

# *PQCD Analysis of Parton-Hadron Duality*

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Jefferson Lab  
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`` QCD nowadays has a split personality. It embodies 'hard' and 'soft' physics, both being hard subjects and the softer the harder."

Y. Dokshitzer

# Outline

- Introduction
- Review DGLAP evolution at large Bjorken (z-dependent scale)
- Describe interplay of TMCs, Large Bjorken x evolution, HigherTwists
- Extract  $\alpha_s \rightarrow$  exploring interplay between perturbative and non perturbative effects (AdS/CFT ideas ?)  
in collaboration with J.P Chen and A. Deur
- Briefly mention nuclear corrections at large Bjorken x
- Conclusions/Outlook for the 12 GeV program and beyond...

# Parton-Hadron Duality $\rightarrow$ an Important Element in Large $x_{Bj}$ Studies

## Practical Aspects:

- ✓ Precise determination of PDFs at large  $x_{Bj}$  needed to extend the domain of validity of PDF global analyses (importance of large  $x$  gluons, ...)  
(Jlab + CTEQ studies, PRL 2011)
- ✓ Tests of QCD predictions at  $x_{Bj}=1$   
(Ratio  $F_2^n/F_2^p$ , W. Melnitchouk et al....)

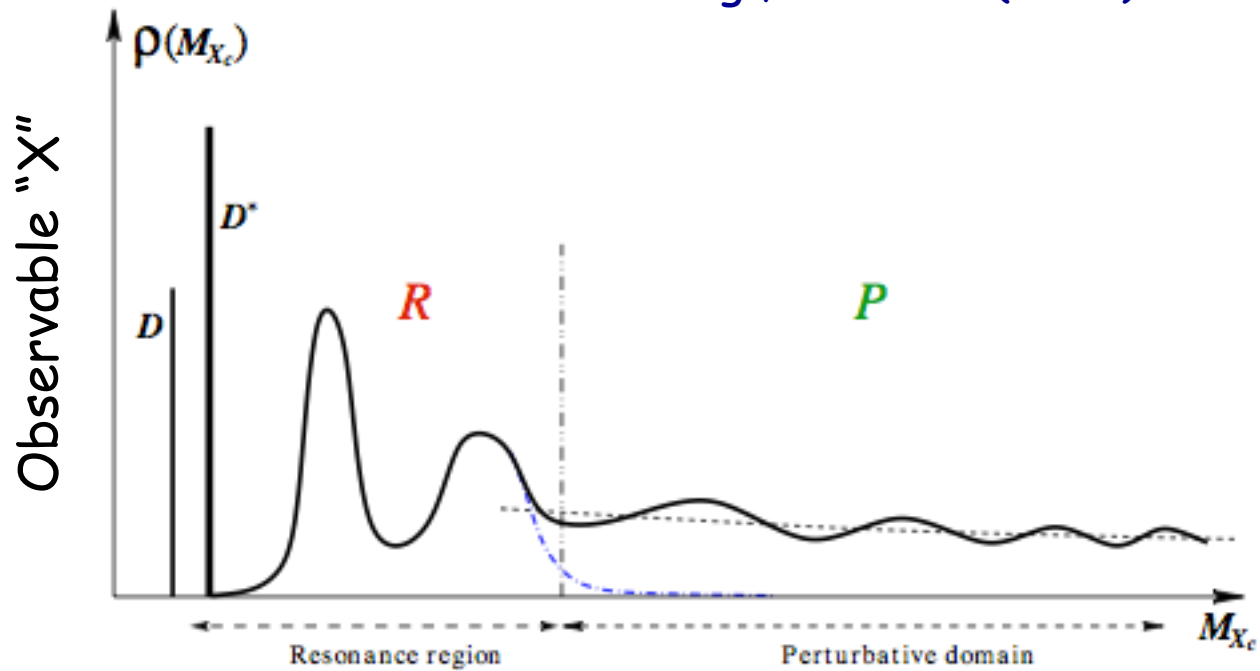
## Theoretical Aspects:

- ✓ Monitoring the transition in QCD between the “perturbative” region, where factorization applies to the “non-perturbative” region: interpretation within factorization theorems in QCD?  
(J. Collins)
- ✓ Possibility of extracting  $\alpha_s$  at low scale  
(GDH sum rule analyses by J.P. Chen, A. Deur)
- ✓ Understanding the mechanism of hadron formation

# In a nutshell

*A Vademecum on Quark-Hadron Duality* 5219

Bigi, Uraltsev (2001)



FS Invariant Mass

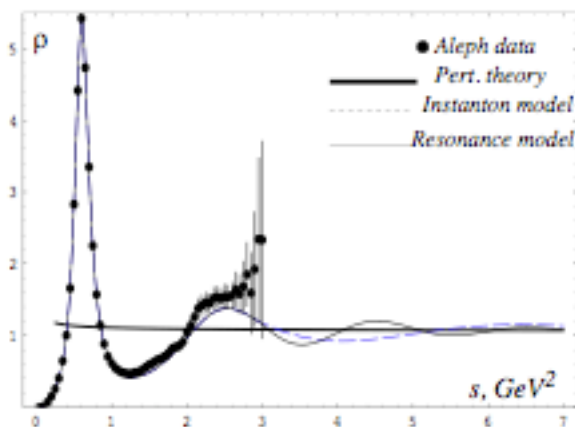
# Duality is Ubiquitous

(from A. Fantoni's talks)

## Data (1)

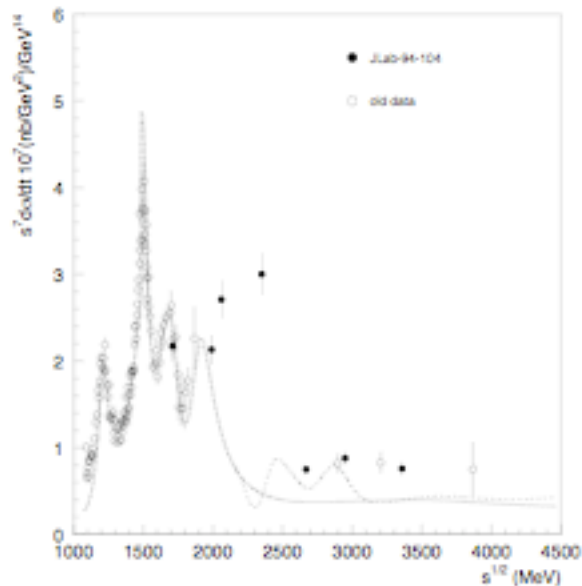
$$\tau \rightarrow \nu + \text{hadrons}$$

M. Shifman, hep-th/0009131



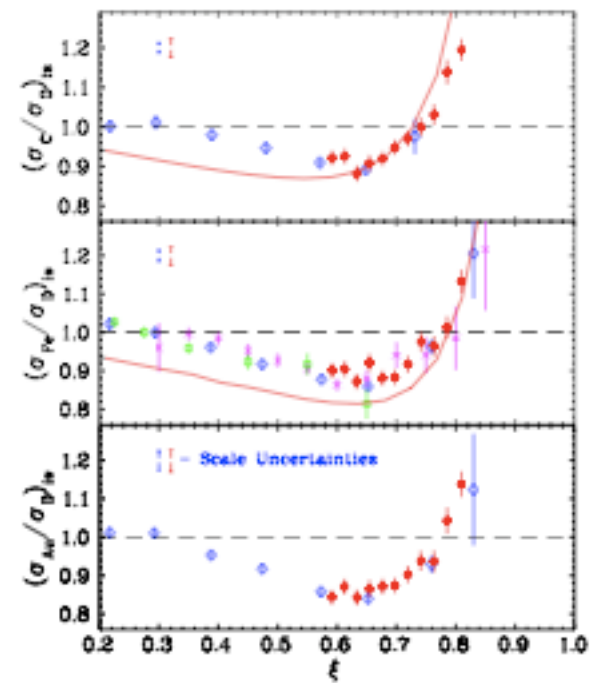
$$\gamma p \rightarrow \pi^+ n$$

L.Y. Zhu *et al.*, PRL 91 (2003) 022003,  
L.Y. Zhu *et al.*, PRC 71 (2005) 044603



$$eA \rightarrow eX$$

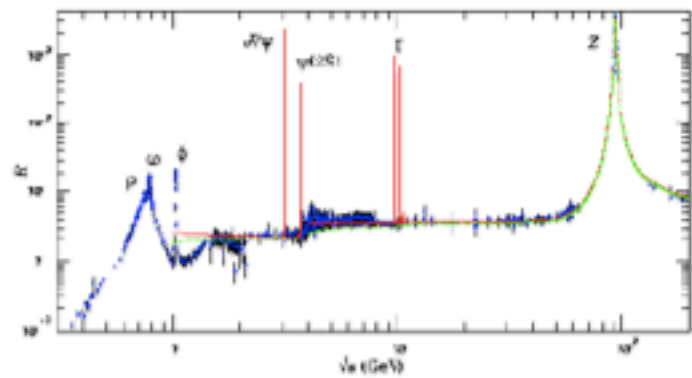
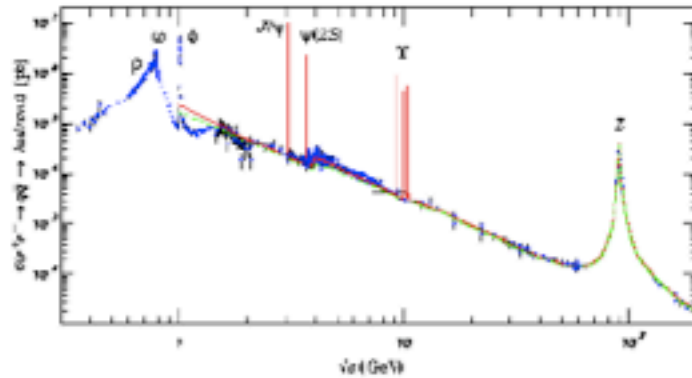
J. Arrington *et al.* (submitted)



## Data (2)

$$e^+ - e^- \rightarrow \text{hadrons}$$

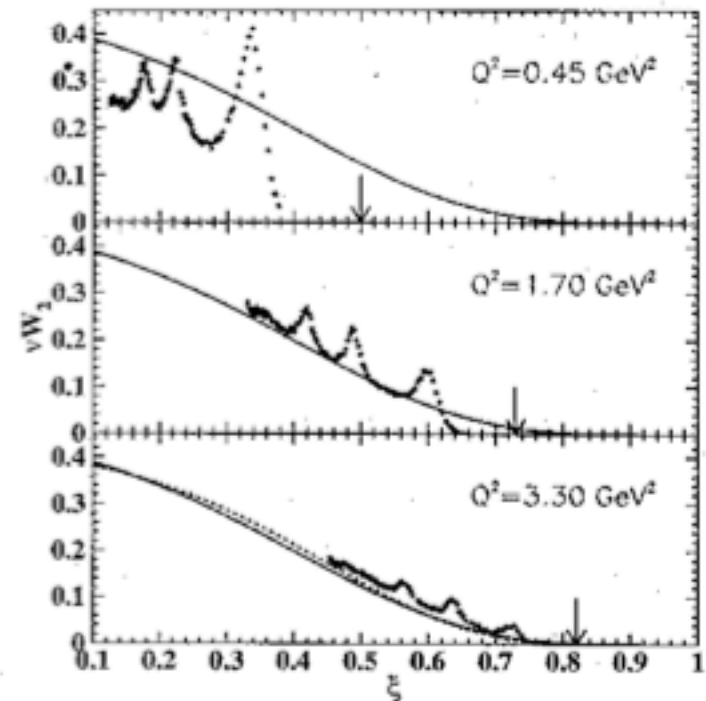
$\sigma$  and  $R$  in  $e^+e^-$  Collisions



$$ep \rightarrow eX$$

I. Niculescu *et al.*, PRL 85 (2000) 1182,

I. Niculescu *et al.*, PRL 85 (2000) 1186





## Focus on DIS: What defines the smooth curve that fits the data?

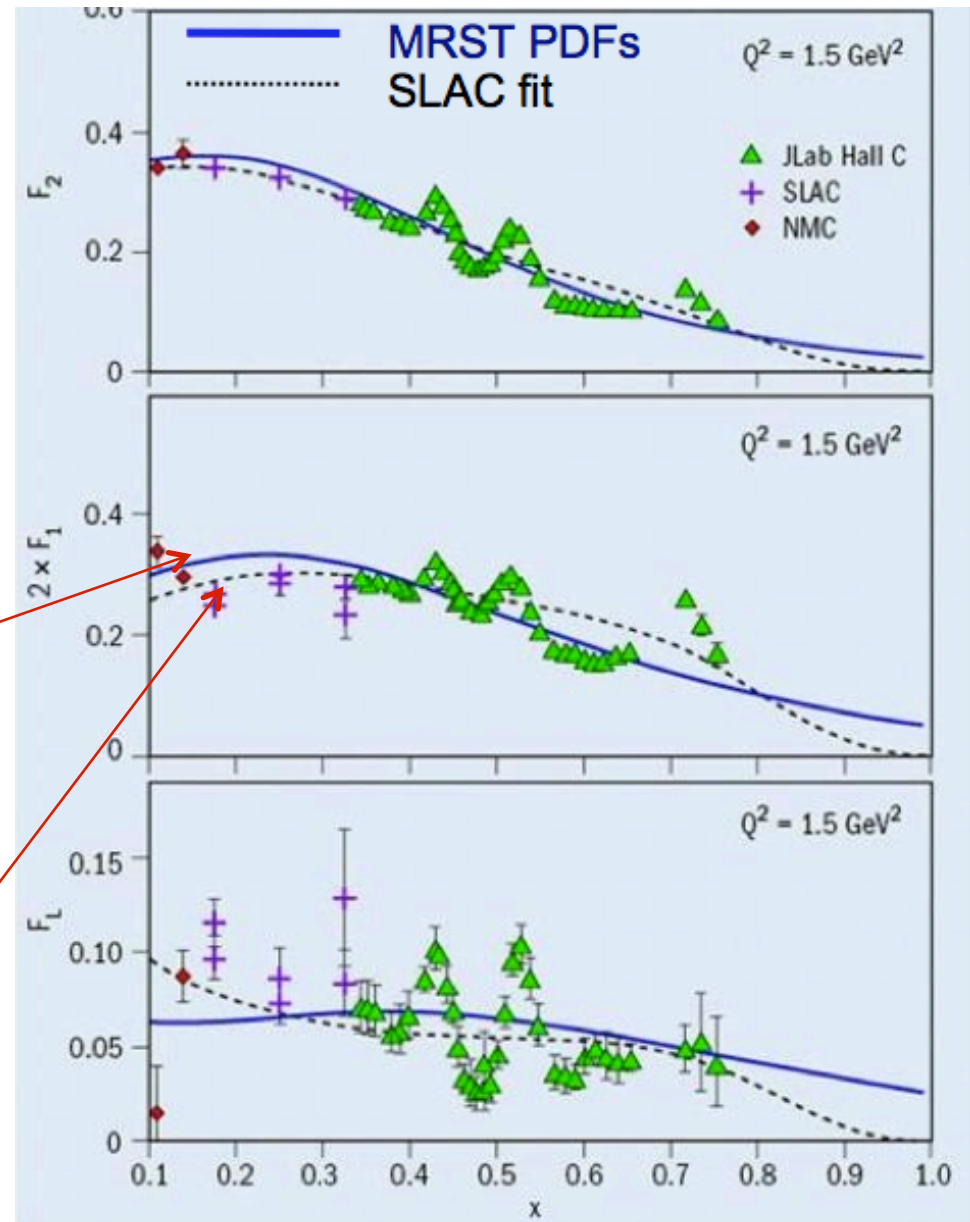
Large  $x_{Bj}$  at fixed  $Q^2$  implies a continuation of the "PQCD curve" into the resonance region

Hall C data

$$W^2 = Q^2 \left( \frac{1}{x_{Bj}} - 1 \right) + M^2$$

PQCD

Data fit



## Two Complementary Approaches

### Quantum Mechanics based Model of Resonances Excitation

Duality arises from Quantum Mechanical properties of quarks confined in “empirically inspired” linear potential: excitation of resonances of opposite parity interfere in all but the leading twist.


(Close, Isgur, Melnitchouk, Jeschonnek, Van Orden...)

### QCD-based phenomenological analyses

OPE-treatable reactions are two step processes:

(1) **Hard dynamics** scattering at large momentum/small distance scale ( $1/Q^2$ ).

(2) **Soft dynamics** process turns the partons into hadrons at much larger distance scales in ( $1/\Lambda_{\text{QCD}}$ ).

First step  Gross features (total cross sections, jet directions, ...)

Second step  Detailed structure of the final state,

In a phenomenological context, duality studies how a number of properties defined from the beginning of the **hard scattering** process, are predetermined and persist in the **non-perturbative** stage.

Where does the separation start?

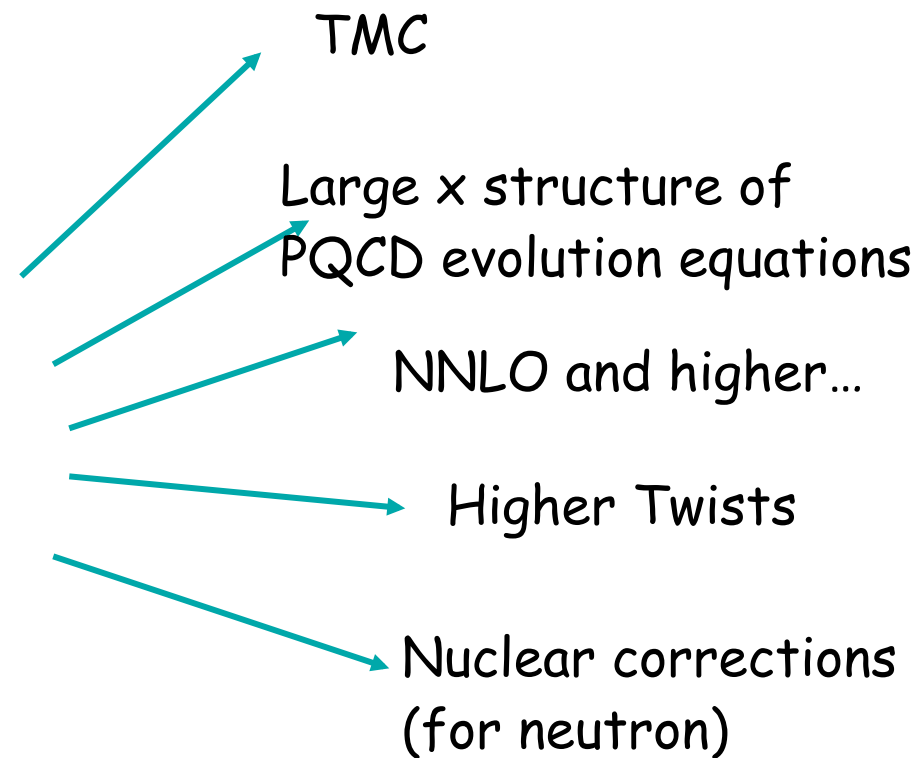
The **separation and yet coexistence** of long distance and short distance structure in QCD has become naturally accepted as part of a ``common wisdom framework'' underlying the interpretation of most experiments

## Systematic Analysis

- S.L., R. Ent, C.Keppel, I. Niculescu, PRL 2000
- N. Bianchi, A. Fantoni, S.L., PRD2003
- A. Fantoni, S.L. 2005
- J.P. Chen, A. Deur, S.L., in preparation 2011

1) Fix the order of the analysis to NLO and extend curve to low  $W^2$

2) Corrections arise that are more important than at low  $x_{Bj}$  and that point at interesting physics (duality)



All effects need to be taken into account simultaneously.

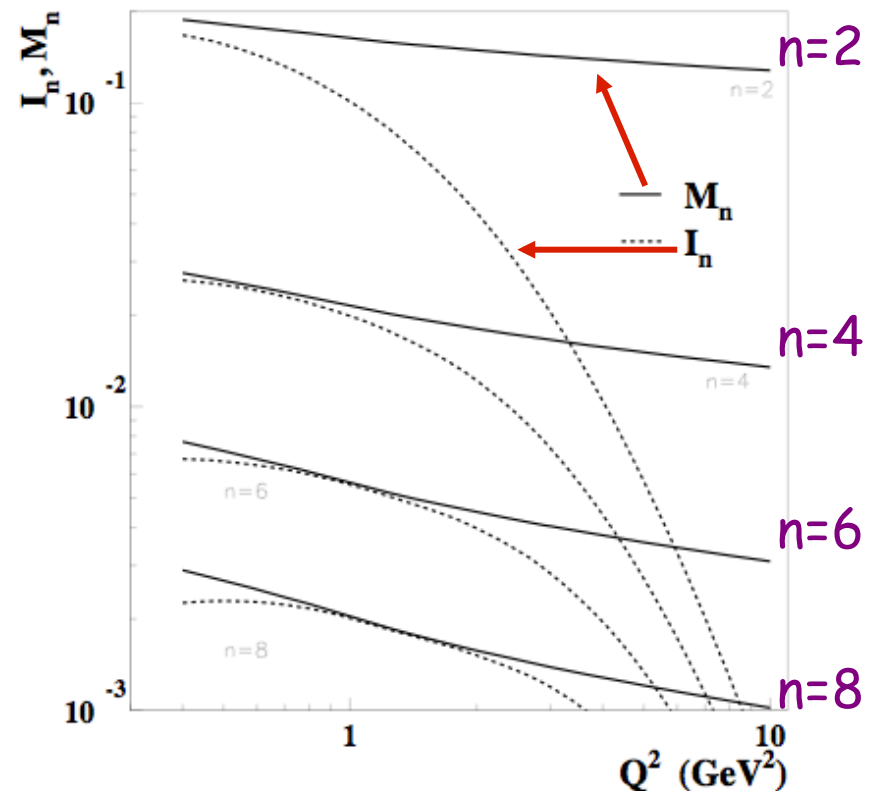
Bianchi, Fantoni, S.L. (PRD, 2003) and Fantoni, S.L. (2006)

$$I^{\text{res}}(Q^2) = \int_{x_{\text{min}}}^{x_{\text{max}}} F_2^{\text{res}}(x, Q^2) dx$$

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2),$$

$$x_{\text{min}} = \frac{Q^2}{Q^2 + W_{\text{max}}^2 - M^2}$$

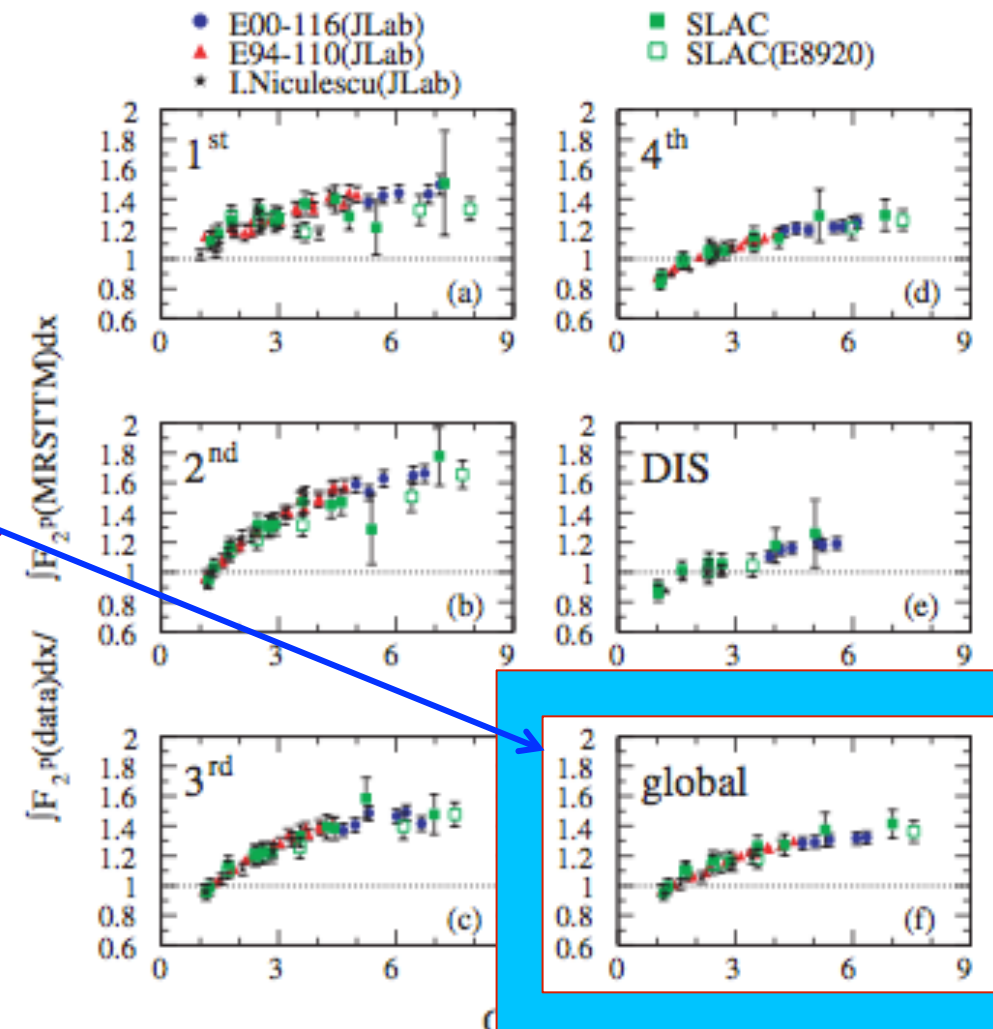
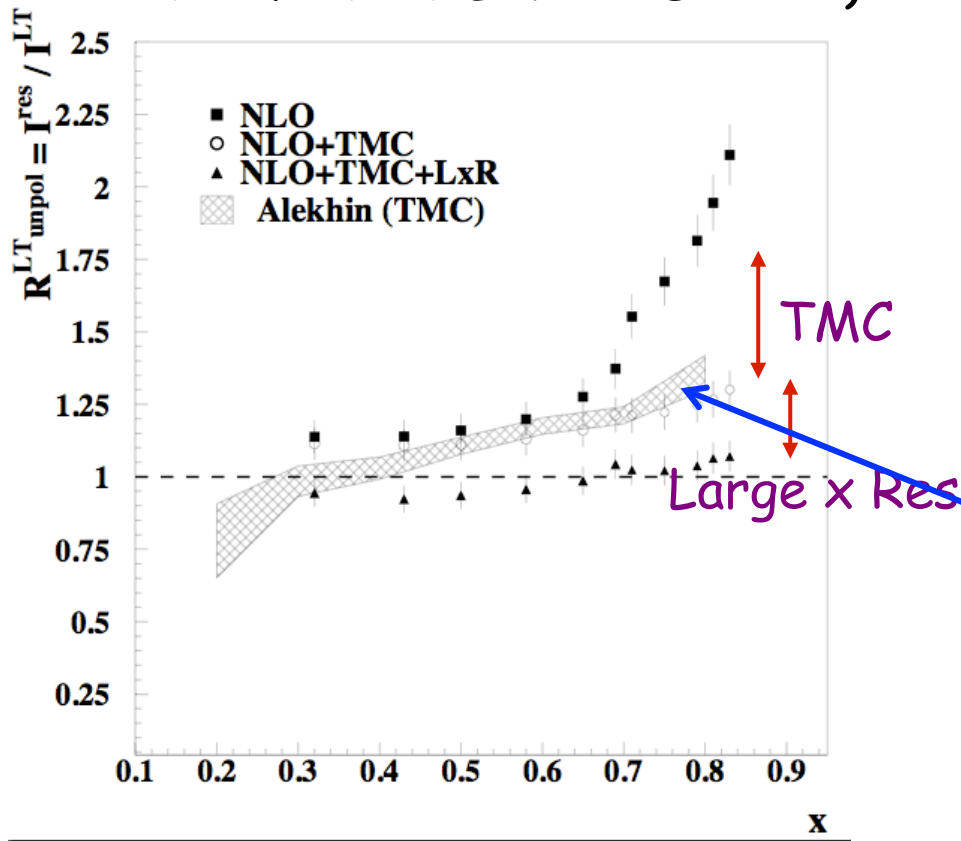
pQCD prediction  
 $M_n \approx \ln Q^2 + \text{NLO orders} \dots$



$I_n$  and  $M_n$  calculated using CTEQ

Unpolarized Jlab+SLAC data  
 Bianchi Fantoni Liuti PRD 2003)

S. P. MALACE *et al.* PRC 2010



$$R = \frac{\int_{x_{\min}}^{x_{\max}} F_2^{res}(x, Q^2) dx}{\int_{x_{\min}}^{x_{\max}} F_2^{LT,param}(x, Q^2) dx}$$

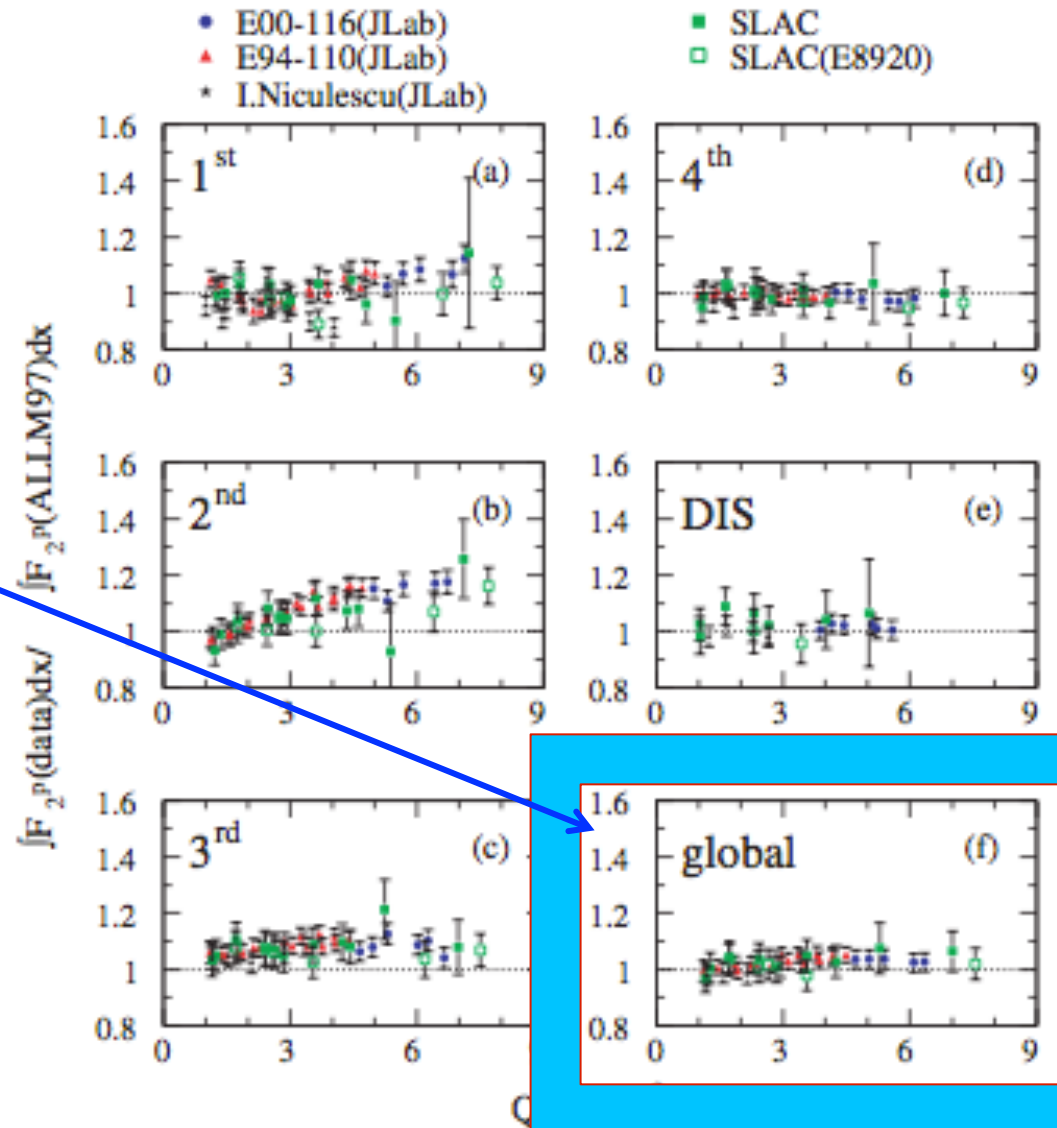
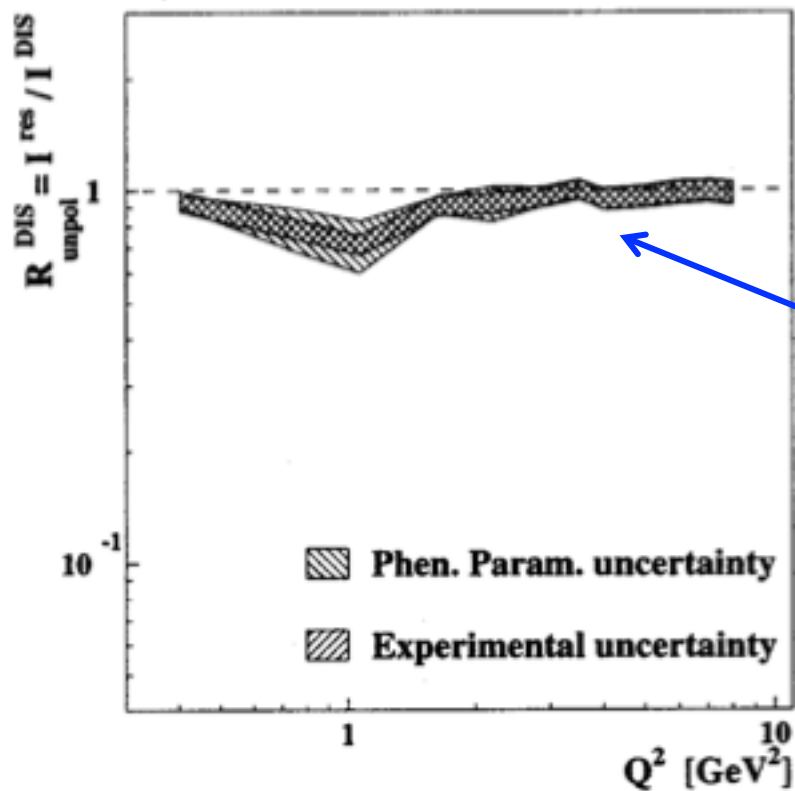
# Using ALLM → extra NP $Q^2$ dependence

Unpolarized Jlab+SLAC data

Bianchi Fantoni Liuti PRD 2003)

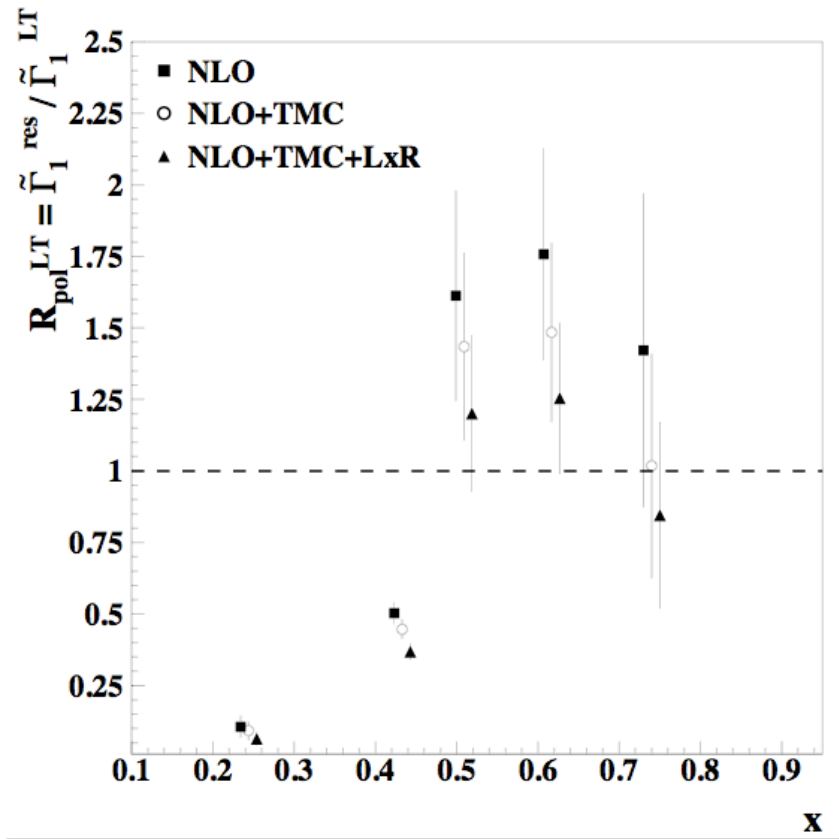
S. Malace et al., PRC 2010

PARTON-HADRON DUALITY IN UNPOLARIZED AND ...



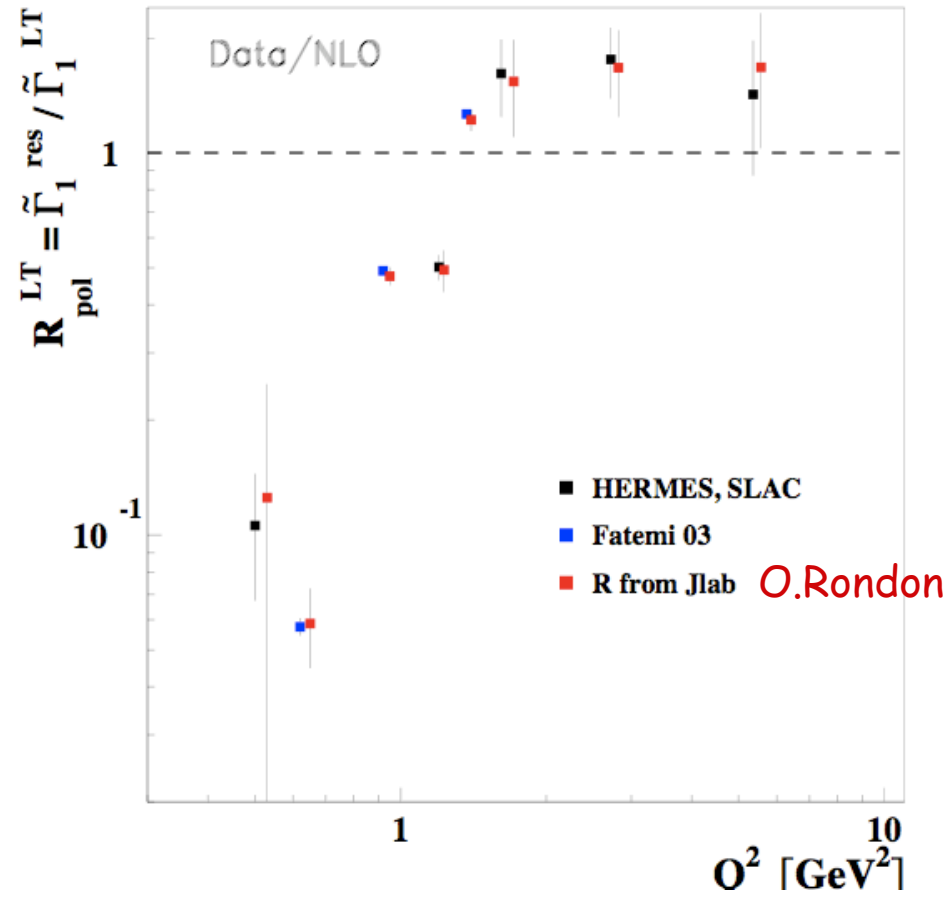
$$R = \frac{\int_{x_{\min}}^{x_{\max}} F_2^{\text{res}}(x, Q^2) dx}{\int_{x_{\min}}^{x_{\max}} F_2^{\text{LT,param}}(x, Q^2) dx}$$

# Polarized HERMES+Jlab+SLAC data



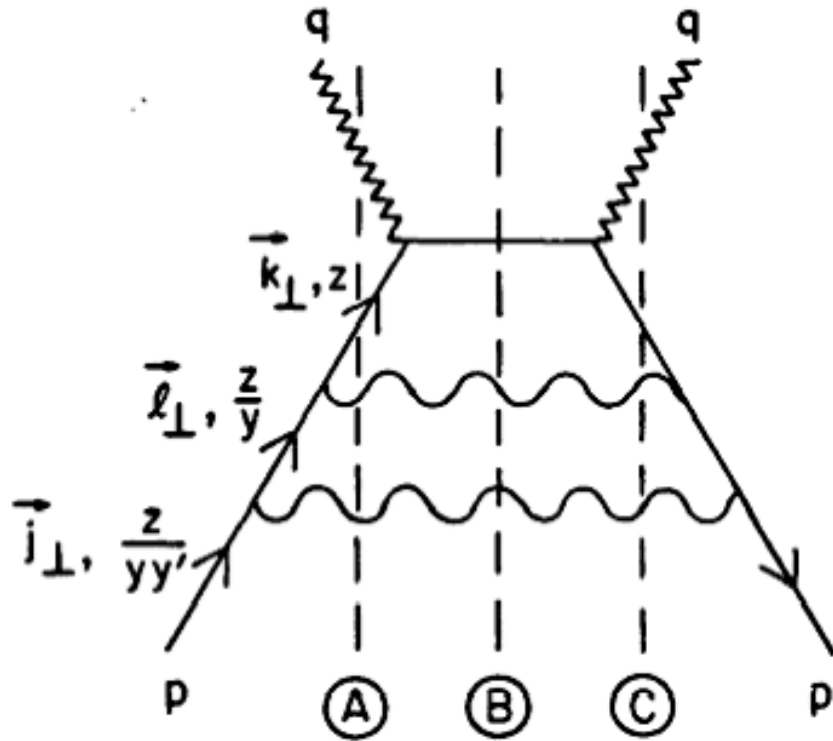


## More recent polarized data (O. Rondon et al.)



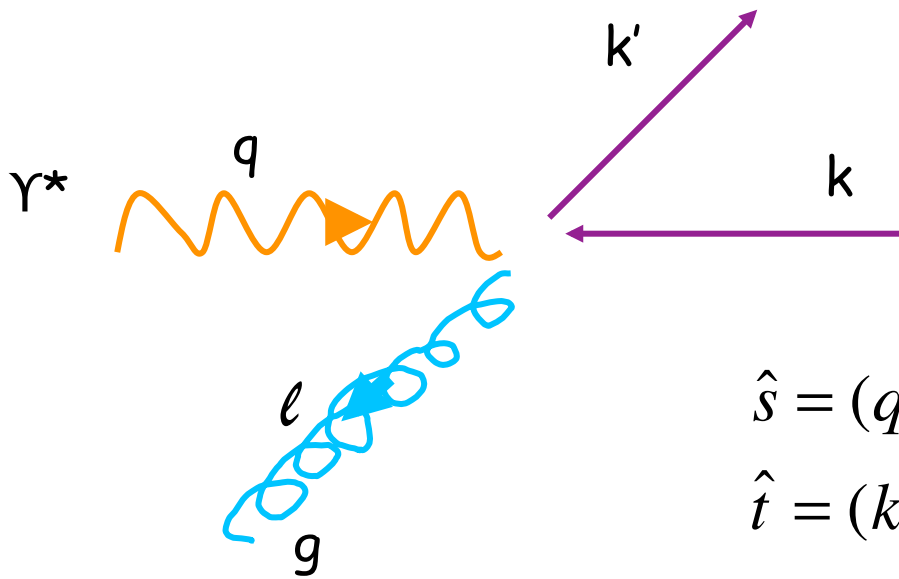
## Large $x_{Bj}$ evolution

(S. Brodsky, SLAC lectures (1979), D. Amati et al., NPB(1980), R. Roberts, "Structure of the Proton")



$\alpha_S = \alpha_S(k^2)$  at each vertex

$$\alpha_S(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}$$



$$\gamma^* q \rightarrow q' g$$

$$\hat{s} = (q + k)^2 = 4k^2$$

$$\hat{t} = (k - k')^2 = -2qk'(1 - \cos\theta)$$

$$\hat{u} = (q - k')^2 = -2qk'(1 + \cos\theta)$$

$$k_T^2 = \frac{\hat{s} (-\hat{t}) \hat{u}}{\hat{s} (\hat{s} + Q^2)} = \frac{\hat{s} \sin^2 \theta}{4}$$

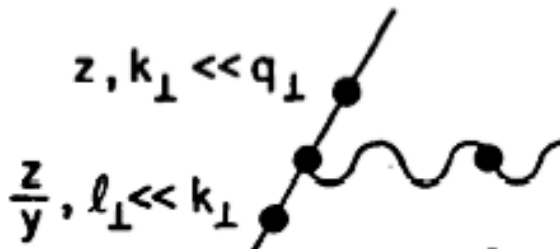
$$(k_T^{MAX})^2 = \frac{\hat{s}}{4}$$

Invariant mass!

In terms of LC variables

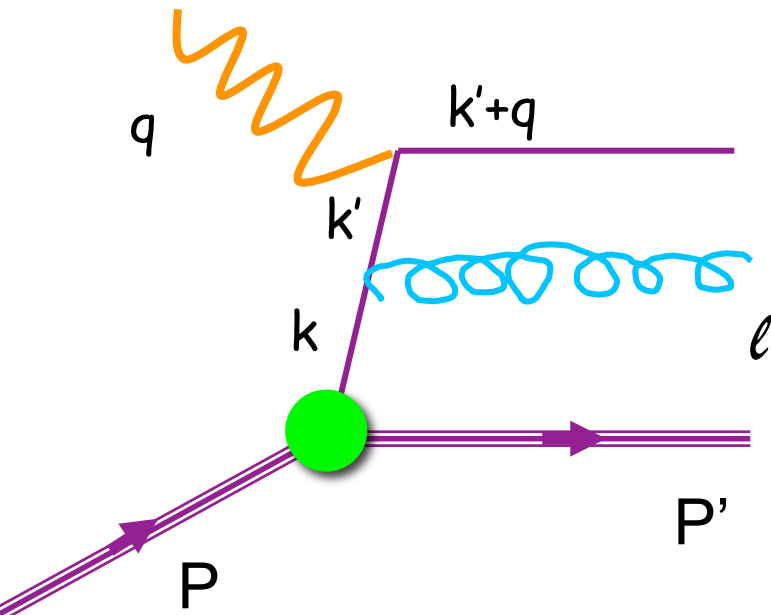
$$k = (k^+ = zP^+, k^- = P^- - \ell^-, k_T)$$

$$\rightarrow (k_T^{MAX})^2 = \frac{Q^2(1-z)}{4z}$$



Next, write amplitude for

$$\gamma^* P \rightarrow (\text{final quark}) + g + X$$

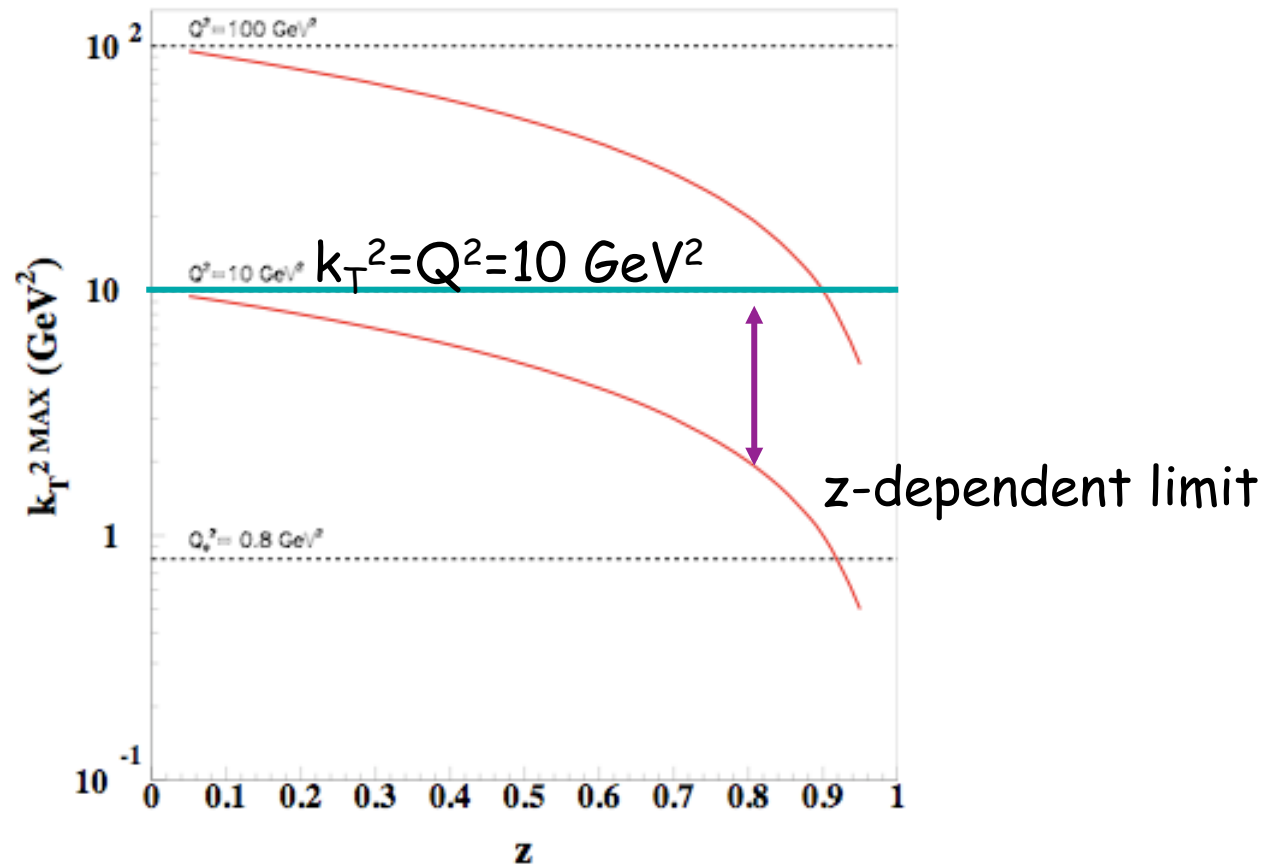


|Amplitude|<sup>2</sup> for  $\gamma^*P \rightarrow$  (final quark) + g + X 

$$q(x, Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2 \frac{1-z}{4z}} dk_T^2 \alpha_S(k_T^2) P_{qq}(z) q\left(\frac{x}{z}, k_T^2\right)$$

Disregarding z-dependence in  $k_T$  integration limit

$$\frac{dq(x, Q^2)}{d \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{qq}(z) q\left(\frac{x}{z}, Q^2\right)$$



It matters at large  $x$ !

As a consequence...

$$\alpha_S(Q^2) \rightarrow \alpha_S[Q^2(1-z)] \approx \alpha_S(Q^2) - \frac{1}{2}\beta_0 \ln(1-z) (\alpha_S(Q^2))^2$$

This takes care of the **large log term** in the Wilson coefficient  $f$ .  
(NLO, MS-bar)

$$F_2^{NS}(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q \int_x^1 dz \underline{C_{NS}(z)} q_{NS}(x/z, Q^2), \quad (24)$$

$$C_{NS}(z) = \delta(1-z) + \left\{ C_F \left( \frac{1+z^2}{1-z} \right)_+ \left[ \ln \left( \frac{1-z}{z} \right) - \frac{3}{2} \right] + \frac{1}{2} (9z+5) \right\}$$

The scale that allows one to annihilate the effect of the large  $\ln(1-z)$  terms at large  $x$  at NLO is the invariant mass,  $W^2$

Equivalent to a resummation of these terms up to NLO

## Target Mass Corrections (TMC)

$$F_2^{LT(TMC)}(x, Q^2) = \frac{x^2}{\xi^2 \gamma^3} F_2^\infty(\xi, Q^2) + 6 \frac{x^3 M^2}{Q^2 \gamma^4} \int_\xi^1 \frac{d\xi'}{\xi'^2} F_2^\infty(\xi', Q^2),$$

## Target Mass Corrections (TMC)

Work in progress based on recent analysis by [A. Accardi, J. Qiu, JHEP \(2008\)](#) that extends range of validity of TMCs approach without introducing mismatches between the  $x$  and  $\xi$  ranges

$$F_{T,L}(x_B, Q^2, m_N^2) = \int_{\xi}^{\xi/x_B} \frac{dx}{x} h_{f|T,L}(\tilde{x}_f, Q^2) \varphi_f(x, Q^2) . \quad (18)$$

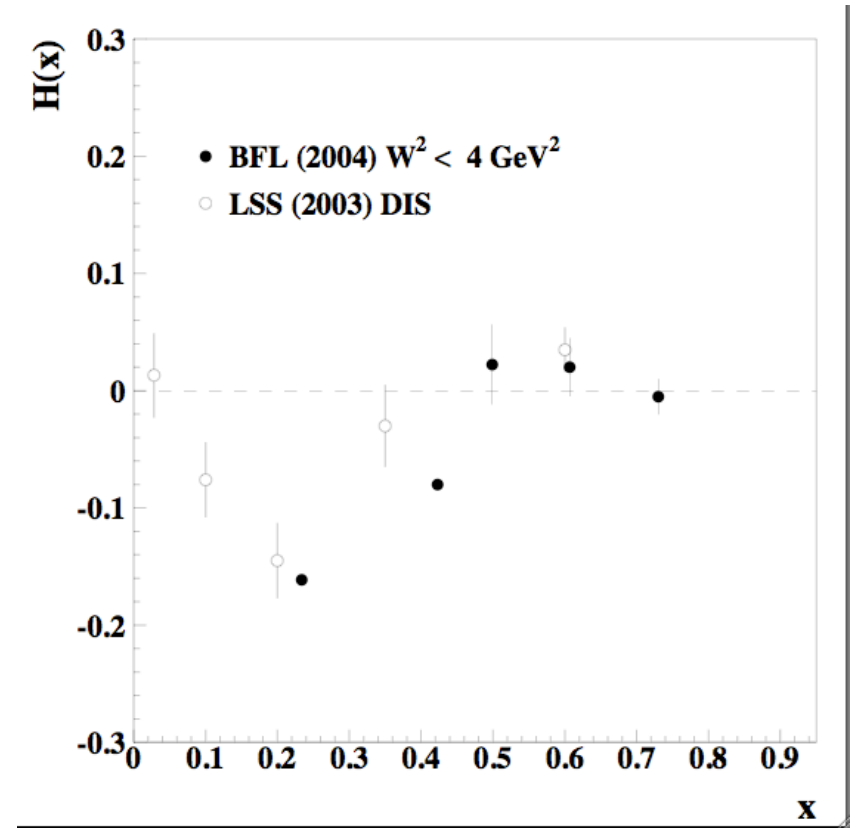
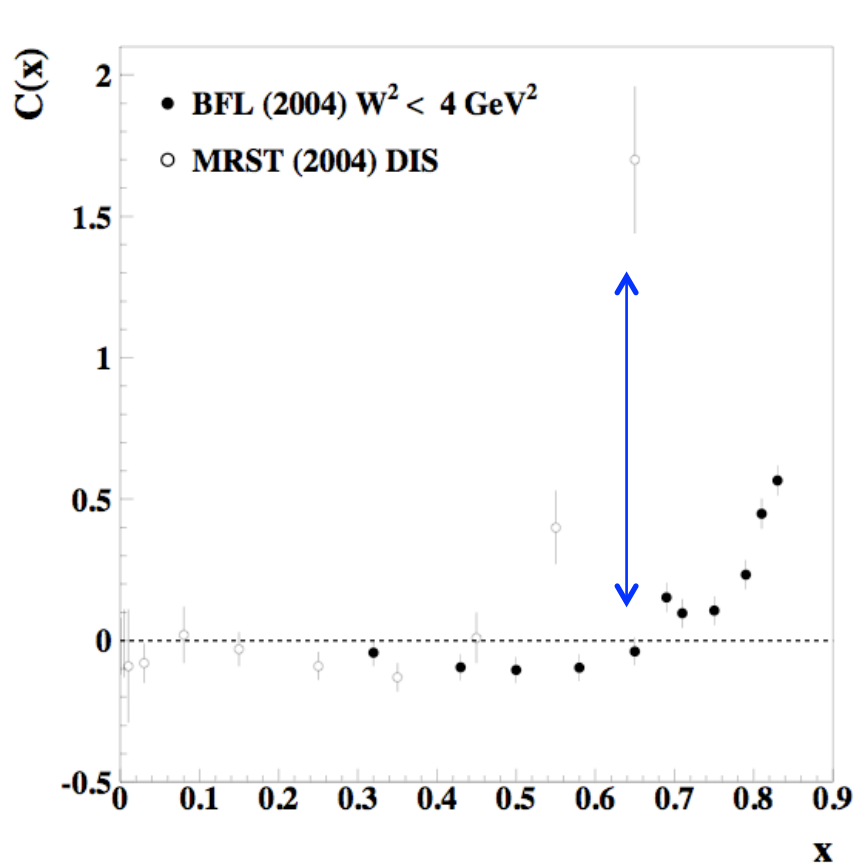
Instead of 1

Joint large  $x$  evolution and new TMCs approach

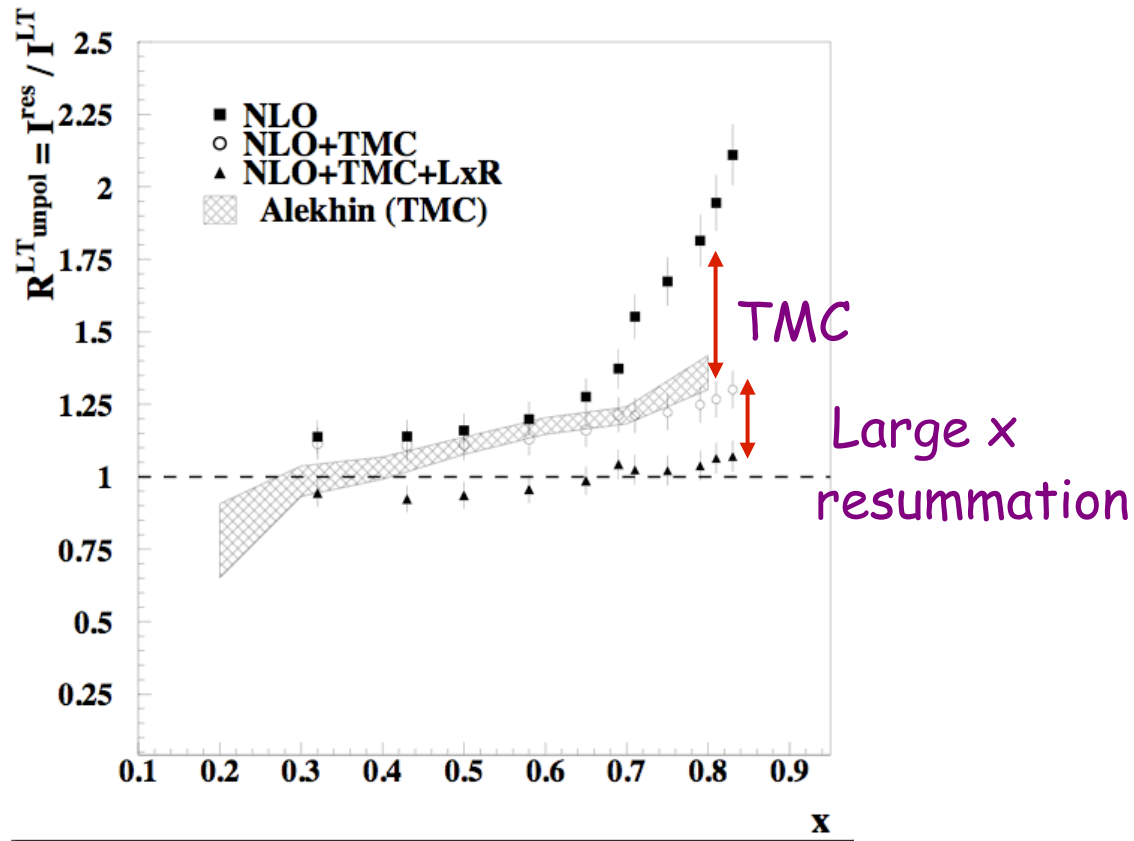


All of these effects can be taken into account using "reasonable" Parameters → No need to introduce large HT component at Large x

$$H(x) = F^{LT}(x)C_{HT}(x) \rightarrow \text{additive form}$$



$$F_2^{\text{exp}} = F_{\text{PQCD}}(1 + C(x)/Q^2)$$

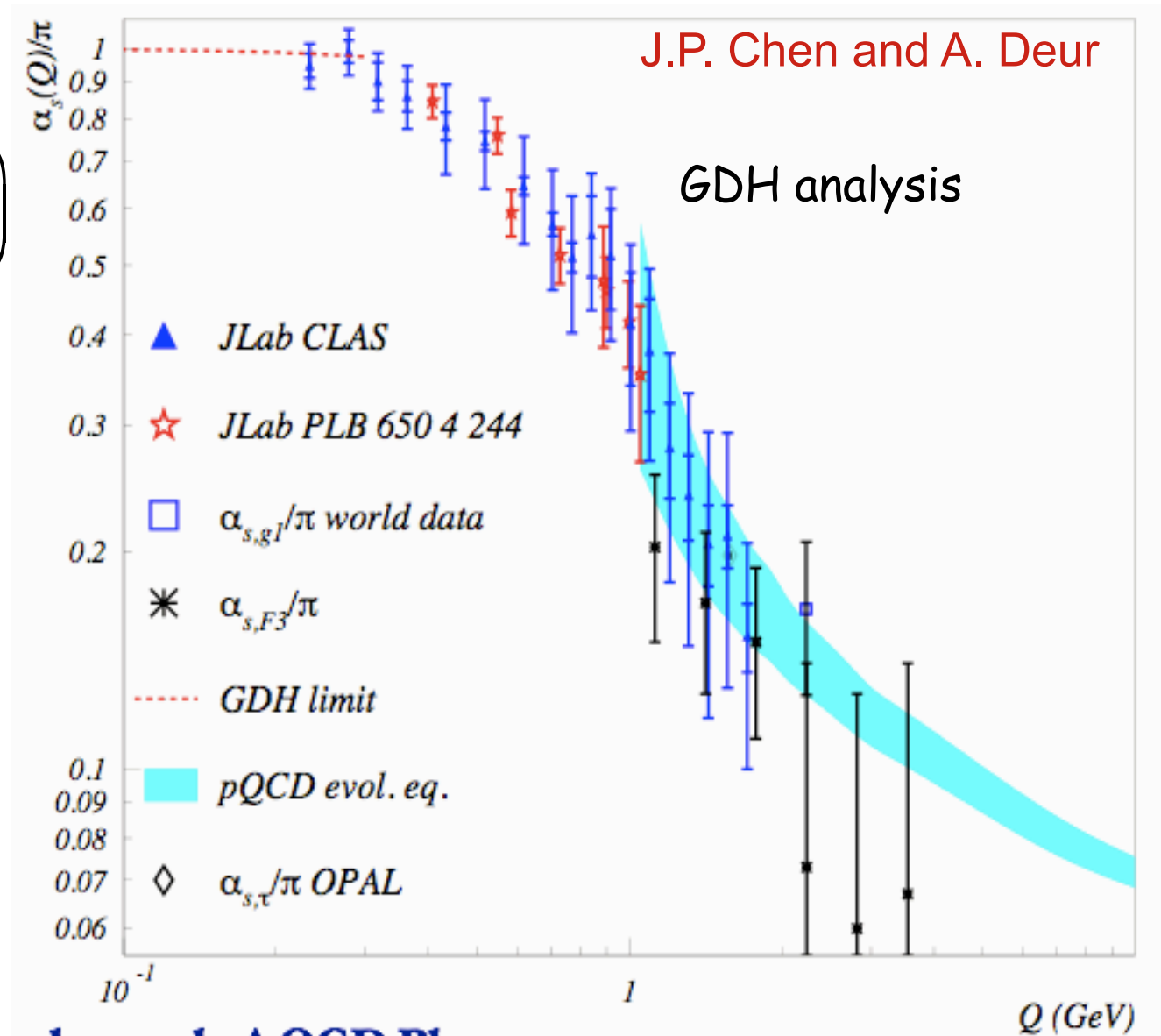


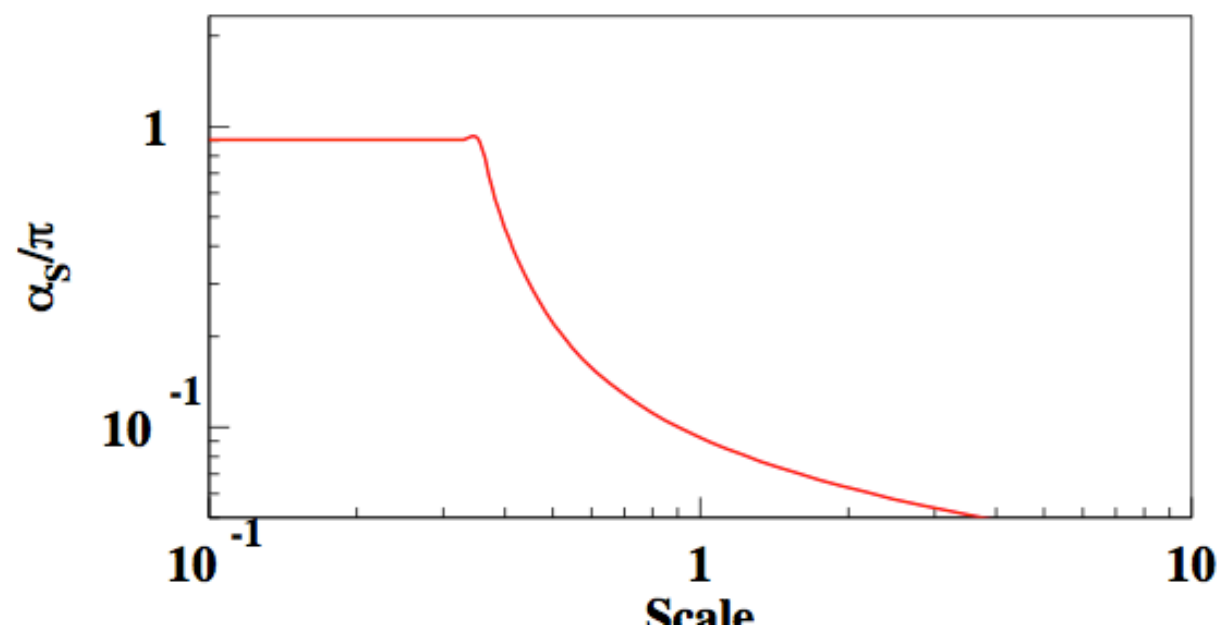
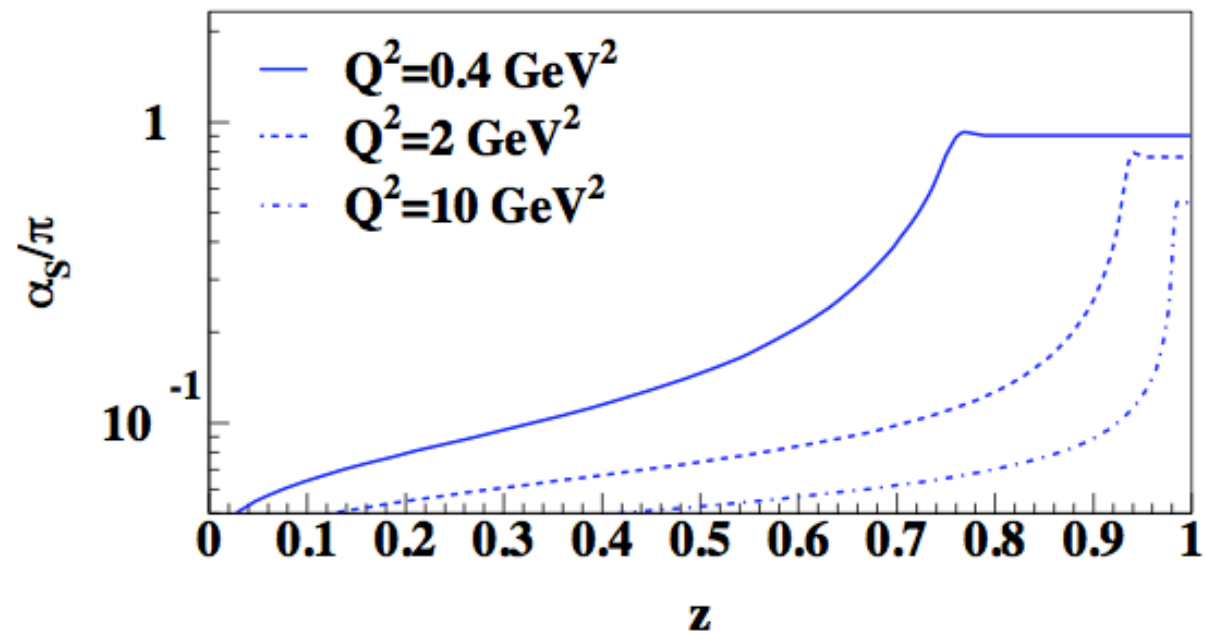
What is happening is that (part) of the HTs contributions have been absorbed in  $\alpha_s$

This is correlated with the fact that at large  $x$ ,  $\alpha_s$  needs to be continued at very low  $Q^2$

$$\alpha_s(Q^2) \rightarrow \alpha_s \left( Q^2 \frac{1-z}{z} \right)$$

Work in progress:  
Use this "positively" to  
Extract  $\alpha_s$  at low scale





# What is the physics behind this?

Pioneering Work

Pennington, Roberts, Ross 80's

+ far reaching extensions

related to discussion at this W'shop

Extension of  $\alpha_s$  in timelike region:  
going through the Landau pole

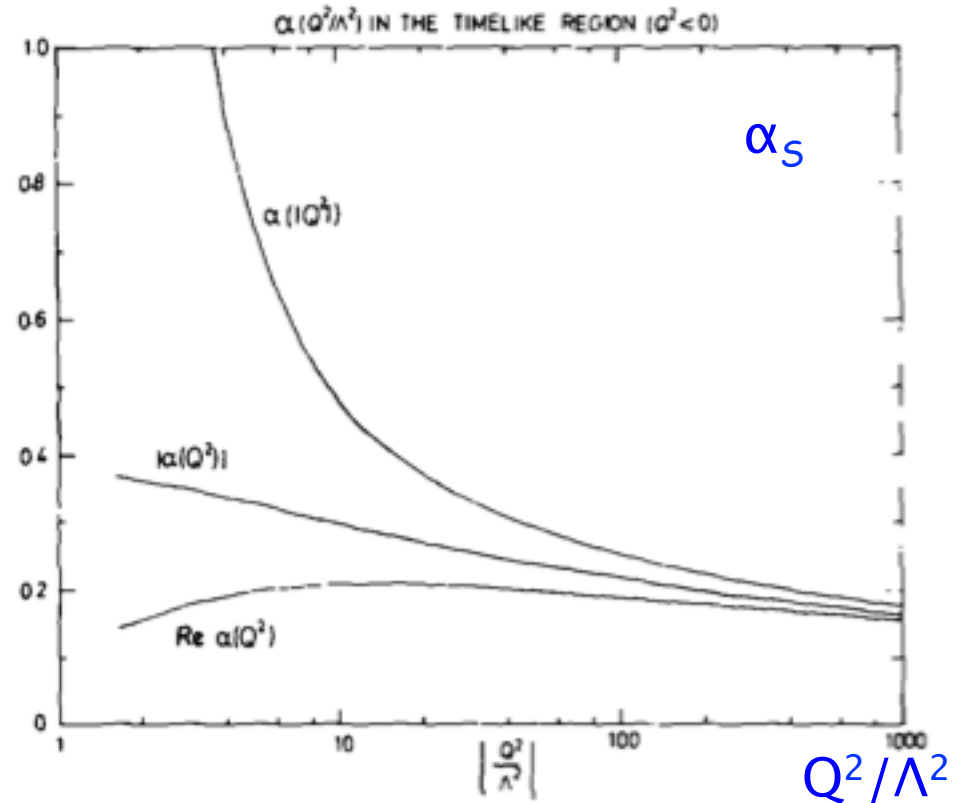


Fig. 1. The three expansion parameters  $\alpha(|Q^2|)$ ,  $|\alpha(Q^2)|$  and  $\text{Re } \alpha(Q^2)$  in the timelike region ( $Q^2 < 0$ ) are shown as functions of  $|Q^2|/\Lambda^2$ .  $\alpha(Q^2)$  is defined by eq. (2) with  $\beta_0, \beta_1$  evaluated for  $n_f = 4$ , but with all higher coefficients zero.  $\alpha(|Q^2|)$  is given by its commonly used, next-to-leading order, expression

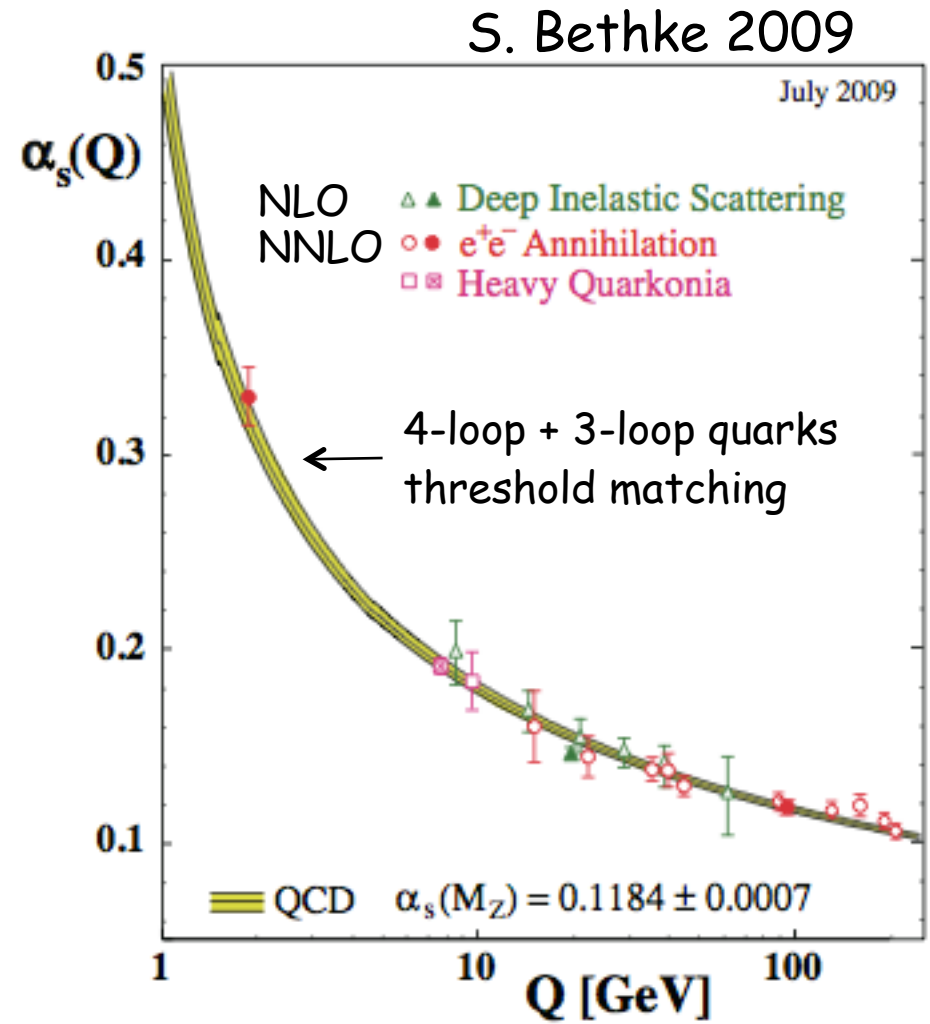
$$\frac{\alpha(|Q^2|)}{4\pi} = \frac{1}{\beta_0 \ln|Q^2/\Lambda^2|} - \frac{\beta_1 \ln \ln|Q^2/\Lambda^2|}{\beta_0^3 \ln^2|Q^2/\Lambda^2|}$$

Note, however, that, for  $|Q^2/\Lambda^2| < 100$ , representing  $|\alpha(Q^2)|$ ,  $\text{Re } \alpha(Q^2)$  by similar power series in  $(\ln|Q^2/\Lambda^2|)^{-1}$  is very inaccurate, unless many terms are included.  $|\alpha(Q^2)$ ,  $\text{Re } \alpha(Q^2)$  have therefore been evaluated from eq. (2) by expressing  $\ln Q^2$  as a series in  $\alpha$ . The curves shown reflect terms up to  $O(|\alpha|^6)$  in  $\text{Im } \alpha$ , eq. (5), and so are accurate even at low  $|Q^2|$ .

# Two Points of Interest for Jlab Community

1) The coupling's behavior as one approaches the Landau pole, matters for predictions at the LHC

Long known systematic difference between  $e^+e^-$  (smaller  $\alpha$ ) and structure functions (larger  $\alpha$ )



2) Study of the interplay between perturbative and non-perturbative effects is important: formulate it in a way where comparison with lattice results is easier and attack the question of process dependent  $\alpha_s$  predictions

### Outline of a Phenomenology Study

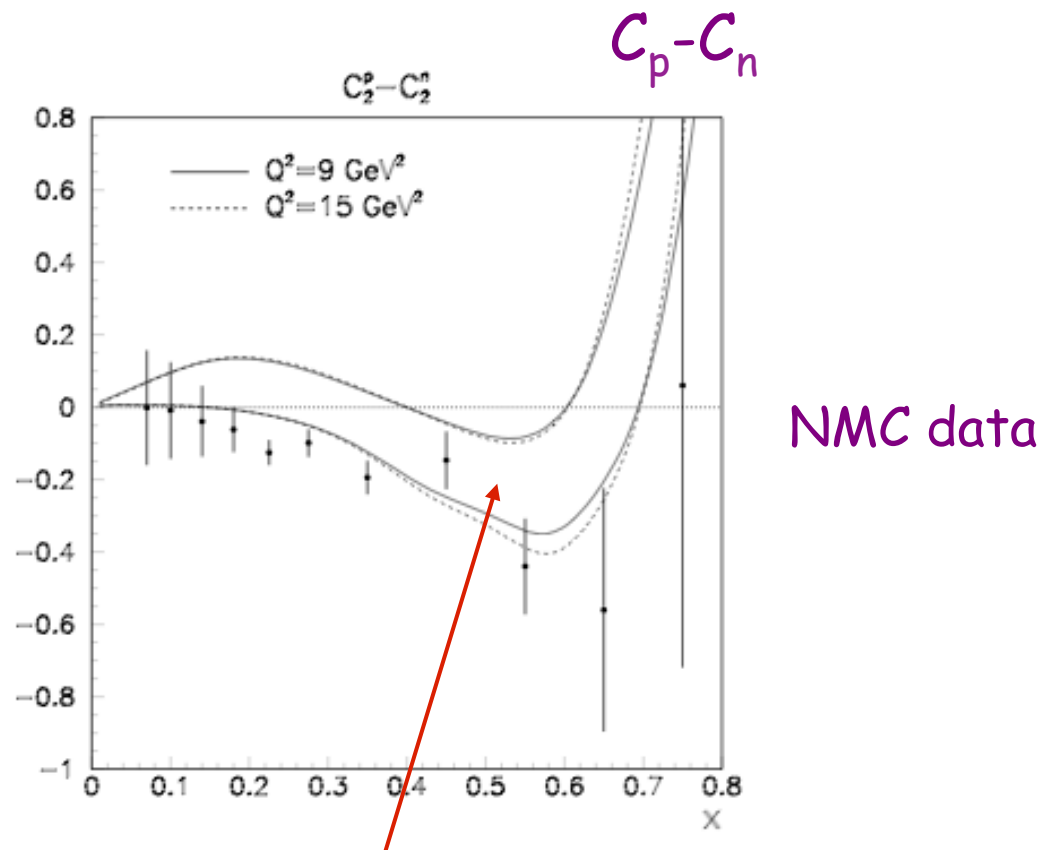
Start from the point that the Landau pole in  $\alpha_s$  is unphysical

Introduce effective couplings/charges that can be extracted directly from data

Define scheme to connect and compare different extractions (commensurate scale relations, Brodsky, Deur, deTeramond)

## Nuclei

✓ Are HTs isospin dependent? Deviations from PQCD effects



Alekhin, Kulagin and S.L., PRD (2003)



## Towards the Jlab 12 GeV/EIC program....

The very large and accurate Jlab Hall C set of data has shown that parton hadron duality can be studied in detail:  $Q^2, W^2$  and longitudinal variables dependences have been analyzed thoroughly

⇒ Observation of similarity between "high" and "low" energy cross sections at the core of strong interaction theory: would love to see study of large  $x$  and larger  $W^2$  to see how HTs behave

⇒ Theoretical background: starts from Finite Energy Sum Rules

Dolen, Horn and Schmid, PR166(1968)

$$S_n \equiv \frac{1}{N^{n+1}} \int_0^N \nu^n \text{Im}F d\nu = \sum \frac{\beta N^\alpha}{(\alpha+n+1)\Gamma(\alpha+1)}$$

⇒ Is there an interpretation within QCD?

Shifman (2005), Bigi and Uraltsev (2004)

Is there a more general implication from factorization theorems of QCD?

⇒ Interplay of ISI and FSI Frankfurt, Collins

## Conclusions

PQCD analysis of highly precise large  $x_{Bj}$  Jefferson Lab data in resonance region is interesting for both studying the transition of partons into hadrons and for studies of  $\alpha_s$