Beyond Dipoles:

exploring small x evolution

via

two hadron correlations

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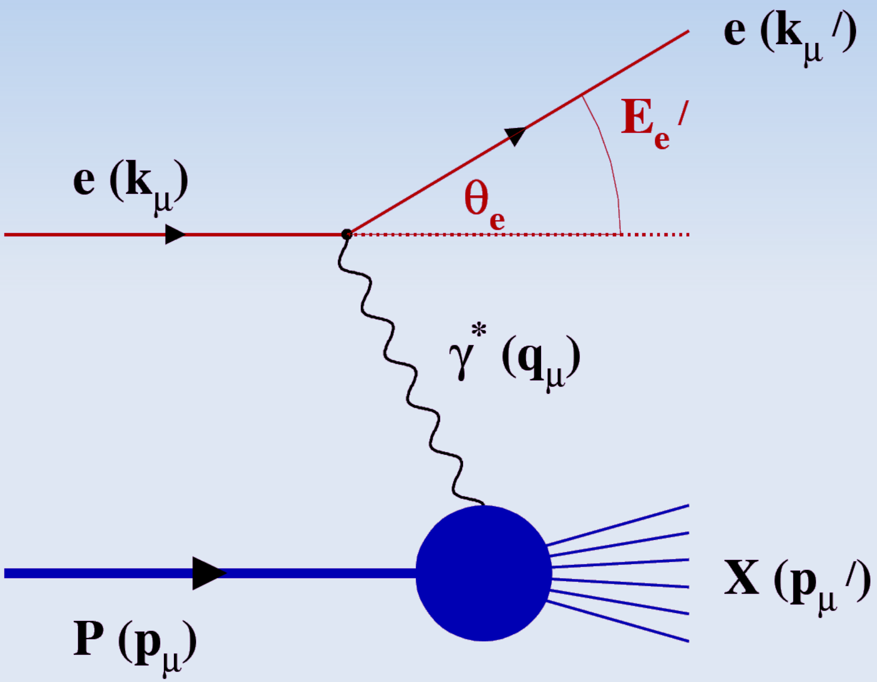
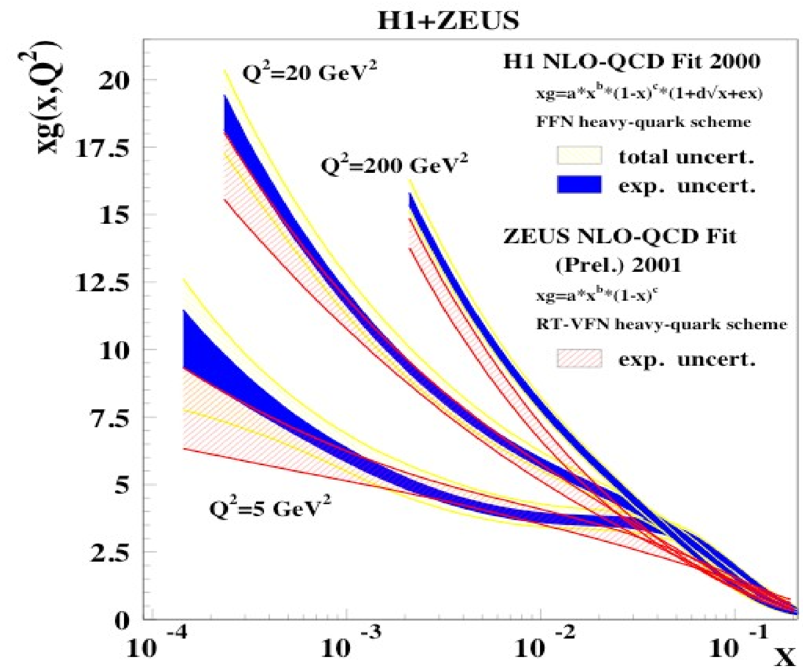
AD-JJM, PRD82 (2010) 074023, PRD81 (2010) 094015

A hadron at small x

DIS: $\mathrm{e\,p \to e\,X}$



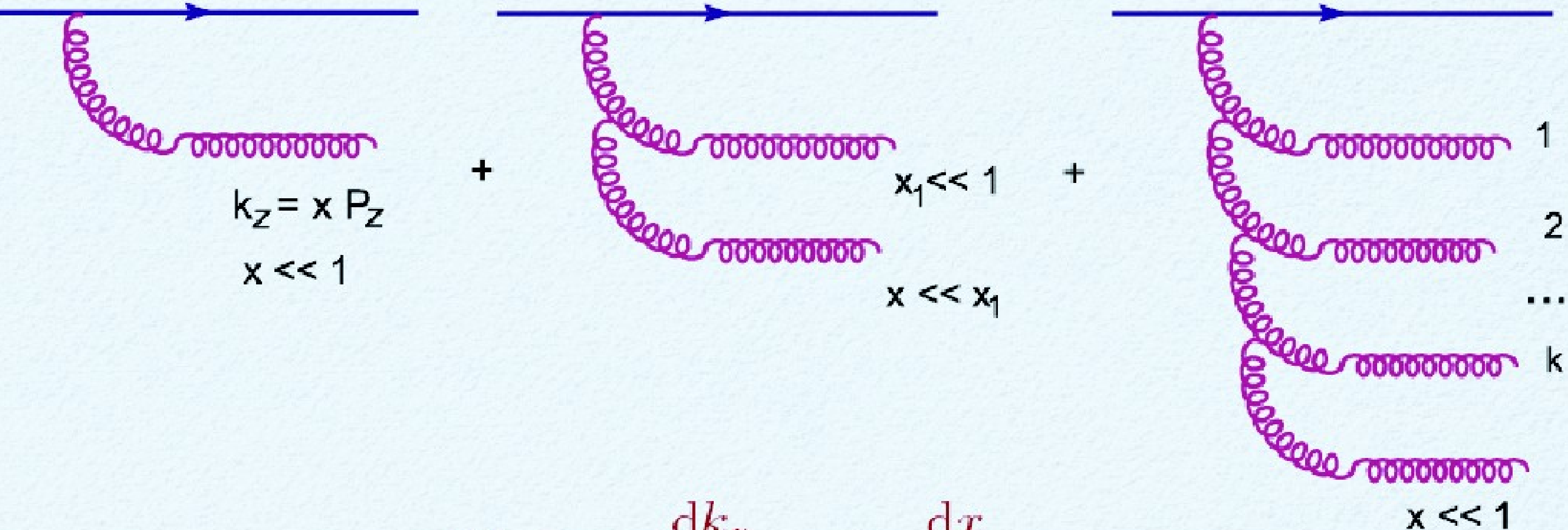


$$\mathrm{x} = \frac{\mathrm{p}^+}{\mathrm{P}^+}$$ is the fraction of hadron energy carried by a parton

there are a lot of gluons at small x

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons



$$\mathrm{d}\mathcal{P} \;\propto\; \alpha_s \frac{\mathrm{d}k_z}{k_z} \;=\; \alpha_s \frac{\mathrm{d}x}{x}$$

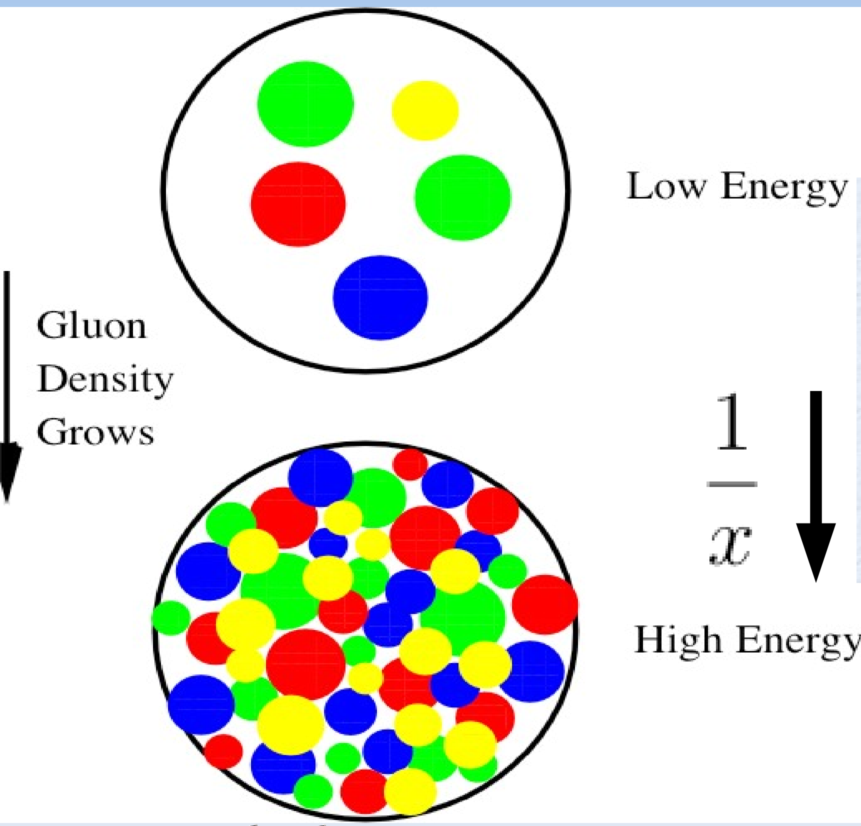
The 'price' of an additional gluon:

$$\mathcal{P}(1) \;\propto\; \alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} \;=\; \alpha_s \ln \frac{1}{x}$$

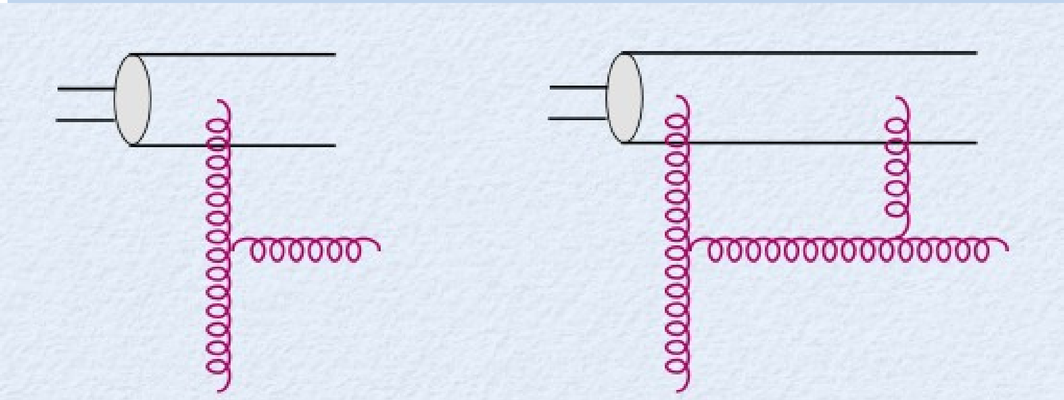
number of gluons grows fast

$$n \sim e^{\alpha_s ln 1/x}$$

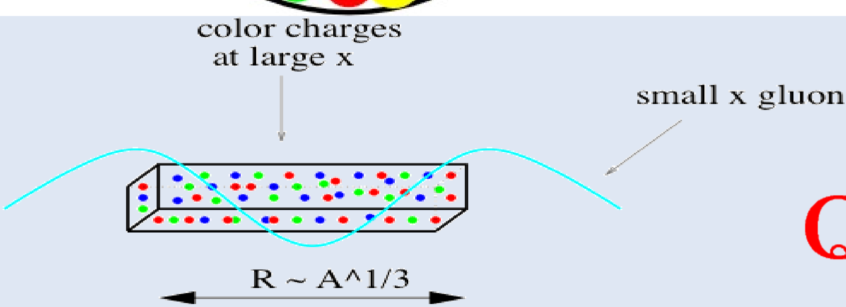
Gluon saturation



“attractive” bremsstrahlung
vs. “repulsive” recombination



$$\frac{\alpha_s \, xG(x, b_t, Q^2)}{S_\perp Q^2} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Effective Action + RGE

$$\mathbf{S[A,\rho] = -\frac{1}{4}\int d^4x\, F^2_{\mu\nu} + \frac{i}{N_c}\int d^2x_t dx^-\, \delta(x^-) Tr[\rho(x_t) U(A^-)]}$$

Large x: color source ρ small x: gluon field $\mathbf{A}^\mu$

$$\mathbf{U(A^-) = \hat{P}\, Exp\left[ig\int dx^+\, A_a^-\, T_a\right]}$$

$$\mathbf{Z[j] = \int [D\rho]\, W_{\Lambda^+}[\rho]\left[\frac{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]-\int j\cdot A}}{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]}}\right]}$$

weight functional:

$\mathbf{W}_{\Lambda^+}[\rho]$ probability distribution of color source ρ at longitudinal scale Λ^+

invariance under change of Λ^+ ⟶ RGE for $\mathbf{W}_{\Lambda^+}[\rho]$

QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields A^μ

Sources ρ

$W_x[\rho]$

0 x 1

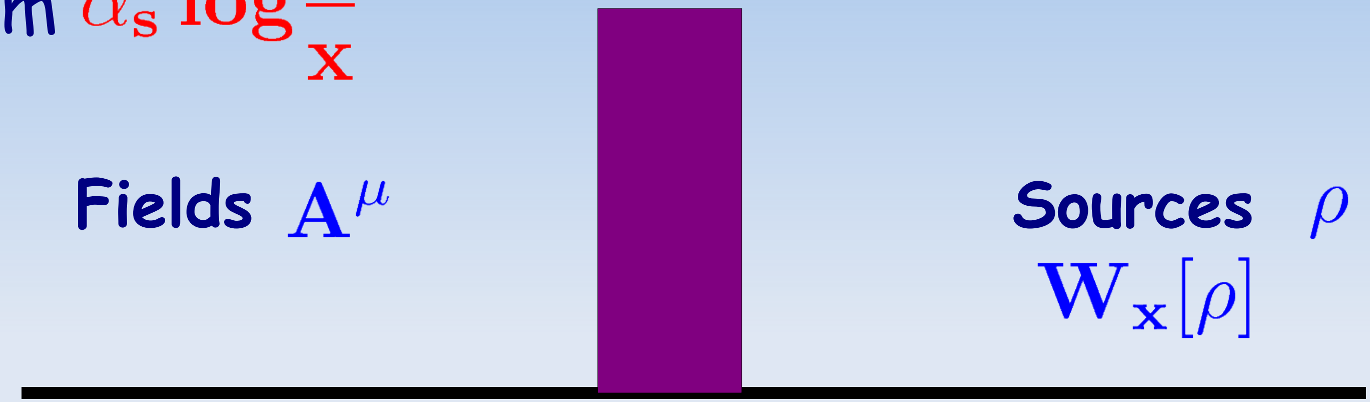
QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields A^μ

Sources ρ

$W_x[\rho]$



0 1

$$A^\mu = A^\mu_{\text{class}} + \delta A^\mu$$

integrate out field fluctuations quadratically

$$\rho \to \rho' = \rho + \delta\rho$$

QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields $\mathbf{A}^\mu$

Sources ρ

$\mathbf{W_x}[\rho]$

$\mathbf{0}$ $\mathbf{x}$ $\mathbf{1}$

$$\frac{\partial \mathbf{W}[\rho]}{\partial \mathbf{ln1/x}} = \frac{1}{2}\int_{\mathbf{x_t},\mathbf{y_t}} \frac{\delta}{\delta\rho^{\mathbf{a}}(\mathbf{x_t})}\chi^{\mathbf{ab}}(\mathbf{x_t},\mathbf{y_t})[\rho]\frac{\delta}{\delta\rho^{\mathbf{a}}(\mathbf{y_t})}\mathbf{W}[\rho]$$

JIMWLK eq. describes x evolution of observables

Beyond dipole + large Nc

JIMWLK evolution equation

$$\frac{d}{dy}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\, \eta_{xy}^{bd}\, \frac{\delta}{\delta\alpha_y^d}\, O \right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[1+U_x^\dagger U_y - U_x^\dagger U_z - U_z^\dagger U_y\right]^{bd}$$

$$U(x_t) = \hat{P} e^{ig\int dx^- \, \alpha^a(x_t)\, T^a}$$

Dipole (2-pt function) evolution

Basic building block in DIS, pA processes

$$\frac{d}{dy} < \mathrm{Tr} V_x^\dagger V_y > = -\frac{\bar{\alpha}_s}{2\pi} \int d^2z \, \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \times$$
$$\left[< \mathrm{Tr} V_x^\dagger V_y > - \frac{1}{N_c} < \mathrm{Tr} V_x^\dagger V_z \, \mathrm{Tr} V_z^\dagger V_y > \right]$$

Evolution of 2-point function depends on 4-point function

$$\frac{d}{dy} < \mathrm{Tr} V_x^\dagger V_z \, \mathrm{Tr} V_z^\dagger V_y > \sim < V^4 + \cdots >$$

Infinitely many coupled equations!

Dipole evolution

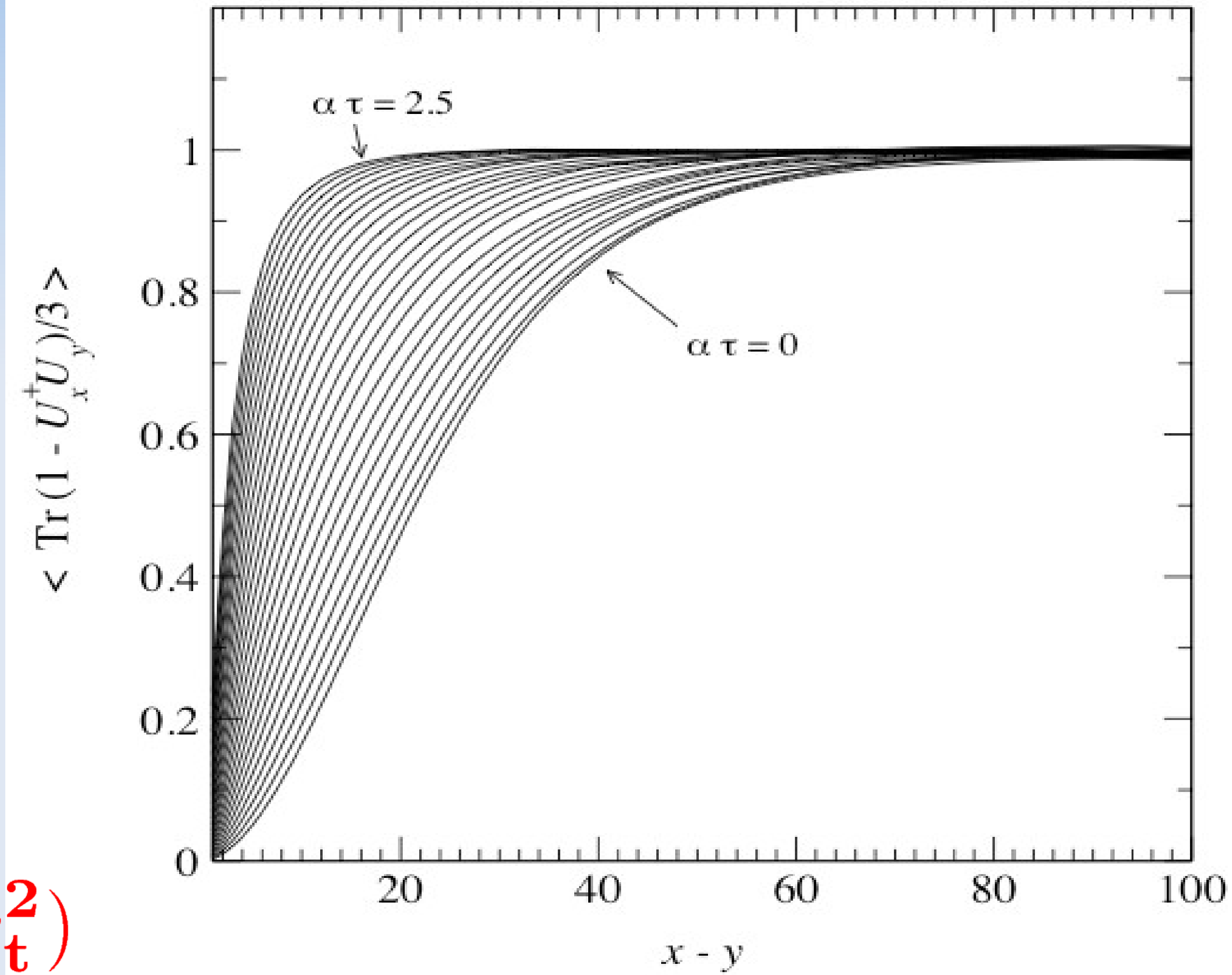
saturation region

dilute region

$\sim [\mathrm{r_t^2\, Q_s^2}]^{\gamma}$

pQCD region

$\sim \mathrm{r_t^2\, xG(x, 1/r_t^2)}$



RW, NPA739 (2004) 183

BK equation

$$\frac{\mathbf{d}}{\mathbf{dy}} < \mathbf{TrV}^\dagger_\mathbf{x}\,\mathbf{V}_\mathbf{y} >= -\frac{\bar{\alpha}_\mathbf{s}}{2\pi}\int \mathbf{d}^2\mathbf{z}\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}\times$$

$$\left[< \mathbf{TrV}^\dagger_\mathbf{x}\,\mathbf{V}_\mathbf{y} > -\frac{1}{\mathbf{N_c}} < \mathbf{TrV}^\dagger_\mathbf{x}\,\mathbf{V}_\mathbf{z} >< \mathbf{TrV}^\dagger_\mathbf{z}\,\mathbf{V}_\mathbf{y} > \right]$$

higher point functions are expressed in terms of the dipole (2-point function)

NLO: B-KW-G-BC (2007-2008)

Running coupling BK

- Output: Modified evolution kernel:

⇒ Leading order:

$$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, K^{LO}(\underline{r},\underline{r}_1,\underline{r}_2)\ [S(\underline{x},\underline{z})\, S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})]$$

↓

⇒ Running coupling:

$$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, \tilde{K}(\underline{r},\underline{r}_1,\underline{r}_2)\ [S(\underline{x},\underline{z})\, S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})]$$

$$\tilde{K}_{Bal}(\underline{r},\underline{r}_1,\underline{r}_2) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

CGC:QCD at high gluon density

multiple scatterings → **p_t broadening**

"Cronin" effect

evolution with ln (1/x) → **suppression**

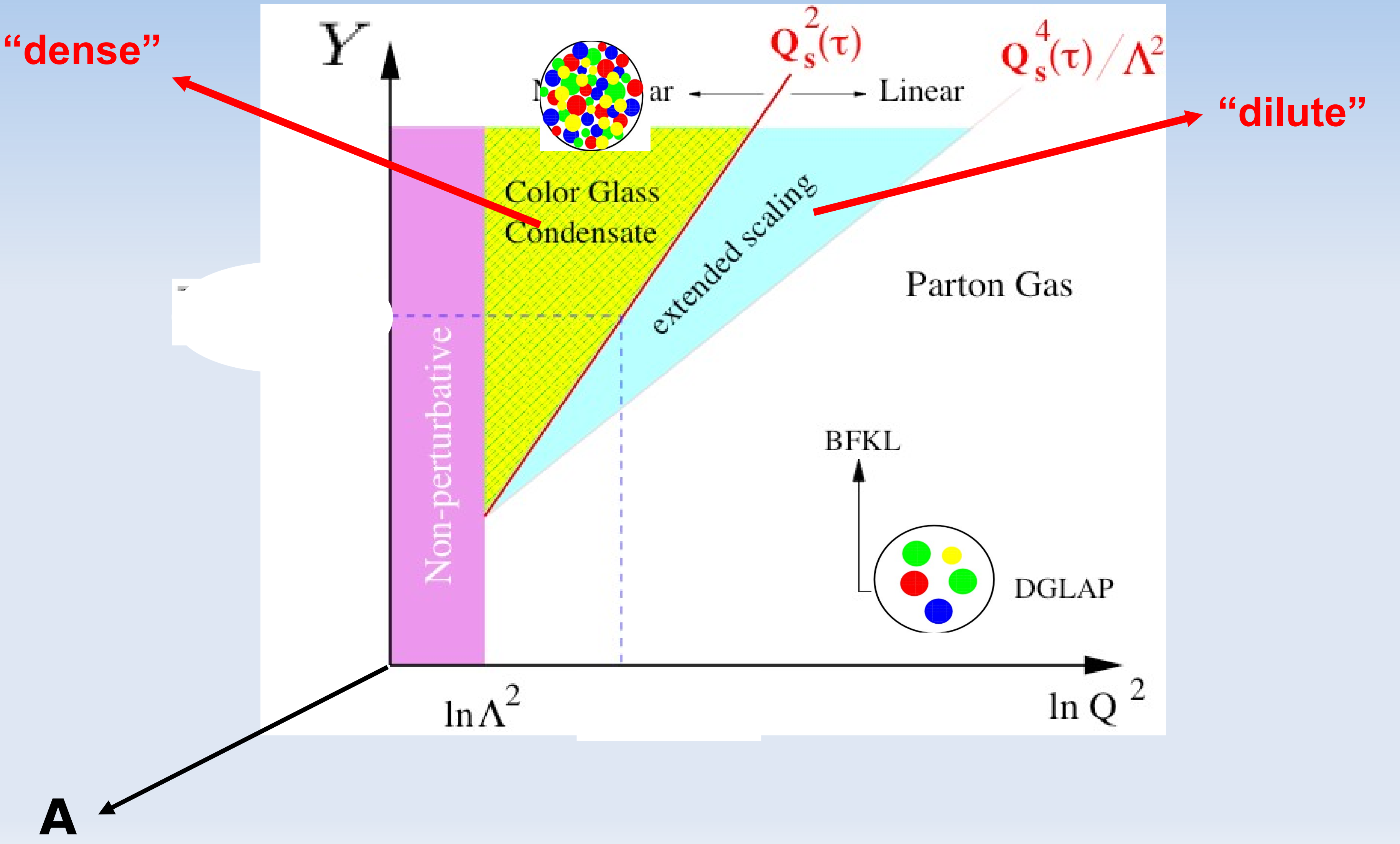
"Leading twist" nuclear shadowing

effective degrees of freedom: Wilson line V (x_t)

In CGC observables are expressed in terms of

$< tr\, V \,.......\, V >$

Road Map of QCD Phase Space



Probing CGC

Inclusive: structure functions, multiplicities

Single inclusive production

transverse momentum, rapidity, centrality dependence

Double inclusive production

azimuthal angular correlations

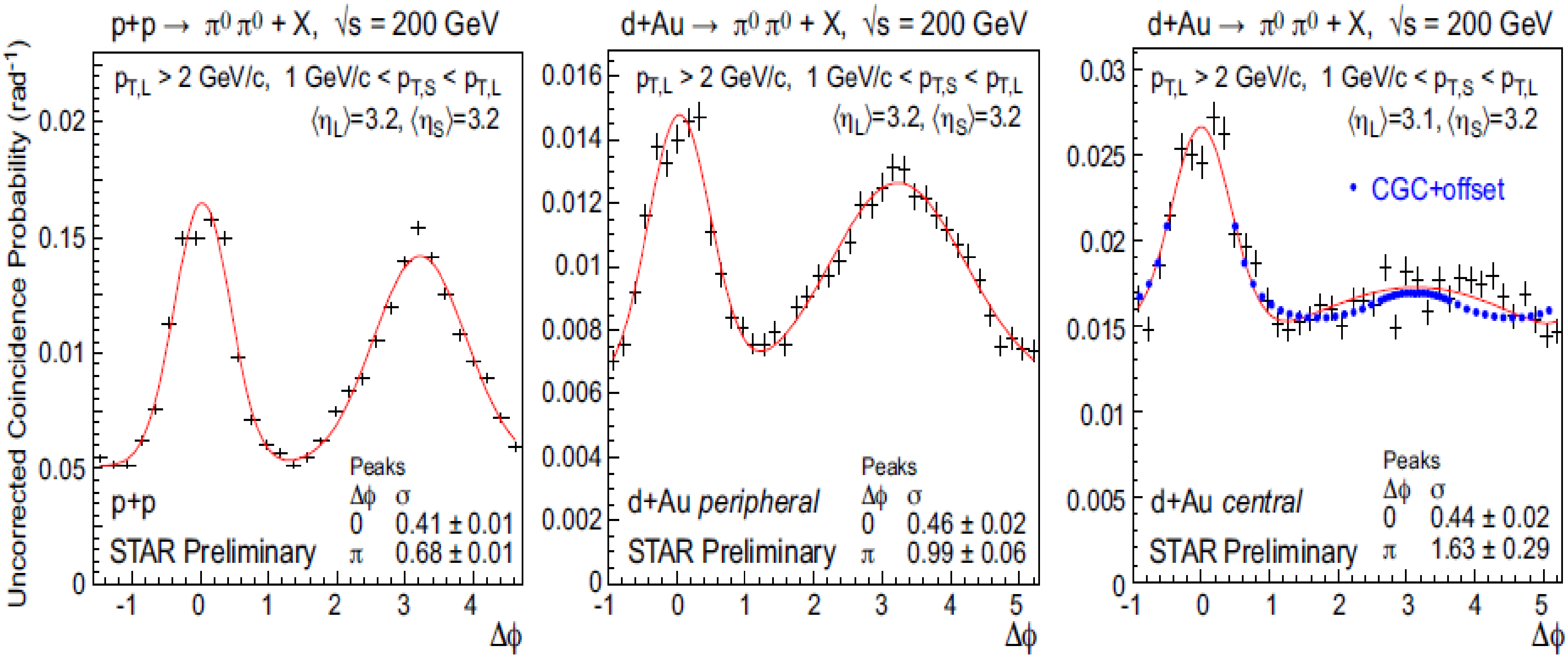
*long range rapidity correlations - **The Ridge***

centrality dependence

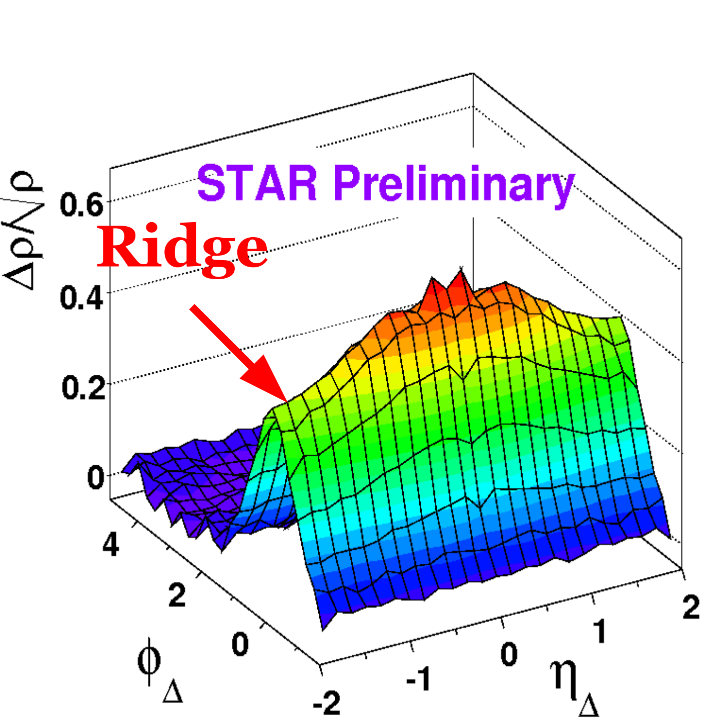
....

azimuthal angular correlations

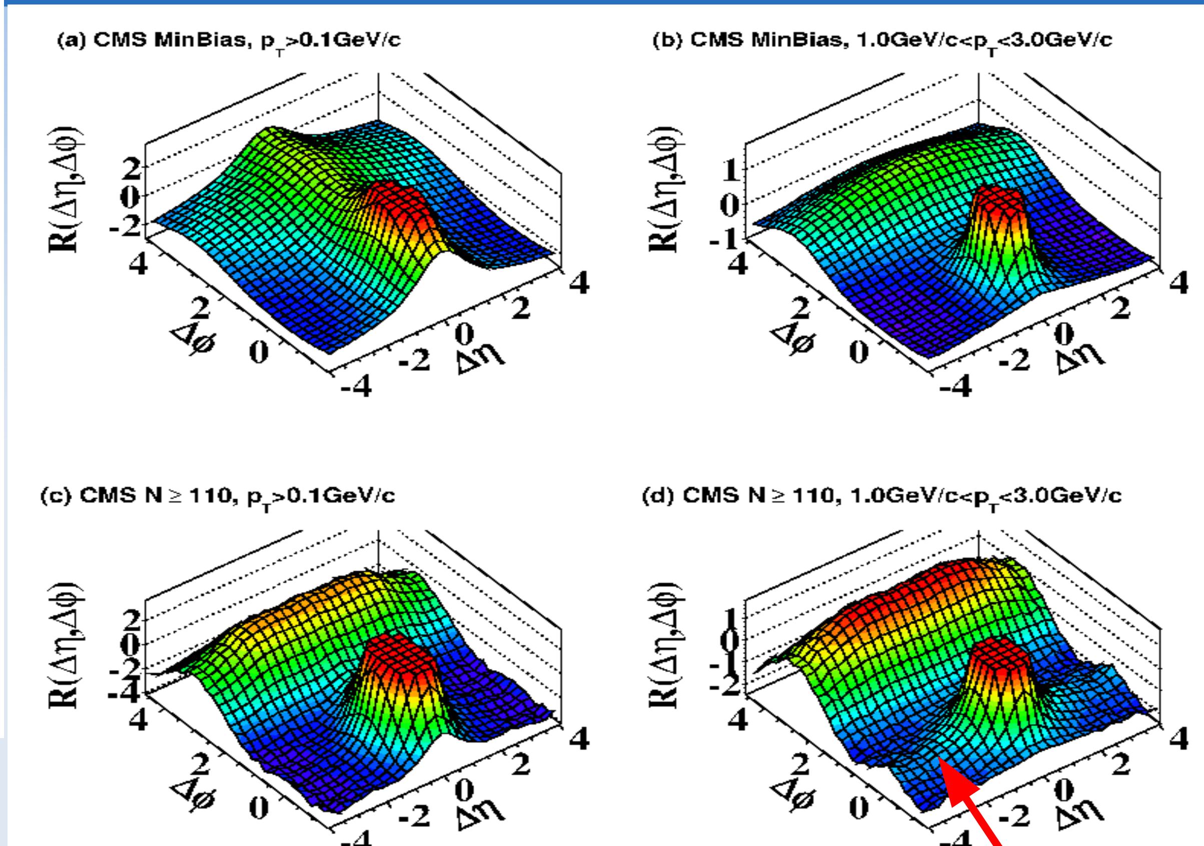
Recent STAR measurement (arXiv:1008.3989v1):



The Ridge



AA at RHIC



PP at LHC

Ridge

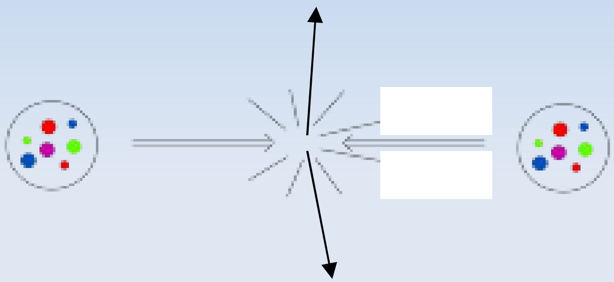
long-range rapidity correlations

Di-hadron production in pA: CGC

produced partons: $k_1, y_1 \qquad k_2, y_2$

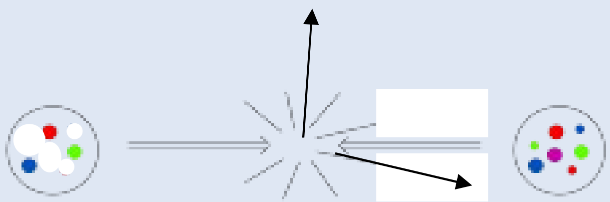
$$x_p = \frac{k_1\, e^{y_1} + k_2\, e^{y_2}}{\sqrt{s}} \qquad x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}}$$

scanning the wave-functions $\quad k_1 \sim k_2 \sim k \sim 2\, GeV$



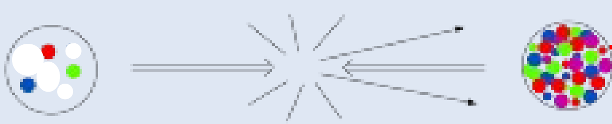
mid-mid correlations
probe moderate x

$$x_p \sim x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$



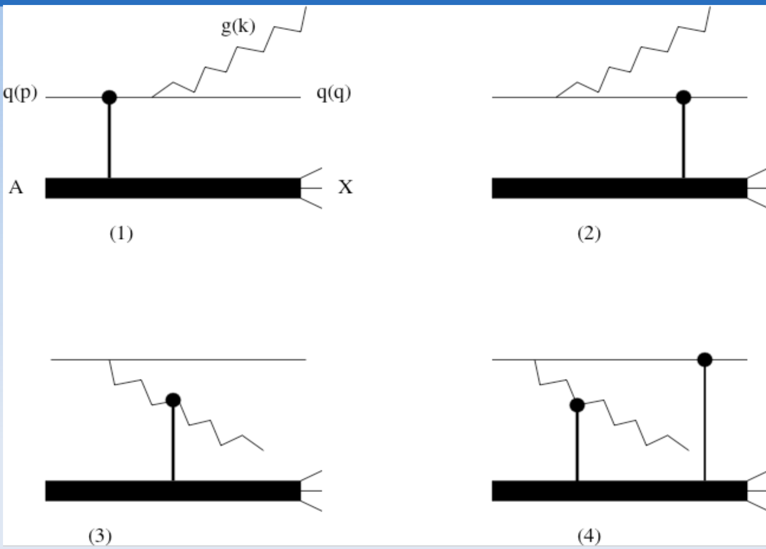
mid-forward

$$x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \to 1 \quad x_A \sim \frac{k}{\sqrt{S}} \sim 10^{-2}$$



forward-forward $\quad x_p \sim \frac{k}{\sqrt{S}}\, e^{y_1} \to 1 \quad x_A \sim \frac{k}{\sqrt{S}}\, e^{-y_2} \sim 10^{-4}$

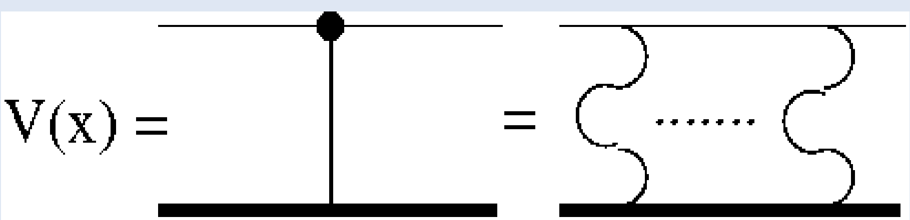
Di-jet production: pA

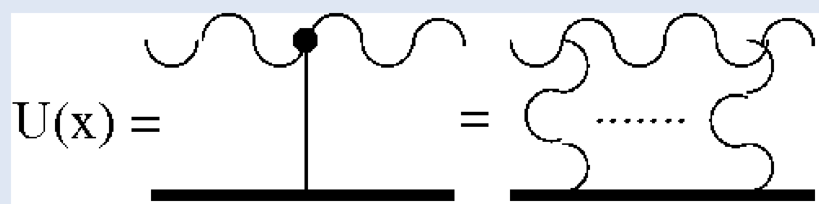


$$\frac{d\sigma^{dA\to q\,g\,X}}{d\,p_t^2\,dy_1\,d\,q_t^2\,dy_2} \sim \int K \otimes$$

$$[< \mathrm{Tr}V^\dagger\, V > + < \mathrm{Tr}V^\dagger\, V\, V^\dagger\, V >$$

JJM and YK, PRD70 (2004), AK and ML, JHEP (2006), FGV, NPA (2006), CM, NPA796 (2007) , KT, NPA (2010), DMXY (2011)

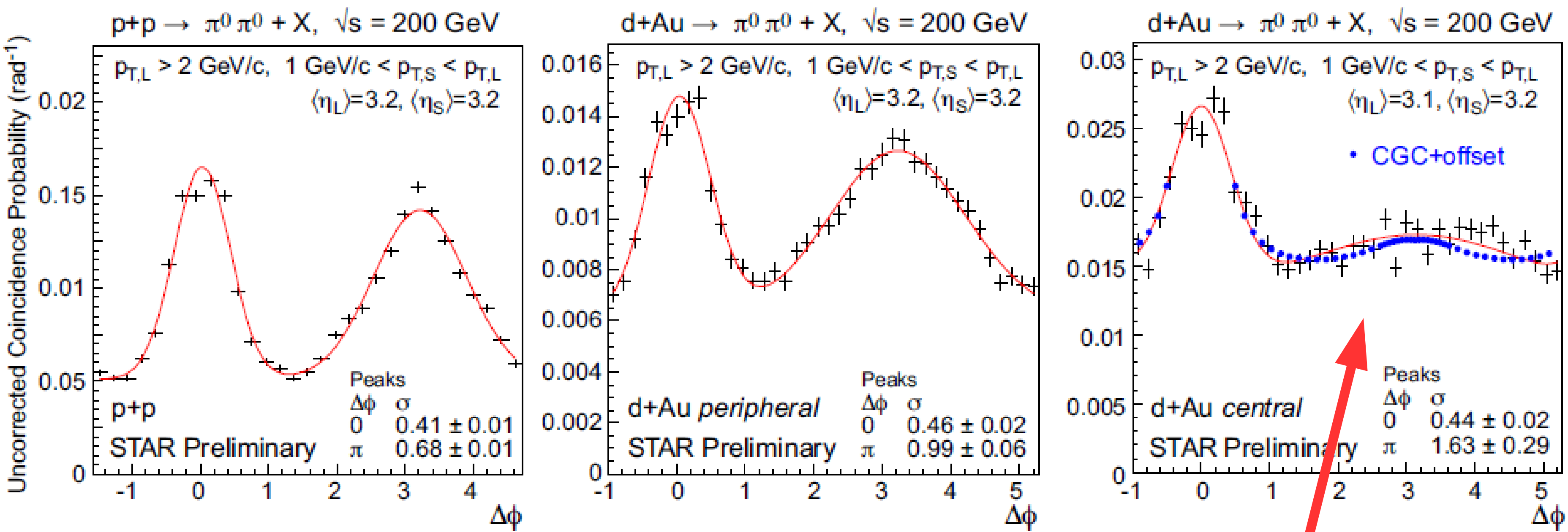




$$U^{ab}(x_t)\, t^b = V^\dagger(x_t)\, t^a\, V(x_t)$$

disappearance of back to back jets

Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from
Albacete + Marquet, PRL (2010)
using running coupling BK solution,
Also by Tuchin, NPA846 (2010)

multiple soft scatterings de-correlate the hadrons

Di-jet production: pA

$O_2(r,\bar{r}) \equiv Tr V_r\, V^\dagger_{\bar{r}}$ ← **F2 in DIS, single hadron in pA**

$$O_4(r,\bar{r}:s) \equiv Tr V^\dagger_r\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\left[Tr V^\dagger_r\, V_s\;\; Tr V_{\bar{r}}\, V^\dagger_s - \frac{1}{N_c} Tr V^\dagger_r\, V_{\bar{r}}\right]$$

$$O_6(r,\bar{r}:s,\bar{s}) \equiv Tr V_r\, V^\dagger_{\bar{r}}\, t^a\, t^b\, [U_s\, U^\dagger_{\bar{s}}]^{ba} = \frac{1}{2}\left[Tr V_r\, V^\dagger_{\bar{r}}\, V_{\bar{s}}\, V^\dagger_s\; Tr V_s\, V^\dagger_{\bar{s}} - \frac{1}{N_c} Tr V_r\, V^\dagger_{\bar{r}}\right]$$

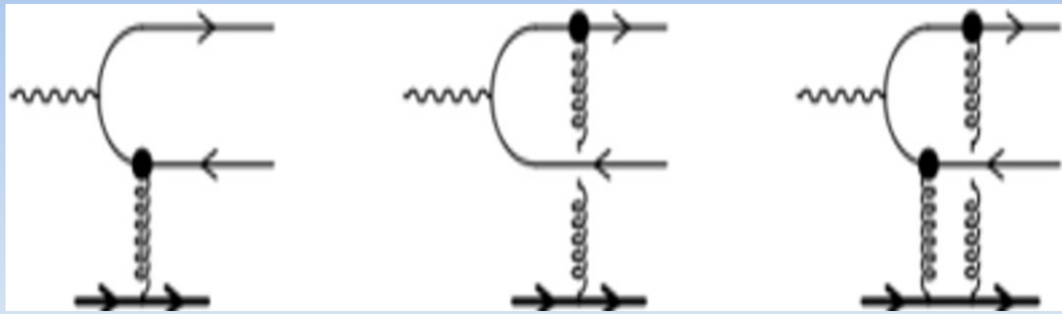
Dipole + large N_c approximation?

$$\begin{aligned}
\langle O_4(r,\bar{r}:s)\rangle &\simeq \langle O_2(r-s)\rangle\, \langle O_2(s-\bar{r})\rangle \\
\langle O_6(r,\bar{r}:s,\bar{s})\rangle &\simeq \langle O_2(r-s)\rangle\, \langle O_2(\bar{r}-\bar{s})\rangle\, \langle O_2(s-\bar{s})\rangle \\
&+ \langle O_2(r-\bar{r})\rangle\, \langle O_2(\bar{s}-s)\rangle\, \langle O_2(s-\bar{s})\rangle
\end{aligned}$$

Di-jet production: DIS

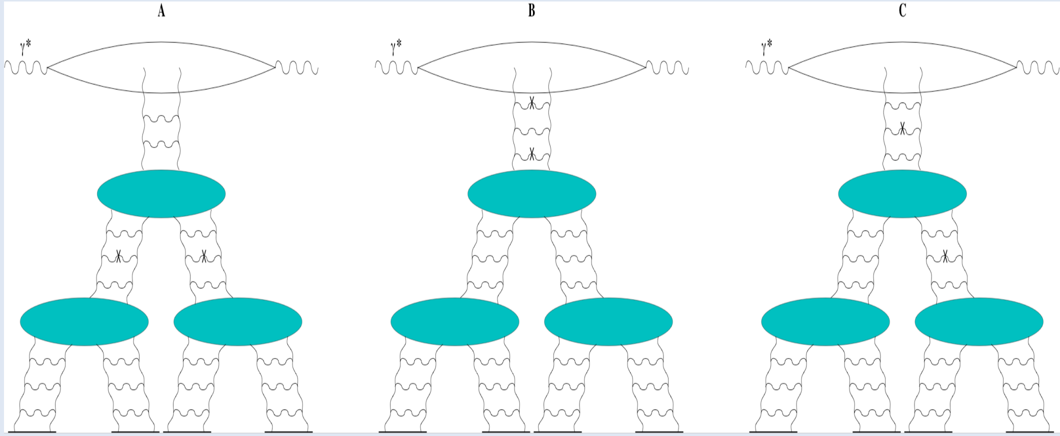
$$\gamma^\star\, \mathbf{p}(\mathbf{A}) \rightarrow \mathbf{q}\, \bar{\mathbf{q}}\, \mathbf{X}$$

FG & JJM, PRD67 (2003)
DMXY (2011)



$$\gamma^\star\, \mathbf{p}(\mathbf{A}) \rightarrow \mathbf{g}\, \mathbf{g}\, \mathbf{X}$$

JJM & YK, PRD70 (2004)
AK & ML, JHEP (2006)



di-jet production in DIS probes higher point functions

Beyond dipole + large Nc approximation

Recall evolution of O2 is sensitive to O4 only

$$\frac{d}{dy}\langle O_4(r,\bar{r}:s)\rangle = -\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\,\Big\langle 2\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right]O_4(r,\bar{r}:s)$$
$$-\frac{1}{N_c}\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}\,TrV_r^\dagger V_z\;\;TrV_s^\dagger V_{\bar{r}}\;\;TrV_z^\dagger V_s\right.$$
$$\left.+\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}TrV_r^\dagger V_s\;\;TrV_z^\dagger V_{\bar{r}}\;\;Tr\,V_s^\dagger V_z\right]\Big\rangle+\cdots\cdots$$

$$with\quad S_4\equiv\frac{1}{C_A\,C_F}\,\langle O_4\rangle\quad and\quad S_2\equiv\frac{1}{C_A}\,\langle O_2\rangle$$

$$\mathbf{\frac{d}{dy}S_4(r,\bar{r}:s)\simeq\frac{d}{dy}[S_2(s-\bar{r})\,S_2(r-s)]+O(\frac{1}{N_c^2})}$$

DIS structure functions,single inclusive production in pA probe dipoles

Beyond dipole + large N_c approximation

$$\frac{d}{dy}\langle O_6(r,\bar r:s,\bar s)\rangle = -\frac{N_c\,\alpha_s}{2(2\pi)^2}\int d^2z\Big\langle 2\,[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}$$

$$+3\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}]\,O_6(r,\bar r:s,\bar s)-\frac{1}{N_c}\Big[$$

$$[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(s-\bar r)^2}{(s-z)^2(\bar r-z)^2}]\,TrV_z\,V_{\bar r}^\dagger\,V_{\bar s}\,V_s^\dagger\;\;TrV_r\,V_z^\dagger\;\;TrV_s\,V_{\bar s}^\dagger$$

$$+[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}]\,TrV_r\,V_z^\dagger\,V_{\bar s}\,V_s^\dagger\;\;TrV_z\,V_{\bar r}^\dagger\;\;TrV_s\,V_{\bar s}^\dagger$$

$$+[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}]\,TrV_r\,V_{\bar r}^\dagger\,V_{\bar s}\,V_z^\dagger\;\;TrV_z\,V_s^\dagger\;\;TrV_s\,V_{\bar s}^\dagger$$

$$+[\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}]\,TrV_r\,V_{\bar r}^\dagger\,V_z\,V_s^\dagger\;\;TrV_{\bar s}\,V_z^\dagger\;\;TrV_s\,V_{\bar s}^\dagger$$

$$+2\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}\,TrV_r\,V_{\bar r}^\dagger\,V_{\bar s}\,V_s^\dagger\;\;TrV_s\,V_z^\dagger\,TrV_z\,V_{\bar s}^\dagger\;+$$

$$[\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}-\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}]TrV_r\,V_s^\dagger\;\;Tr\,V_{\bar r}^\dagger\,V_{\bar s}\;\;TrV_s\,V_{\bar s}^\dagger$$

$$+[\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}+\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}-\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}-\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}]TrV_r\,V_{\bar r}^\dagger\,TrV_{\bar s}\,V_s^\dagger\;\;TrV_s\,V_{\bar s}^\dagger$$

these are the leading N_c pieces

Beyond dipole + large N_c approximation

$$-\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]TrV_r\,V^\dagger_{\bar{r}}\,V_{\bar{s}}\,V^\dagger_s\,V_z\,V^\dagger_{\bar{s}}\,V_s\,V^\dagger_z$$

$$+\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]TrV_r\,V^\dagger_{\bar{r}}\,V_z\,V^\dagger_{\bar{s}}\,V_s\,V^\dagger_z\,V_{\bar{s}}\,V^\dagger_s$$

$$+\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]$$

$$TrV_r\,V^\dagger_z\,V_s\,V^\dagger_{\bar{s}}\,V_z\,V^\dagger_{\bar{r}}\,V_{\bar{s}}\,V^\dagger_s-2\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\,TrV_r\,V^\dagger_{\bar{r}}\,V_{\bar{s}}\,V^\dagger_z\,V_s\,V^\dagger_{\bar{s}}\,V_z\,V^\dagger_s$$

$$-\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]$$

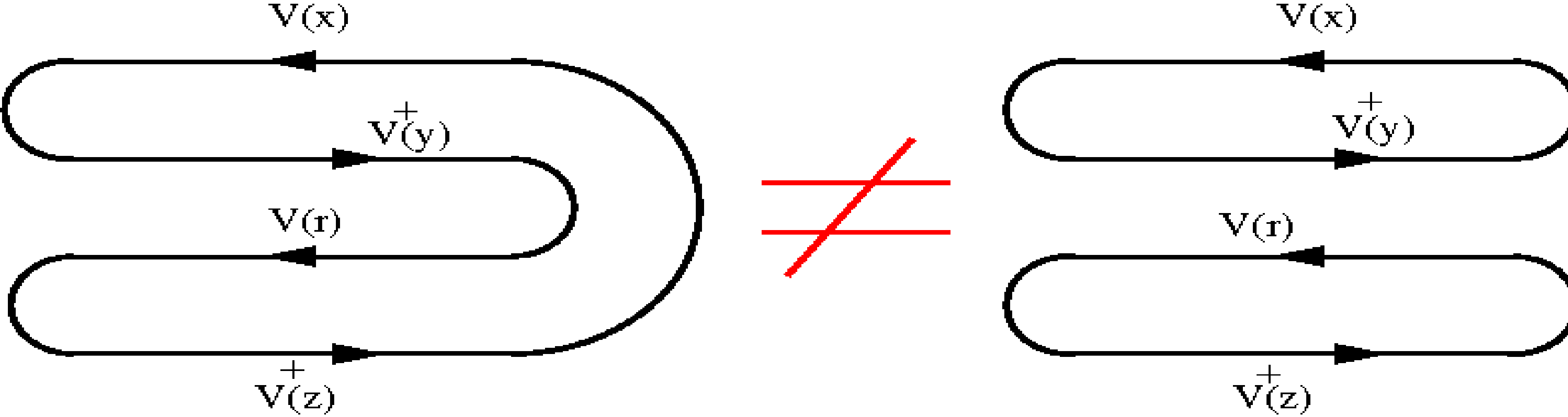
$$TrV_r\,V^\dagger_{\bar{s}}\,V_s\,V^\dagger_{\bar{r}}\,V_{\bar{s}}\,V^\dagger_s$$

$$-\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]\,TrV_r\,V^\dagger_{\bar{r}}\Bigg]$$

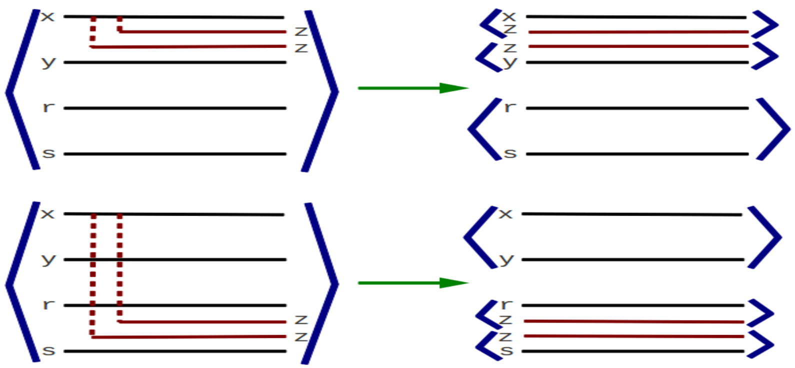
$$+\quad\frac{1}{N_c^2}\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}\,TrV^\dagger_r\,V_z\;\;TrV^\dagger_z\,V_{\bar{r}}\Bigg\rangle-\frac{1}{4\,N_c}\frac{d}{dy}\;<TrV^\dagger_r\,V_{\bar{r}}>$$

these are the $\mathrm{O}(\frac{1}{\mathrm{N_c^2}})$ *pieces*

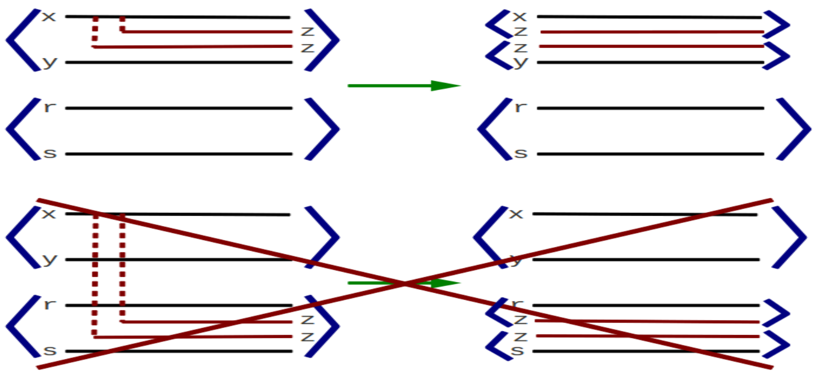
Quadrupoles vs. dipoles



and they evolve differently even at large N_c



JIMWLK



Dipole approximation

JIMWLK: Beyond dipole + large Nc

Di-jet production probes quadrupoles

$$\mathbf{S_6} \equiv \frac{\mathbf{1}}{\mathbf{C_A\, C_F}} \langle \mathbf{O_6} \rangle$$

$$\frac{d}{dy} S_6(r, \bar{r} : s, \bar{s}) \neq \frac{d}{dy} \left[S(r-s)\; S(\bar{s}-\bar{r})\; S(s-\bar{s}) + \cdots\cdots \right]$$

dipole approximation is not valid!

simplifies in some kinematics
dijet momentum imbalance: WW gluons

F. Dominguez et al.

JIMWLK: Beyond dipole + large Nc

Dijet production poses new challenges to CGC
but
every challenge can become an opportunity

Quadrupoles

What is their energy dependence ?
How large are the N_c suppressed terms ?
....

solving JIMWLK: in progress

B. Schenke

High energy QCD: Color Glass Condensate

A new region of QCD phase space

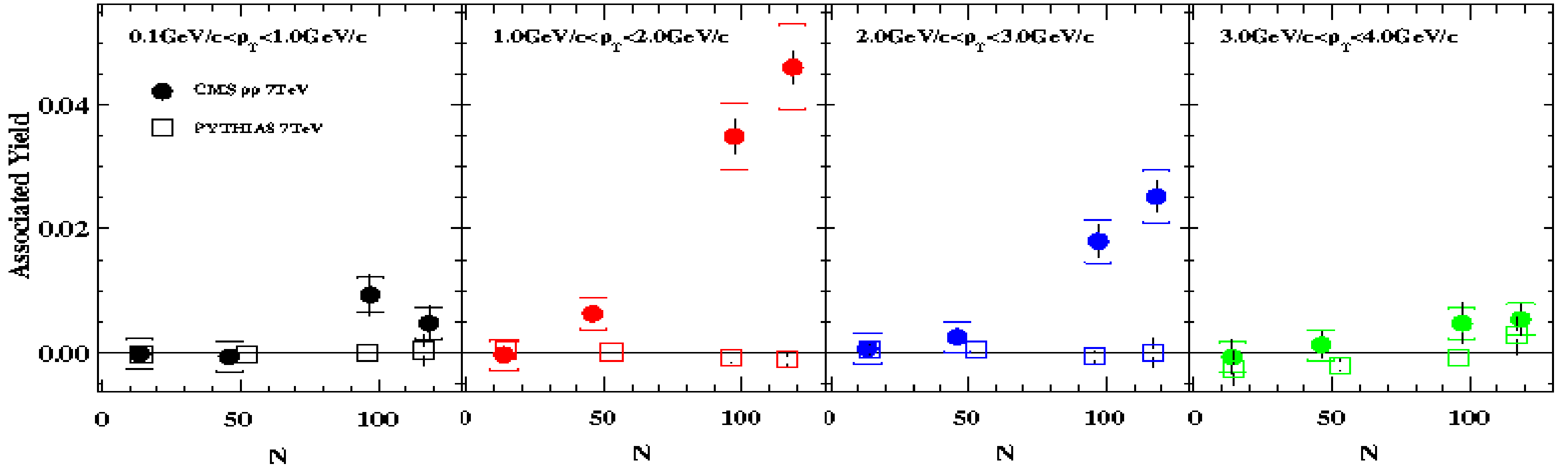
A systematic approach with controlled approximations

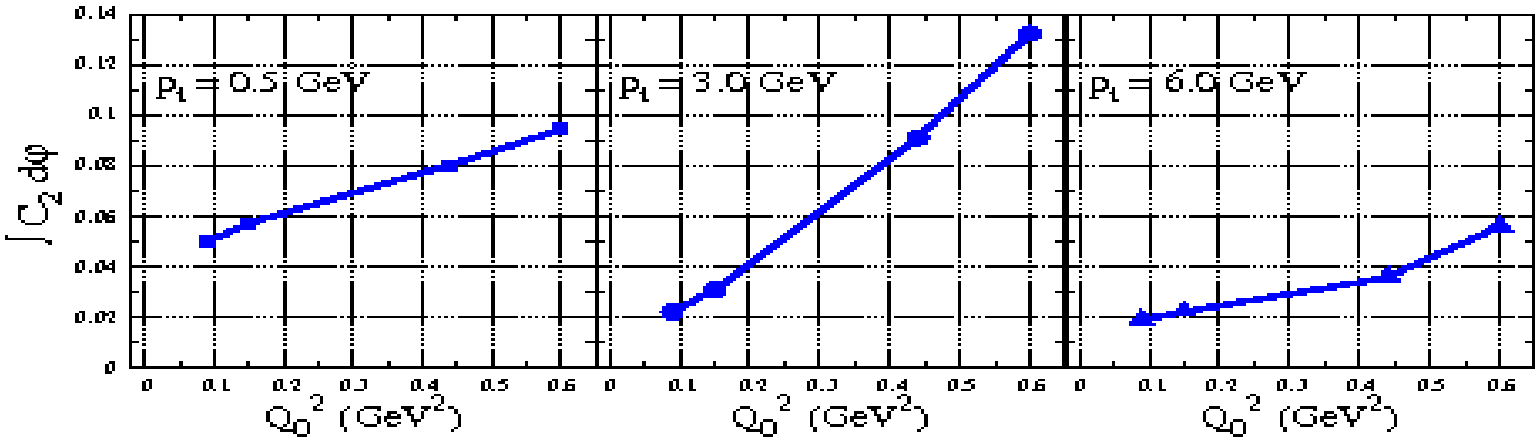
Q_s: a dynamical semi-hard scale

Evidence for CGC at HERA, RHIC and now at LHC

2-hadron correlations probe the dynamics of CGC beyond dipoles

The CMS ridge at LHC





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