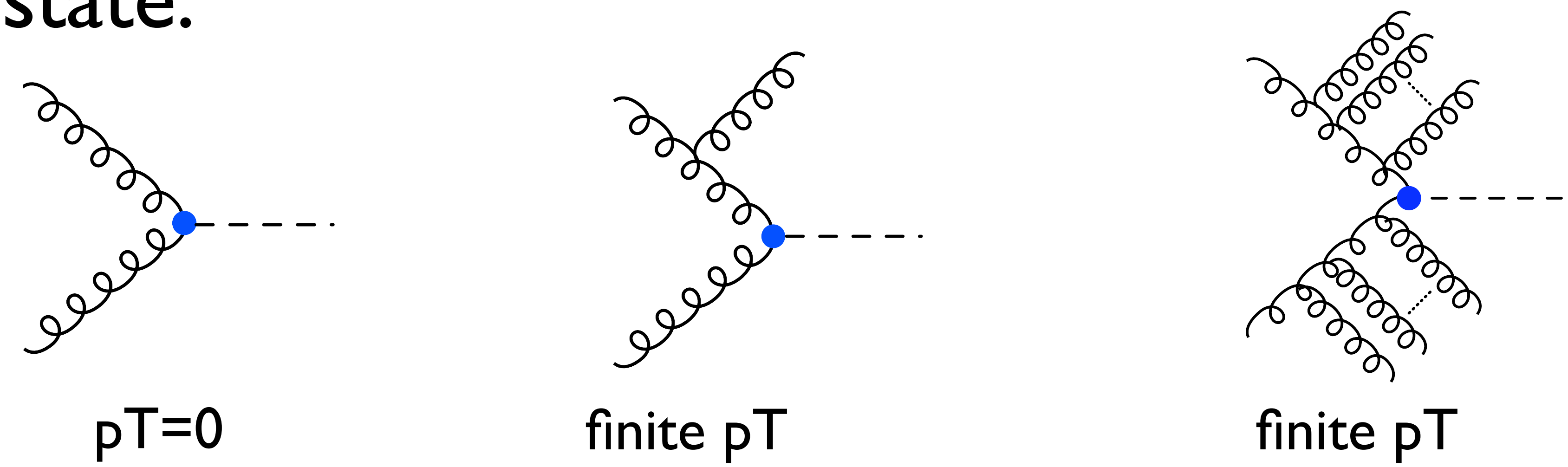
$$\frac{d\sigma^{(\mathrm{Y})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} = \frac{d\sigma^{(\mathrm{pert})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2} - \frac{d\sigma^{(\mathrm{asym})}_{AB\to CX}}{dQ^2\,dy\,dQ_T^2}$$

- Obtained from fixed order calculation.
- Less Singular terms.
- Important in region of large Q_T.

EFT Framework

EFT framework

- Low pT region dominated by soft and collinear emissions from initial state:



- Soft and Collinear emissions dominate the low pT distribution:

$$p_n \sim m_h(\eta^2, 1, \eta), \quad p_{\bar{n}} \sim m_h(1, \eta^2, \eta), \quad p_s \sim m_h(\eta, \eta, \eta),$$

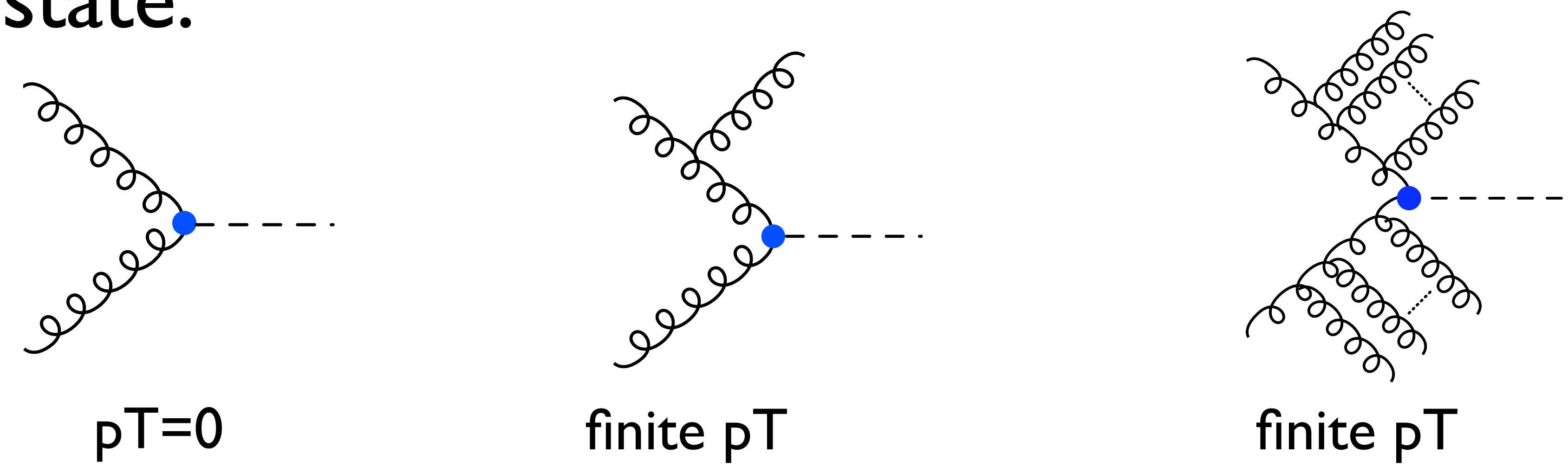
$$\eta \sim \frac{p_T}{m_h}$$

- Hierarchy of scales suggests EFT approach with well defined power counting.

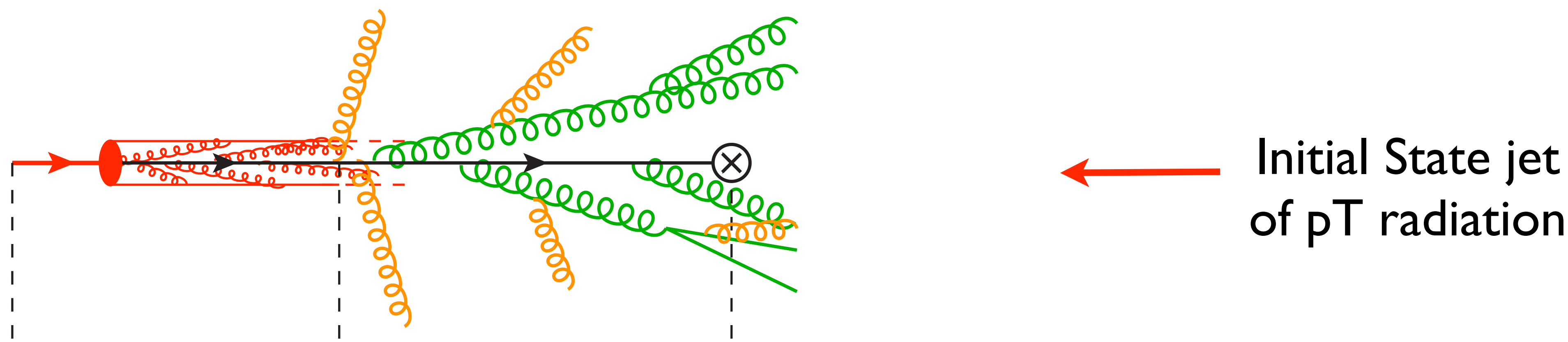
$$m_h \gg p_T \gg \Lambda_{QCD}\,, \qquad p_T \sim \Lambda_{QCD}$$

EFT framework

- Low pT region dominated by soft and collinear emissions from initial state:



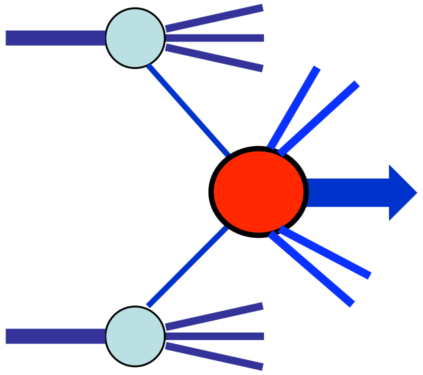
- Colliding parton is part of initial state pT radiation beam jet:



- Gives rise to impact-parameter Beam Functions (iBFs). (SM,Petriello) Analogous beam functions arise in other processes:

(Stewart, Tackmann, Waalewijin; Fleming, Leibovich, Mehen)

- Soft recoil radiation is restricted. Gives rise to a soft function.



EFT framework

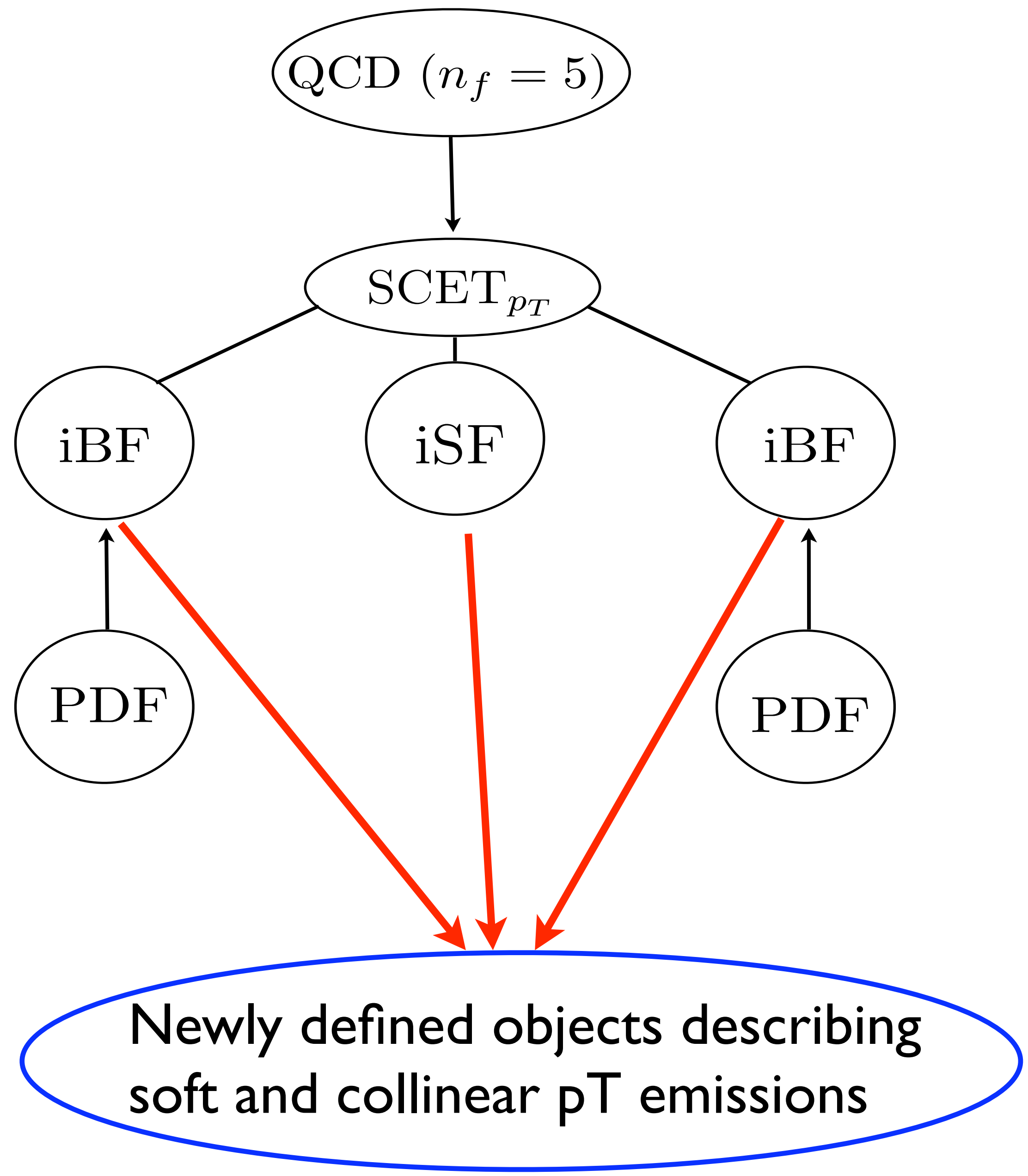
$$\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{p_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$$

Top quark integrated out.

Matched onto SCET.

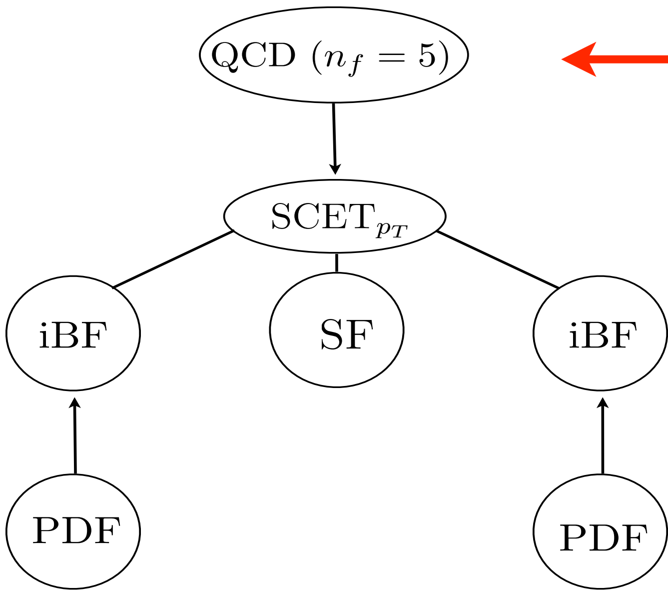
Soft-collinear factorization.

Matching onto PDFs.

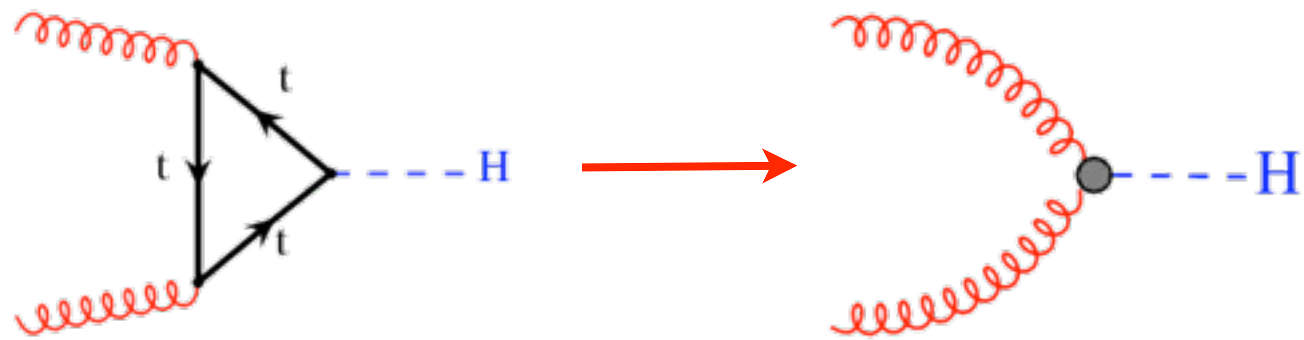


$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Integrating out the top



We are here



- Effective Higgs production operator

$$\mathcal{L}_{m_t} = C_{GGh}\,\frac{h}{v}\,G^{a}_{\mu\,\nu}\,G^{\mu\,\nu}_{a} \quad , \qquad C_{GGh} = \frac{\alpha_s}{12\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right\}$$

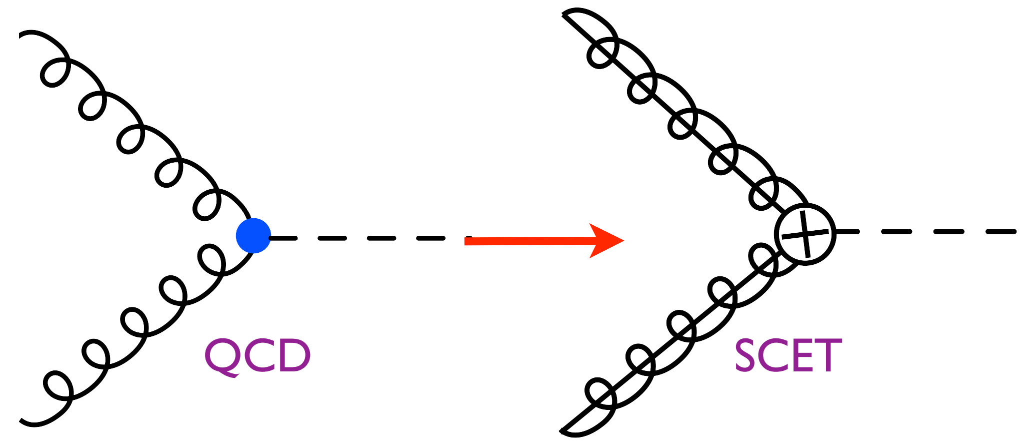
Two loop result for Wilson coefficient.

(Chetyrkin, Kniehl, Kuhn, Schroder, Steinhauser, Sturm)

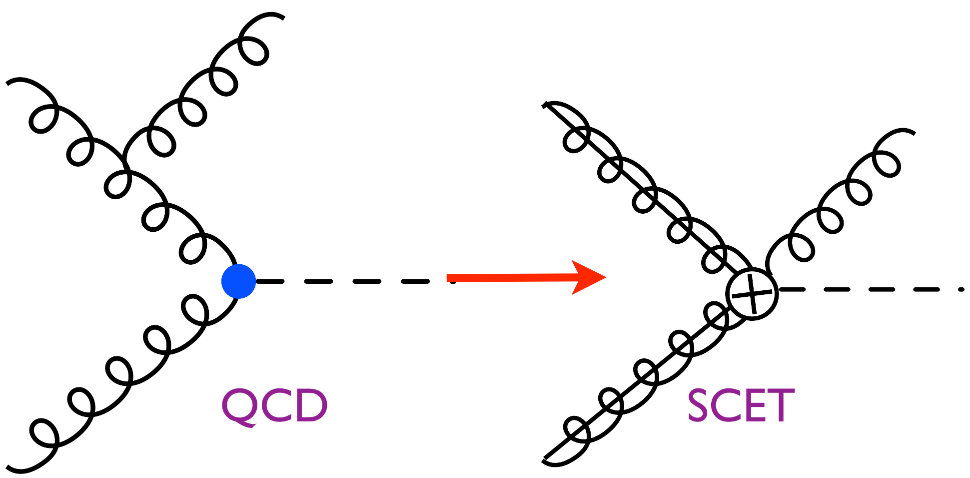
Matching onto SCET

- **Matching equation:**

$$O_{QCD} = \int d\omega_1 \int d\omega_2\, C(\omega_1, \omega_2)\, \mathcal{O}(\omega_1, \omega_2)$$

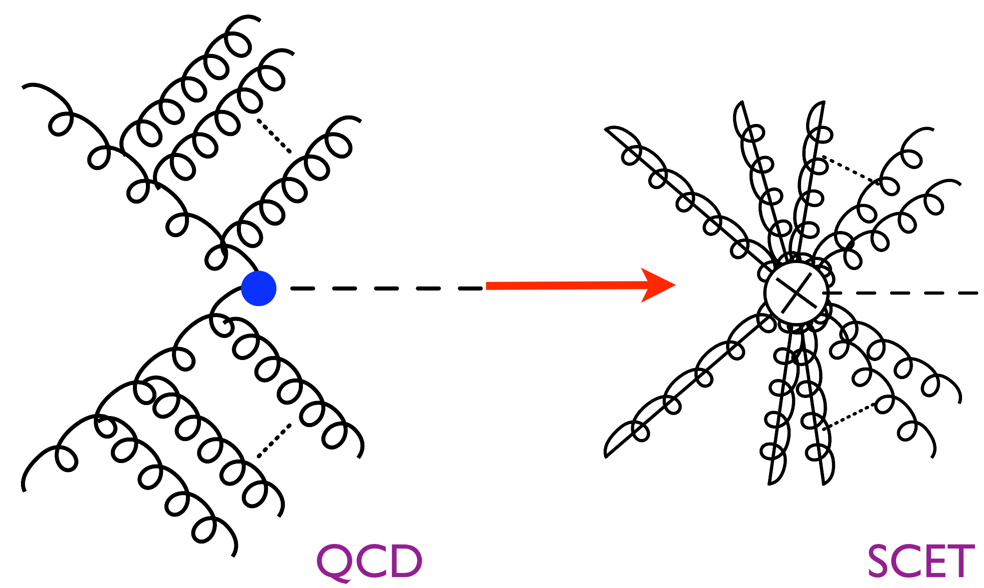


Tree level matching



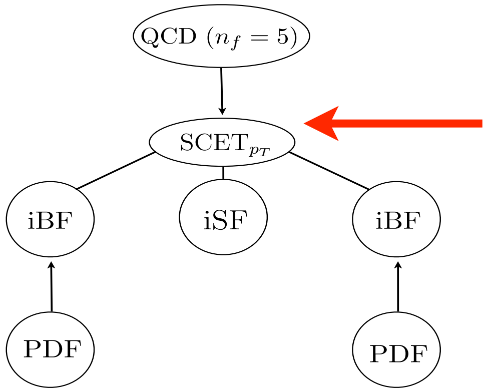
Matching real emission graphs

Soft and Collinear emissions build into Wilson lines determined by soft and collinear gauge invariance of SCET.



- **Effective SCET operator:**

$$\mathcal{O}(\omega_1, \omega_2) = g_{\mu\nu} h\, T\{\mathrm{Tr}\left[S_n (gB^{\mu}_{n\perp})_{\omega_1} S_n^{\dagger} S_{\bar{n}} (gB^{\nu}_{\bar{n}\perp})_{\omega_2} S_{\bar{n}}^{\dagger}\right]\}$$



We are here

SCET Cross-Section

- SCET differential cross-section:

$$\frac{d^2\sigma}{du\,dt} = \frac{1}{2Q^2}\left[\frac{1}{4}\right]\int \frac{d^2p_{h_\perp}}{(2\pi)^2}\int \frac{dn\cdot p_h d\bar{n}\cdot p_h}{2(2\pi)^2}(2\pi)\theta(n\cdot p_h + \bar{n}\cdot p_h)\delta(n\cdot p_h\bar{n}\cdot p_h - \vec{p}^{\,2}_{h_\perp} - m_h^2)$$
$$\times\ \delta(u-(p_2-p_h)^2)\delta(t-(p_1-p_h)^2)\sum_{\text{initial pols.}}\sum_X \left|C(\omega_1,\omega_2)\otimes\langle hX_nX_{\bar{n}}X_s|\mathcal{O}(\omega_1,\omega_2)|pp\rangle\right|^2$$
$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-P_{X_n}-P_{X_{\bar{n}}}-P_{X_s}-p_h),$$

- Schematic form of SCET cross-section:

$$\frac{d^2\sigma}{dp_T^2dY} \sim \int PS\,|C\otimes\langle\mathcal{O}\rangle|^2$$

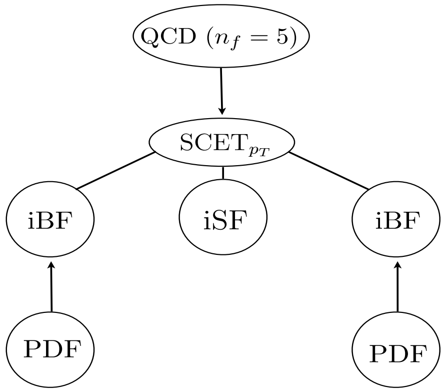
Phase space integrals.

Hard matching coefficient.

SCET matrix element.

Apply soft-collinear decoupling

SCET Cross-Section



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

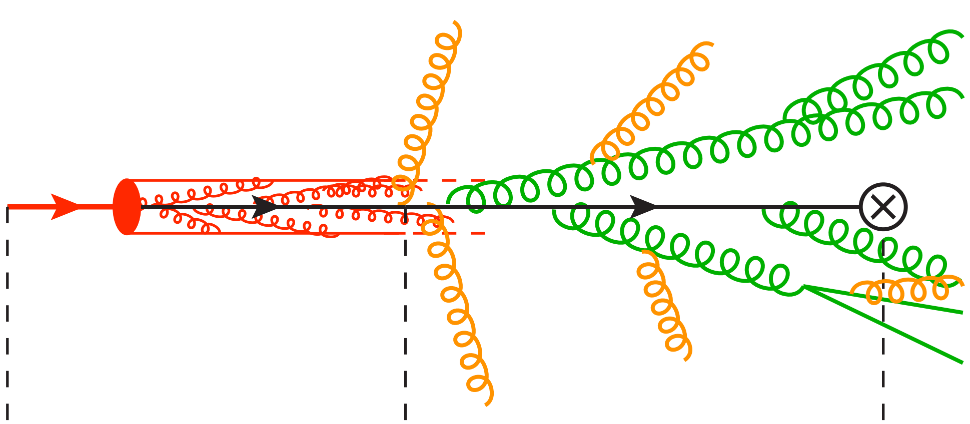
Hard function

Impact-parameter Beam Functions (iBFs)

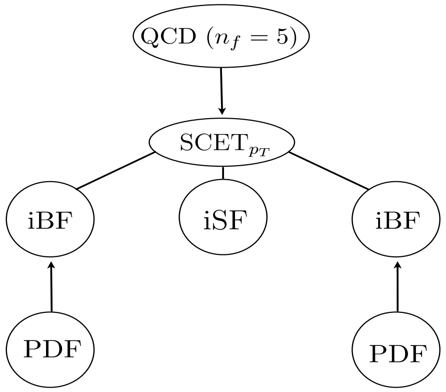
Soft function

Describes collinear pT emissions

Describes soft pT emissions



SCET Cross-Section



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Hard function

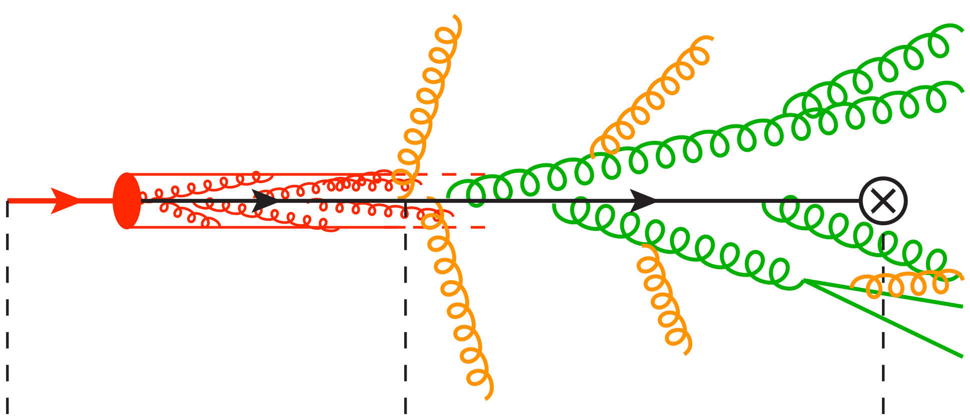
Impact-parameter Beam Functions (iBFs)

Soft function

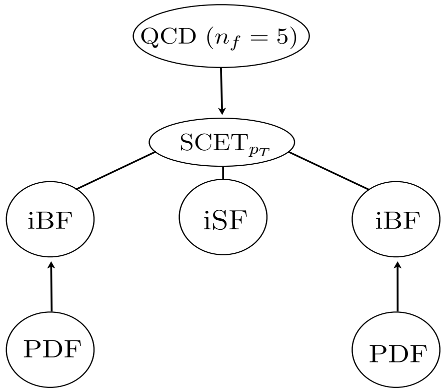
Describes collinear pT emissions

Describes soft pT emissions

Unintegrated nucleon distribution functions



SCET Cross-Section



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

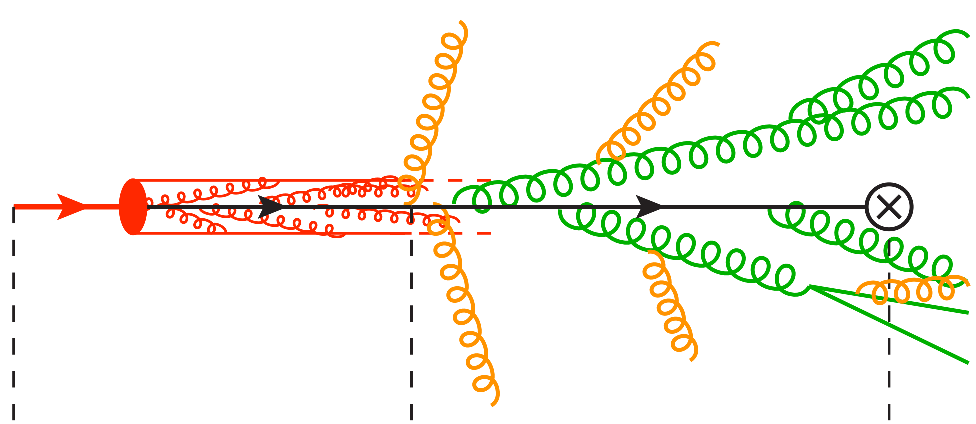
Hard function

Impact-parameter Beam Functions (iBFs)

Describes collinear pT emissions

Soft function

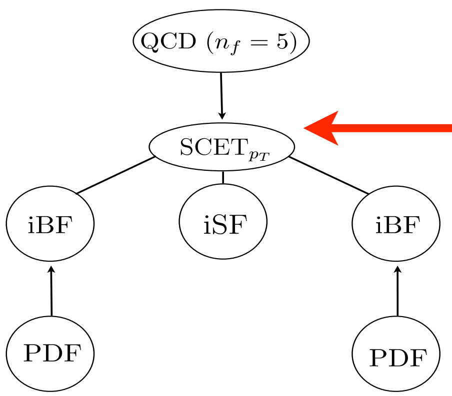
Describes soft pT emissions



Presence of soft function. Plays an important role in the structure of factorization.

(Differs from Becher, Neubert)

SCET Cross-Section



We are here

- Formula in detail:

Hard

$$\frac{d^2\sigma}{du\,dt} = \frac{(2\pi)}{(N_c^2-1)^2 8Q^2}\int dp_h^+ dp_h^- \int d^2k_h^\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{k}_h^\perp\cdot\vec{b}_\perp}$$

$$\times\ \delta\left[u-m_h^2+Qp_h^-\right]\delta\left[t-m_h^2+Qp_h^+\right]\delta\left[p_h^+p_h^- - \vec{k}_{h\perp}^2 - m_h^2\right]\int d\omega_1 d\omega_2 |C(\omega_1,\omega_2,\mu)|^2$$

$$\times \int dk_n^+ dk_{\bar{n}}^- \, B_n^{\alpha\beta}(\omega_1,k_n^+,b_\perp,\mu)\, B_{\bar{n}\alpha\beta}(\omega_2,k_{\bar{n}}^-,b_\perp,\mu)\, \mathcal{S}(\omega_1-p_h^- - k_{\bar{n}}^-, \omega_2 - p_h^+ - k_n^+, b_\perp,\mu)$$

n-collinear iBF

bn-collinear iBF

Soft

- iBFs and soft functions field theoretically defined as the fourier transform of:

$$J_n^{\alpha\beta}(\omega_1,x^-,x_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_1|\left[gB^A_{1n\perp\beta}(x^-,x_\perp)\delta(\bar{\mathcal{P}}-\omega_1)gB^A_{1n\perp\alpha}(0)\right]|p_1\rangle$$

$$J_{\bar{n}}^{\alpha\beta}(\omega_1,y^+,y_\perp,\mu) = \sum_{\text{initial pols.}} \langle p_2|\left[gB^A_{1n\perp\beta}(y^+,y_\perp)\delta(\bar{\mathcal{P}}-\omega_2)gB^A_{1n\perp\alpha}(0)\right]|p_2\rangle$$

$$S(z,\mu) = \langle 0|\bar{T}\left[\mathrm{Tr}\left(S_{\bar{n}}T^D S_{\bar{n}}^\dagger S_n T^C S_n^\dagger\right)(z)\right] T\left[\mathrm{Tr}\left(S_n T^C S_n^\dagger S_{\bar{n}} T^D S_{\bar{n}}^\dagger\right)(0)\right]|0\rangle.$$

Equivalence of Zero-Bin & Soft Subtractions

- Zero-bin iBF reproduces soft graphs. This is the equivalence of zero-bin and soft subtractions in SCET. (Stewart, Hoang; Lee, Sterman; Idilbi, Mehen; Chiu, Fuhrer, Kelly, Hoang, Manohar;...)

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes B_n \otimes B_{\bar{n}} \otimes S$$

Zero-bin Subtraction in order to avoid double counting the soft region.

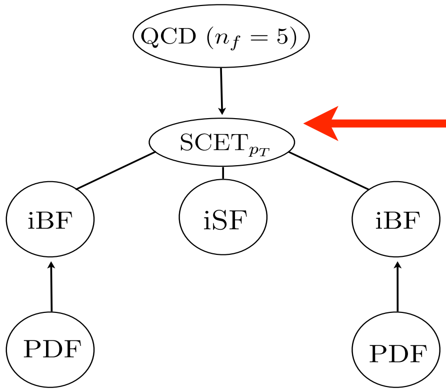
$$B^{\alpha\beta}_{n,\bar{n}}(\omega, k^{\pm}, b_{\perp}, \mu) = \tilde{B}^{\alpha\beta}_{n,\bar{n}}(\omega, k^{\pm}, b_{\perp}, \mu) - B^{\alpha\beta}_{\{n0,\bar{n}0\}}(\omega, k^{\pm}, b_{\perp}, \mu)$$

Purely Collinear iBF

"Naive" iBF

Zero-bin iBF
Equivalent to soft graphs

Factorization in SCET



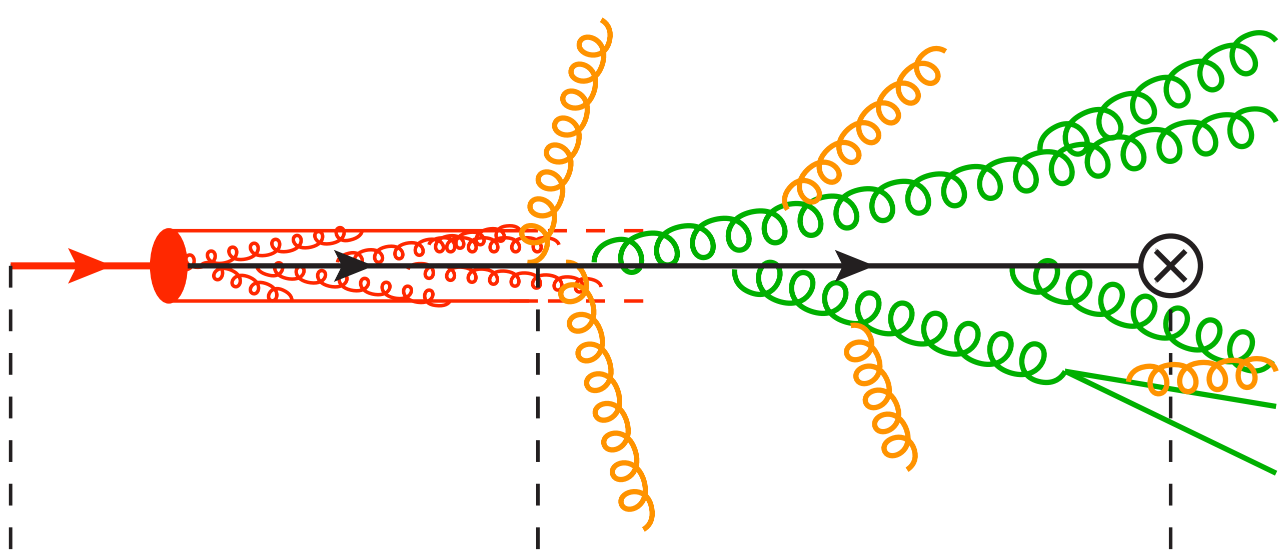
We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Hard function → Physics of hard scale. Sums logs of mh/pT.

Impact-parameter Beam Functions (iBFs) → Describes collinear pT emissions

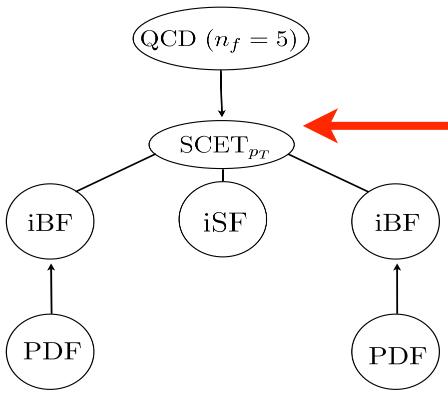
Inverse soft function (iSF) → Describes soft pT emissions



Analogous factors and soft-subtractions also appear in TMD-factorization formalism: Drell-Yan, SIDIS

(J.C.Collins, F. Hautmann; X.-d.Ji, J.P.Ma, F.Yuan; Belitsky; Aybat, Rogers,...)

Factorization in SCET



We are here

$$
\begin{aligned}
\frac{d^2\sigma}{du\,dt} &= \frac{(2\pi)}{(N_c^2-1)^2 8Q^2} \int dn\cdot p_h \int d\bar{n}\cdot p_h \int d^2k_h^\perp \int dk_n^+ d^2k_n^\perp \int dk_{\bar{n}}^- d^2k_{\bar{n}}^\perp \int d^4k_s \\
&\times \int \frac{dx^- d^2x_\perp}{2(2\pi)^3} \int \frac{dy^- d^2y_\perp}{2(2\pi)^3} \int \frac{d^4z}{(2\pi)^4} e^{\frac{i}{2}k_n^+ x^- - i\vec{k}_n^\perp\cdot x_\perp} e^{\frac{i}{2}k_{\bar{n}}^- y^+ - i\vec{k}_{\bar{n}}^\perp\cdot y_\perp} e^{ik_s\cdot z} \\
&\times \delta\left(u - m_h^2 + Q\bar{n}\cdot p_h\right) \delta\left(t - m_h^2 + Qn\cdot p_h\right) \delta\left(\bar{n}\cdot p_h n\cdot p_h - \vec{k}_{h\perp}^2 - m_h^2\right) \\
&\times \int d\omega_1 d\omega_2 |C(\omega_1,\omega_2,\mu)|^2 J_n^{\alpha\beta}(\omega_1, x^-, x_\perp, \mu)\, J_{\bar{n}\alpha\beta}(\omega_2, y^+, y_\perp, \mu)\, S(z,\mu) \\
&\times \delta\left(\omega_1 - \bar{n}\cdot p_h - k_{\bar{n}}^- - k_s^-\right) \delta(\omega_2 - p_h^+ - k_n^+ - k_s^+) \delta^{(2)}(k_s^\perp + k_n^\perp + k_{\bar{n}}^\perp + k_h^\perp),
\end{aligned}
$$

Residual light-cone momenta regulate spurious rapidity divergences.

- iBFs and iSF are regulated by kinematics of the process and free of rapidity divergences.
- iBFs are fully unintegrated nucleon distributions instead of TMD pdfs.

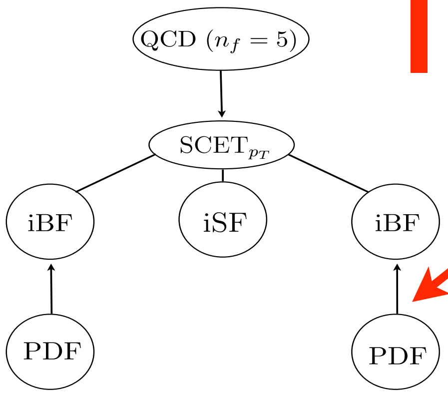
(see talks by M. Aybat, I. Cherednikov, J.C. Collins)

- In singular gauges, transverse gauge links can be added

(Garcia-Echevarria, Idilbi, Scimemi; Belitsky, Ji, Yuan)

Perturbative pT

Integrating Out the pT Scale



We are here

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

iBFs are proton matrix elements
and sensitive to the
non-perturbative scale

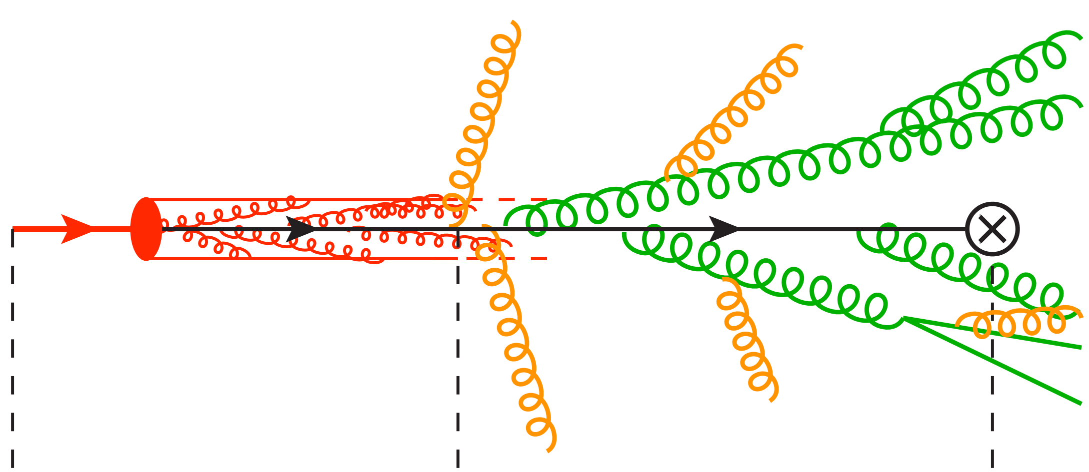
- The iBFs are matched onto PDFs to separate the perturbative and non-perturbative scales:

$$\tilde{B}_n = \mathcal{I}_{n,i} \otimes f_i,$$

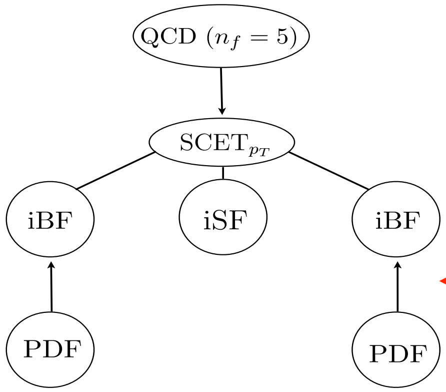
iBF

Matching coefficient

PDF



iBFs to PDFs



We are here

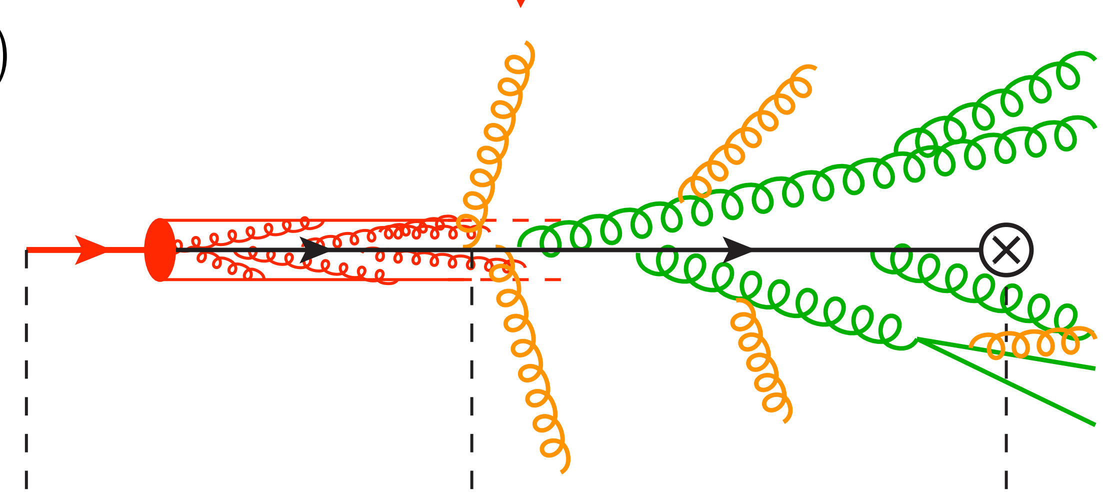
- iBF is matched onto the PDF with matching coefficient defined as:

$$\tilde{B}_n^{\alpha\beta}(z, t_n^+, b_\perp, \mu) = -\frac{1}{z} \sum_{i=g,q,\bar{q}} \int_z^1 \frac{dz'}{z'} \mathcal{I}_{n;g,i}^{\alpha\beta}(\frac{z}{z'}, t_n^+, b_\perp, \mu) f_{i/P}(z', \mu)$$

Proton fragments into pT radiation beam jet

- Tree level matching

$$\mathcal{I}_{n;g,i}^{(0)\beta\alpha}(\frac{z}{z'}, t_n^+, b_\perp, \mu) = g^2 g_\perp^{\alpha\beta} \delta(t_n^+) \delta(1 - \frac{z}{z'})$$



- Finite part of iBF in dim-reg gives matching coefficient at higher orders.

Factorization Formula

- Factorization formula in full detail:

$$\frac{d^2\sigma}{d\mathrm{p}_T^2\, dY} = \frac{\pi^2}{4(N_c^2-1)^2 Q^2} \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \int_{x_1}^1 \frac{dx_1'}{x_1'} \int_{x_2}^1 \frac{dx_2'}{x_2'}$$
$$\times\; H(x_1, x_2, \mu_Q; \mu_T)\mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) f_{i/P}(x_1', \mu_T) f_{j/P}(x_2', \mu_T)$$

Hard function. (H) — Transverse momentum function. ($\mathcal{G}^{ij}$) — PDFs. ($f_{i/P}$, $f_{j/P}$)

- The transverse momentum function is a convolution of the iBF matching coefficients and the soft function:

$$\mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) = \int dt_n^+ \int dt_{\bar{n}}^- \int \frac{d^2 b_\perp}{(2\pi)^2} J_0(|\vec{b}_\perp| p_T)$$

Collinear pT emissions →

$$\times\; \mathcal{I}_{n;g,i}^{\beta\alpha}\Big(\frac{x_1}{x_1'}, t_n^+, b_\perp, \mu_T\Big)\, \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}\Big(\frac{x_2}{x_2'}, t_{\bar{n}}^-, b_\perp, \mu_T\Big)$$

Soft pT emissions →

$$\times\; \mathcal{S}^{-1}\Big(x_1 Q - e^{Y}\sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y}\sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_n^+}{Q}, b_\perp, \mu_T\Big)$$

Factorization Formula

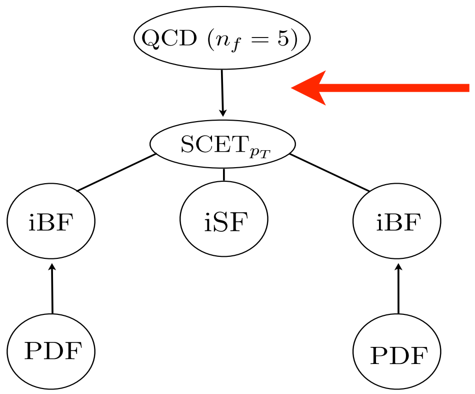
$$\frac{d^2\sigma}{d\mathrm{p}_T^2\, dY} = \frac{\pi^2}{4(N_c^2-1)^2 Q^2} \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \int_{x_1}^1 \frac{dx_1'}{x_1'} \int_{x_2}^1 \frac{dx_2'}{x_2'}$$
$$\times\; H(x_1, x_2, \mu_Q; \mu_T) \mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) f_{i/P}(x_1', \mu_T) f_{j/P}(x_2', \mu_T)$$

- One can express the formula entirely in momentum space:

$$\mathcal{G}^{ij}(x_1, x_1', x_2, x_2', p_T, Y, \mu_T) = \frac{1}{2\pi} \int dt_n^+ \int dt_{\bar{n}}^- \int d^2k_n^\perp \int d^2k_{\bar{n}}^\perp \int d^2k_s^\perp \frac{\delta(p_T - |\vec{k}_n^\perp + \vec{k}_{\bar{n}}^\perp + \vec{k}_s^\perp|)}{p_T}$$
$$\times\; \mathcal{I}_{n;g,i}^{\beta\alpha}(\frac{x_1}{x_1'}, t_n^+, k_n^\perp, \mu_T)\, \mathcal{I}_{\bar{n};g,j}^{\beta\alpha}(\frac{x_2}{x_2'}, t_{\bar{n}}^-, k_{\bar{n}}^\perp, \mu_T)$$
$$\times\; \mathcal{S}^{-1}(x_1 Q - e^Y \sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_{\bar{n}}^-}{Q}, x_2 Q - e^{-Y}\sqrt{\mathrm{p}_T^2 + m_h^2} - \frac{t_n^+}{Q}, k_s^\perp, \mu_T)$$

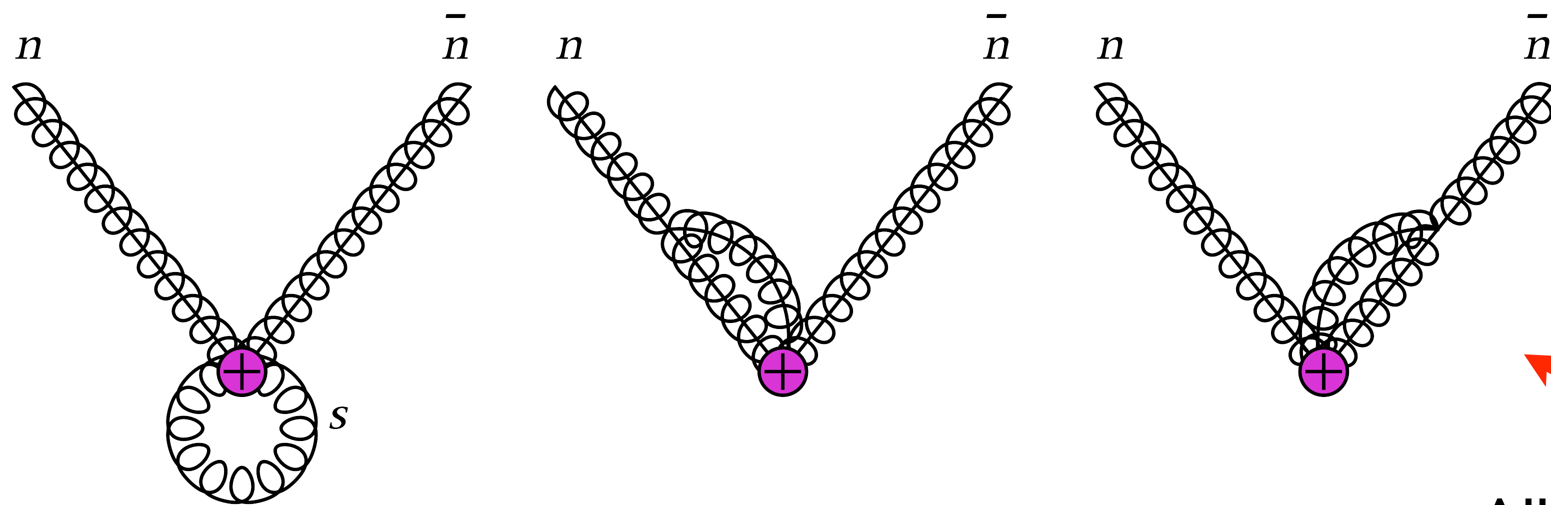
Fixed order and Matching Calculations

One loop Matching onto SCET



We are here

$$O_{QCD} = \int d\omega_1 \int d\omega_2 \, C(\omega_1, \omega_2) \, \mathcal{O}(\omega_1, \omega_2)$$



One loop SCET graphs

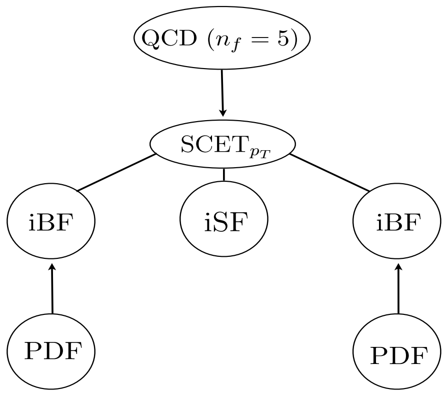
All graphs scaless and vanish in dimensional regularization.

- Wilson Coefficient obtained from finite part in dimensional regularization of the QCD result for gg->h. At one loop we have:

$$C(\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2, \mu) = \frac{c\,\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{v}\left\{1 + \frac{\alpha_s}{4\pi}C_A\left[\frac{11}{2} + \frac{\pi^2}{6} - \ln^2\left(-\frac{\bar{n}\cdot\hat{p}_1 n\cdot\hat{p}_2}{\mu^2}\right)\right]\right\}$$

(Ahrens, Becher, Neubert, Yang; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, Van Neerven)

iBFs

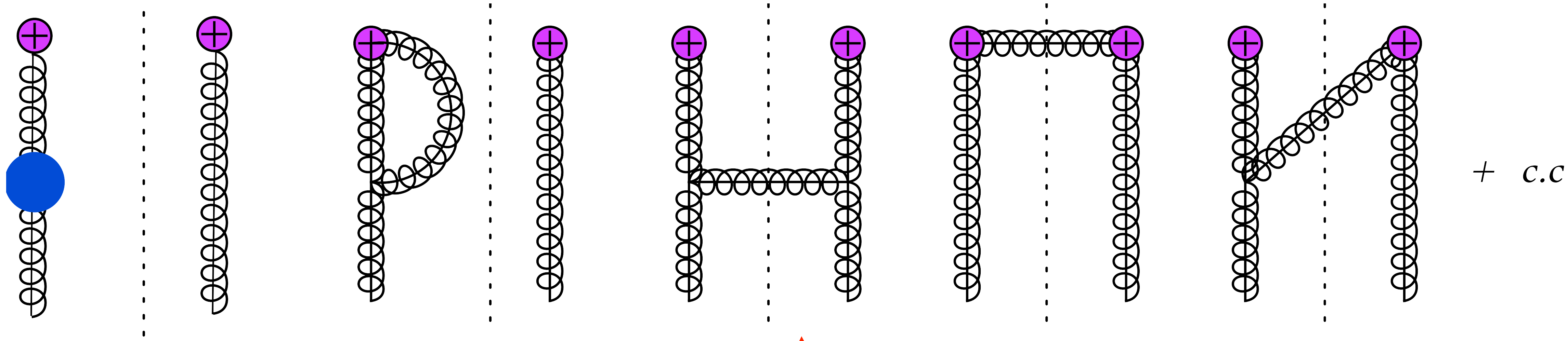


We are here

- Definition of the iBF:

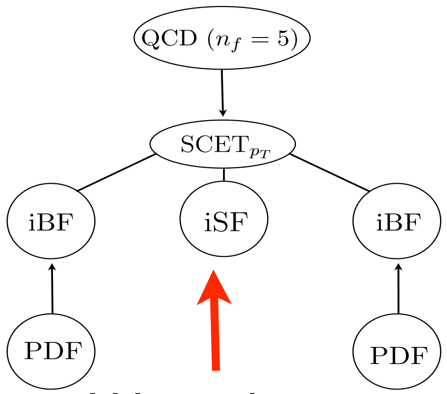
$$\tilde{B}_n^{\alpha\beta}(x_1, t_n^+, b_\perp, \mu) = \int \frac{db^-}{4\pi} e^{\frac{i}{2}\frac{t_n^+ b^-}{Q}} \sum_{\text{initial pols.}} \sum_{X_n} \langle p_1 | \left[g B^A_{1n\perp\beta}(b^-, b_\perp) | X_n \rangle \right.$$

$$\times \left. \langle X_n | \delta(\bar{\mathcal{P}} - x_1 \bar{n} \cdot p_1) g B^A_{1n\perp\alpha}(0) \right] | p_1 \rangle,$$



One loop graphs

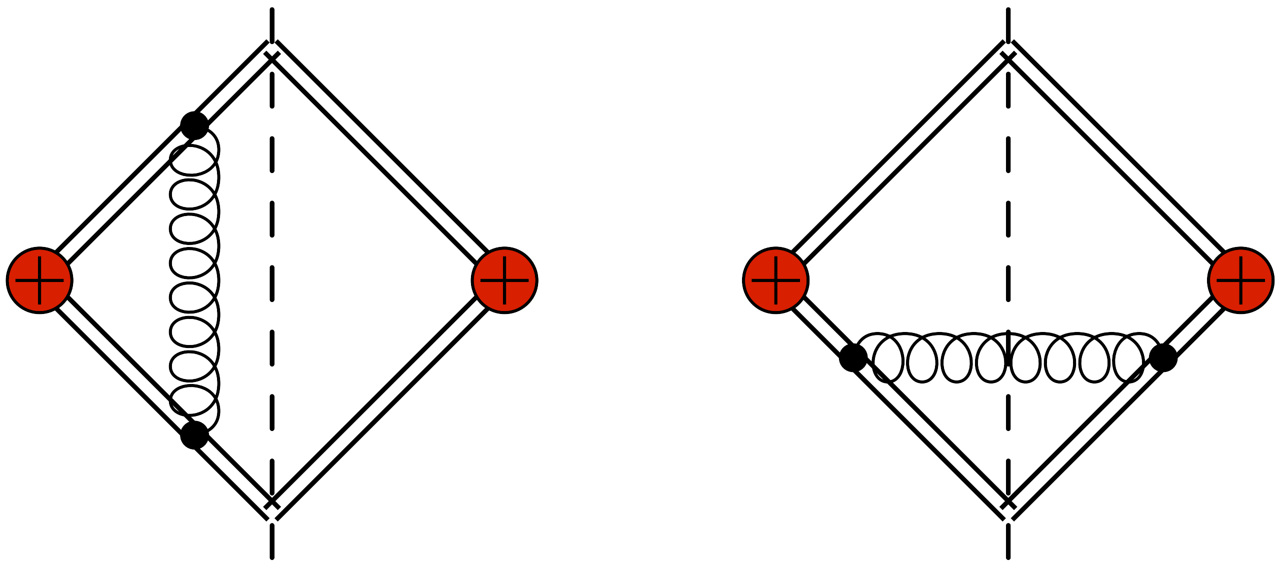
Soft function



We are here

- Soft function definition:

$$S(z) = \langle 0|\mathrm{Tr}\Big(\bar{T}\{S_{\bar{n}}T^D S^\dagger_{\bar{n}} S_n T^C S^\dagger_n\}\Big)(z)\mathrm{Tr}\Big(T\{S_n T^C S^\dagger_n S_{\bar{n}} T^D S^\dagger_{\bar{n}}\}\Big)(0)|0\rangle$$



One loop graphs

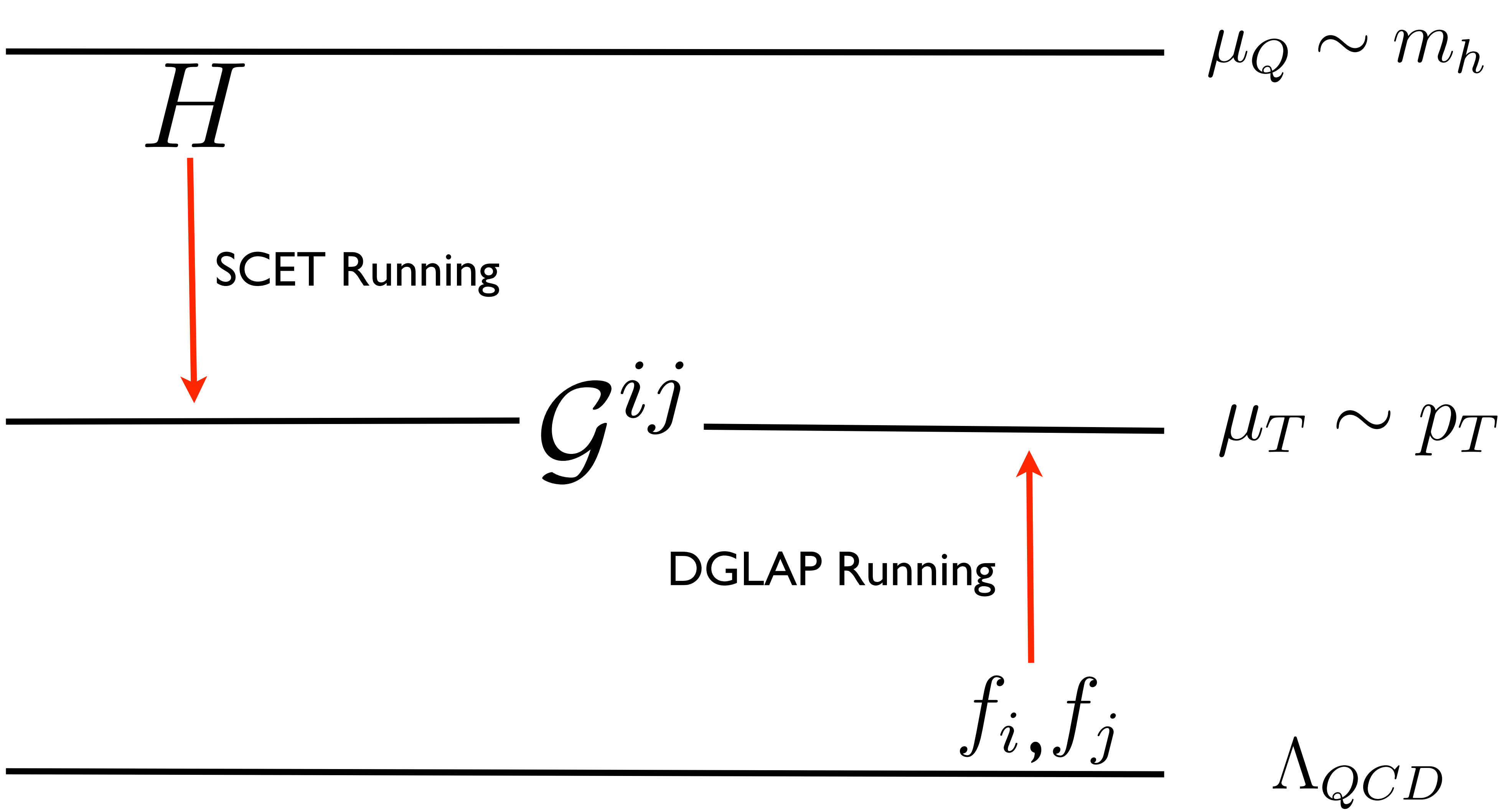
Running

Running

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Schematic picture of running:



Running

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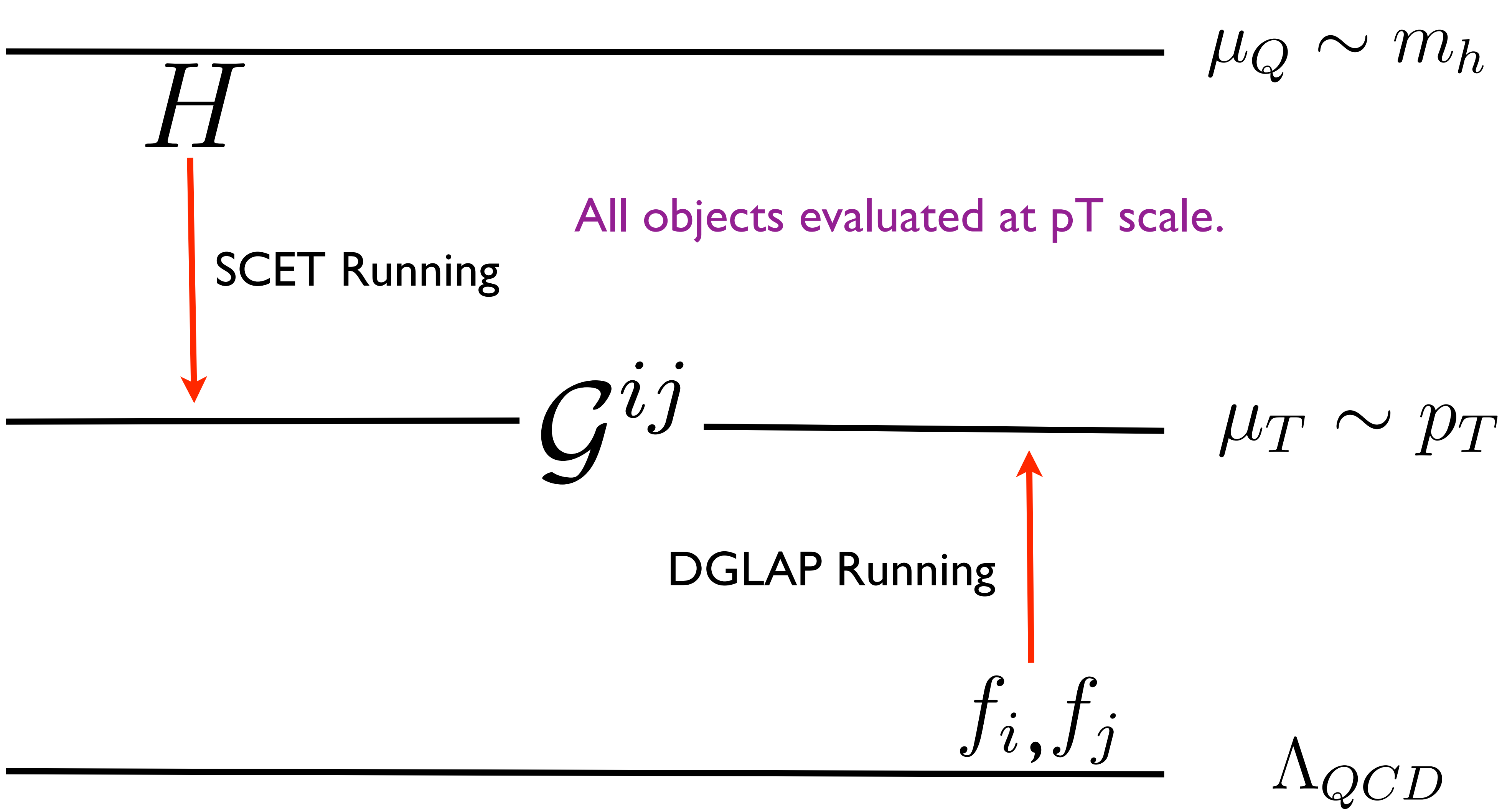
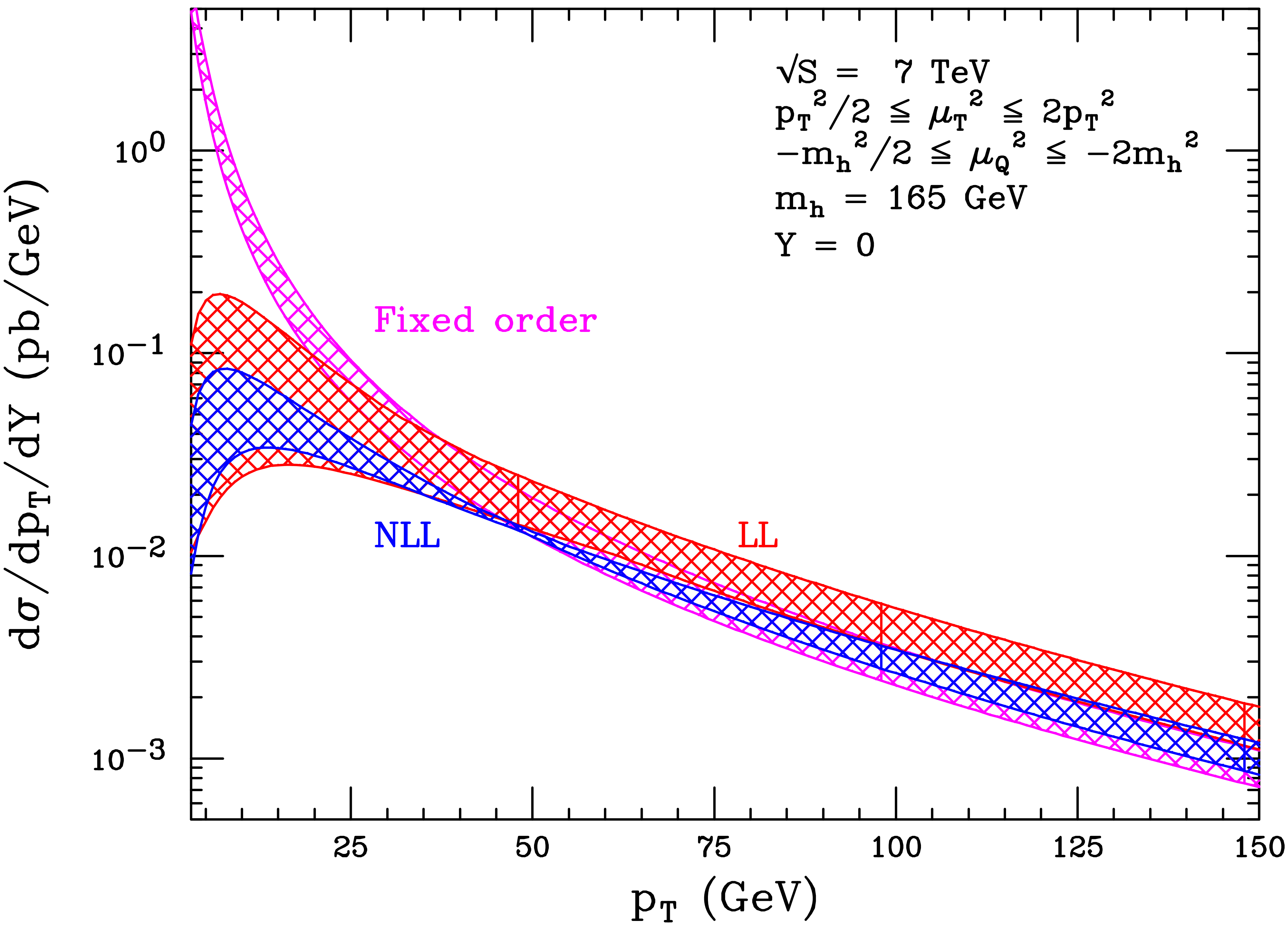


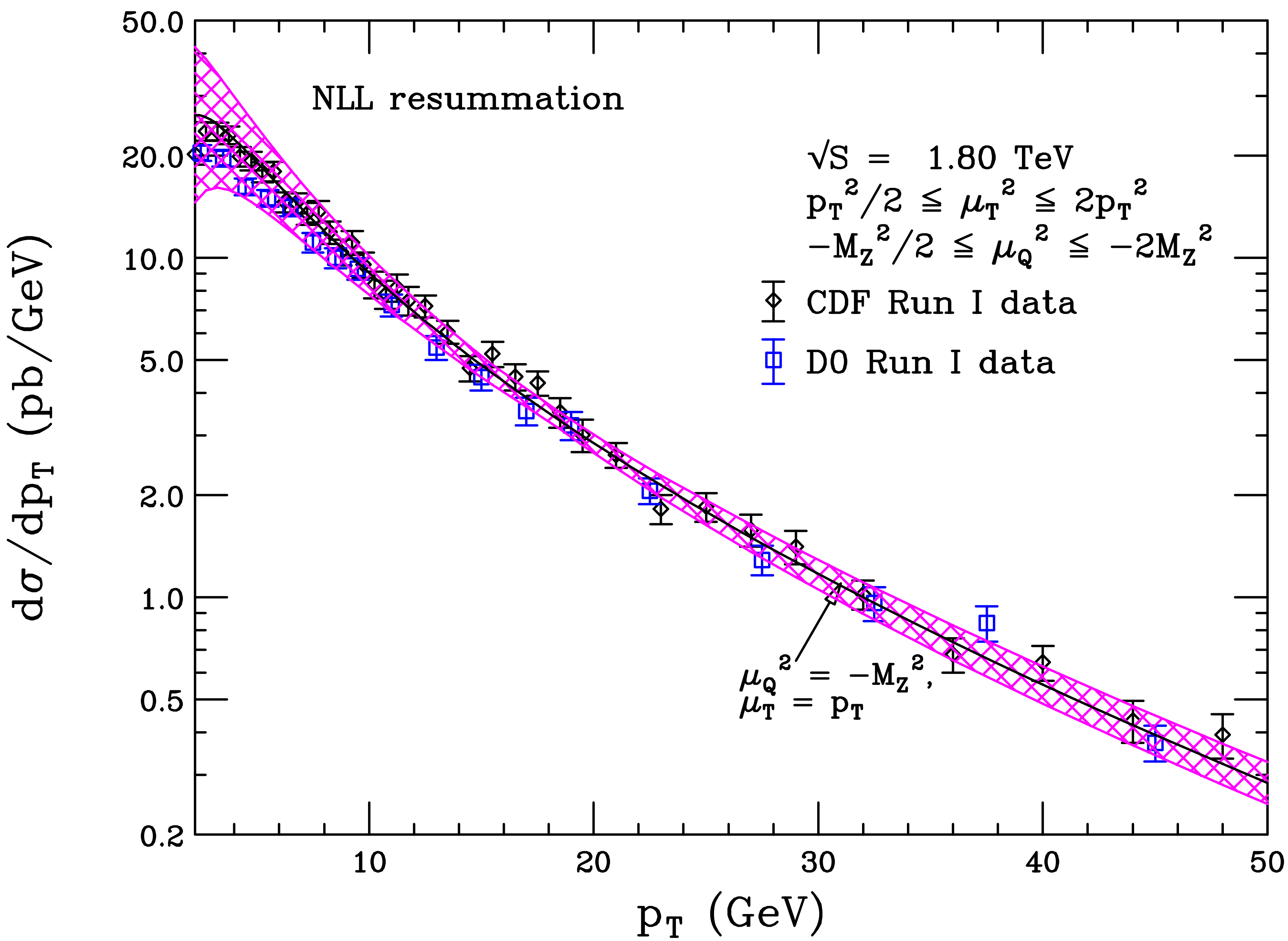
Numerical Results

Higgs pT Distribution



- Prediction for Higgs boson pT distribution.

Z-production: Comparison with Data

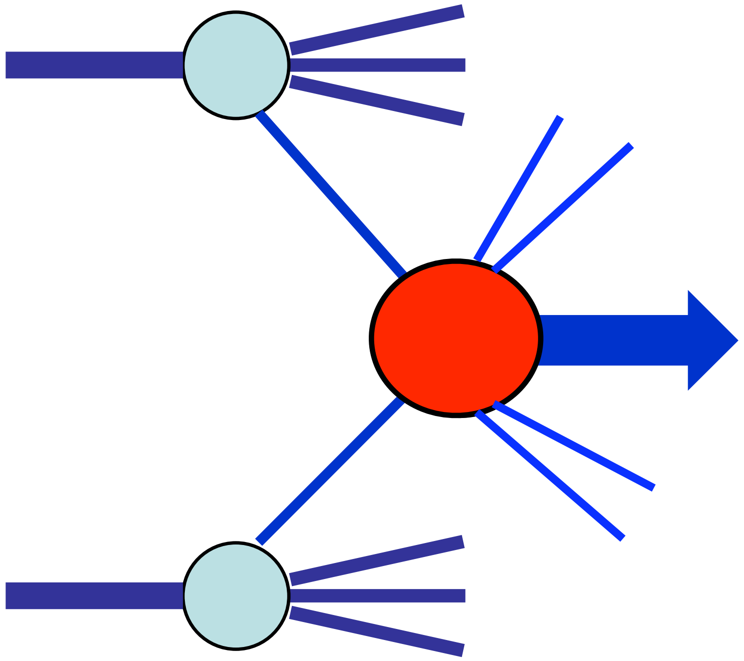


- Good agreement with data.
- Theory curve determined completely by perturbative functions and standard PDFs.

Non-Perturbative pT Region

Non-Perturbative pT Region

- Non-perturbative region of pT:



$$p_T \sim \Lambda_{QCD}$$

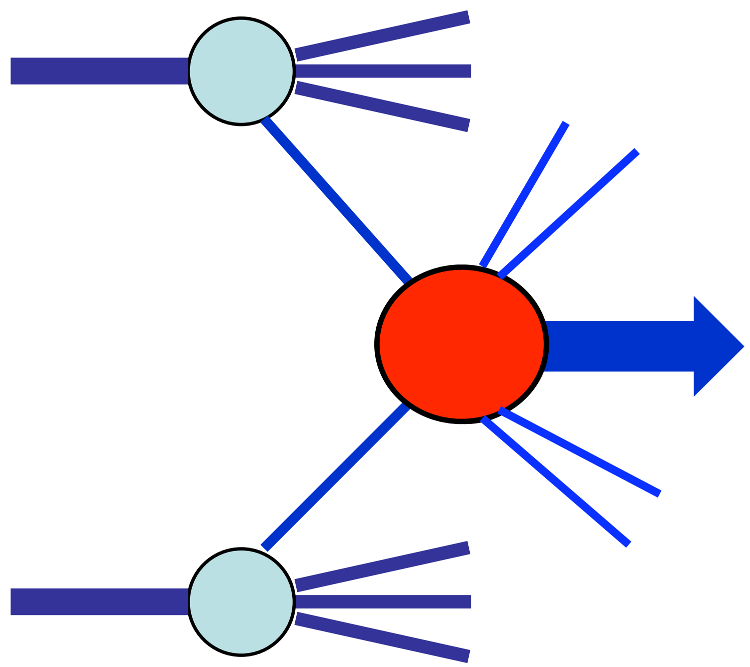
Distribution sensitive to transverse momentum dynamics in nucleon

- iBFs and iSF are non-perturbative:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Non-Perturbative pT Region

- Non-perturbative region of pT:



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Distribution sensitive to transverse momentum dynamics in nucleon

- iBFs and iSF are non-perturbative:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$

Unintegrated nucleon distribution amplitudes (iBFs)

+

Inverse Soft function (iSF)

- Soft factor can be absorbed into iBFs.

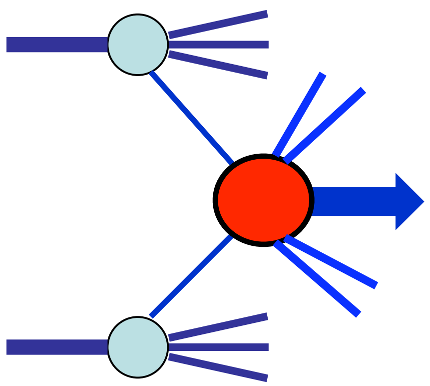
Plays an important role in TMD formalism.

(See talk by M. Aybat, I. Cherednikov, J.C.Collins)

Non-Perturbative pT Region

- Non-perturbative region of pT:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \tilde{B}_n \otimes \tilde{B}_{\bar{n}} \otimes S^{-1}$$



- In order to smoothly connect non-perturbative and perturbative regions, we still write

$$\tilde{B}_n = \mathcal{I}_{n,i} \otimes f_i\,, \qquad \tilde{B}_{\bar{n}} = \mathcal{I}_{\bar{n},j} \otimes f_j$$

$\mathcal{I}_{n,i}$ → non-perturbative

$\mathcal{I}_{\bar{n},j}$ → non-perturbative

Non-Perturbative pT Region

- Transverse momentum function (TMF) is now non-perturbative

$$\frac{d^2\sigma}{dp_T^2\, dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

Hard function.

Transverse momentum function (TMF).

PDFs.

Can make non-perturbative model

Field theoretically defined object

Scale dependence and running known

Model for Non-Perturbative TMF

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

$$\mathcal{G}^{qrs}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T) = \int_0^\infty dp_T' \, \mathcal{G}^{qrs}_{\text{part.}}(x_1, x_2, x_1', x_2', p_T\sqrt{1+(p_T'/p_T)^2}, Y, \mu_T) \times G_{mod}(p_T', a, b, \Lambda),$$

Model function

Partonic function

(Hoang, Ligeti, Stewart, Tackmann)

- Model function:

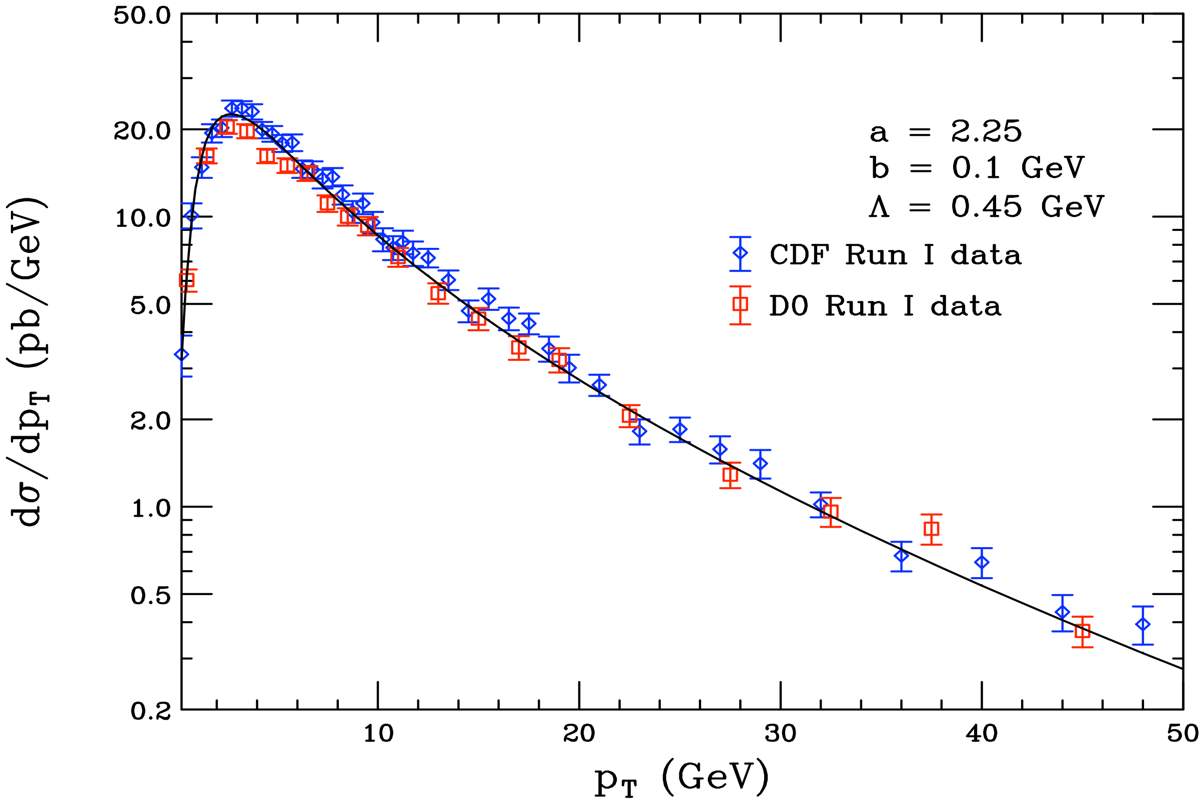
$$G_{mod}(p_T', a, b, \Lambda) = \frac{N}{\Lambda^2}\left(\frac{p_T'^2}{\Lambda^2}\right)^{a-1} \exp\left[-\frac{(p_T'-b)^2}{2\Lambda^2}\right], \qquad \int_0^\infty dp_T' \, G_{mod}(p_T', a, b, \Lambda) = 1.$$

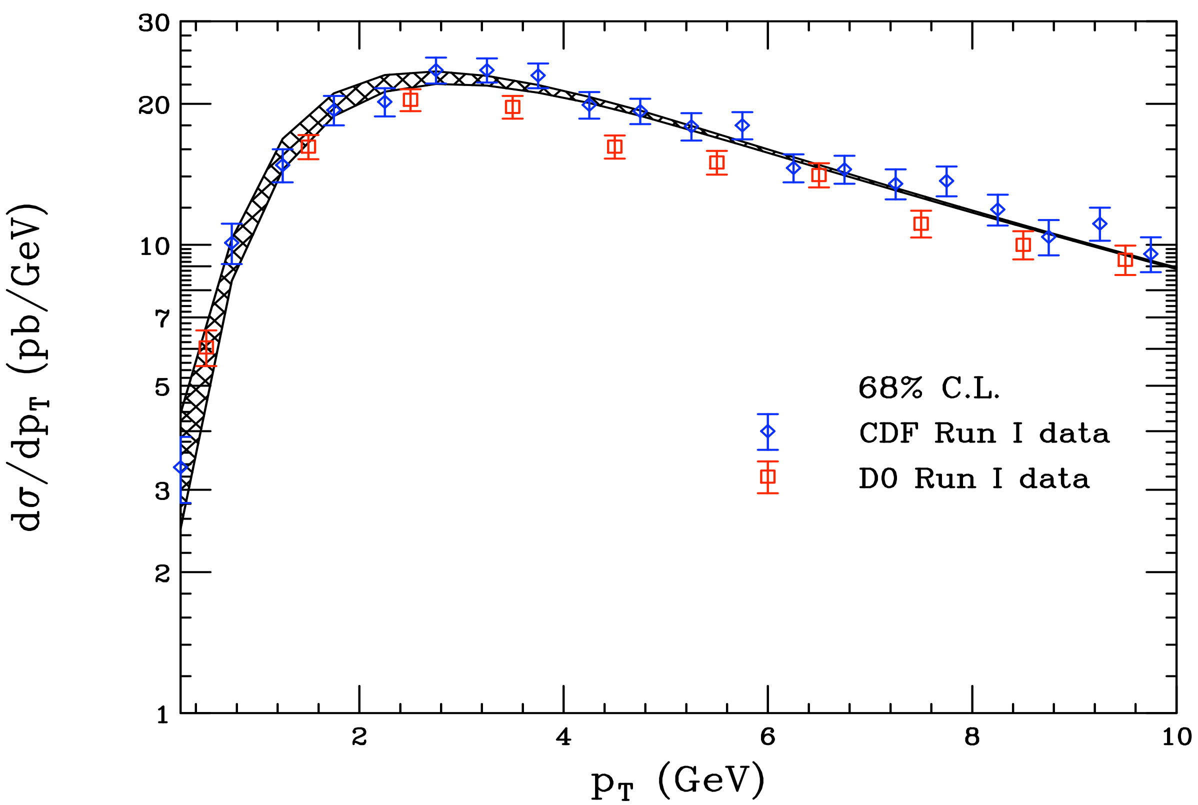
- Model reduces to the perturbative result for large pT:

$$\mathcal{G}^{qrs}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T)\Big|_{p_T \gg \Lambda_{QCD}} = \mathcal{G}^{qrs}_{\text{part.}}(x_1, x_2, x_1', x_2', p_T, Y, \mu_T) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{p_T}\right).$$

- Similar to analysis done in CSS with “bmax”.

Including the Non-Perturbative Region





- pT spectrum including the non-perturbative region

- Model dependence restricted only to non-perturbative region as expected.

Summary

- Factorization formula:

$$\frac{d^2\sigma}{dp_T^2 dY} \sim H \otimes \mathcal{G}^{ij} \otimes f_i \otimes f_j$$

- Perturbative pT distribution given in terms of perturbatively calculable functions and the standard PDFs.

- Non-perturbative pT region determined by unintegrated nucleon distributions (iBFs) and inverse soft function (iSF).

- Smooth transition for spectrum from non-perturbative pT to perturbative pT and large pT.

- Performed NLL resummation and found good agreement with data.

Backup Slides

$$\frac{d^2\sigma_{Z,q\bar{q}}}{dp_T^2 dY} = \frac{4\pi^2}{3}\frac{\alpha}{\sin^2\theta_W} e_{q\bar{q}}^2 \frac{1}{s\, p_T^2} \sum_{m,n} \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^n {}_nD_m \ln^m \frac{M_Z^2}{p_T^2}$$

$$\begin{aligned}
\text{leading logarithmic} &: \alpha_s^n L^{2n-1}, \\
\text{next-to-leading logarithmic} &: \alpha_s^n L^{2n-2}, \\
\text{next-to-next-to-leading logarithmic} &: \alpha_s^n L^{2n-3}.
\end{aligned}$$

$$\begin{aligned}
{}_1D_1 &= A^{(1)} f_A f_B, \\
{}_1D_0 &= B^{(1)} f_A f_B + f_B \left(P_{qq}\otimes f\right)_A + f_A \left(P_{qq}\otimes f\right)_B, \\
{}_2D_3 &= -\frac{1}{2}\left[A^{(1)}\right]^2 f_A f_B, \\
{}_2D_2 &= -\frac{3}{2}A^{(1)}\left[f_B \left(P_{qq}\otimes f\right)_A + f_A \left(P_{qq}\otimes f\right)_B\right] - \left[\frac{3}{2}A^{(1)}B^{(1)} - \beta_0 A^{(1)}\right] f_A f_B, \\
{}_2D_1 &= \Big\{ -A^{(1)} f_B \left(P_{qq}\otimes f\right)_A \ln\frac{\mu_F^2}{M_Z^2} - 2B^{(1)} f_B \left(P_{qq}\otimes f\right)_A - \frac{1}{2}\left[B^{(1)}\right]^2 f_A f_B \\
&\quad + \frac{\beta_0}{2}A^{(1)} f_A f_B \ln\frac{\mu_R^2}{M_Z^2} + \frac{\beta_0}{2}B^{(1)} f_A f_B - \left(P_{qq}\otimes f\right)_A \left(P_{qq}\otimes f\right)_B \\
&\quad - f_B \left(P_{qq}\otimes P_{qq}\otimes f\right)_A + \beta_0 f_B \left(P_{qq}\otimes f\right)_A \Big\} + \left[A \leftrightarrow B\right].
\end{aligned}$$

$$
\begin{aligned}
\frac{d^2\sigma}{du\,dt} \;=\; & \sum_{qijKL} \frac{\pi F^{KL;q}}{4Q^4 N_c^2} \int d^2k_\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \delta\Big[\omega_u\omega_t - \vec{k}_\perp^2 - M_z^2\Big]\, H_Z^{KL;ijq}(\omega_u,\omega_t,\mu_Q;\mu_T) \\
& \times\; J_n^q(\omega_u,0,b_\perp,\mu_T) J_{\bar{n}}^{\bar{q}}(\omega_t,0,b_\perp,\mu_T) S_{qq}(0,0,b_\perp,\mu_T)
\end{aligned}
$$