

# **Non-perturbative momentum dependence of the coupling constant and hadronic models**

Pre-DIS Wokshop  
QCD Evolution Workshop: from collinear to non collinear case  
April 8-9, 2011  
JLab

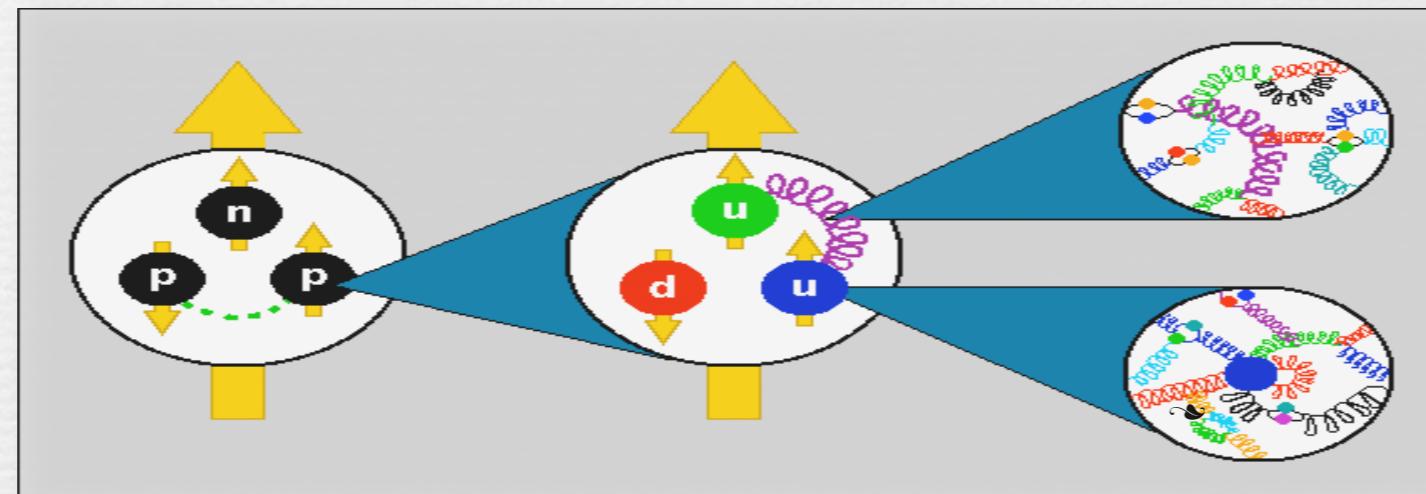
**Aurore Courtoy**  
INFN-Pavia

# Outline

- ❖ Hadronic Models
- ❖ Finding the **Hadronic Scale**
- ❖ Rôle of the **Coupling Constant**: non-perturbative approach
- ❖ ‘Non-perturbative evolution’ and final-state interactions

# Hadronic Physics at Intermediate Energies: Hadronic Models

Hadron  $\Leftrightarrow$  Constituent quarks  $\Leftrightarrow$  Current quarks



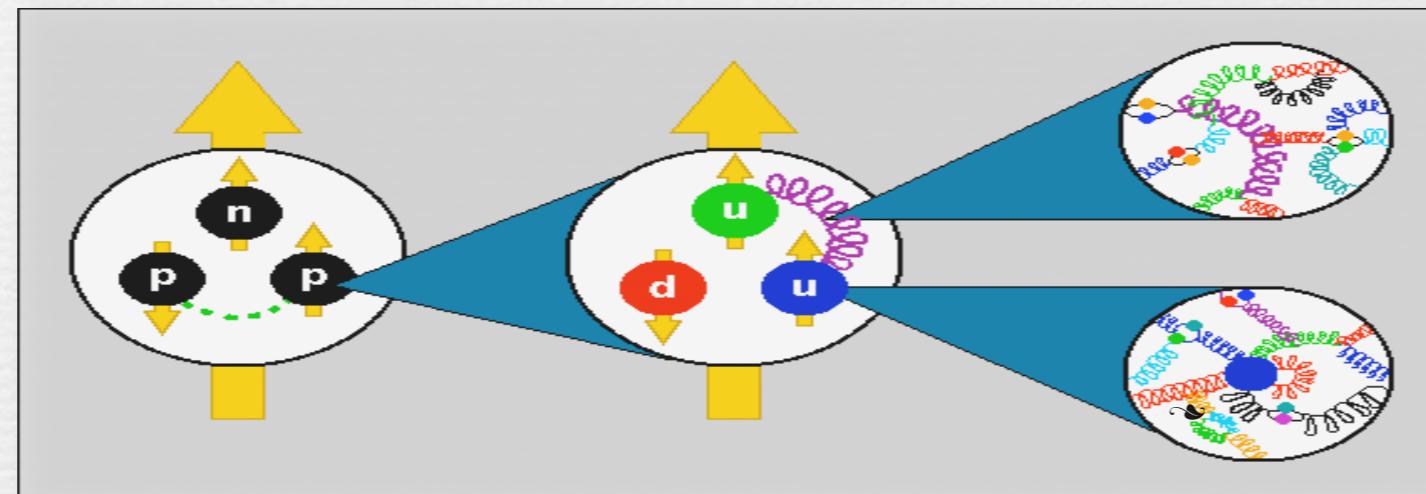
Nonperturbative vs. Perturbative QCD

Models of Hadron Structure

Renormalization Group Eqs.

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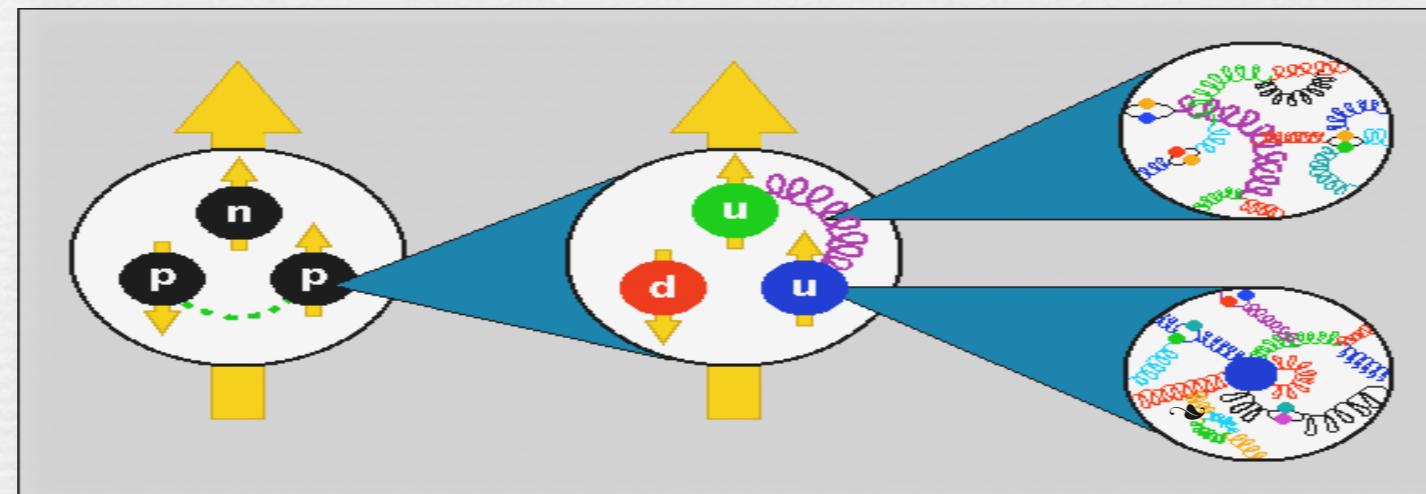
Renormalization Group Eqs.

## Observable

- ❖ calculated in hadronic model
- ❖ at scale  $\mu_0$
- ❖ switch on QCD evolution

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?

# Hadronic Scale: Standard Approach

There exists a scale at which there is no sea and no gluon:

$$\langle (u_v + d_v) (\mu_0^2) \rangle_{n=2} = 1$$

QCD evolution introduces gluons and sea quarks:

e.g. CTEQ parameterization PRD51 :

$$\langle (u_v + d_v) (Q^2 = 10 \text{ GeV}^2) \rangle_{n=2} = 0.36$$

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**Evolve downward high energy data until 2<sup>nd</sup> moment=1  
Find  $\mu_0^2$**

# Hadronic Scale: Standard Approach

## Models scenarios in MSbar scheme

- ~ quark model

- ~  $\mu_0^2 = 0.1 \text{ GeV}^2$

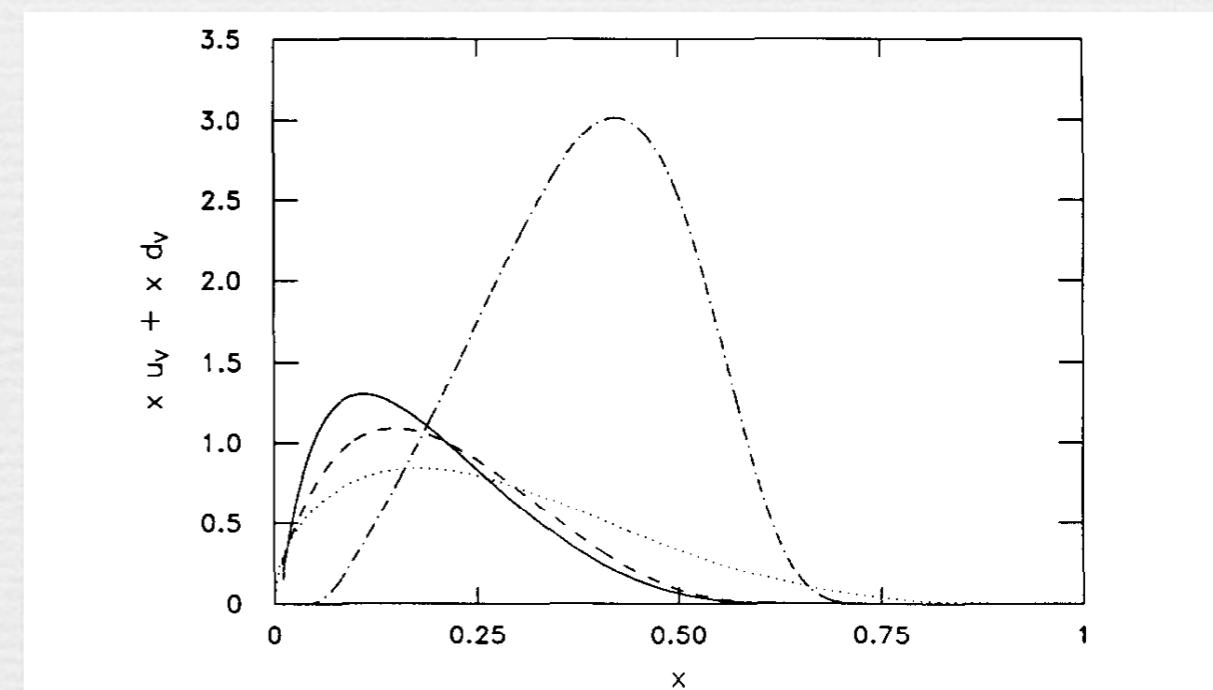
- ~  $\Lambda_{\text{LO}} = .27 \text{ GeV}$ ;  $\Lambda_{\text{NLO}} = .2 \text{ GeV}$

- ~  $\alpha_s^{\text{LO}} = 4\pi \times .32$ ;  $\alpha_s^{\text{NLO}} = 4\pi \times .13$

- ~ partonic scenario

- ~  $\mu_0^2 = 0.2 \text{ GeV}^2$

e.g. Isgur-Karl model :  
valence distribution



..... IK at  $\mu_0^2$

----- LO evolution to  $Q^2=10 \text{ GeV}^2$

----- NLO evolution to  $Q^2=10 \text{ GeV}^2$

..... CTEQ parametrization



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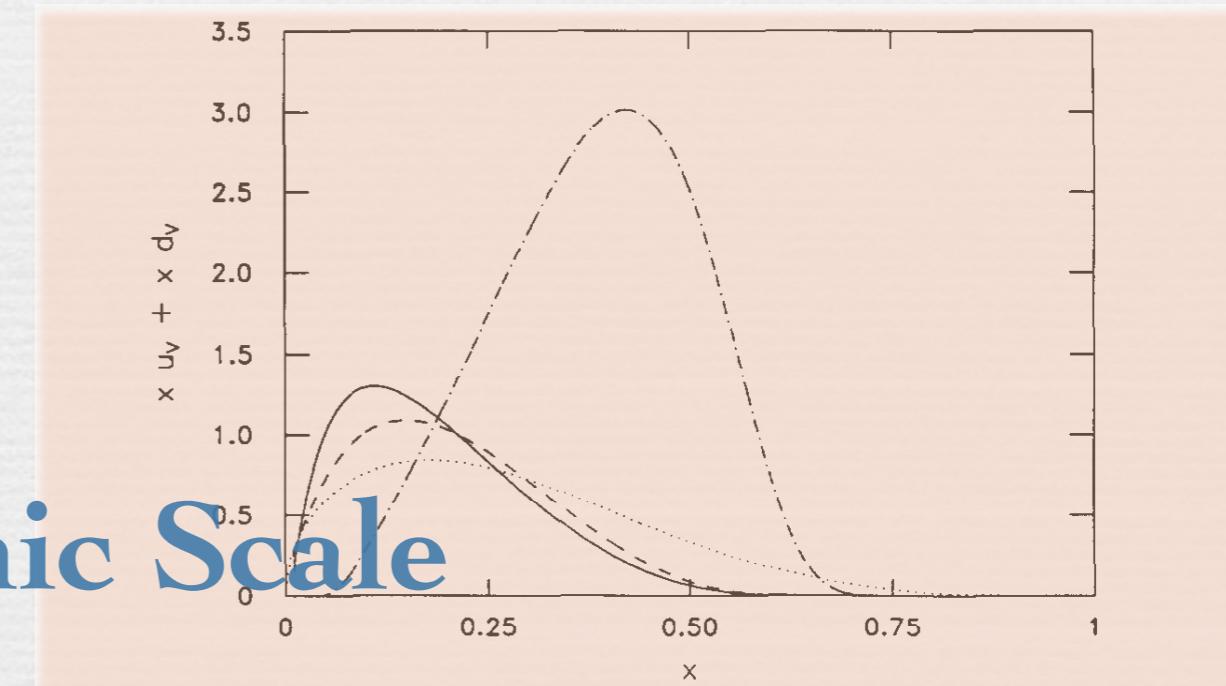
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e.g. Isgur-Karl model :  
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## Low Hadronic Scale

- IK at  $\mu_0^2$
- L0 evolution to  $Q^2=10 \text{ GeV}^2$
- NLO evolution to  $Q^2=10 \text{ GeV}^2$
- CTEQ parametrization

# ‘Perturbative’ Coupling Constant

$$\frac{d a(Q^2)}{d(\ln Q^2)} = \beta_{N^m LO}(\alpha) = \sum_{k=0}^m a^{k+2} \beta_k$$

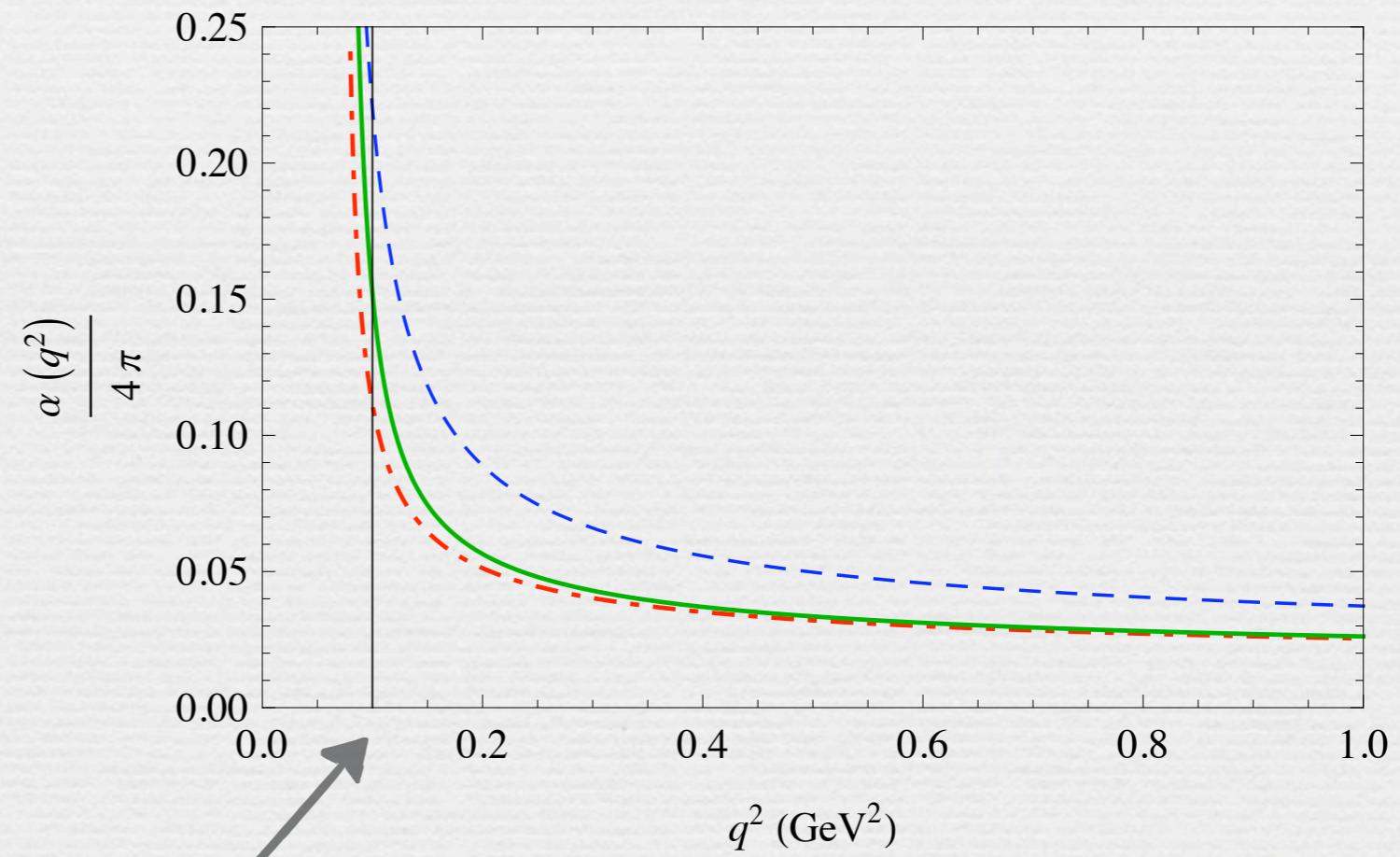
$\overline{MS}$  scheme

L0 exact perturbative solution  $\Lambda=250$  MeV

NLO exact perturbative solution  $\Lambda=250$  MeV

NNLO exact perturbative solution  $\Lambda=250$  MeV

Hadronic scale



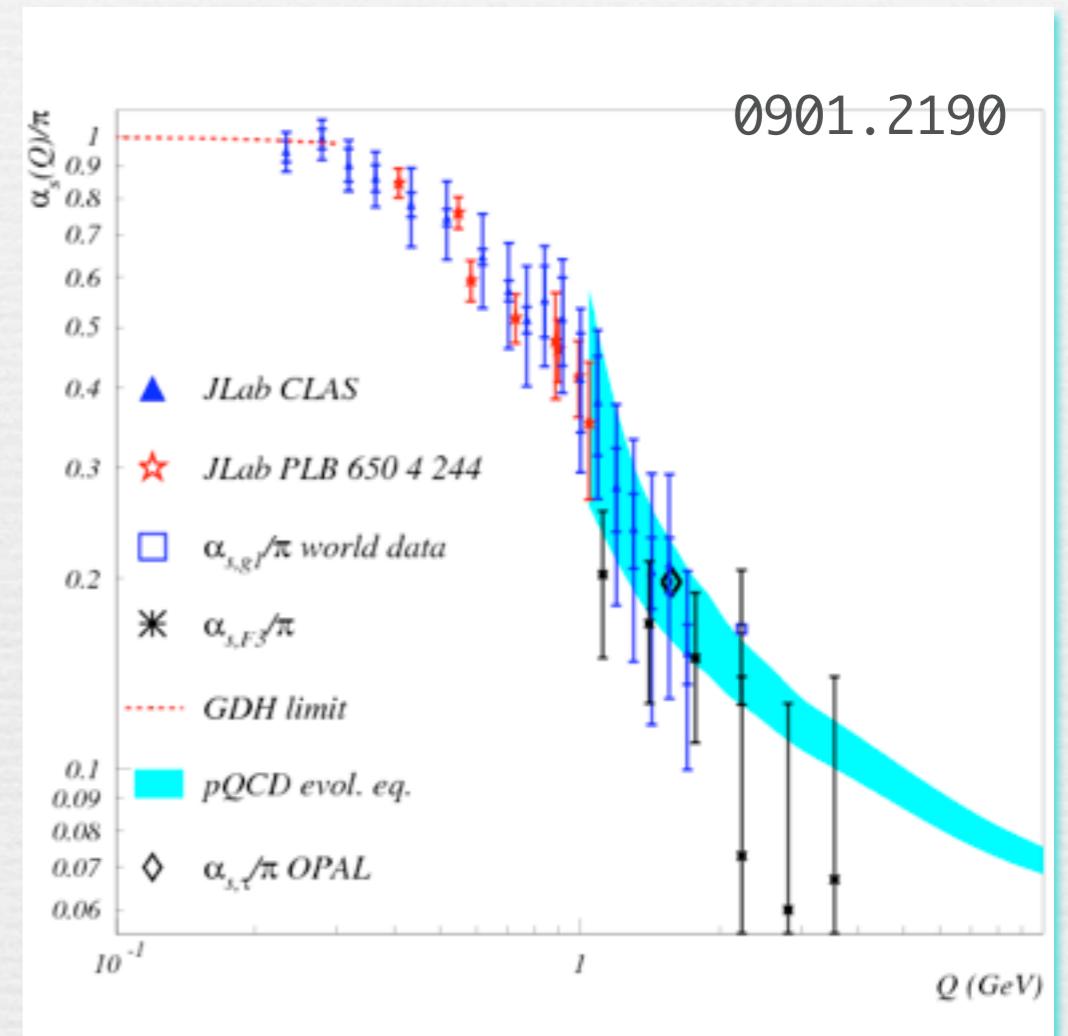
# Infrared Freezing of $\alpha_s$

## Non-perturbative approaches:

- ❖ Importance of finite couplings
- ❖ Taming the Landau pole

e.g. :

- Cornwall, Phys.Rev.D26, 1453 (1982)  
Mattingly & Stevenson, Phys.Rev.D49, 437 (1994)  
Dokshitzer, Marchesini & Webber, Nucl.Phys.B469 (1996) 93  
Cornwall & Papavassiliou, Phys.Rev.Lett.79, 1209 (1997)  
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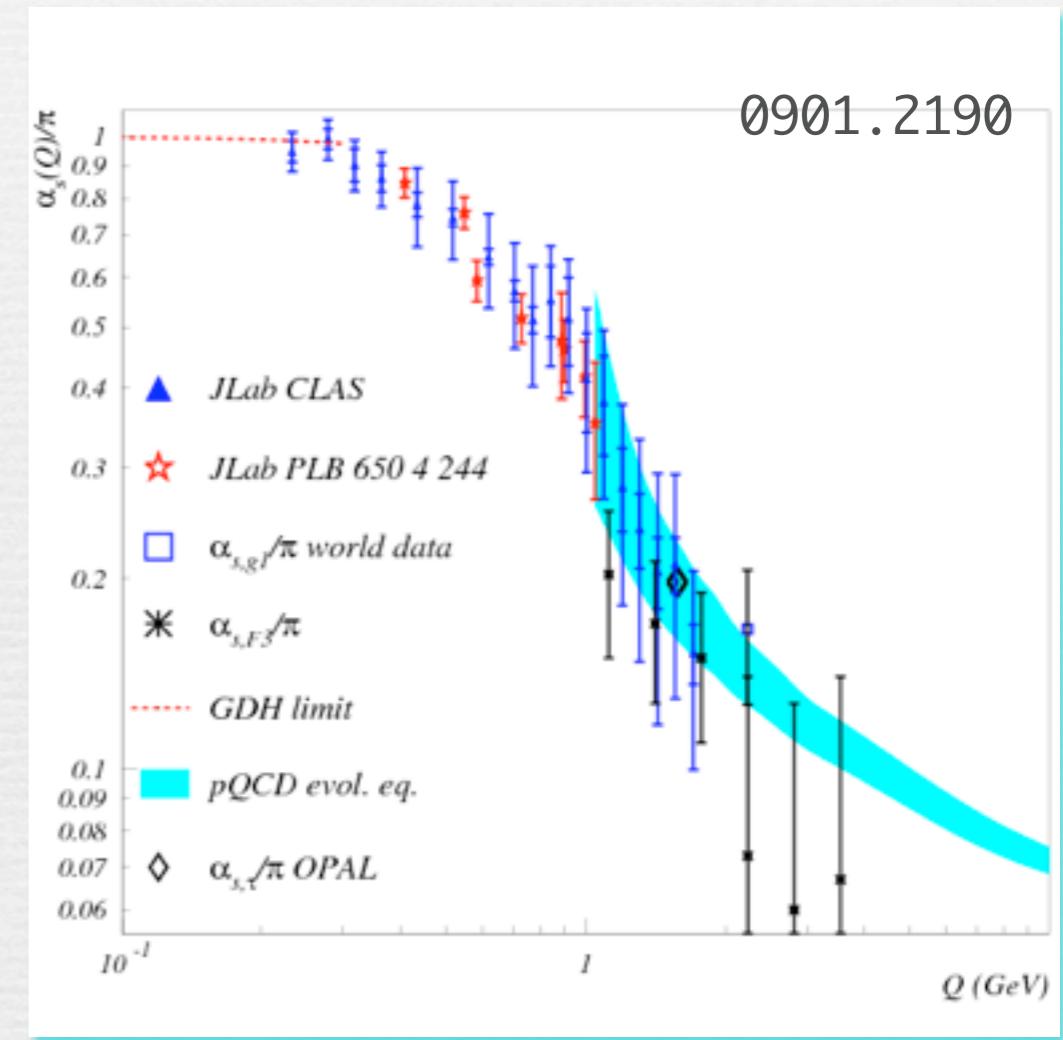
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1<sup>st</sup> step: Qualitative analysis

Implications of IR finite  $\alpha_s$  in hadronic physics

# NP Gluon Propagator: Gluon Mass as IR Regulator

Solving the Schwinger-Dyson eqs ...

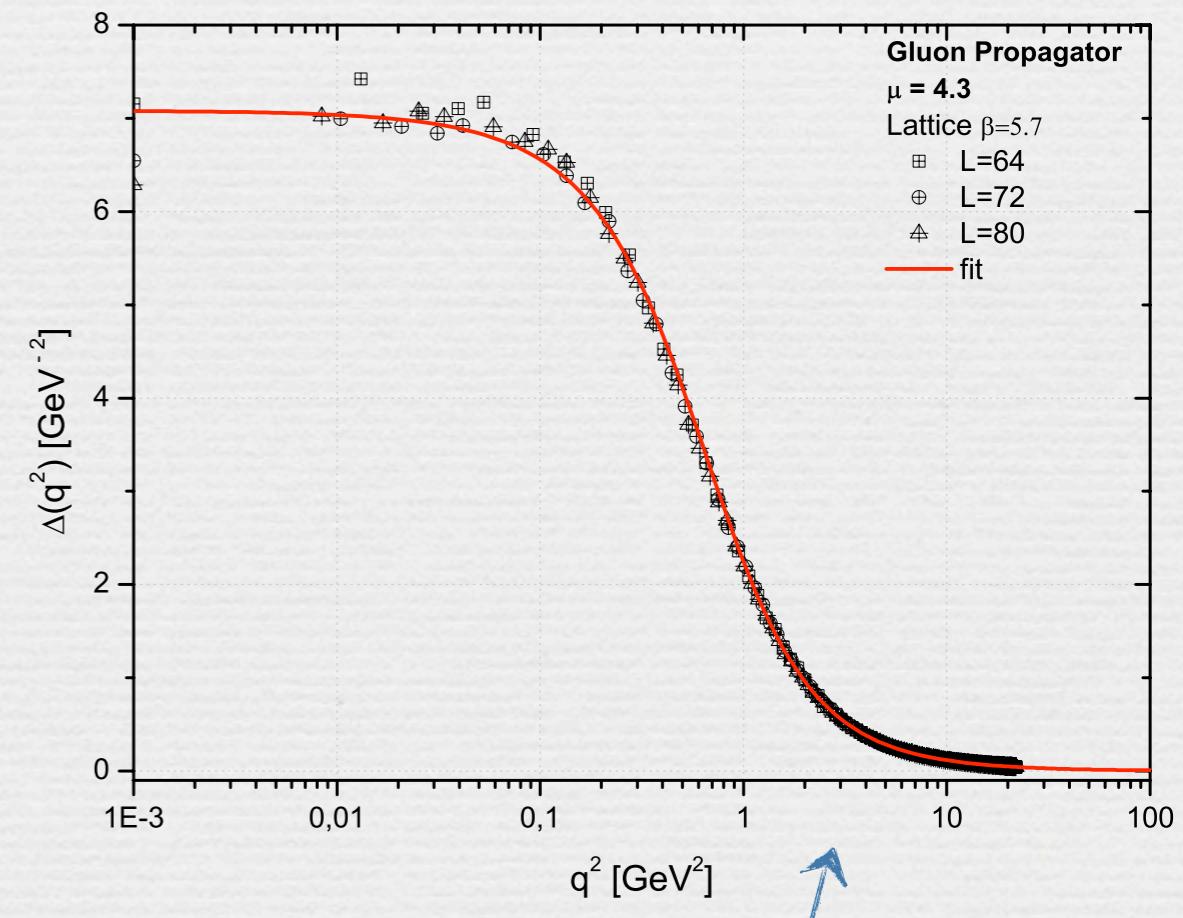
$$\Delta^{-1}(Q^2) = Q^2 + m^2(Q^2)$$

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A. C. Aguilar and J. Papavassiliou, JHEP0612, 012 (2006)

$$m^2(Q^2) = m_0^2 \left[ \ln \left( \frac{Q^2 + \rho m_0^2}{\Lambda^2} \right) \Big/ \ln \left( \frac{\rho m_0^2}{\Lambda^2} \right) \right]^{-1-\gamma}$$

effective gluon mass  
phenomenological estimates

$$m_0 \sim \Lambda - 2\Lambda$$

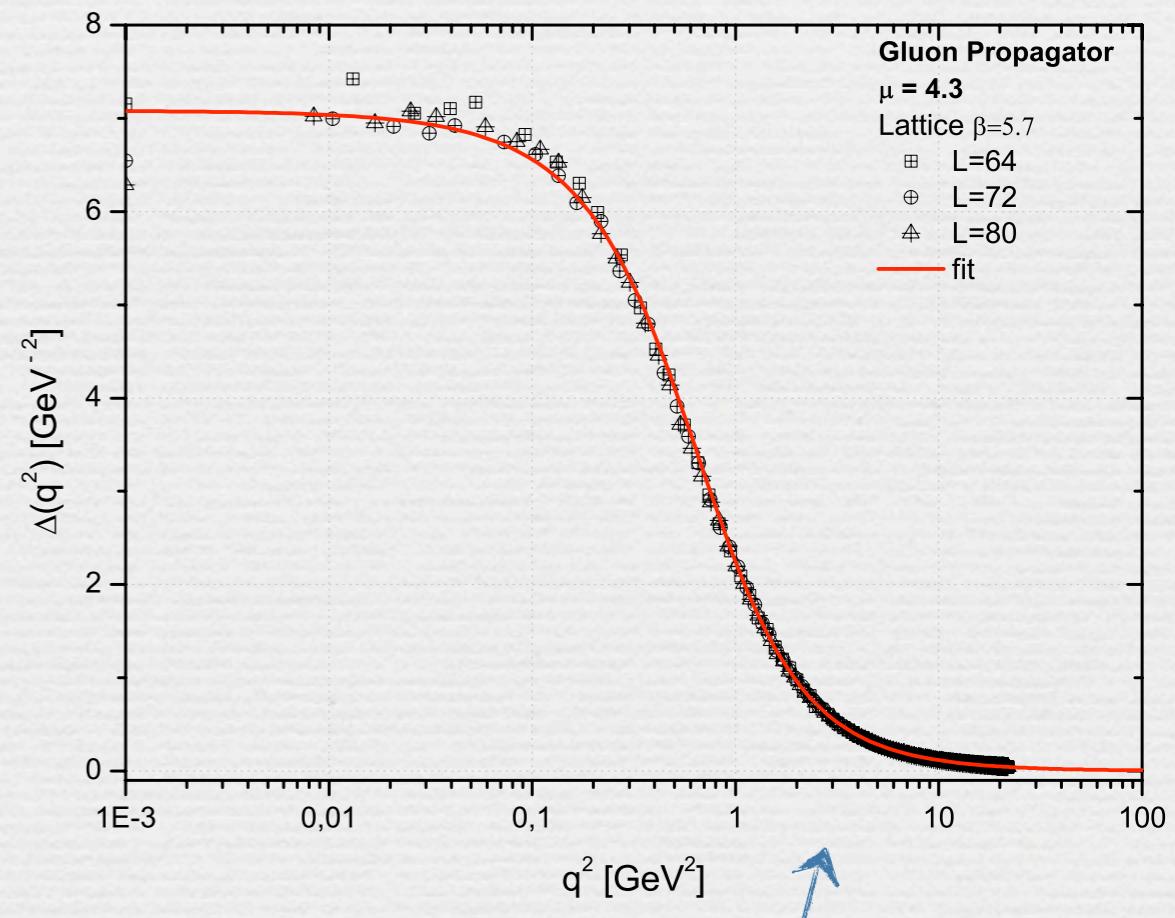


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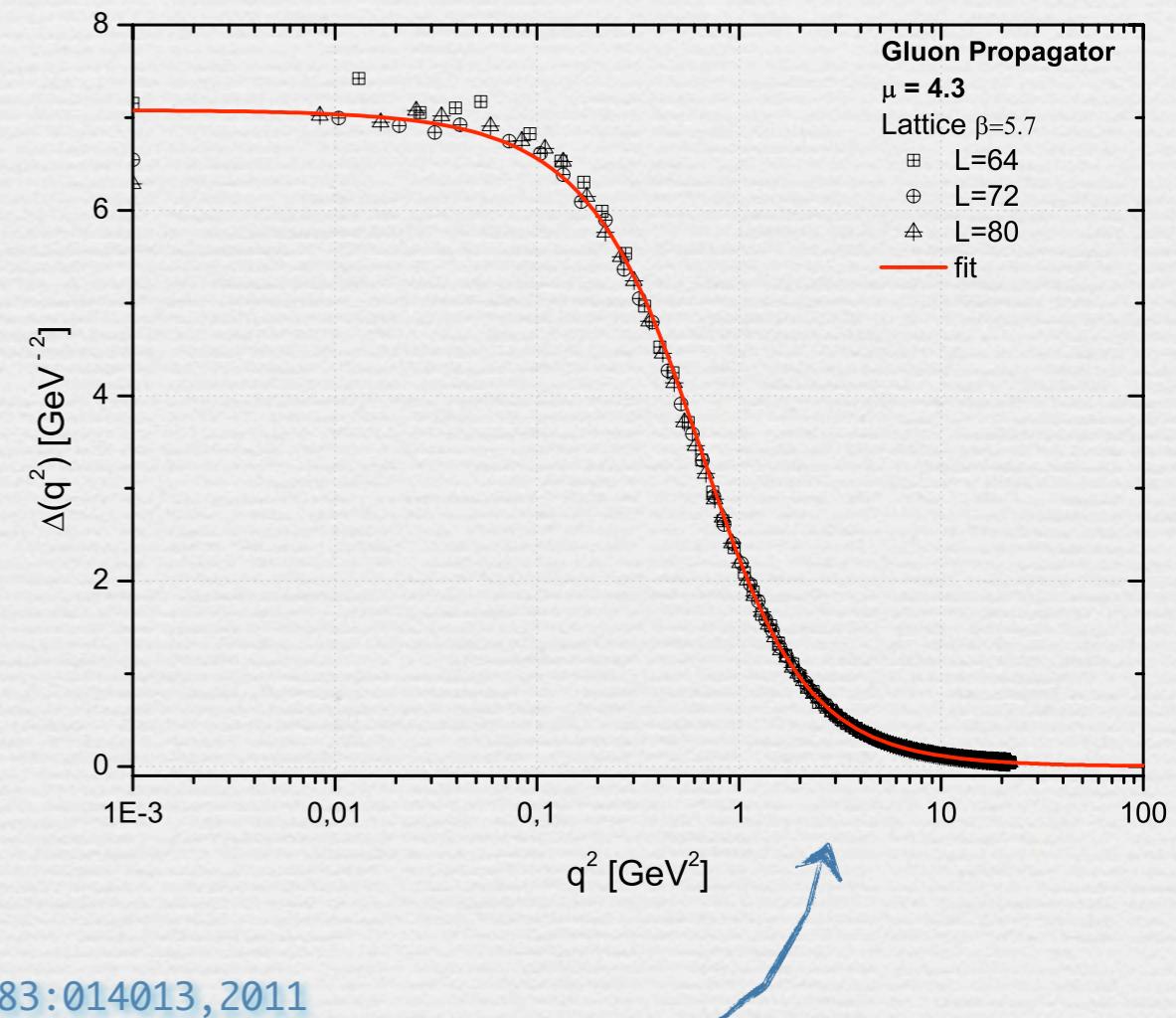
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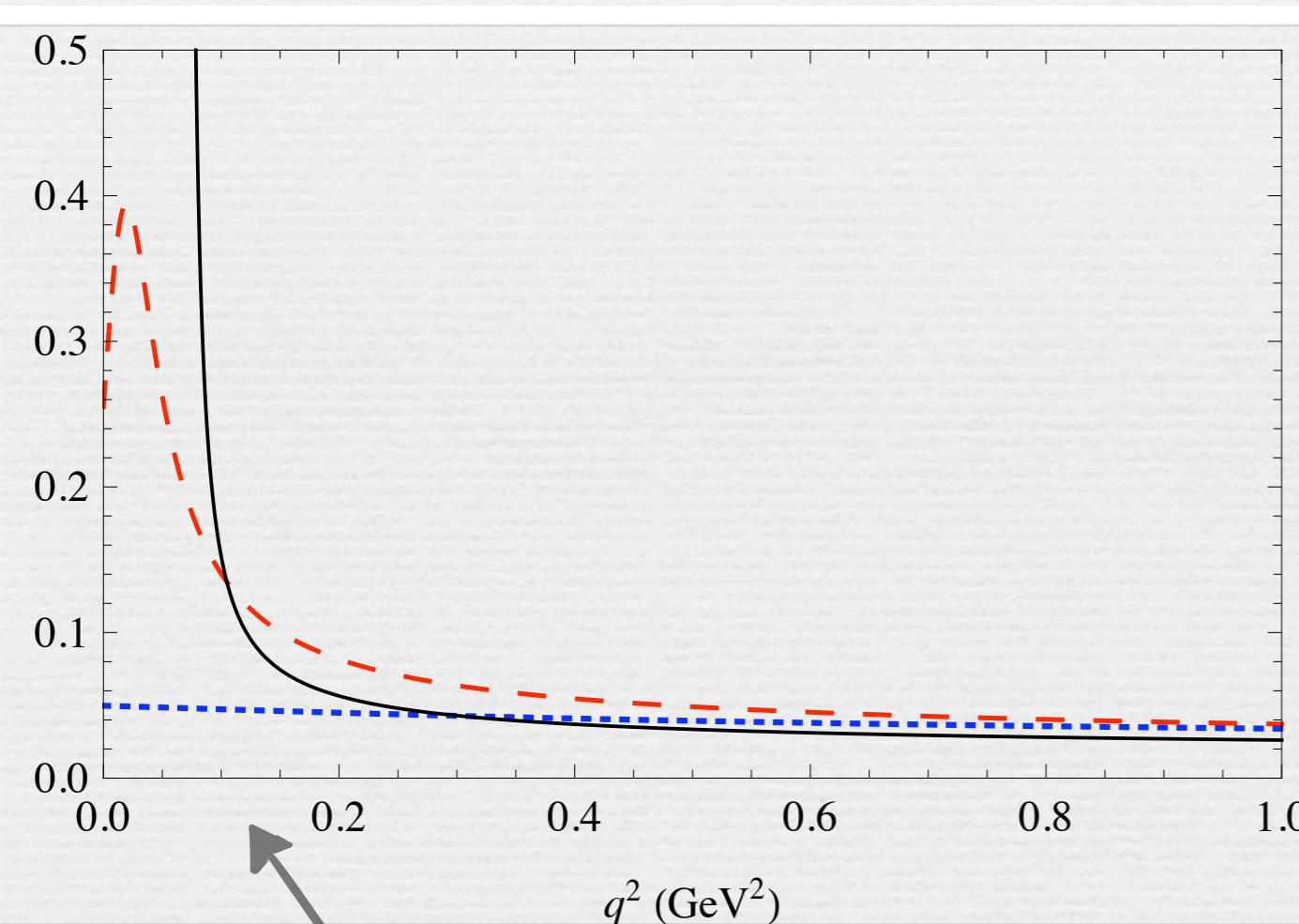
Solution free of Landau pole

Freezes in the IR



# NP Momentum-dependence of the Coupling Constant

$$\frac{\alpha_{\text{NP}}(Q^2)}{4\pi} = \left[ \beta_0 \ln \left( \frac{Q^2 + \rho m^2(Q^2)}{\Lambda^2} \right) \right]^{-1}$$



L0 perturbative evolution  
 $\Lambda=250$  MeV ;  $\overline{MS}$  scheme

Low mass scenario NP coupling constant  
 $m_0=250$  MeV ;  $\Lambda=250$  MeV ;  $\rho=1.5$

High mass scenario NP coupling constant  
 $m_0=500$  MeV ;  $\Lambda=250$  MeV ;  $\rho=2.$

Hadronic scale

# Perturbative vs. NP ‘evolution’: Fixing the hadronic scale

2nd moment of  $f_1$

$$\langle q_v(Q^2) \rangle_n = \langle q_v(\mu_0^2) \rangle_n \left( \frac{\alpha(Q^2)}{\alpha(\mu_0^2)} \right)^{d_{NS}^n}$$

L0 perturbative evolution

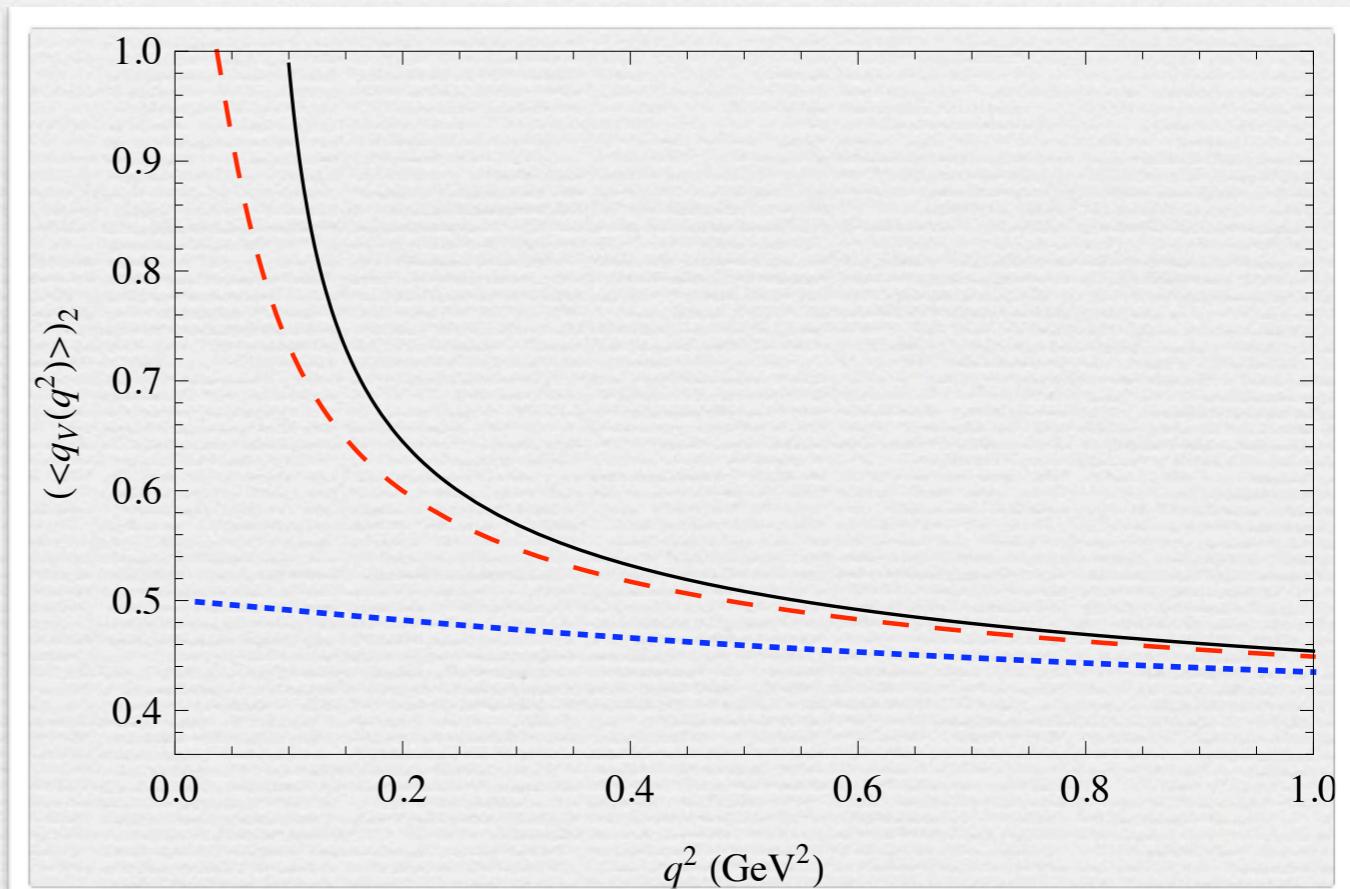
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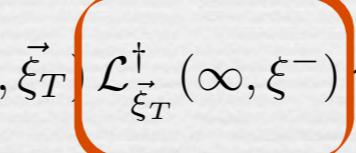
# Final State Interactions and NP evolution

T-odd TMDs :

- ♦ Matrix element of low twist operator



$$\begin{aligned} f_{1T}^{\perp q}(x, k_T) &= -\frac{M}{2k_x} \int \frac{d\xi^- d^2 \vec{\xi}_T}{(2\pi)^3} e^{-i(xp^+ \xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \\ &\times \frac{1}{2} \sum_{S_y=-1,1} S_y \langle PS_y | \bar{\psi}_q(\xi^-, \vec{\xi}_T) \mathcal{L}_{\vec{\xi}_T}^\dagger(\infty, \xi^-) \gamma^+ \mathcal{L}_0(\infty, 0) \psi_q(0, 0) | PS_y \rangle + \text{h.c.} \end{aligned}$$



- ♦ Importance of gauge link



$$\mathcal{L}_{\vec{\xi}_T}(\infty, \xi^-) = \mathcal{P}\exp \left( -ig \int_{\xi^-}^{\infty} A^+(\eta^-, \vec{\xi}_T) d\eta^- \right)$$

# Final State Interactions and NP evolution

Twofold problem :

- ◆ FSI mimicked by a one-gluon-exchange
  - gluon propagator
- ◆ Explicit dependence on the coupling constant
  - relevance of NP scheme for model calculations

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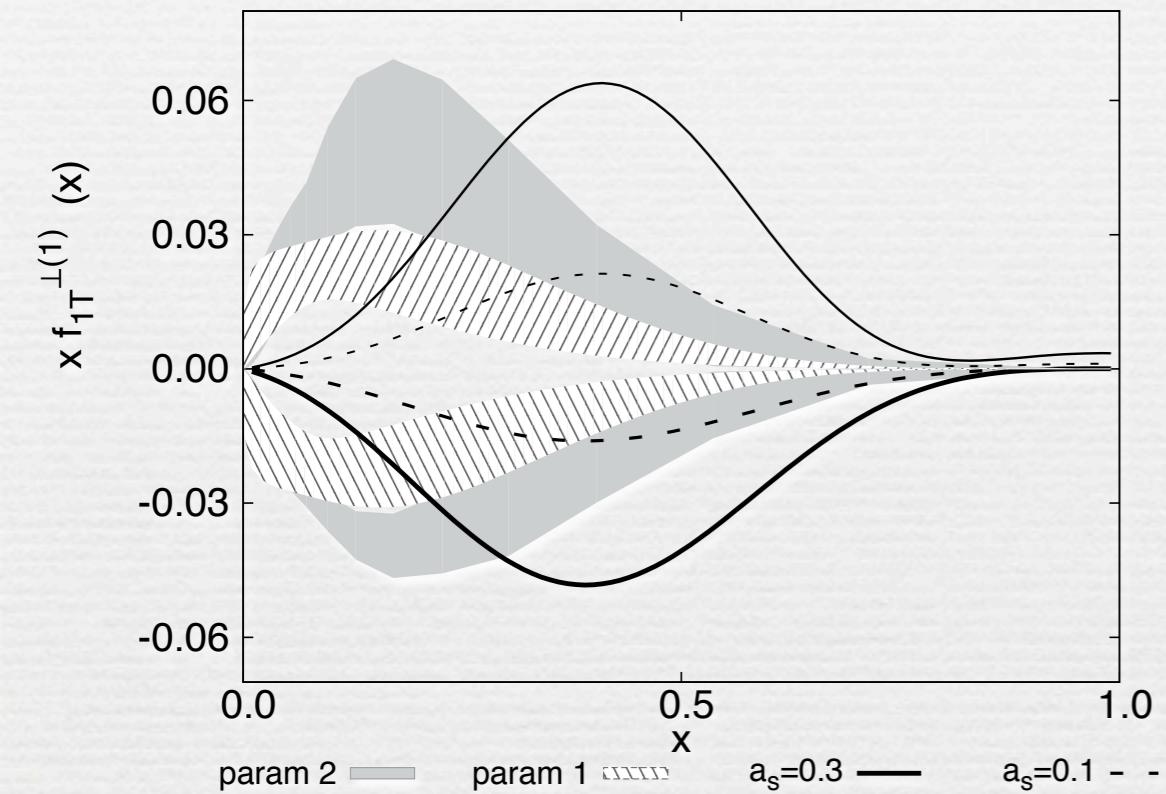
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Example :

- ♦ MIT bag model calculation
  - perturbative QCD governs the dynamics inside the confining region
  - no need for NP gluon propagator
  - NP scheme → change of hadronic scale
- ♦ Other model calculations? e.g. L. Gamberg and M. Schlegel, Phys. Lett. B 685 (2010) 95

# Sivers & Boer-Mulders functions

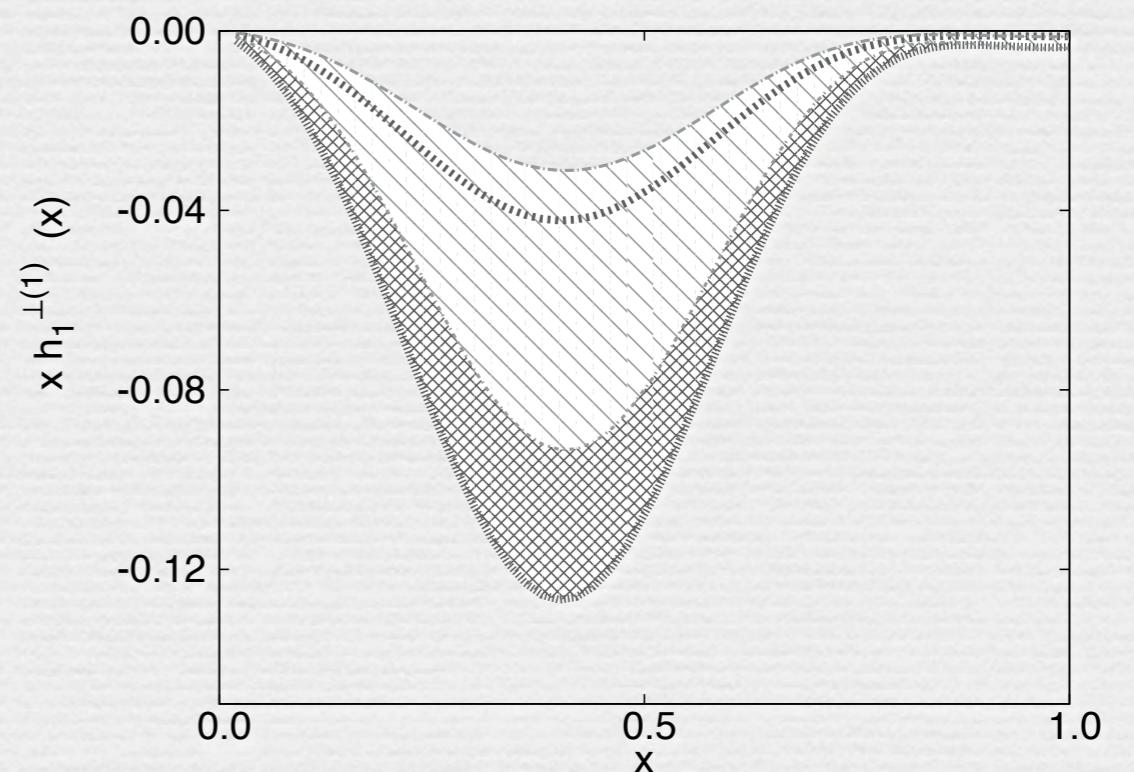


Bag Model:

rescaling of  $f_{1T}^{\perp}$  &  $h_1^{\perp}$

$$0.1 < \frac{\alpha_s(\mu_0^2)}{4\pi} < 0.3$$

dashed  $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.1$   
 solid  $\frac{\alpha_s(\mu_0^2)}{4\pi} = 0.3$



# Conclusions

- ❖ Low hadronic scale validated by IR behavior of  $\alpha_s$
- ❖ Good description of perturbative dynamics by ‘standard scheme’: now supported by NP scheme
- ❖ Set of parameters needs to be pushed towards ‘pure valence’ s hadronic scale:  
**NP scheme favors scenarios valence quarks + sea + gluons**
- ❖ Quantitative analysis:  
would depend on HOW the IR freezing is obtained.
- ❖ Impact on Phenomenology:  
Value of coupling constant  $\rightarrow$  *theoretical errorband*  
NP gluon propagator  
**QCD evolution equations at low  $Q^2$  ?**