Gluonic Pole Matrix elements & Universality of TMD Fragmentation Fncts

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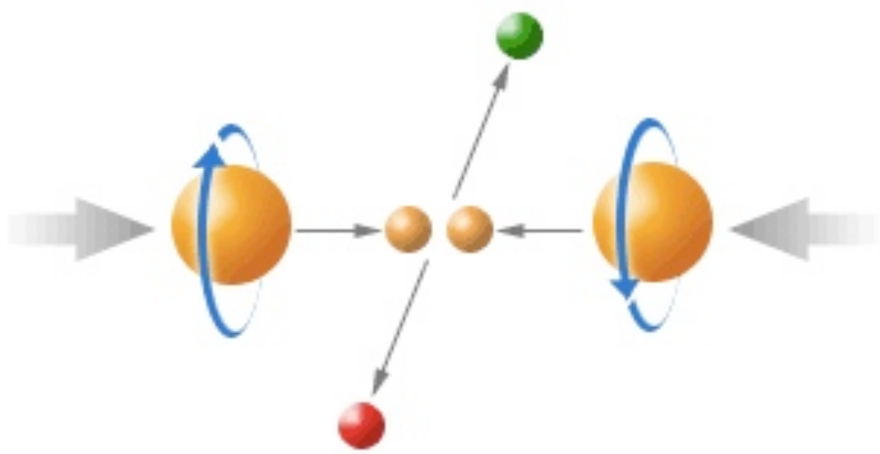
with A. Mukherjee & P. Mulders to appear PRD 2011---arXiv:1010.4556

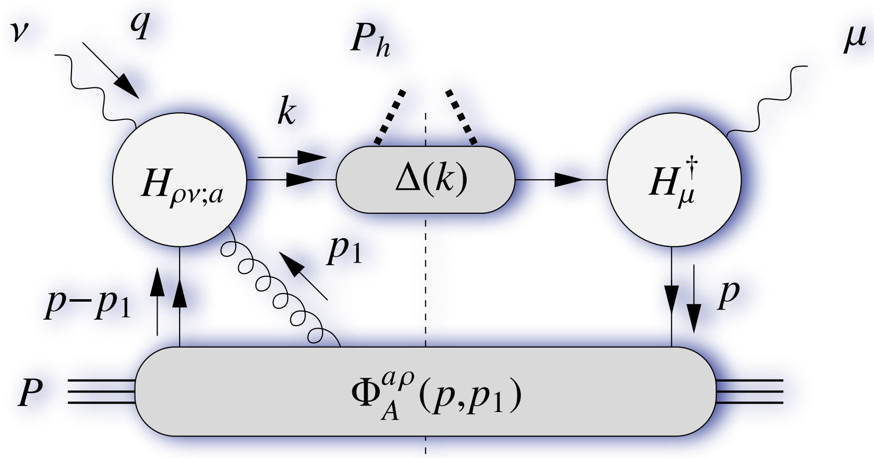
~~Outline~~

Pre-Summary (Pre-DIS)

- We use general properties of scattering amplitudes in QCD to study support properties of parton correlation functions in particular gluonic pole matrix elements
- Assuming analyticity and unitarity to hold for forward offshell-parton hadron scattering we uncover the singularity structure which determines support properties for PDFs and PFFs
- We show that single & mutiple gluon pole matrix elements vanish in the limit when the momentum of these gluons go to zero for fragmentation
- These techniques applied
  - Quark quark correlation (collinear) Landshoff Polkinghorne Short NPB 71, PRpts. 72 ...
  - Multi-parton correlators (collinear) Jaffe NPB 83
  - GPDs collinear correlators Diehl Gousset PLB 98, Belitsky & Radyushkin PRpts. 2005, Goldstein & Liuti arXive hep/ph 2010 ...

Hi-Energy Scattering Factorization





- Importance of "Transverse Structure" of Hadrons is accounted for in terms of quark and gluon correlators which are sensitive to k_T
- Observables are built from these correlations
- e.g. TSSAs & AAs

$$W_{\mu\nu} = \int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k) \Delta(k) H_\nu(p,k)\right]$$

$$\Phi^{\mathcal{U}^{[C]}}_{ij}(x,k_T) = \int \frac{d\xi^- \, d^2\xi_T}{(2\pi)^3} \, e^{ik\cdot\xi} \langle P| \overline{\psi}_j(0) \, \mathcal{U}^{[C]}_{[0;\xi]} \, \psi_i(\xi) \, |P\rangle\big|_{\xi^+=0=\mathrm{LF}}$$

$$\mathcal{U}^{[C]}_{[0;\xi]} = \mathcal{P} \mathrm{e}^{-ig\int_C ds\cdot A(s)}$$

Path ordered exponential Gauge link which ensures color GI

Factorization & Sensitivity to $P_T \sim k_\perp$ → TMDs

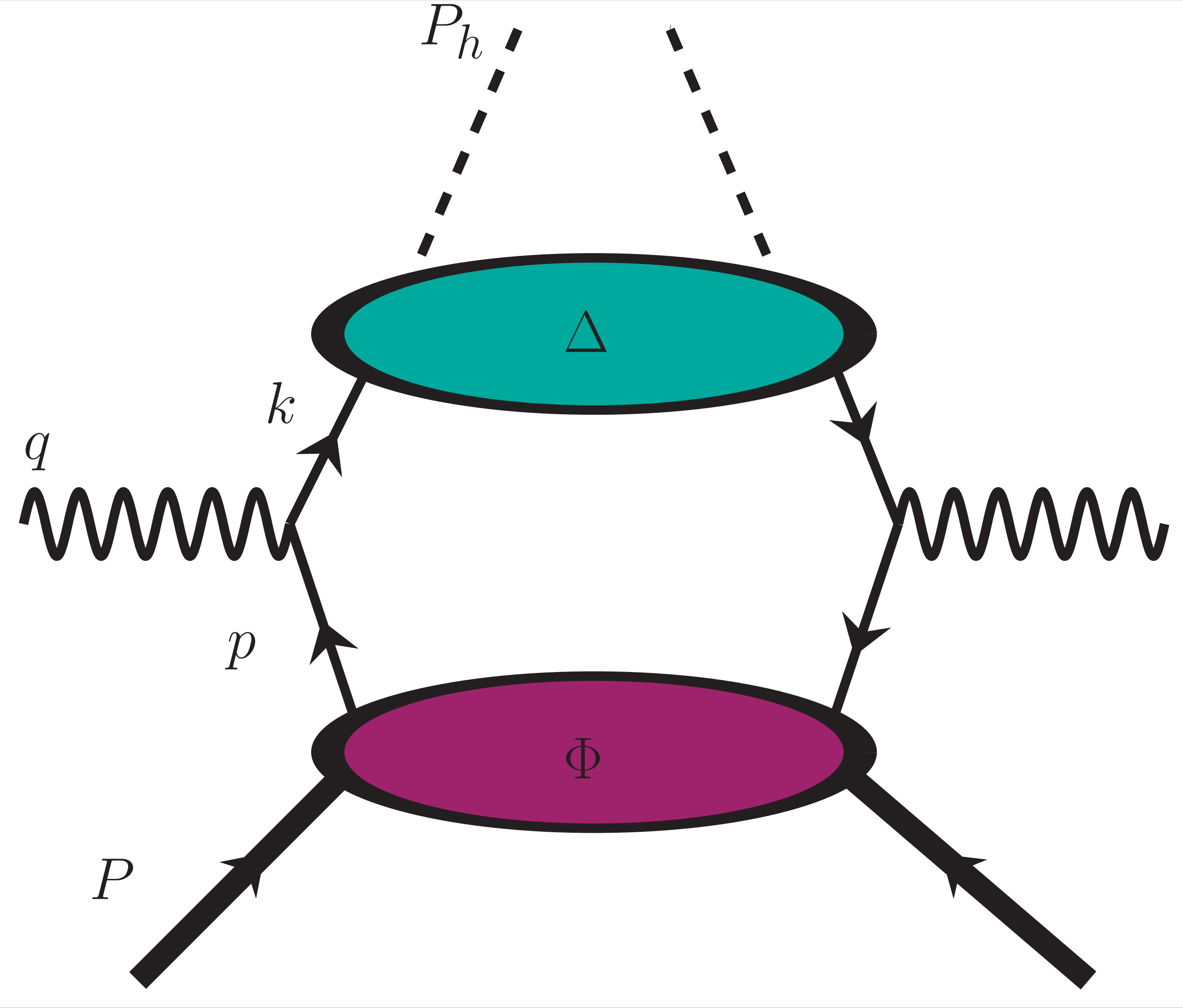
Tree level-Factorization Ralson & Soper NPB 1979-Drell Yan,
Mulders Tangerman NPB 1996-SIDIS

TSSAs in SIDIS

$$d^6\sigma = \hat{\sigma}_{\rm hard}\, \mathcal{C}[w f D]$$

Structure functions that are extracted

$$\mathcal{F}_{AB} = \mathcal{C}[w\, f\, D]$$



$$\mathcal{C}[wfD] = \sum_a x e_a^2 \int d^2\mathbf{p}_T d^2\mathbf{k}_T \delta^{(2)}\left(\mathbf{p}_T - \mathbf{k}_T - \frac{P_{h\perp}}{z}\right) f^a(x, p_T^2) D^a(z, k_T^2)$$

where $\boldsymbol{K}_T \equiv -\boldsymbol{k}_T z$

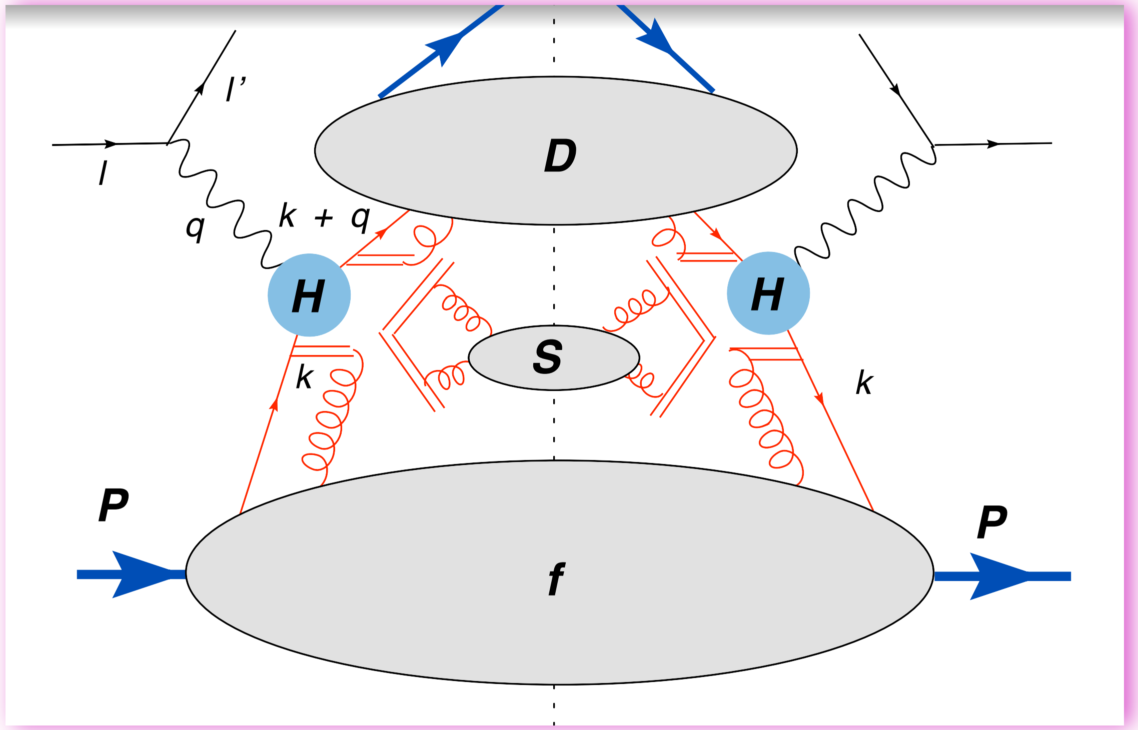
8 Leading Twist TMDs: Correlation Matrix Dirac space

$$\Phi^{[\gamma^+]}(x,\boldsymbol{p}_T) \equiv f_1(x,\boldsymbol{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x,\boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,\boldsymbol{p}_T) \equiv S_T^i\, h_{1T}(x,\boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, h_{1T}^{\perp}(x,\boldsymbol{p}_T^2)\right) + \frac{\epsilon_T^{ij} p_T^j}{M}\, h_1^{\perp}(x,\boldsymbol{p}_T^2)$$

		quark		
		U	L	T
nucleon	U	f_1		$h_1^{\perp}$ -
	L		g_1 -	$h_{1L}^{\perp}$ -
	T	$f_{1T}^{\perp}$ -	$g_{1T}^{\perp}$ -	h_1 - $h_{1T}^{\perp}$ -



More systematically beyond leading order ("tree level")

CS NPB 81, Collins Hautman PLB 00, Ji Ma Yuan PRD 05
see also Cherednikov Karanikas Stefanis NPB 10

See talks of Cherednikov, Collins and Abyat

soft gluon radiation

- Extra divergences at one loop and higher
- Various strategies to address them at one loop/higher
- Extra variables needed to regulate divergences
- Modifies convolution integral by introduction soft factor
- Will show cancels in certain weighted asymmetries

$$\mathcal{C}\left[H; w f S D\right] \equiv x_B H(Q^2,\mu^2,\rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(z p_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, S(\ell_T^2, \mu^2, \rho)\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

Hard

TMD

Soft

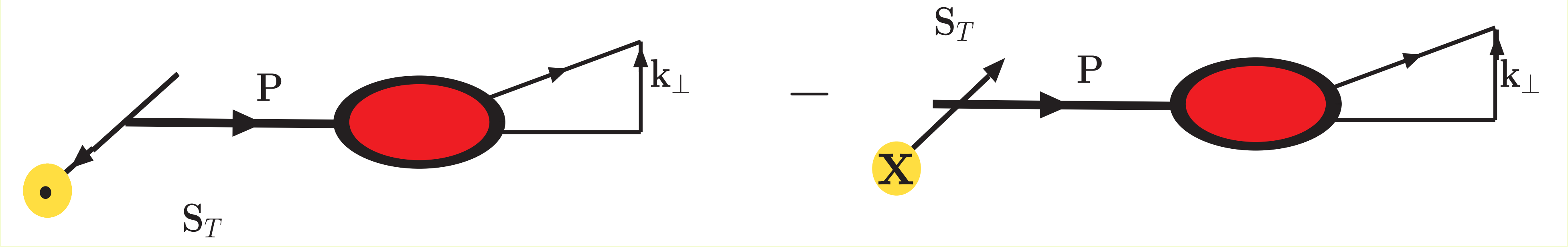
FF

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2005, also Bacchetta Boer Diehl Mulders JHEP 2008

Observable Effects
TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

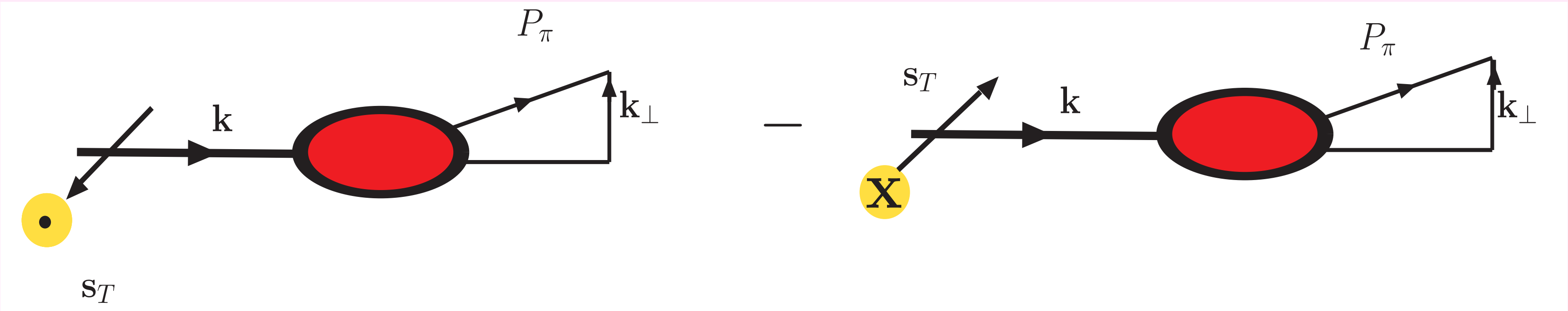
- Sivers PRD: 1990 TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron $\Longrightarrow$



$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born}$ $\quad$ $\boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp})}\, f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$

.... Fragmentation...

- Collins NPB: 1993 **TSSA is associated with** *transverse* **spin of fragmenting quark and transverse momentum of final state hadron**

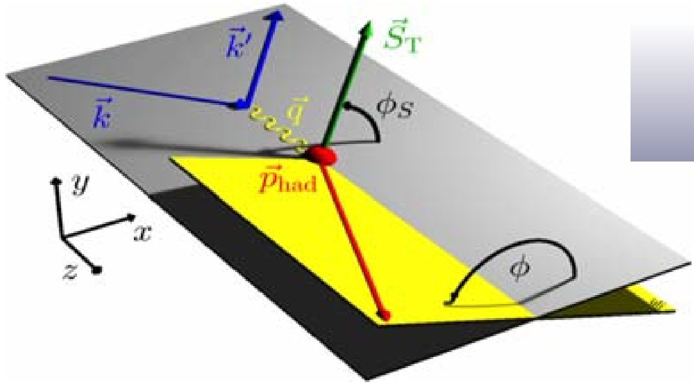


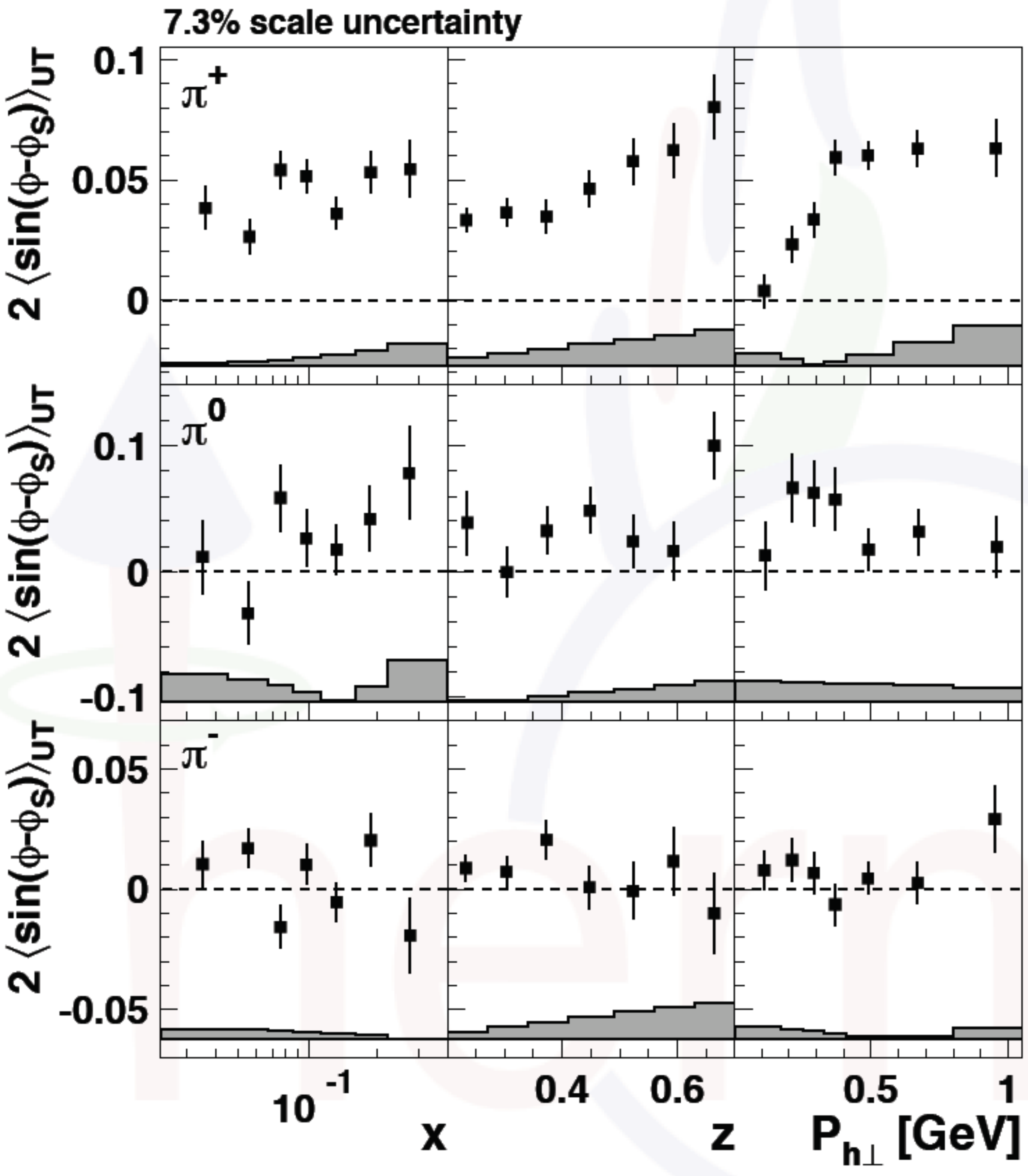
$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born}$$

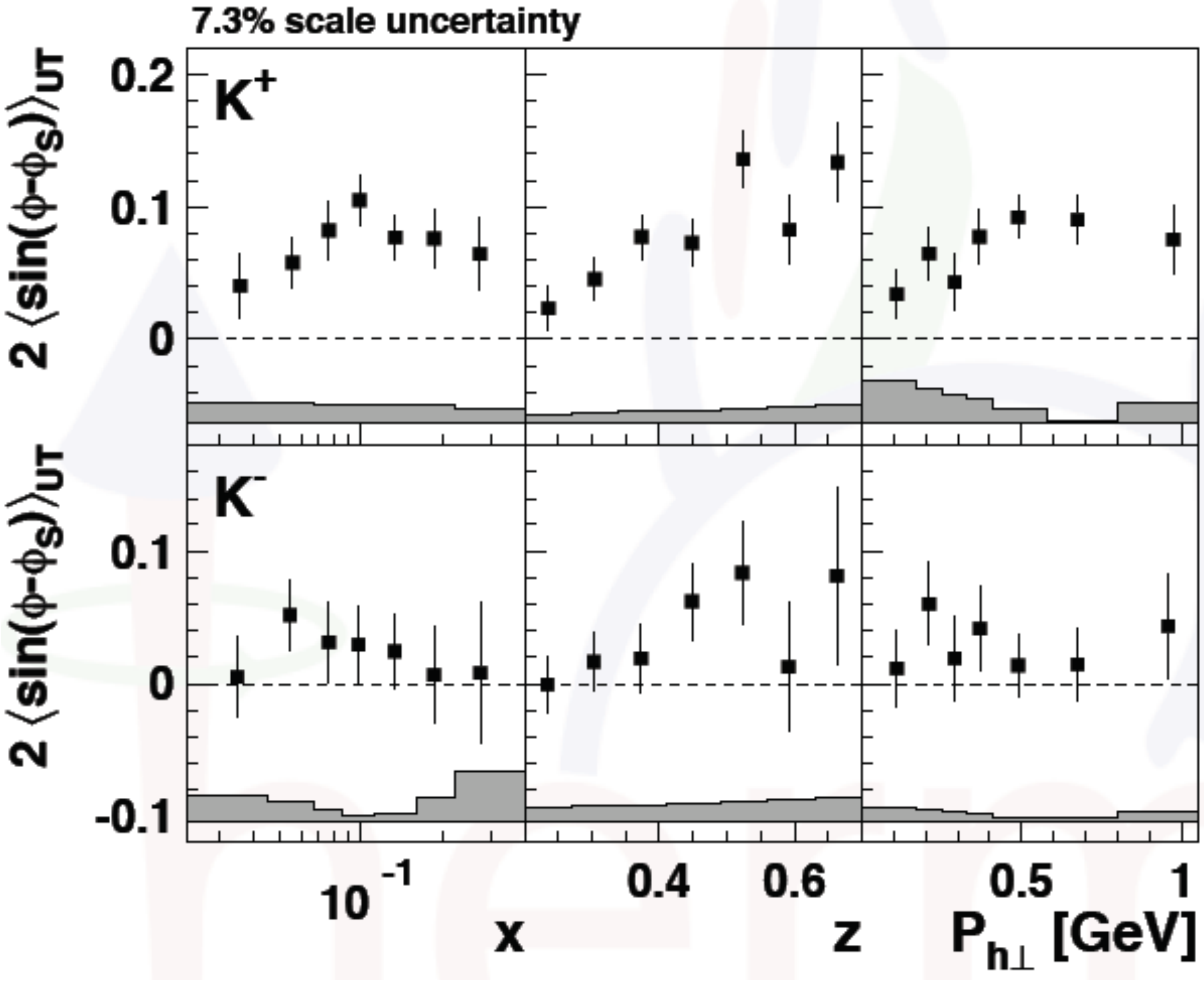
$$\boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)}\, H_1^\perp(x, \boldsymbol{p}_\perp)$$

Hermes PRL 2009

$$\ell p \to \ell' \pi X$$







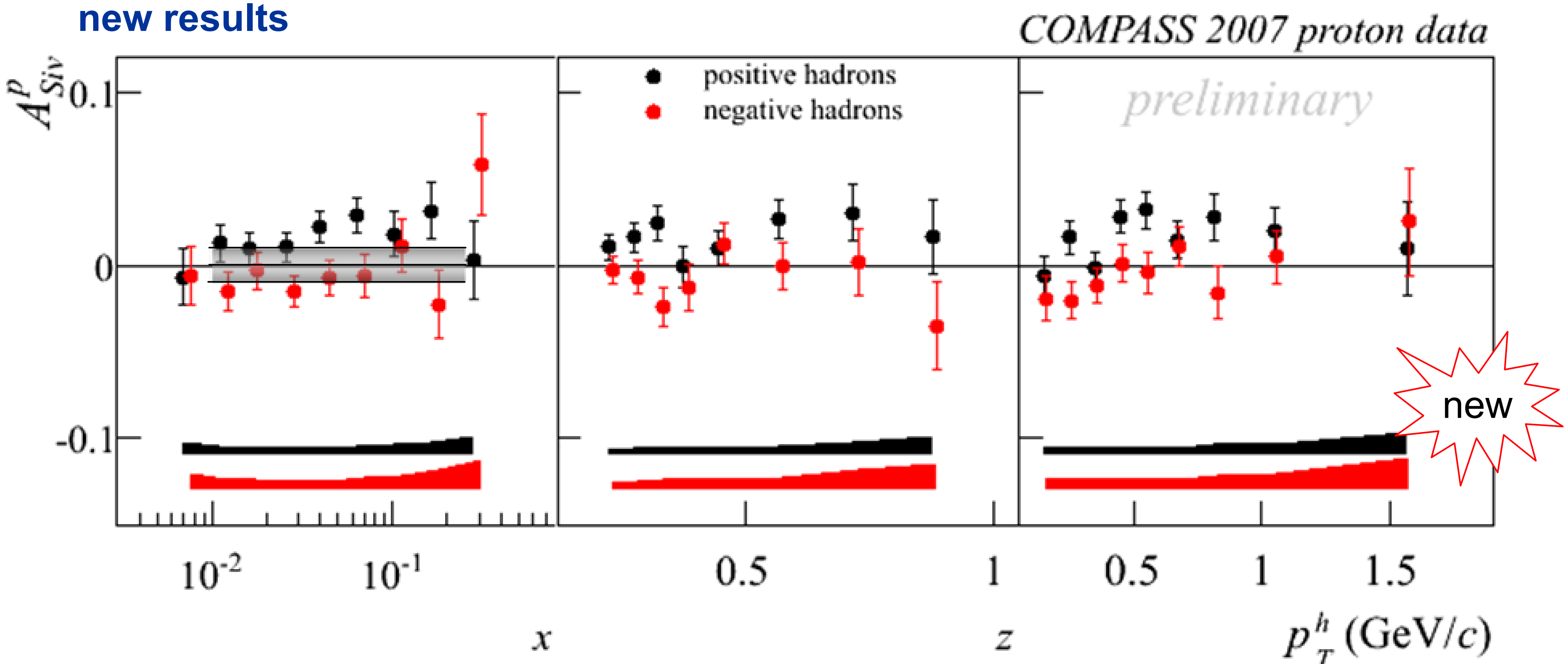
From Anna Martin DIS 2010

See talk of Krysz Kurek

Sivers asymmetry – proton data

the analysis of the 2007 data is over

new results



evidence for a positive signal for h^+,
which extends to small x, in the region not measured before

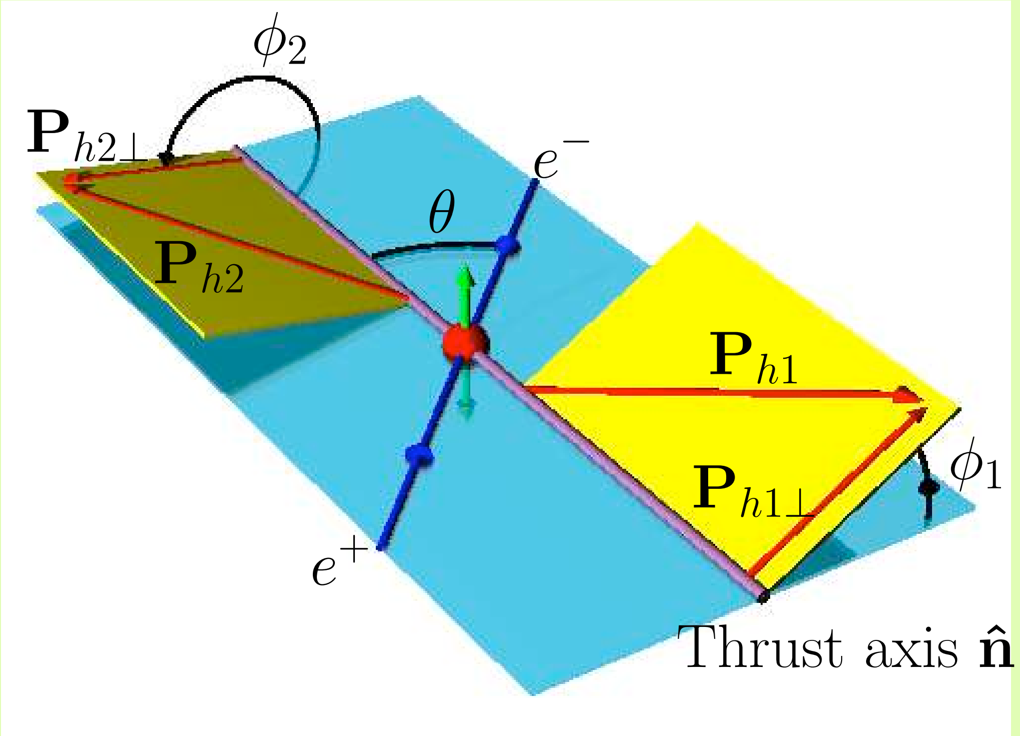
Reliability of Transversity Extraction Universality of Collins Fragmentation Function

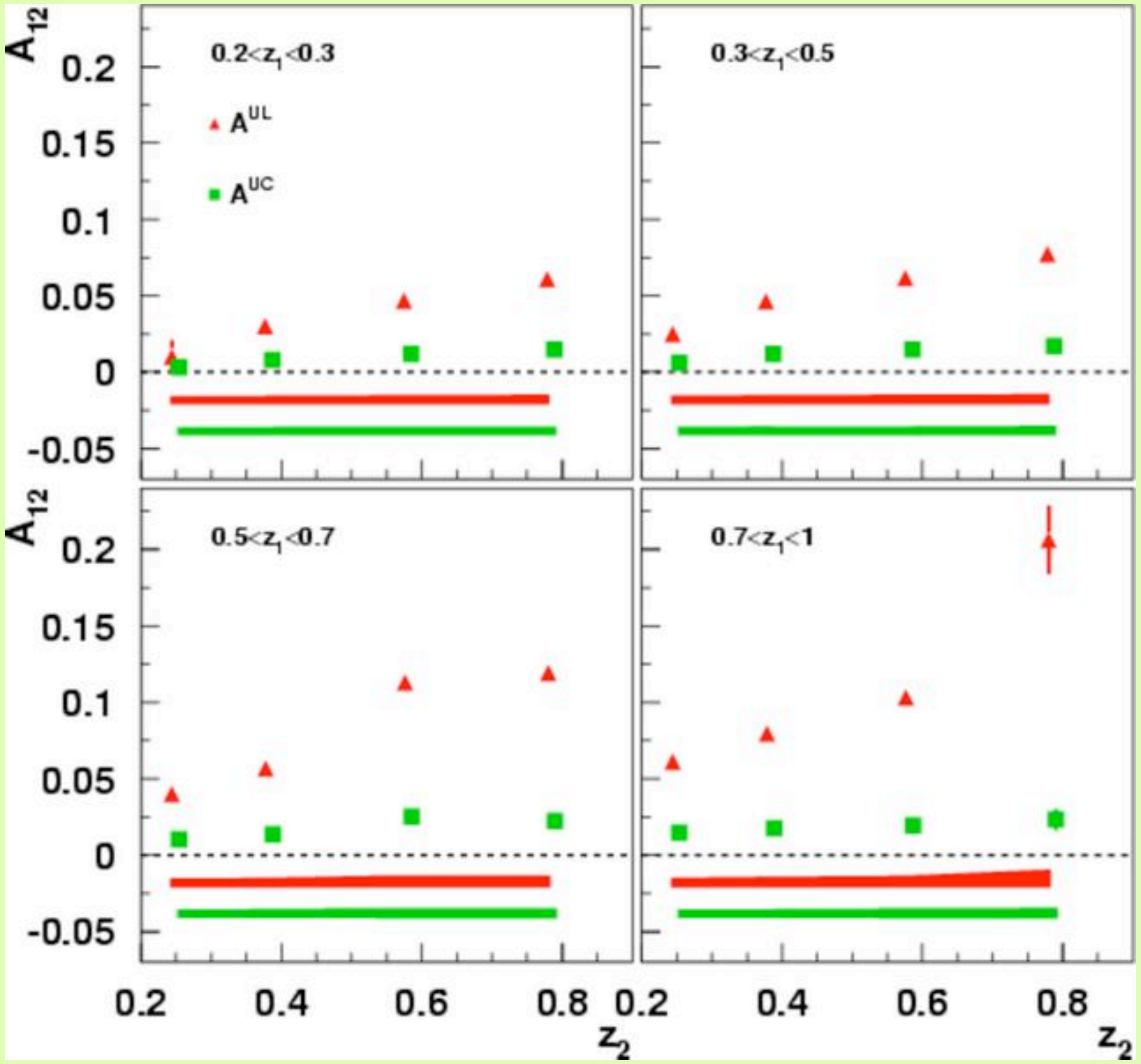
Belle KEKB measurement of the Collins Frag. Function PRL 2006 & arXiv:0805.2975

From talk of Ralf Seidl

$$\frac{d\sigma\left(e^+e^- \to h_1 h_2 X\right)}{d\Omega dz_1 dz_2 d^2 q_T} = \cdots B(\Theta)\cos(\varphi_1 + \varphi_2) H_1^{\perp[1]}(z_1)\overline{H}_1^{\perp[1]}(z_2)$$

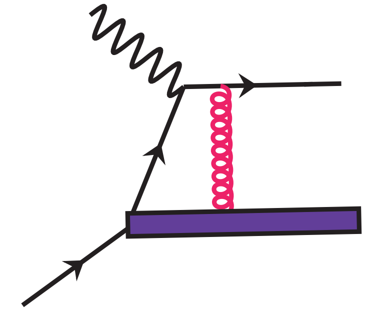
$$B(\Theta) \overset{cm}{=} \frac{1}{4}\sin^2\Theta$$





Ralf Seidl EIC Workshop, Hampton, VA May 08

Sivers



Anselmino et al. PRD 05, EPJA 08

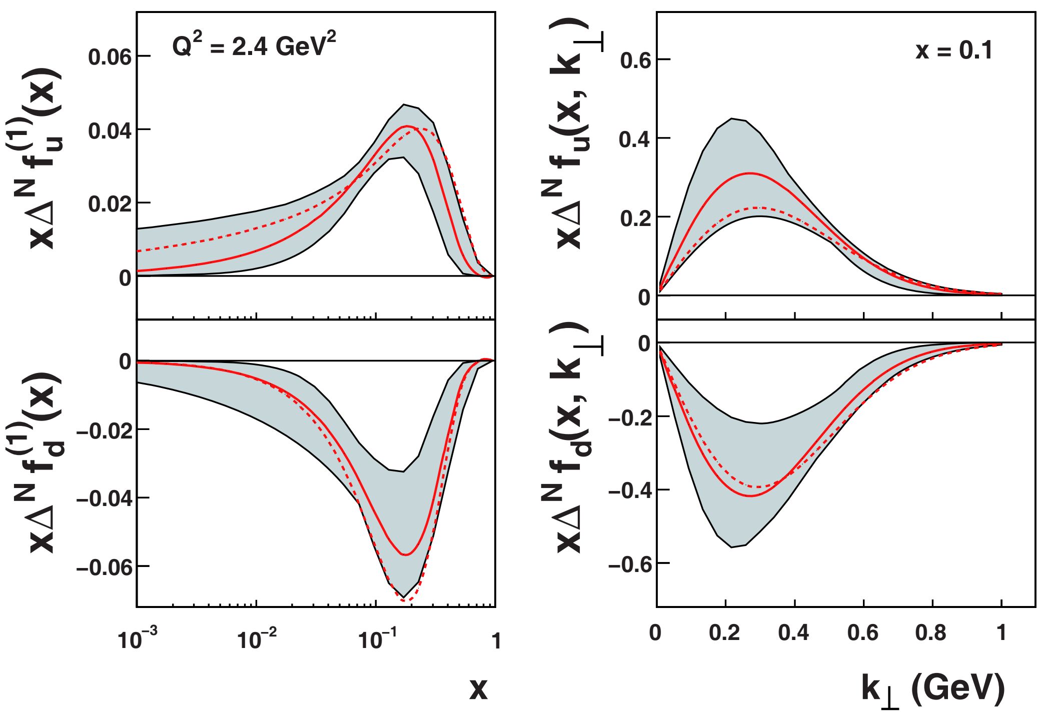
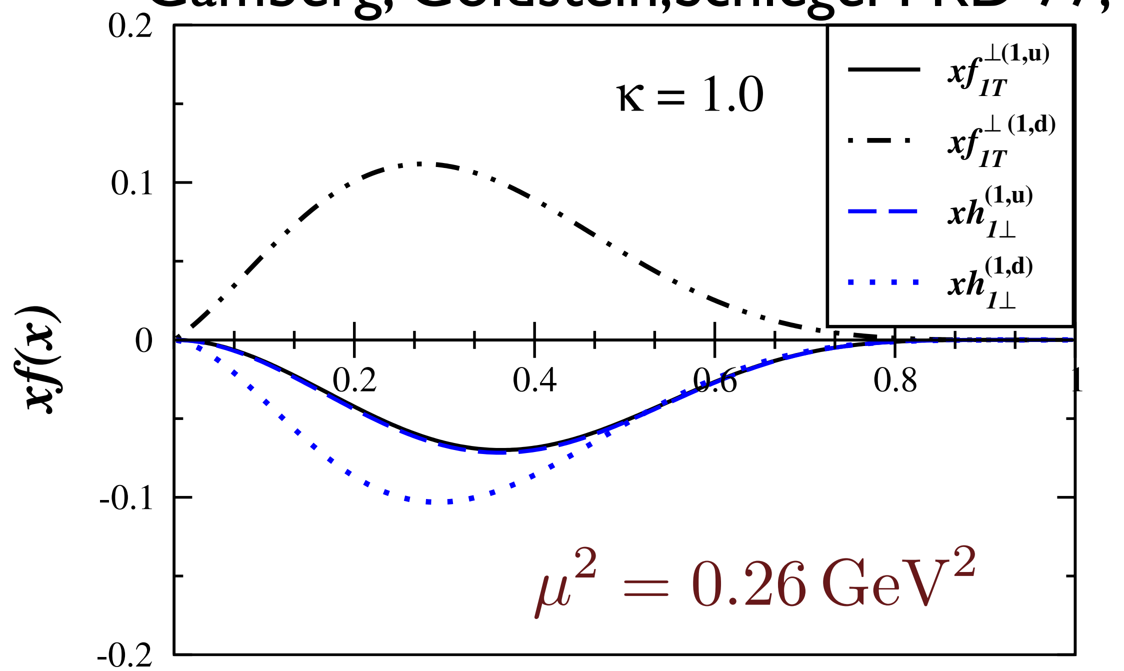
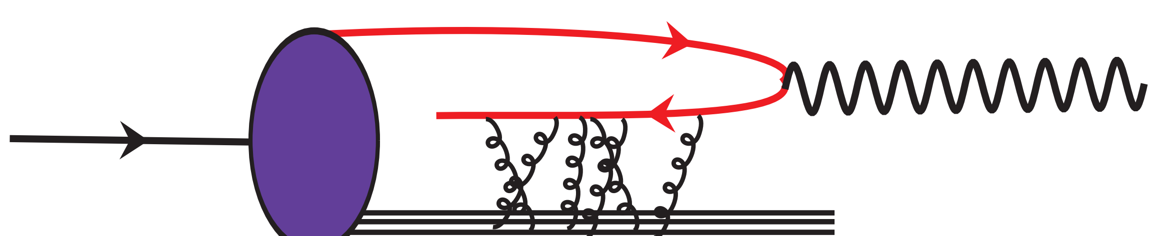
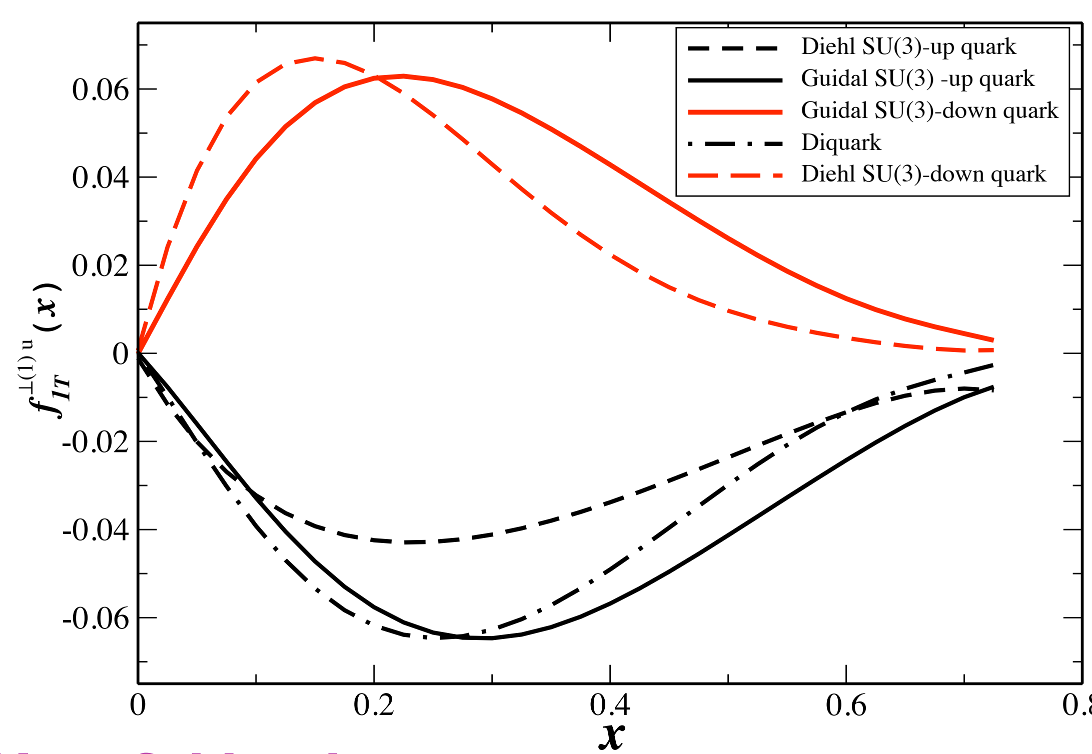


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Gamberg, Goldstein,Schlegel PRD 77, 2008







L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

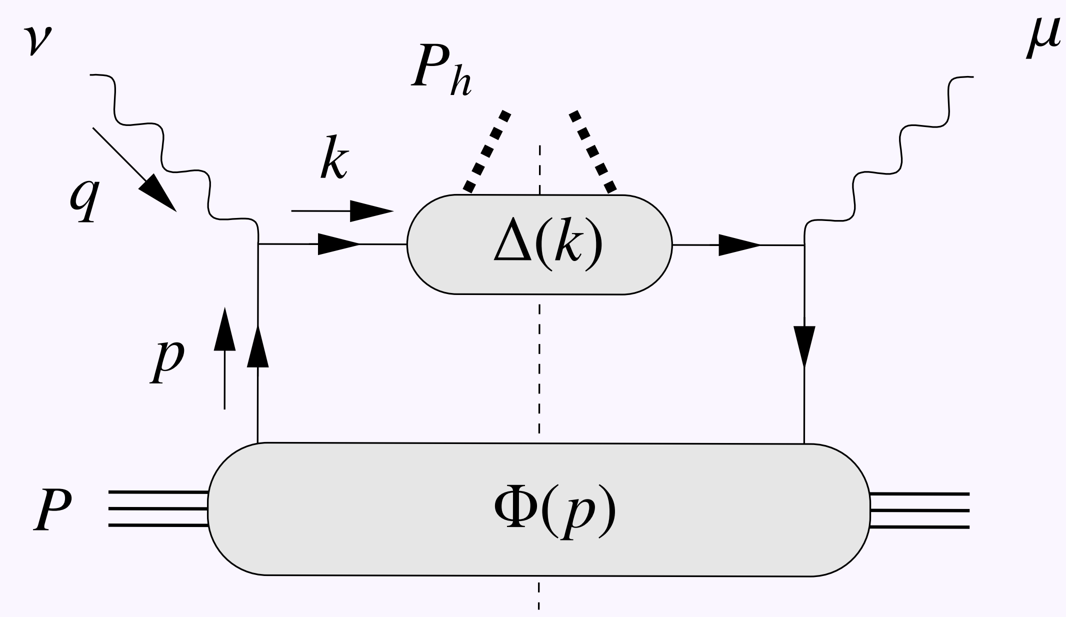
Explore this in Correlator

They are F.T. of forward ME of non-local quark and gluon ops. btwn hadrons states.

$$\Phi^{\mathcal{U}^{[C]}}_{ij}(x,k_T) = \int \frac{d\xi^- \, d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi} \langle P|\,\overline{\psi}_j(0)\,\mathcal{U}^{[C]}_{[0;\xi]}\,\psi_i(\xi)\,|P\rangle\big|_{\xi^+=0=\mathrm{LF}}$$

$$\Delta^{[\mathcal{U}]}_{ij}(z,k_T) = \sum_X \int \frac{d\xi^+ \, d^2\xi_T}{(2\pi)^3}\; e^{i\,k\cdot\xi} \langle 0|\mathcal{U}_{[0,\xi]}\psi_i(\xi)|P,X\rangle\langle P,X|\bar{\psi}_j(0)|0\rangle|_{LF}$$

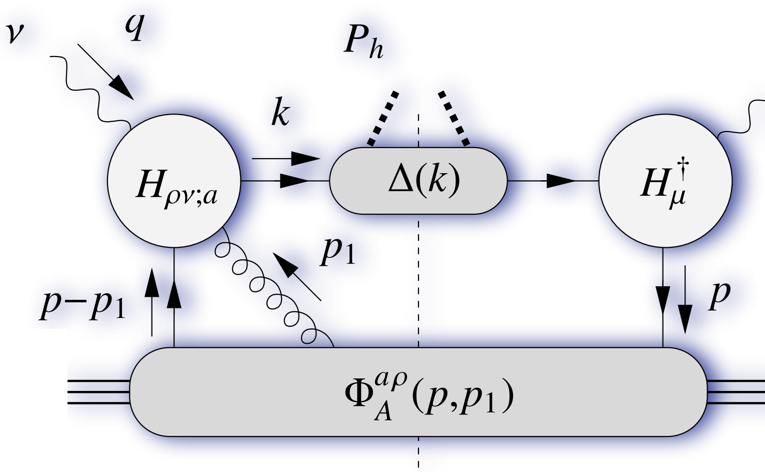
For k_T dependent quantities non-locality restricted to the light front $\xi^+ = 0$



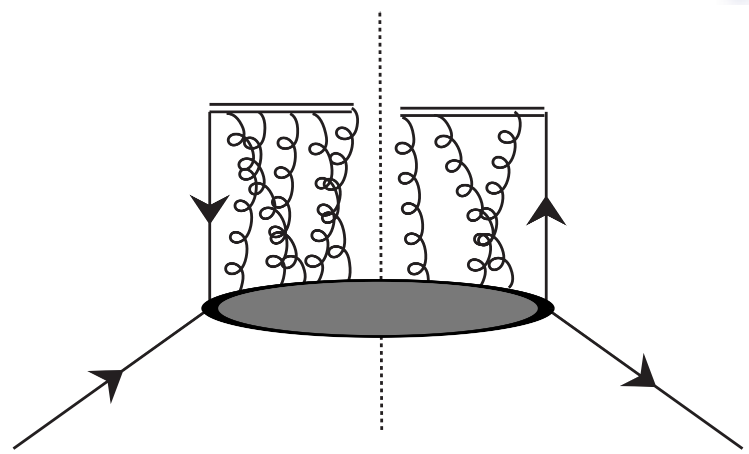
$$W_{\mu\nu} = \int d^4p d^4k \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

Gauge link ensures Color Gauge Inv.

Gauge link determined by summing leading gluon interactions
Efremov,Radyushkin *Theor. Math. Phys.* 1980, Belitsky, Ji, Yuan NPB 2003,
***Boer, Bomhof, Mulders Pijlman, et al.* 2003 - 2008- NPB, PLB, PRD**



$$\mathcal{U}_{[0;\xi]}^{[C]} = \mathcal{P}\mathrm{e}^{-ig\int_C ds\cdot A(s)}$$

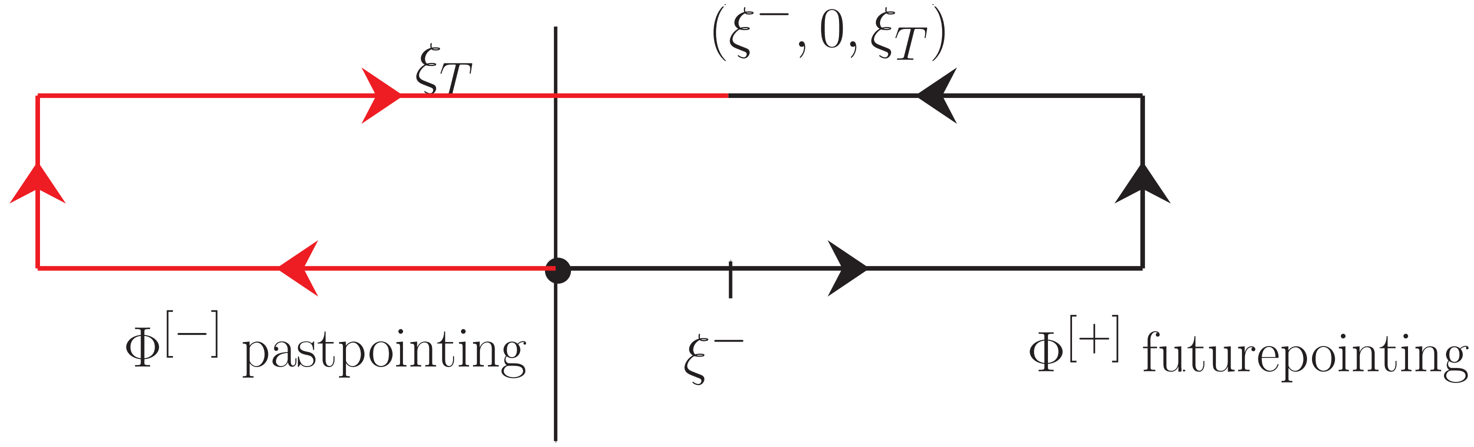


Some models …
Belitsky, Ji, Yuan NPB 2002,
Summing gauge link with color
LG, M. Schlegel PLB 2010

$$\langle P|\,\overline{\psi}_j(0)\,\mathcal{U}_{[0;\xi]}^{[C]}\,\psi_i(\xi)\,|P\rangle\big|_{\xi^+=0=\mathrm{LF}}$$

The path ***C*** is fixed by the hard subprocess within the full hadronic reaction

$$W_{\mu\nu} = \int d^4p d^4k \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{\mathcal{U}_{[\infty;\xi]}^{[C]}}(p)H_\mu^\dagger(p,k)\Delta(k)H_\nu(p,k)\right]$$



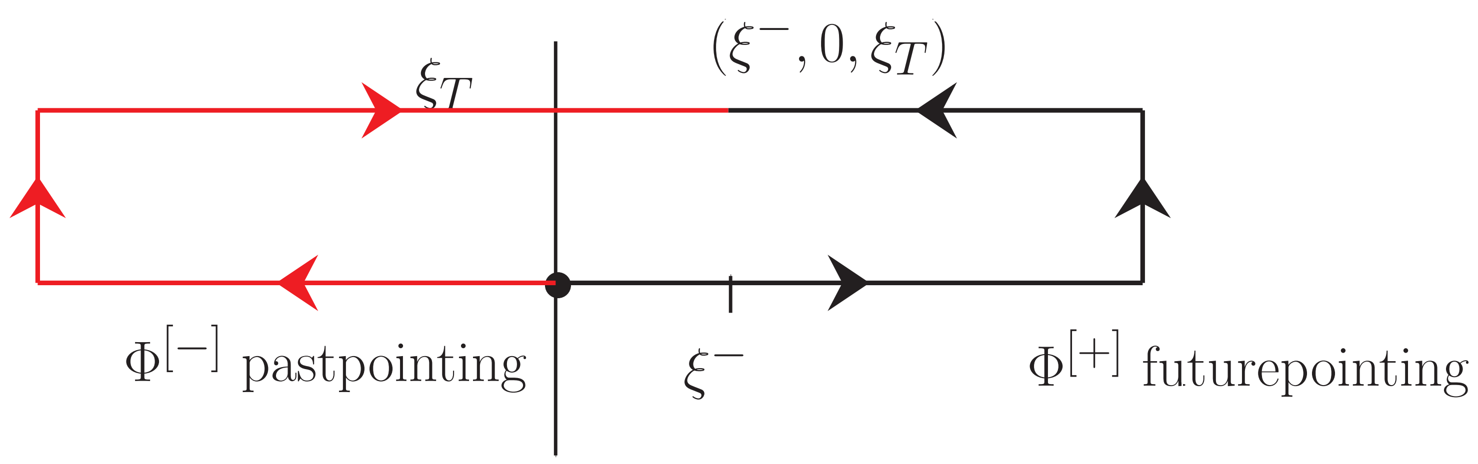
- The integration path of the gauge-link between the two parton fields, which involves re-summing collinear and transverse gluon interactions

$$\langle P|\,\overline{\psi}_j(0)\,\mathcal{U}^{[C]}_{[0;\xi]}\,\psi_i(\xi)\,|P\rangle\big|_{\xi^+=0=\mathrm{LF}}$$

- Depends on the hard partonic subprocess. In particular it depends on the color-flow through the subprocess

$$W_{\mu\nu}=\int d^4p\,d^4k\,\delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p)H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

The path ***C*** is fixed by the hard subprocess within the full hadronic reaction

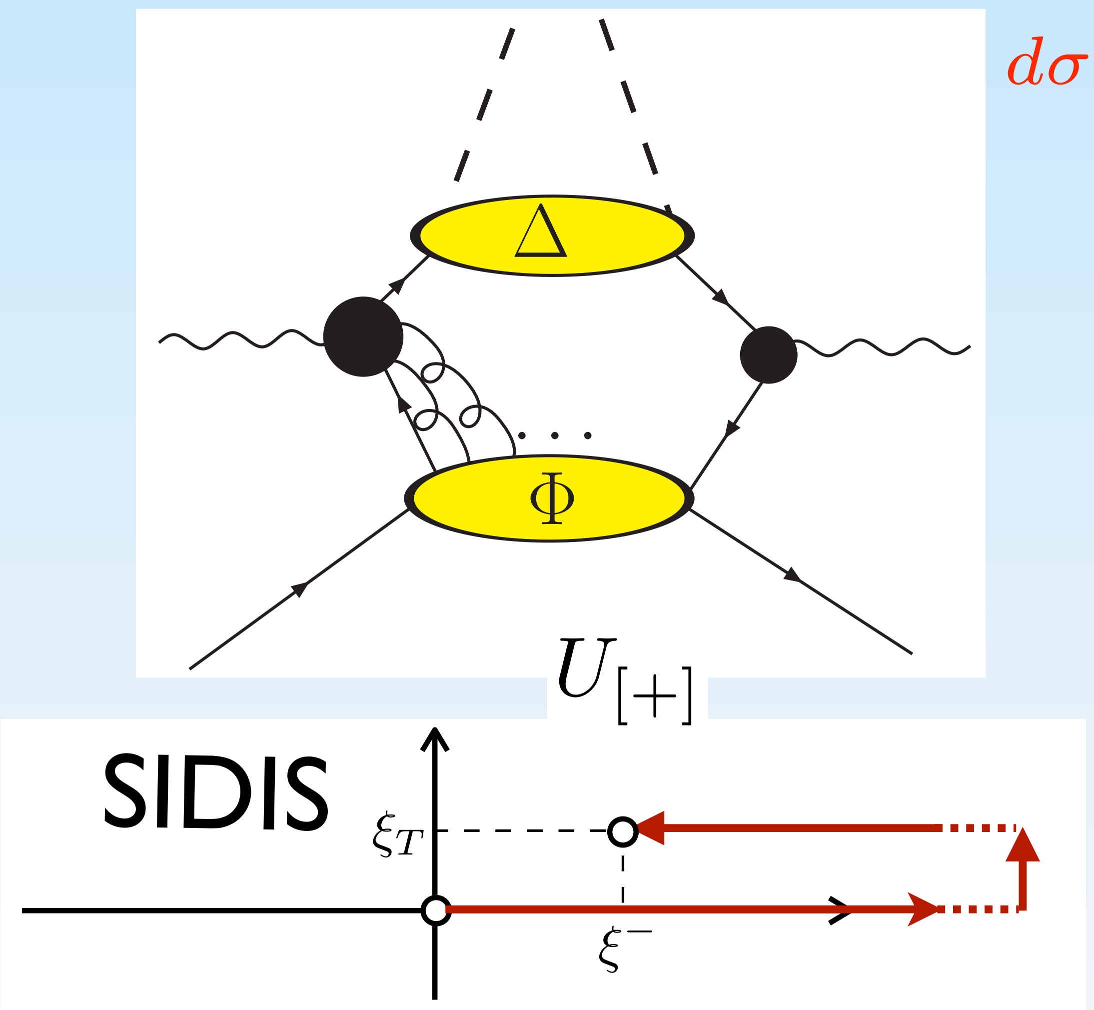


“Generalized Universality” Fund. Prediction of QCD Factorization

$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

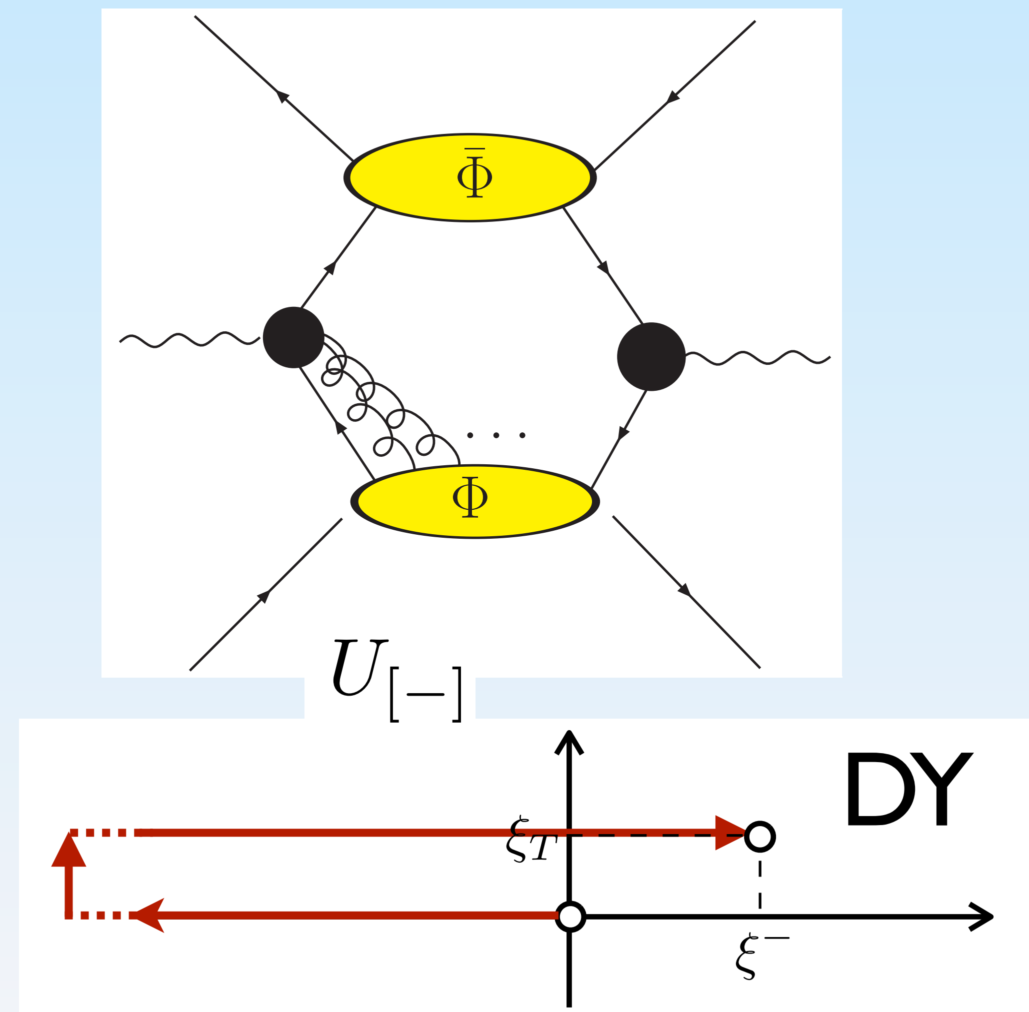
Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



$d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow$

P&T

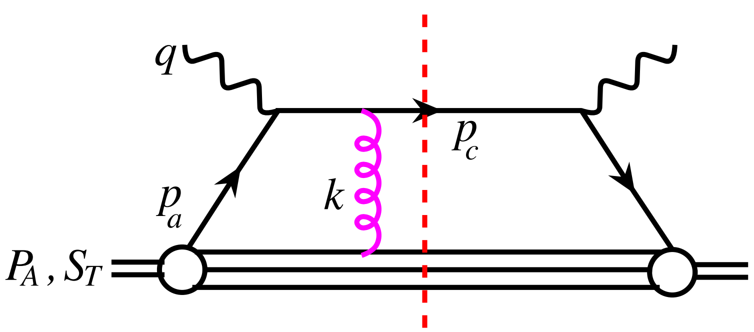
$\longleftrightarrow$

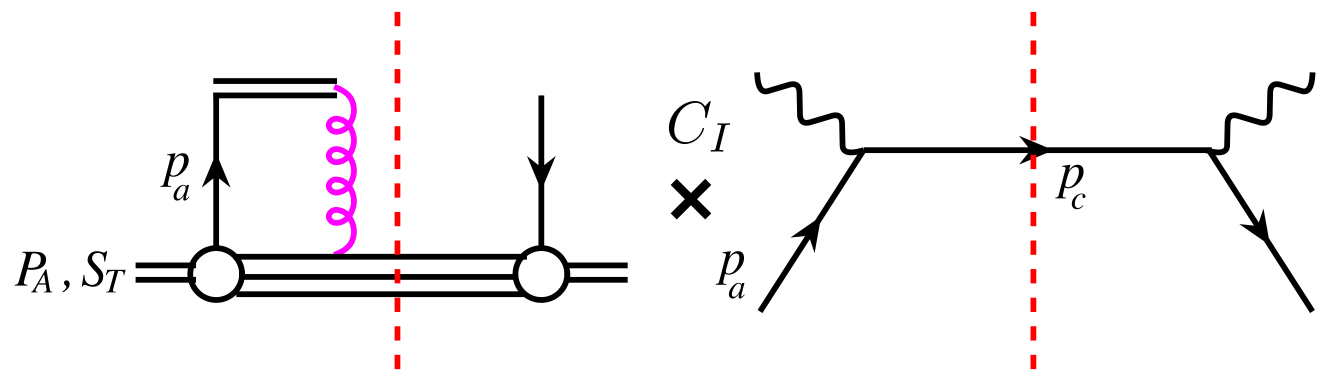


$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

Classic example SIDIS and DY

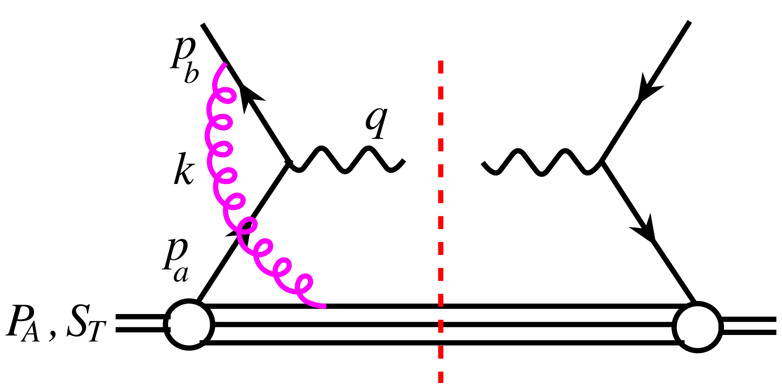
Final-state interaction in SIDIS

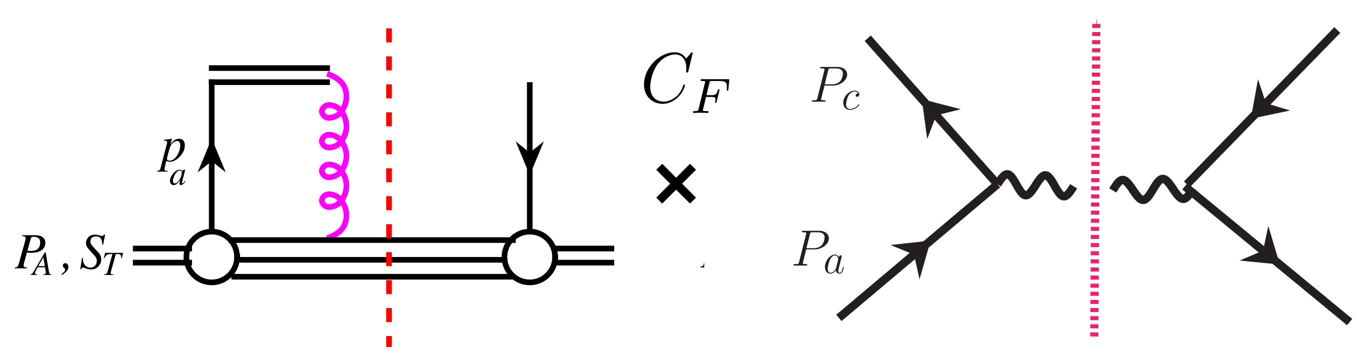




$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

and initial-state interaction in DY





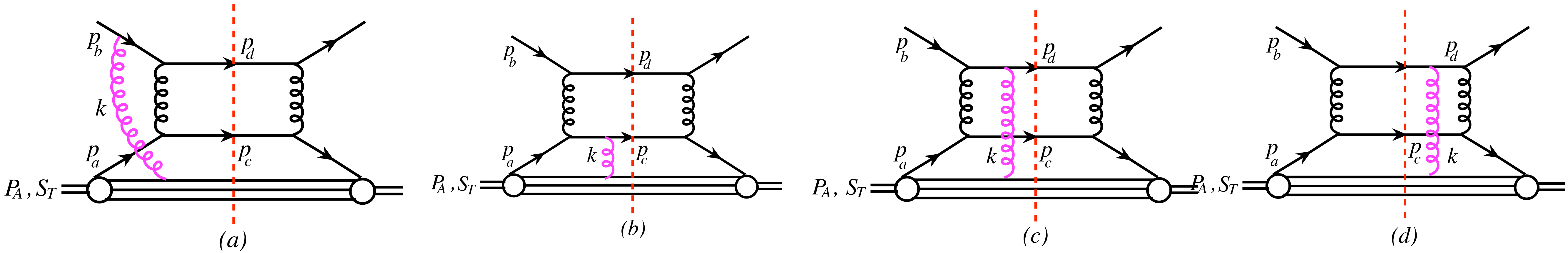
$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right],$$

Collins PLB 02
BHW NPB 02

Different color factors

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- consider channel $qq' \to qq'$

One gluon exchange approx for ISI and FSI

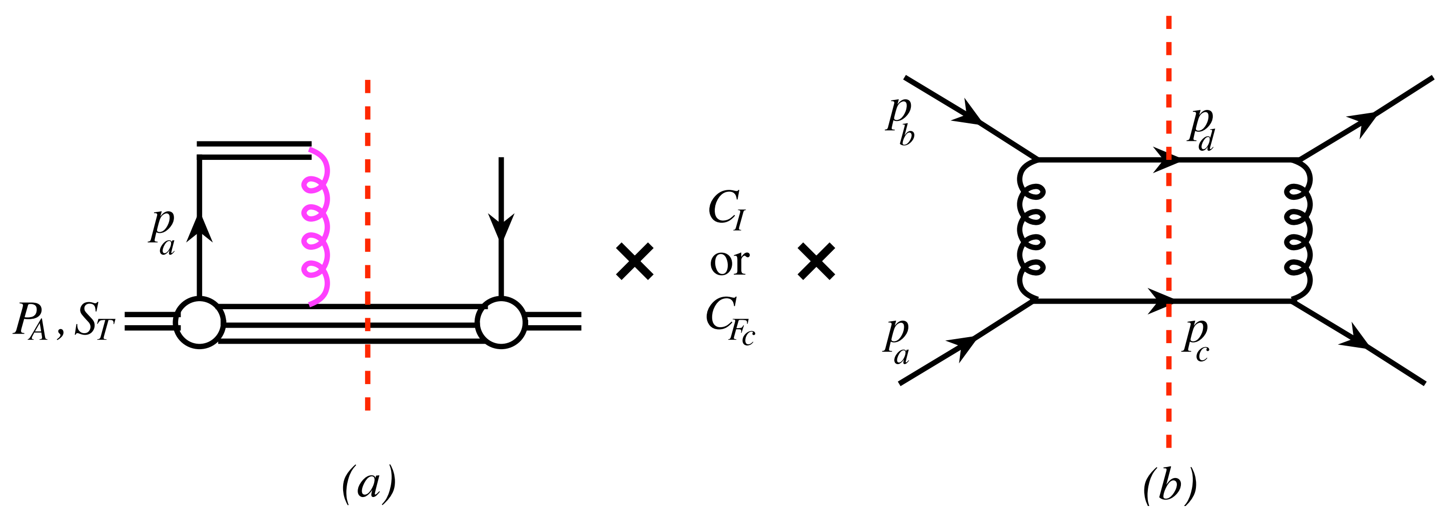


$$\left[\frac{-g}{-k^+ - i\epsilon} T^a\right]$$

$$\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

$$C_I = -\frac{1}{2N_c^2},$$

$$C_{F_c} = -\frac{1}{4N_c^2},$$

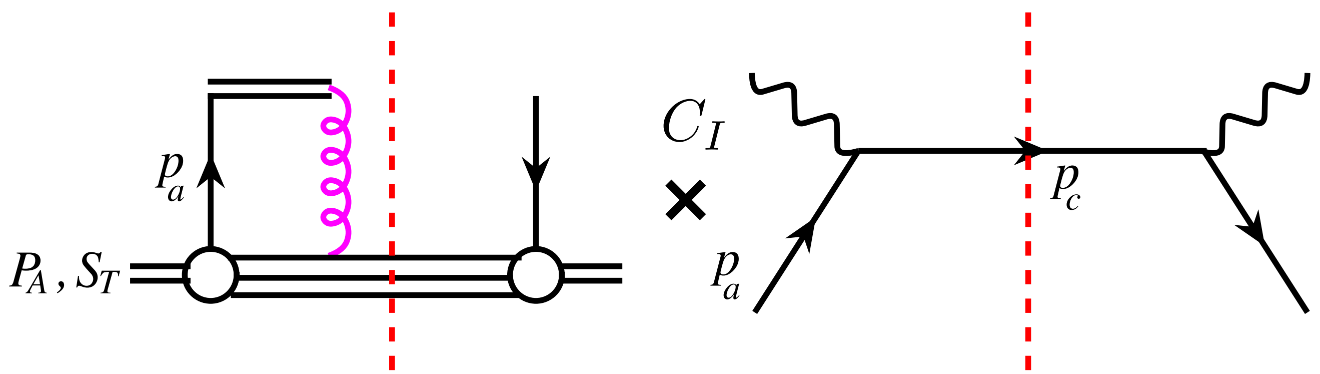


Note unpolarized color factor

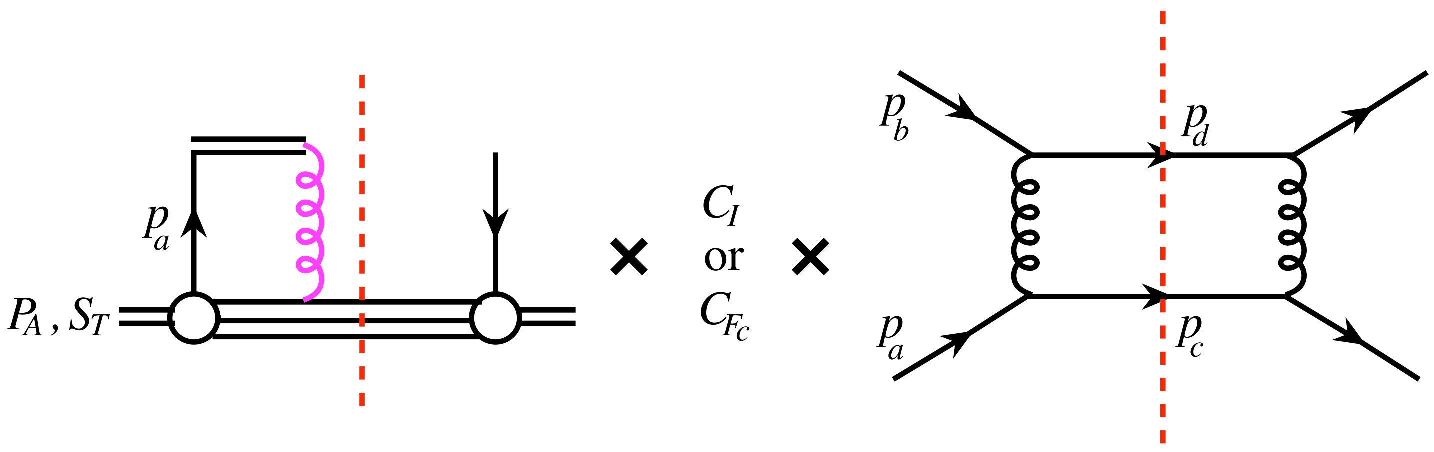
$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

Comparing imag. pt of eikonal propagators for subprocess in SIDIS and inclusive single particle production

Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS



see e.g.
L.G. & Kang PLB 2010



(a) (b)

$$\Delta^N f_{a/A}^{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS}.$$

Process dependence in TMDs Prediction of Factorization

- Process dependence or gluonic pole factors arise in azimuthal asymmetries
- In particular in weighted asymmetries

$$\Phi_{\partial}^{\alpha[\mathcal{U}]}(x) = \int d^2k_T k_T^{\alpha} \Phi^{[\mathcal{U}]}(x, k_T).$$

Decomposes $\Phi_{\partial}^{\alpha\,[\mathcal{U}]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[\mathcal{U}]}\,\pi\Phi_G^{\alpha}(x,x),$

T-even → $\tilde{\Phi}_{\partial}^{\alpha}(x)$

T-odd → $\Phi_G^{\alpha}(x,x)$

Thus one way to study process dependence through the first moment of the correlator

$$\epsilon_T^{ij} k_\perp^i S_T^j f_{1T}^{\perp(1)}(x, k_\perp^2) \sim \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big[\mathrm{Tr}[\gamma^+ \Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+ \Phi](-\vec{S}_T)\Big]$$

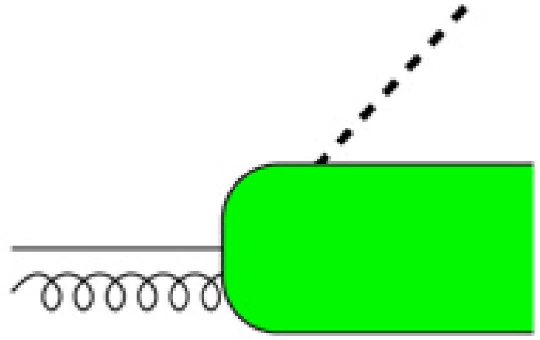
$$\pi \Phi_G^\alpha(x, x; P) = \frac{1}{2} M \left(i h_1^{\perp(1)}(x) \frac{1}{2}[\not{P}, \gamma^\alpha] + \epsilon_T^{\alpha\beta} S_T^\beta \not{P} f_{1T}^{\perp(1)}(x) \right)$$

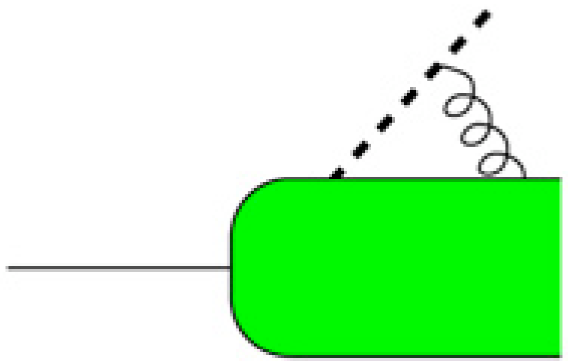
Weighted Cross Sections contain ETQS Functions LINK BTW collinear and TMD Pictures

Phases in soft poles propagators-hard subprocesses Efremov & Teryaev Yad. Fiz & PLB 1984-1985
Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..???, Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ...
Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan 2007, Bacchetta et al. 2007

“T-odd Effects” Fragmentation

- In fragmentation the discussion slightly more complicated, since the gauge-links are not the only potential source of *T-odd effects*. As pointed out by Collins NPB93, also the *internal* final state interactions of the observed outgoing hadron with its accompanying jet, in matrix elements appearing as the one-particle inclusive out-state $|P_h\, X\rangle$ can produce T-odd effects





- Thus due to the explicit appearance of outstates, time-reversal symmetry does not constrain the parametrization of the fragmentation correlators (as does for pdfs)
- Hence *T-odd* fragmentation effects could arise from both FSI and gauge-links

- In BPM NPB 03 it was shown that the first moment of the quark fragmentation correlator with future (e^+e^-) and past (*SIDIS*) pointing Wilson lines can be decomposed

$$\Delta^{[\pm]\alpha}(\frac{1}{z},\frac{1}{z}) = \tilde{\Delta}^{\alpha}_{\partial}(\frac{1}{z}) \pm \pi\Delta^{\alpha}_{G}(\frac{1}{z},\frac{1}{z})$$

- Due to the presence of out states $\tilde{\Delta}^{\alpha}_{\partial}$ in $\pi\Delta^{\alpha}_{G}$ both contain T-even and T-odd fragmentation functions

- The parametrization of both these matrix elements contain, for instance, a Collins-effect-like fragmentation function

$$H_1^{\perp(1)} - \tilde{H}_1^{\perp(1)} \leftrightarrow \text{SIDIS}$$

$$H_1^{\perp(1)} + \tilde{H}_1^{\perp(1)} \leftrightarrow e^+e^-\text{annihilation}$$

More complicated Processes

- In other processes one may again encounter fragmentation correlators with more complicated gaugelinks than the simple future or past pointing Wilson lines. In those cases one can also make a decomposition such as described above, but with different factors in front of the gluonic pole matrix element

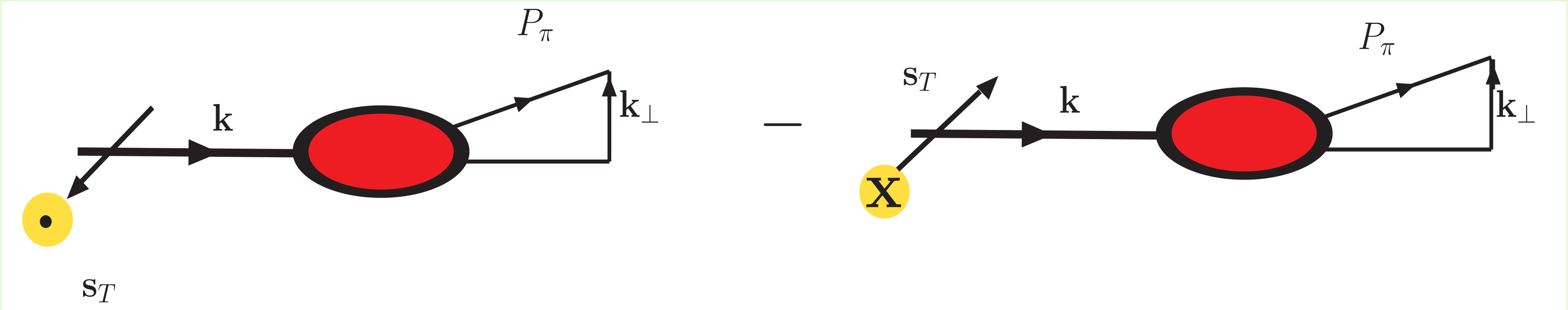
$$\Delta^{[\mathcal{U}]\alpha}(\frac{1}{z},\frac{1}{z}) = \tilde{\Delta}^{\alpha}_{\partial}(\frac{1}{z}) + C^{[\mathcal{U}]}\pi\Delta^{\alpha}_{G}(\frac{1}{z},\frac{1}{z})$$

Reliability of Transversity Extraction Universality of Collins Fragmentation Function

- Collins NPB: 1993 **TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron**

$$D_{h/q^\uparrow}\left(z, K_T^2\right) = D_1^q\left(z, K_T^2\right) + H_1^{\perp q}\left(z, K_T^2\right)\frac{(\hat{\boldsymbol{k}} \times \boldsymbol{K}_T)\cdot \boldsymbol{s}_q}{z M_h},$$

$$\Delta_\partial^{\alpha\,[\mathcal{U}]}(z) \quad = \quad \int d^2k_T\; k_T^\alpha \Delta^{[\mathcal{U}]}(z, k_T) = \tilde{\Delta}_\partial^\alpha\left(\tfrac{1}{z}\right) + C_G^{[\mathcal{U}]}\,\pi\Delta_G^\alpha\left(\tfrac{1}{z}, \tfrac{1}{z}\right)$$



$$\frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp\left(z, k_T^2\right) = \frac{1}{2}\,\mathrm{Tr}\left[\Delta(z, k_T)\, i\sigma^{i-}\gamma_5\right].$$

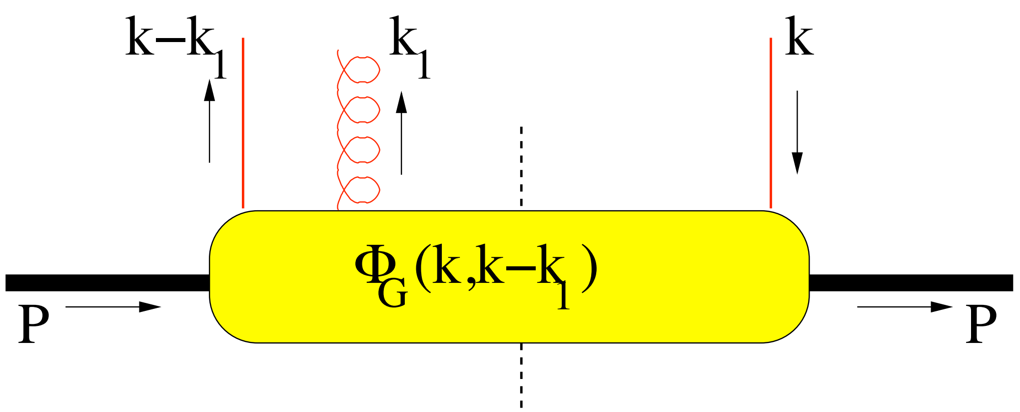
Gluonic Pole Matrix elements

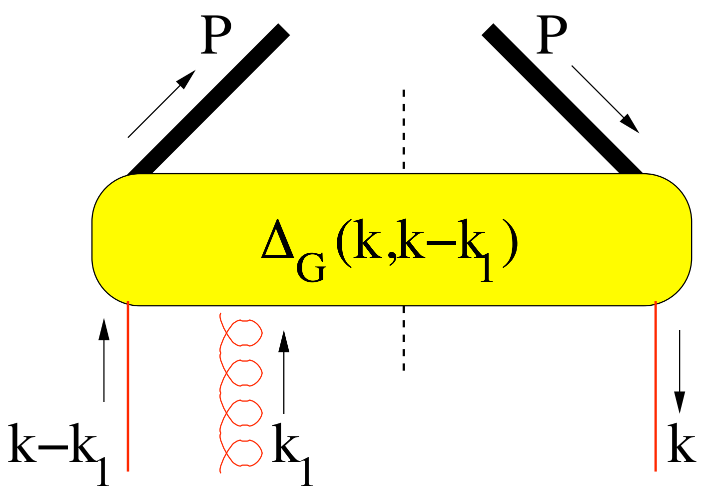
"Model independent"analysis of GPME L.G. A. Mukherjee & P. Mulders PRD 2011

arXiv:1010.4556

Consider correlator multi-particle scattering amplitudes

$$\Phi_G^{\alpha}(x, x-x_1) = \int \frac{d\xi^-}{2\pi}\frac{d\eta^-}{2\pi}\; e^{ix_1\eta\cdot P} e^{i(x-x_1)\xi\cdot P}\,\langle P|\,\overline{\psi}(0)\,U^n_{[0;\eta]}\,gG^{n\alpha}(\eta)\,U^n_{[\eta;\xi]}\,\psi(\xi)\,|P\rangle\,\big]_{\rm LC}$$





The steps in these considerations:

1) The k^- integrations in the multi-parton correlators lead to light-front correlators, for which time-ordering is irrelevant

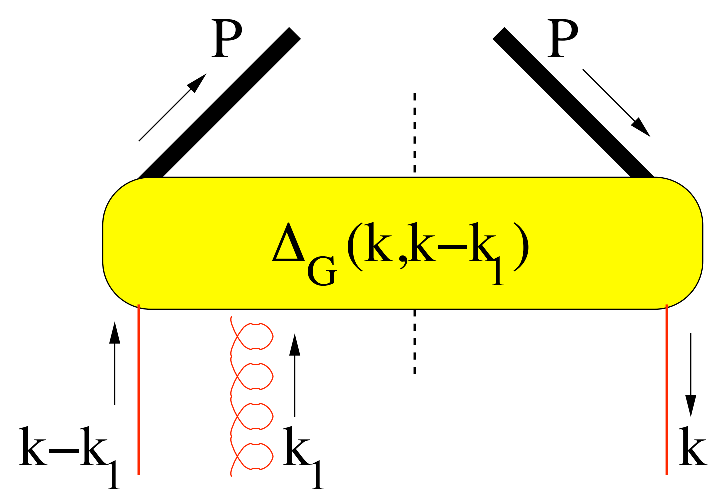
2) Then correlators can be expressed as matrix elements of time-ordered products of operators then using *LSZ* formalism can study analytic structure poles and cuts
GPDs Diehl-Gousset-1998, Radyushkin and Belitsky Phys. Rep. 2005
Jaffe-NPB 1984, quark-quark and multi-parton correlators collinear correlators

The steps in these considerations:

.....

3) These pictures become just hadron-parton amplitudes, e.g. the quark-quark correlator is related to the forward and non-forward antiquark-hadron scattering amplitude. Depending on the precise structure these are un-truncated Greens functions or time ordered products. Can use LSZ formalism to study analytic/singluarity structure

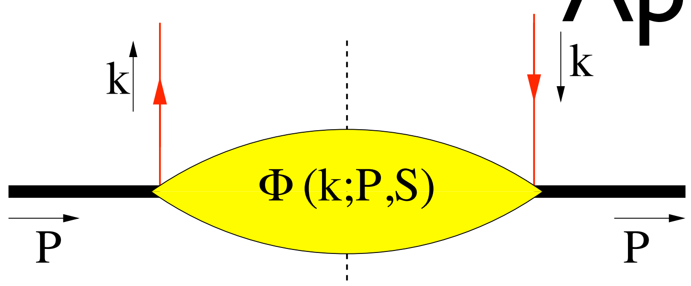
Goal to study support properties in limit $x_1 \to 0$

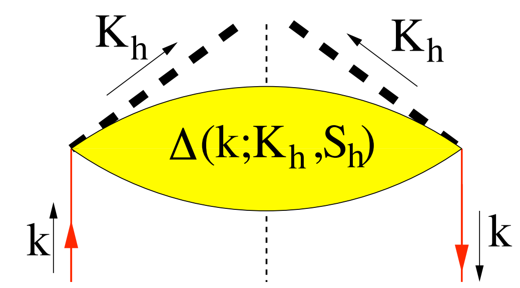


Analysis based on covariant parton model or quark-target amplitude

Landshoff, Polkinghorne, and Short 1971 NPB

Apply to TMDs





Consider scattering amp of an off shell anti-quark and an onshell proton

$$N(P) + \bar{q}(-k) \rightarrow N(P) + \bar{q}(-k)$$

$$A(k^2; s, u) = \int d^2\xi e^{ik\xi} \langle P| T \left(\bar{\psi}(0)\Gamma\psi(\xi)\right) |P\rangle$$

Note $A(k^2; s, u)$ is not truncated in off shell parton legs

$$s = (P-k)^2\,, \quad u = (P+k)^2\,, \quad s + u = 2k^2 + 2M^2$$

- Make the standard assumption that it is possible to use analyticity for QCD-amplitudes
- Cuts for non-negative $\mathrm{Re}\, s\,,\ \mathrm{Re}\, u$
- Singularites for non-negative $\mathrm{Re}\, k^2$

Sudakov Kinematics

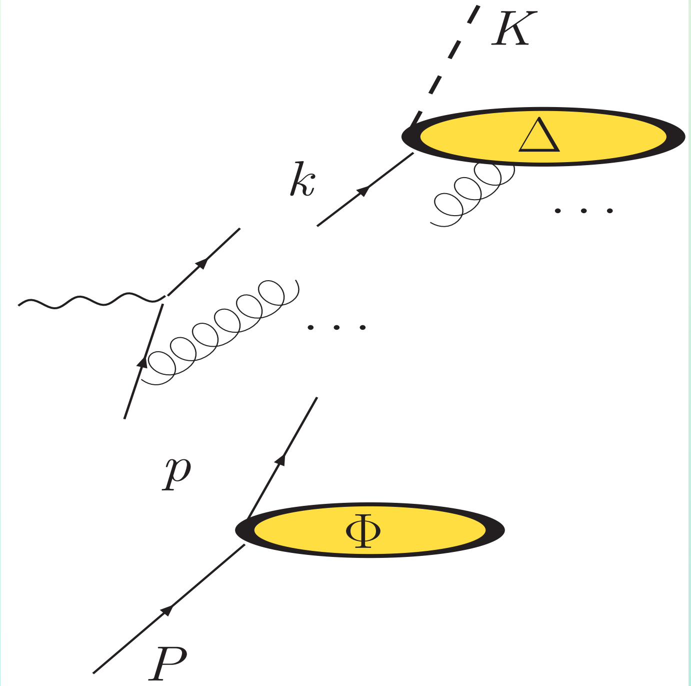
- **Restricts hadrons well sep. in momentum phase-space** $P \cdot K \sim p \cdot k \sim Q^2$
- **Inside correlator momenta are soft** $P \cdot p \sim P^2 = M^2$

Partons involved in hard scattering described "Sudakov" decomposition P **and** n

$$k = \frac{1}{z} K^\mu + k_T^\mu + \sigma_h n_h^\mu$$

$$p = xP^\mu + p_T^\mu + \sigma n^\mu$$

~ Q   ~ M   ~ M²/Q



$$n^2 = 0, \quad P \cdot n = 1, \quad K \cdot n_h = 1, \quad \sigma = p \cdot P \sim M^2, \quad \sigma_h \sim M_h^2 \quad \ldots$$

Integrate over $P \cdot p$

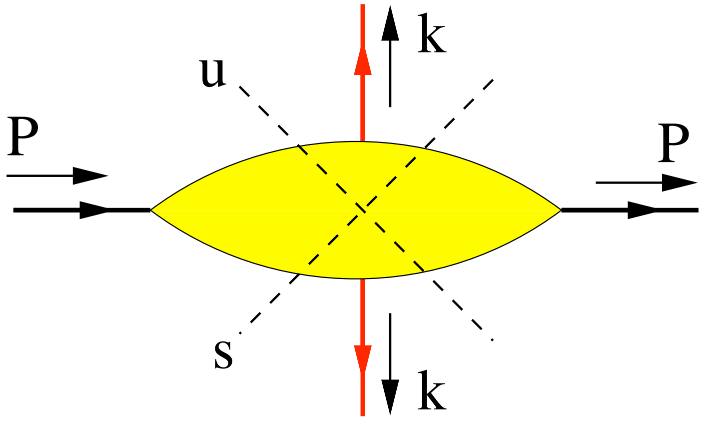
$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int d(p \cdot P) \Phi(p, P)$$

- To see analyticity properties in k^- plane we express k^- thru invariants *s*, *u* and k^2

$$k^- = \frac{s + k_T^2 + i\epsilon}{2(x-1)} + P^-$$

$$k^- = \frac{u + k_T^2 + i\epsilon}{2(x+1)} - P^-$$

$$k^- = \frac{k^2 + k_T^2 + i\epsilon}{2x}$$



Impose DIS kinematics

- Consider the scattering amp. projected on DIS kinematics $k^+ = xP^+$ and integrate over k^- taking into account singularity structure

- singularities located in k^- complex

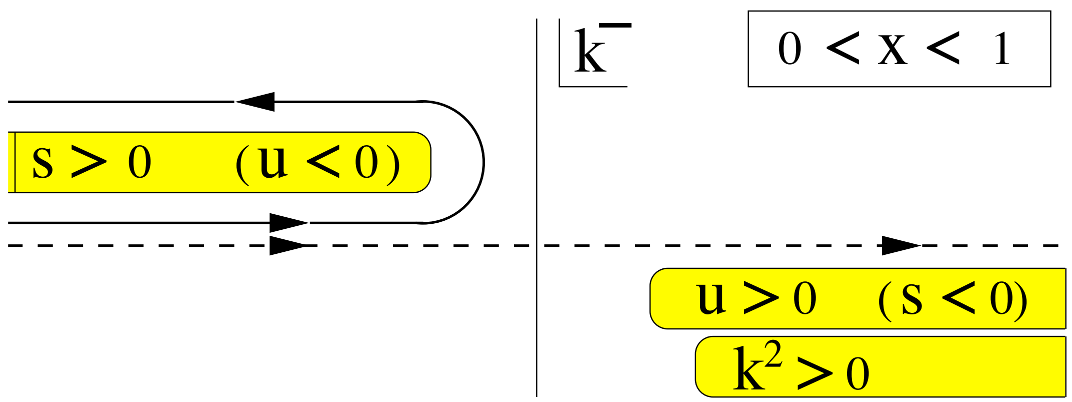
$$k^- = \frac{s + k_T^2 + i\epsilon}{2(x-1)} + P^-$$

$$k^- = \frac{u + k_T^2 + i\epsilon}{2(x+1)} - P^-$$

$$k^- = \frac{k^2 + k_T^2 + i\epsilon}{2x}$$

Singularities move in complex plane depending on value of

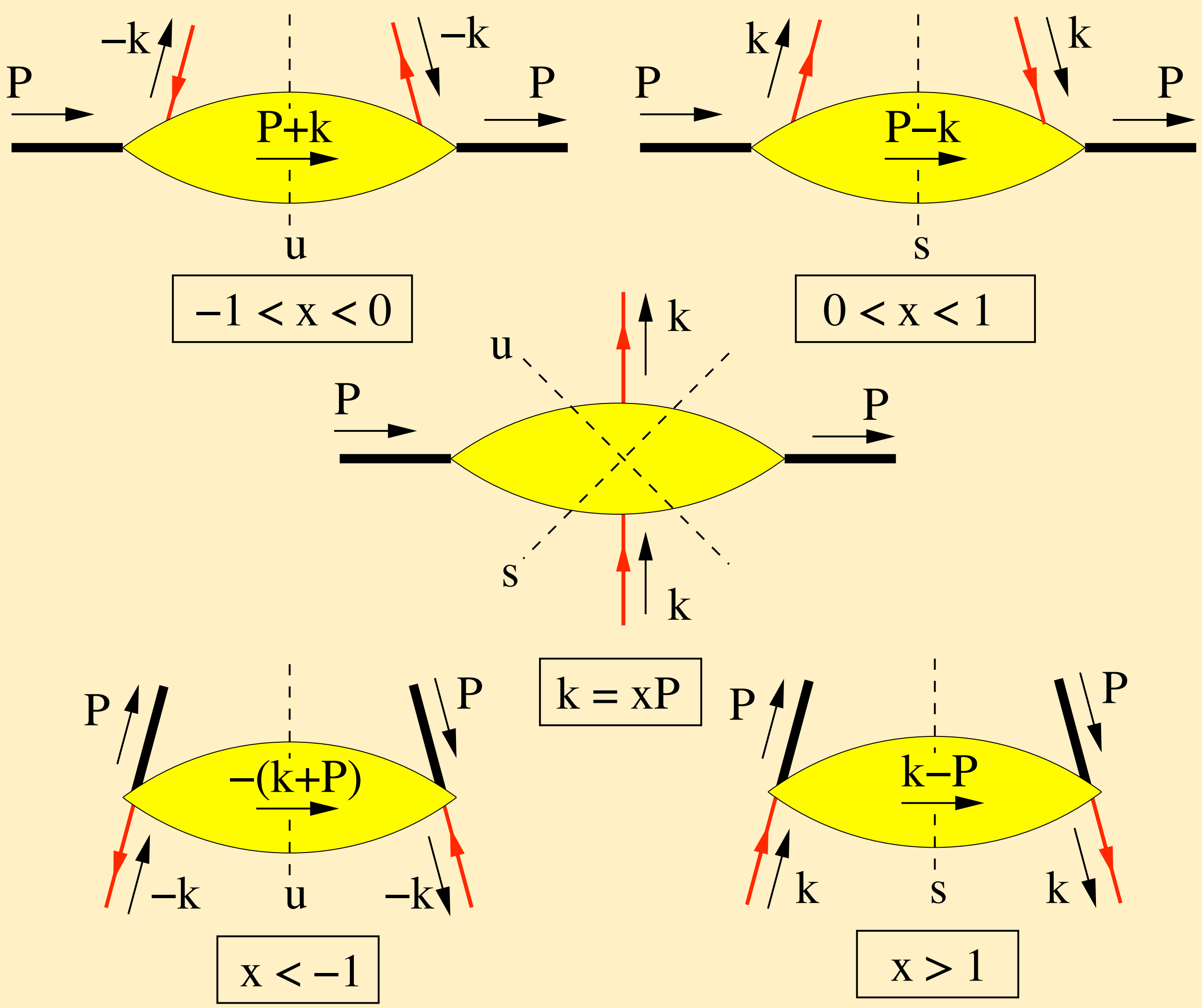
$$-1 \leq x \leq 1$$



TMDs

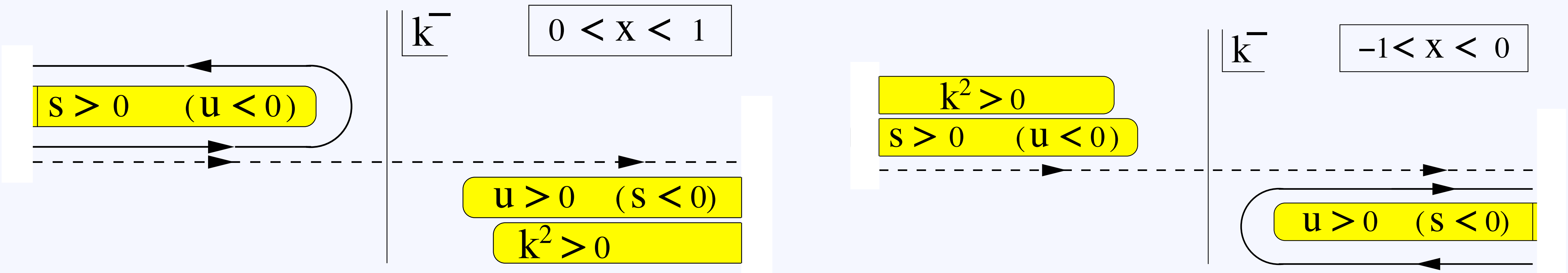
$$\Phi^{\alpha}(x, k_T^2) = \int dk^- \mathcal{A}^{\alpha}(s + i\epsilon, k^2 + i\epsilon, u + i\epsilon)\Big|_{LF}$$

$$= \int dk_1^- \mathcal{A}^{\alpha}(k_1^- + i\epsilon f_a(x))\Big|_{LF}$$



Thus Support in x region PDFs

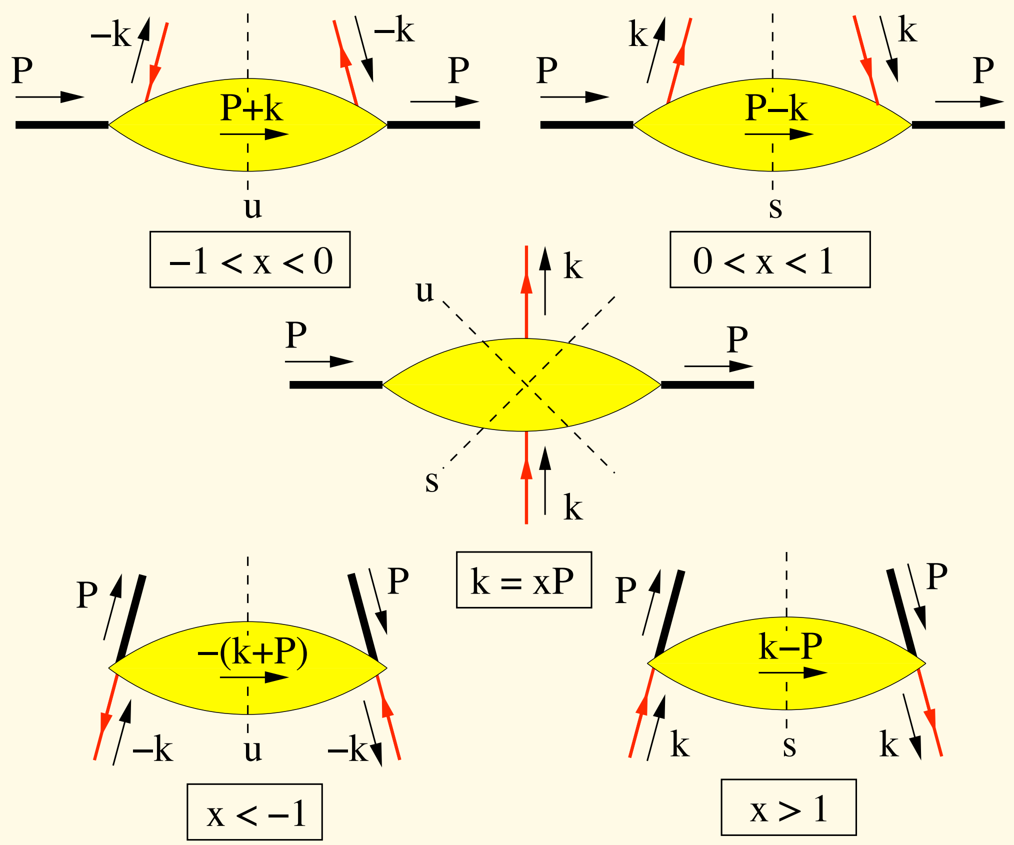
$$\Phi(x) = \theta(x)\,\theta(1-x)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-x)\,\theta(1+x)\,\mathrm{Disc}_{[u]}\mathcal{A}$$



Integration contours k^- wrapped around the s and u cuts for positive and negative values of x iff $-1 \le x \le 1$ yield quark and anti-quark distribution functions

Must assume convergence in the variable k^- or use subtracted relations

$A(k^2; s, u)$ Scattering Amplitudes



•Integrating parton correlators over k^- connects them to the anti-parton -hadron scattering four-point function

•Depending on the value of *x*, the imaginary part of $A(k^2; s, u)$ represents the (anti)-parton distribution or fragmentation correlators.

Support in $x = 1/z$ region Fragmentation

$$\Delta(x) = \theta(x-1)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-1-x)\,\mathrm{Disc}_{[u]}\mathcal{A}$$

$$= \theta(z)\theta(1-z)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-z)\theta(1+z)\,\mathrm{Disc}_{[u]}\mathcal{A}$$

The case for fragmentation is different since the parton propagator has positive k^2, thus contours in x and z not connected by analytic continuation Landshoff and Polkinghorn Phys. Rep. 1972

Extend analyticity study to multi-parton distribution and fragmentation function

$$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$$

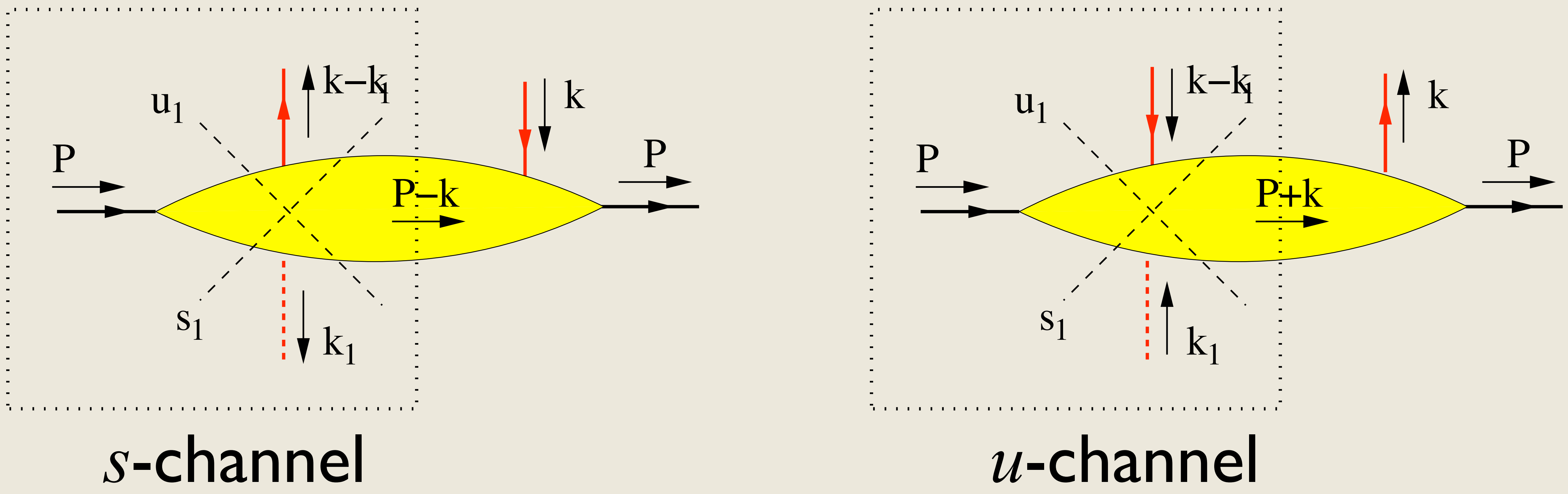
Studying contours of additional integrations

$$\begin{aligned}
\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, k'^2) &= \int d^4\xi d^4\eta e^{ik'\cdot\xi} e^{ik_1\cdot\eta} \langle 0|T\ (F^{n\alpha}(\eta)\psi(\xi))\,|P, X\rangle \\
&= \frac{i\sqrt{Z_q}\,(\not{k}' + m)}{k'^2 - m_q^2 + i\epsilon} \frac{i\sqrt{Z_g}\,T^{n\alpha}}{{k_1}^2 - m_q^2 + i\epsilon} \tilde{G}^\alpha(k', k_1; P, P_X)
\end{aligned}$$

Again use LSZ formalism for off shell partonic lines...

Extend analyticity study to multi-parton distribution and fragmentation function

$$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$$



- The additional invariants for the amplitude

$$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$$

- Relevant for gluonic pole matrix elements for case $s_1 > 0$ and for the case $u_1 > 0$

- Studying contours of additional integrations

The additional invariants for the amplitude

$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$

relevant for gluonic pole matrix elements, for the case s > 0 and for the case u > 0.

$$s_1 = (P \mp k \pm k_1)^2 \quad \text{and} \quad u_1 = (P \mp k_1)^2$$

$$k_1^- = \frac{s + (k_T - k_{1T})^2 + i\epsilon}{2(x_1 - (x \mp 1))} - (P^- - k^-)$$

$$k_1^- = \frac{u_1 + k_{1T}^2 + i\epsilon}{2(x_1 \mp 1)} + P^-$$

$$k_1^- = \frac{k_1^2 + k_{1T}^2 + i\epsilon}{2x_1}$$

← *Here parton virtualities become very important*

$$k_1^- = \frac{(k-k_1)^2 + (k_T - k_{1T})^2 + i\epsilon}{2(x_1 - x)} + k^-$$

Comments

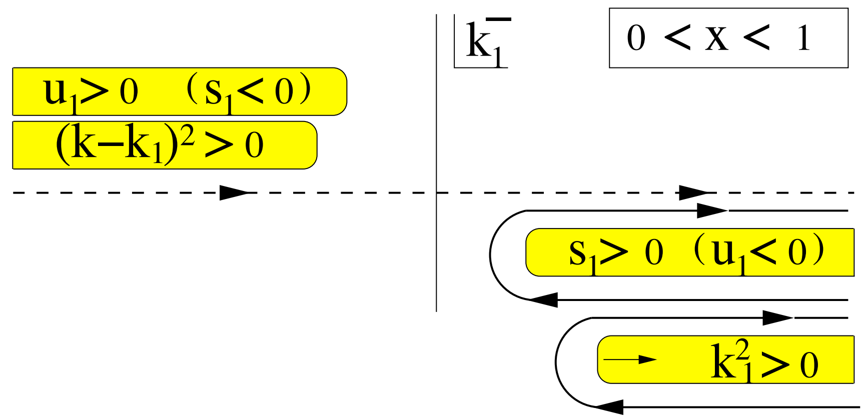
- Depending on the value of x_1 the integration contour in k_1^- bypasses the singularities encountered in the complex plane in a unique way, which dictates the support properties of the quark-gluon-quark correlation functions
- The denominators in the expressions relating k_1^- to s_1 and u_1 tell us that only when $x_1 \in [x-1,1]$ (for positive x) or $x_1 \in [-1, x+1]$ (for negative x) the singularities in s_1 and u_1 are relevant

- Study case of s-channel ($s > 0$) $0 < x < 1$
- Look at the gluonic poles $x_1 \to 0$
- $x_1 \to 0$ is in the interval $0 < x < 1$

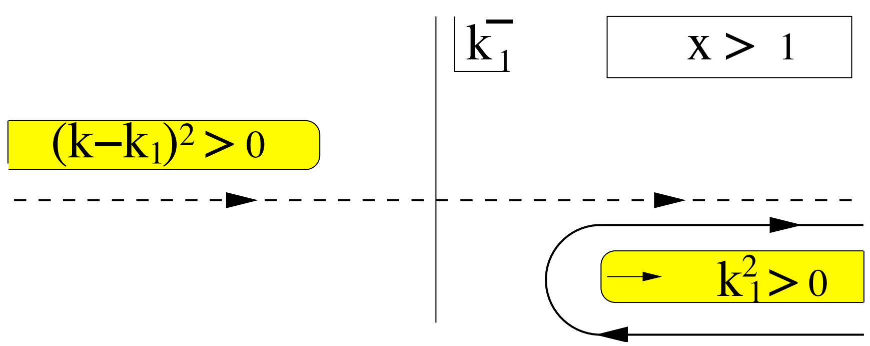
$$k_1^- = \frac{s_1 + i\epsilon}{2(x_1 - (x \mp 1))} + k^- \qquad k_1^- = \frac{u_1 + i\epsilon}{2(x_1 \mp 1)}$$

$$k_1^- = \frac{(k - k_1)^2 + i\epsilon}{2(x_1 - x)} + k^- \qquad k_1^- = \frac{k_1^2 + i\epsilon}{2x_1}$$

$$\lim_{x_1 \to 0} \Delta_G(x, x - x_1) = \Delta_G(x, x) \to 0$$



$$\Phi_G(x, x - x_1)$$



$$\Delta_G(x, x - x_1)$$

For the case $x > 1$ the k_1^- integration can be wrapped around the cut k_1^2 which smoothly vanishes for $x_1 \to 0$ describes the by the arrow inside branch cut indicates that it harmlessly recedes to infinity

$$k_1^- = \frac{k_1^2 + k_{1T}^2 + i\epsilon}{2x_1}$$

Agrees with earlier model analysis Collins, Metz PRL 2004
Agrees with earlier model analysis LG, A. Mukherjee, P. Mulders PRD 2008
Agrees with model independent spectral analysis A. Metz, S. Meissner PRL 2009
Agrees with 1 and 2 gluon exchange calculation from GL in hadron inside jet F. Yuan PRD 2009
Recent ppr. by Boer,Kang,Vogelsang,Yuan-predictions on Lambda polarization in SIDIS & $e^+ e^-$

- Generalize: we show that our arguments for vanishing gluonic pole matrix elements hold for general multi-gluonic and even multi-partonic pole matrix elements.

- Considering the analytic properties of general multi-gluonic pole matrix elements we can proceed inductively

- For two gluons one simply extends the nesting of momenta

$$k - k_1 \quad \text{and} \quad k_1 \quad w/ \quad k - k_1 - k_2,\, k_1 - k_2, \quad \text{and} \quad k_2$$

- adds to the set (s, u, s_1, u_1), (s_2, u_2) without changing the behavior of the others $\Delta_G(x, x, x)$

- Since higher pole ME appear in higher k_T moments of correlator $\Delta_{ij}^{[\mathcal{U}]}(z, k_T)$ we conclude based on general assumptions of analyticity for QCD scattering amplitudes that this TMD correlator is universal

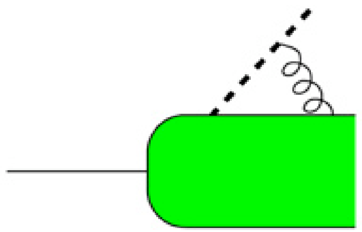
Comments

In wrapping the integration around the s- or u-cut must assume convergence in the variable k^- or use subtracted relations

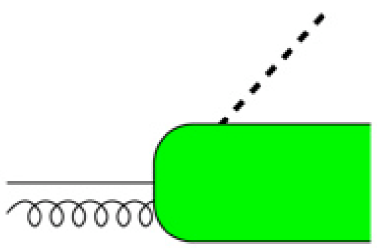
Comments cont.

All "T"-odd effects for frag. in $\tilde{\Delta}^{\alpha}_{\partial}(\frac{1}{z})$ and no "process dependence" $\Delta_G(\frac{1}{z},\frac{1}{z}) = 0$

$$\tilde{\Delta}(\frac{1}{z}) \quad = \tfrac{M}{z} i H_1^{\perp(1)}(z) \tfrac{1}{2}\,[\not{K}, \gamma^{\alpha}] \neq 0$$



$$\pi \Delta_G(\frac{1}{z},\frac{1}{z};K) \quad = \tfrac{M}{z} i \tilde{H}_1^{\perp(1)}(z) \tfrac{1}{2}\,[\not{K}, \gamma^{\alpha}] = 0$$



Process dependence remains in T-odd PDFs

$$C_G^{[\mathcal{U}]} \pi \Phi_G^{\alpha}(x,x)$$

Conclusions

•Study support of multi-parton correlation functions through analytic structure of scattering amplitude
•Gluonic pole contribution to fragmentation function vanishes--model independent result
•Implies universality of Collins function
•Consistent with a number of past studies
•We extend to analysis to all parton insertions
•All insertions vanish when $x_i \to 0$ for fragmentation