Evolution of TMDs in the light-cone gauge

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QCD Evolution Workshop: from collinear to non collinear case
April 8-9, 2011
Thomas Jefferson National Accelerator Facility
Newport News, VA

Based on collaboration with
N.G Stefanis: *BOCHUM*$_{\mathrm{DE}}$

- Unintegrated transverse-momentum dependent parton densities (TMDs) contain information about intrinsic (L & T) motion of partons inside a hadron

- TMD can be defined as a straightforward generalization of collinear PDF: → `Collins; Soper (1979)`

$$P_{[\text{collinear}]}(x, \mu) \ \to \ \mathcal{P}_{[\text{tmd}]}(x, \boldsymbol{k}, \mu, \zeta)$$

—depends on the $x = k^+/p^+$ (L) and $\boldsymbol{k}_\perp$ (T) components of parton's momentum; on the UV scale μ and on the rapidity cutoff ζ

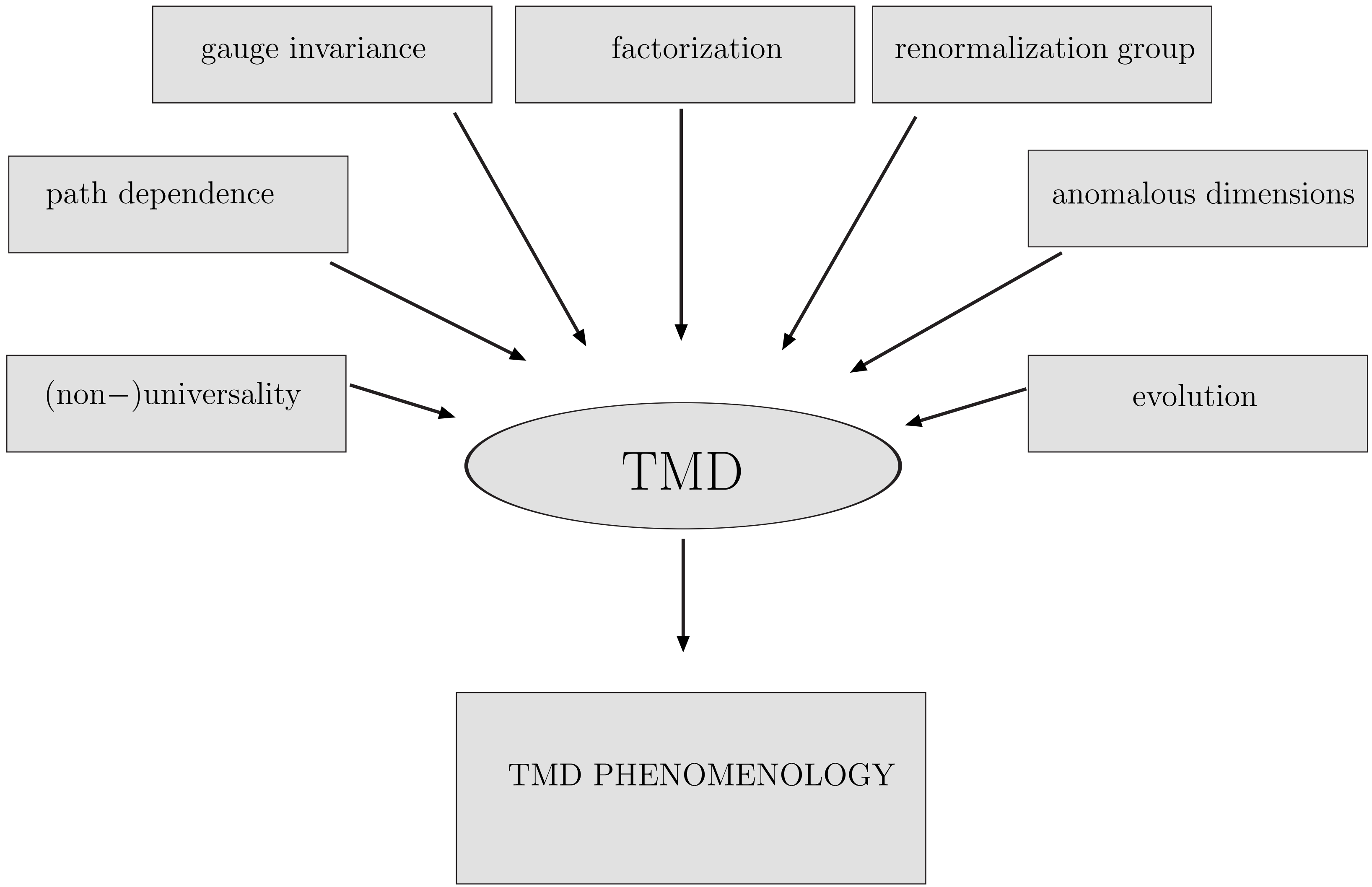
- collinear PDFs *(are expected to)* restore after the $\boldsymbol{k}_\perp$-integration

$$P_{[\text{collinear}]}(x, \mu) \sim \int d^2 k_\perp \ \mathcal{P}_{[\text{tmd}]}(x, \boldsymbol{k}, \mu, \zeta)$$

Nowadays: the issue is much more intricate than *(one could imagine)* some 30 years ago!

TMD ROADMAP

(idea by N. Stefanis)



- INTEGRATED PDFs: DIS

$$W_{\mu\nu} = \frac{1}{2\pi} \Im m \left[i \int d^4\xi \, e^{iq\xi} \langle P | T\{ J_\mu(\xi) J_\nu(0) \} | P \rangle \right]$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{P \cdot q} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left(P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) F_2(x_B, Q^2)$$

- QCD FACTORIZATION in DIS

$$F(x_B, Q^2) = H(x_B, Q^2/\mu^2) \otimes F_D(\mu^2) = \sum_i \int_{x_B}^1 \frac{d\xi}{\xi} C_i \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2} \right) F_D^i(\xi, \mu^2)$$

RENORMALIZATION PROPERTIES: DGLAP

$$\mu \frac{d}{d\mu} Q_{i/h}(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z} \right) Q_{j/h}(x, \mu)$$

- MOMENTS of collinear PDFs are related to matrix elements of the local twist-2 operators within the OPE

GAUGE INVARIANT COLLINEAR PDF

$$Q_{i/h}(x,\mu) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)[\xi^-, 0^-]\gamma^+\psi_i(0^-, \mathbf{0}_\perp)|h\rangle$$

Gauge invariance is saved by the insertion of the light-like gauge link

$$[y, x]_r = \mathcal{P}\exp\left[-ig\int_{\tau_1}^{\tau_2} d\tau r^\mu A_\mu^a(r\tau)t^a\right] \quad r^\mu\tau_1 = x\ ,\ r^\mu\tau_2 = y$$

Distinguish between L- $[\ ,\ ]_{[n,\,v,\,v_0]}$ and T- $[\ ,\ ]_{[\boldsymbol{l}\,]}$ gauge links!

QCD FACTORIZATION for TMD

→ Collins, Soper (1981); Collins, Metz (2004); Ji, Ma, Yuan (2005); Bacchetta, et al: (2005)

Standard factorization expected:

$$F(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2) = \sum_i e_i^2 \cdot H \otimes \mathcal{F}_D \otimes \mathcal{F}_F \otimes S$$

- Extra (rapidity) divergences;
- Complicated structure of gauge links: non-universality (generalized factorization):
  → Bacchetta, Bomhof, Mulders, Pijlman
- Generalized factorization may fail: several counter-examples exist
  → Collins, Qiu; Mulders, Rogers

PROBLEMS of OPERATOR DEFINITION of TMD

TMD with the light-like and transverse gauge links:

$$\mathcal{F}_i(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \, \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} .$$

$$\cdot \langle P, S | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger_{[n]} [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{\boldsymbol{l}} \cdot$$

$$\cdot \gamma^+ \cdot$$

$$\cdot [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]_{\boldsymbol{l}} [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]_{[n]} \psi_i(0^-, 0_\perp) | P, S \rangle \,|_{\xi^+=0}$$

Formally:

$$\int d^2k_\perp \mathcal{F}_i(x, \boldsymbol{k}_\perp) = F_i(x)$$

CLASSIFICATION of SINGULARITIES

1. Ultraviolet poles $\sim \frac{1}{\varepsilon}$: removed by the standard renormalization procedure;

2. Overlapping divergences: contain the UV and rapidity poles simultaneously $\sim \frac{1}{\varepsilon} \ln \theta$: generalized renormalization procedure

3. Pure rapidity divergences: $\sim \ln^{1,2} \theta$: can be safely resummed by means of the Collins-Soper equation.

4. Specific self-energy divergences: stem from the gauge links, do not affect rapidity evolution; treated by modifications of the soft factors

TMD DEFINITIONS COMPENDIUM

- A_v-TMD: axial non-light-like gauge $(v \cdot A) = 0$; L_v-gauge links vanish; rapidity cutoff: $\zeta = (2P \cdot v)^2/|v^2|$ $\longrightarrow$ `Collins, Soper`

- C_v-TMD: covariant gauge; L_v-gauge links survive; rapidity cutoff: $\zeta = (2P \cdot v)^2/|v^2|$ $\longrightarrow$ `Ji, Ma, Yuan`

- A_n-TMD: light-like axial gauge $(n \cdot A) = 0\ ,\ n^2 = 0$; L_n-gauge links vanish; T-gauge links survive; regularization: $\theta = (P \cdot n)/\eta$

$$\frac{1}{[q^+]_\eta} = \frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)$$

light-like gauge links in the soft factor $\longrightarrow$ `ICh, Stefanis`

- C_n-TMD: covariant gauge; L_n-gauge links survive; T-gauge links vanish; regularization $\zeta = (2P \cdot v)^2/|v^2|$, $v^2 \neq 0$ in the soft factor $\longrightarrow$ `Collins, Hautmann`

- L-TMD: lattice simulations; direct connector as gauge link, no regularization parameters, no light-like gauge links $\longrightarrow$ `Haegler, Musch`

RAPIDITY EVOLUTION

Fourier transformed TMD

$$\mathcal{F}\left(x, \mathbf{b}_\perp\right) = \int d^2\mathbf{k}_\perp\, \mathcal{F}\left(x, \mathbf{k}_\perp\right)$$

Collins-Soper rapidity evolution holds for the A_v-TMD and C_v-TMD, $\zeta = (2P \cdot v)^2/|v^2|$

$$\zeta \frac{\partial}{\partial \zeta}\, \mathcal{F}_{[A,C_v]}\left(x, \mathbf{b}_\perp; \mu, \zeta\right) = \left[K_v(\mu, \mathbf{b}_\perp) + G_v(\mu, \zeta)\right] \mathcal{F}_{[A,C_v]}\left(x, \mathbf{b}_\perp; \mu, \zeta\right)$$

$$\mu \frac{d}{d\mu} K_v = -\mu \frac{d}{d\mu} G_v = \gamma_{\rm cusp}$$

$$K_v(\mu, \mathbf{b}_\perp) + G_v(\mu, \zeta) = -\frac{\alpha_s C_F}{\pi} \ln \frac{x^2 \zeta^2 b_\perp^2 e^{2\gamma_E - 1}}{4}$$

x

RAPIDITY EVOLUTION: LIGHT-CONE TMD

Generalized definition of TMD:

$$\mathcal{F}_{[n]}\left(x, \boldsymbol{k}_\perp\right) \cdot R_n^{-1} =$$

$$\frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle h|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]}$$

$$\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]} \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]_{[\boldsymbol{l}]} [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]_{[n]} \psi(0^-, \mathbf{0}_\perp)|h\rangle \times$$

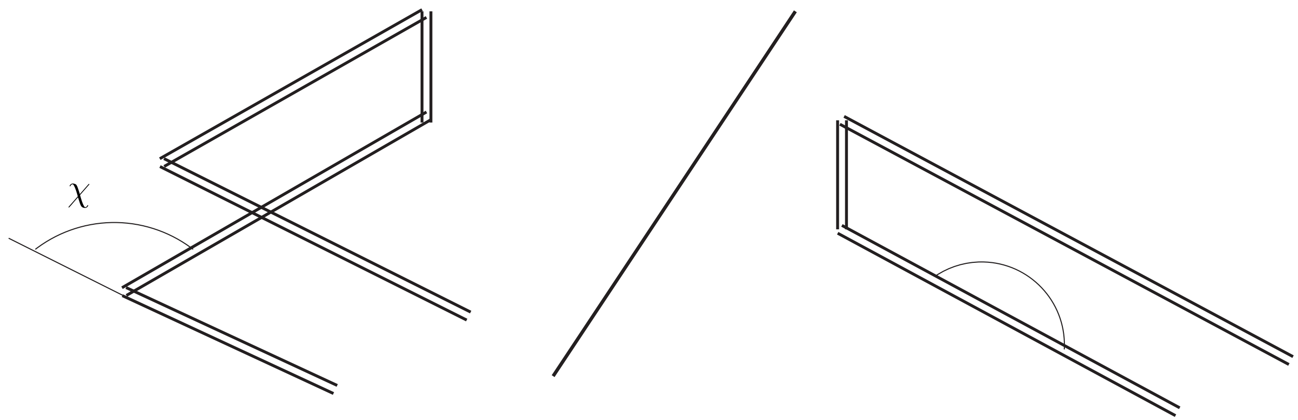
$$\times \left[\text{soft factor}\right]^{-1}$$

Soft factor, which cancels cusp (rapidity) divergency, gives rise to the self-energy, cusp-independent singularity! The soft factor must be changed itself:

soft factor $=$

$$\langle 0| \, \mathcal{P} \exp \left[ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^{\mu} \, t^{a} A^{a}_{\mu}(\zeta) \right] \cdot \mathcal{P}^{-1} \exp \left[-ig \int_{\mathcal{C}'_{\rm cusp}} d\zeta^{\mu} \, t^{a} A^{a}_{\mu}(\xi + \zeta) \right] |0\rangle \; /$$

$$\langle 0| \, \mathcal{P} \exp \left[ig \int_{\mathcal{C}_{\rm smooth}} d\zeta^{\mu} \, t^{a} A^{a}_{\mu}(\zeta) \right] \cdot \mathcal{P}^{-1} \exp \left[-ig \int_{\mathcal{C}'_{\rm smooth}} d\zeta^{\mu} \, t^{a} A^{a}_{\mu}(\xi + \zeta) \right] |0\rangle$$

soft factor $=$ 

However, this change doesn't affect the rapidity evolution: $d/d\theta[\text{self} - \text{energy}] = 0$.

- stay always on the light-cone
- all divergences under control
- systematic procedure to remove all undesirable singularities (to be checked in higher orders)

RAPIDITY EVOLUTION: LIGHT-CONE TMD

Consider: TMD distribution of quark in a quark: $|h\rangle = |p\rangle$

Tree approximation

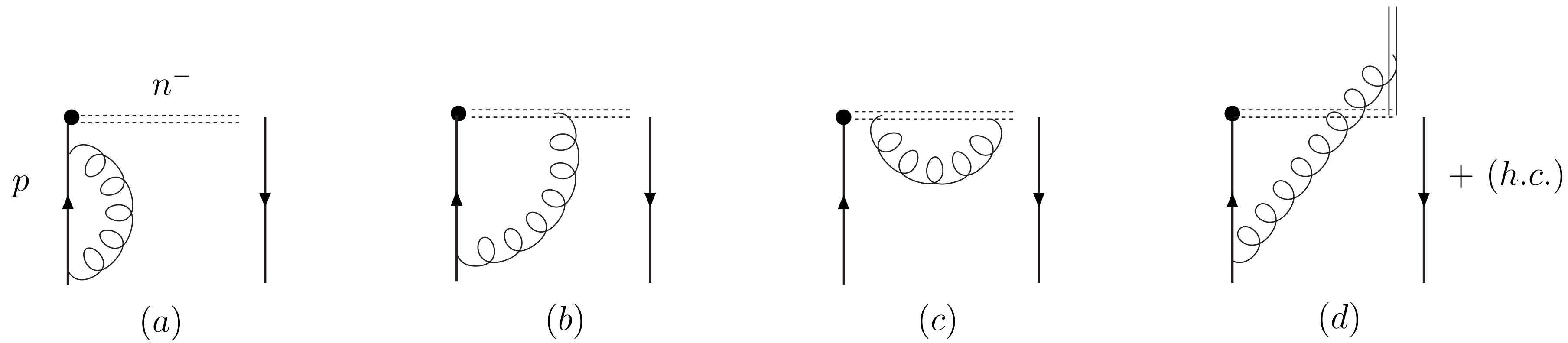
$$\mathcal{F}^{(0)}_{[\mathrm{An}]}(x, \mathbf{k}_\perp) =$$

$$= \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} \langle p|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)\gamma^+\psi(0^-, \mathbf{0}_\perp)|p\rangle =$$

$$= \delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp)$$

RAPIDITY EVOLUTION: LIGHT-CONE TMD

Rapidity dependence stems from the one-gluon graphs



$$\Sigma_\eta^{(a)} = g^2 C_{\rm F} \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \left[(\hat{p}\gamma_\mu\gamma^+ + \gamma^+\gamma_\mu\hat{p}) \frac{(p-q)^\mu}{(p-q)^2} - 2\gamma^+ \right] \frac{1}{(q^2+i0)[q^+]_\eta} \frac{i\hat{p}}{p^2}$$

$$\to \int_0^1 dx \frac{(1-x)^{1-\epsilon} x^{-\epsilon}}{[xp^+]} = \frac{1}{p^+} \int_0^1 dx \frac{(1-x)^{1-\epsilon} x^{-\epsilon}}{x \pm i\frac{\eta}{p^+}} =$$

$$= \frac{1}{p^+} \left[-1 - \ln \frac{\pm i\eta}{p^+} - \epsilon \left(2 - \frac{\pi^2}{3} - \frac{1}{2} \ln^2(\pm i\eta) \right) + O(\epsilon^2) \right]$$

RAPIDITY EVOLUTION: LIGHT-CONE TMD

PRELIMINARY!

$$\theta\frac{\partial}{\partial\theta}\,\mathcal{F}_{[\mathrm{An}]}\left(x,\mathbf{b}_\perp;\mu,\theta\right)=\left[K_{\mathrm{n}}(\mu,\mathbf{b}_\perp)+G_{\mathrm{n}}(\mu,\theta)\right]\mathcal{F}_{[\mathrm{An}]}\left(x,\mathbf{b}_\perp;\mu,\theta\right)$$

$$\mu\frac{d}{d\mu}K_{\mathrm{n}}=-\mu\frac{d}{d\mu}G_{\mathrm{n}}=\gamma_{\mathrm{cusp}}$$

$$K_{\mathrm{n}}(\mu,\mathbf{b}_\perp)+G_{\mathrm{n}}(\mu,\theta)=-\frac{\alpha_s C_{\mathrm{F}}}{\pi}\ln\,\theta^2 b_\perp^2\mathcal{C}_{\mathrm{n}}$$

(COMPLETE) EVOLUTION: LIGHT-CONE TMD

PRELIMINARY!

UV evolution:

$$\mu \frac{d}{d\mu} \left[\mathcal{F}_{[\mathrm{An}]} \cdot R_n^{-1} \right] = \gamma_0 \left[\mathcal{F}_{[\mathrm{An}]} \cdot R_n^{-1} \right]$$

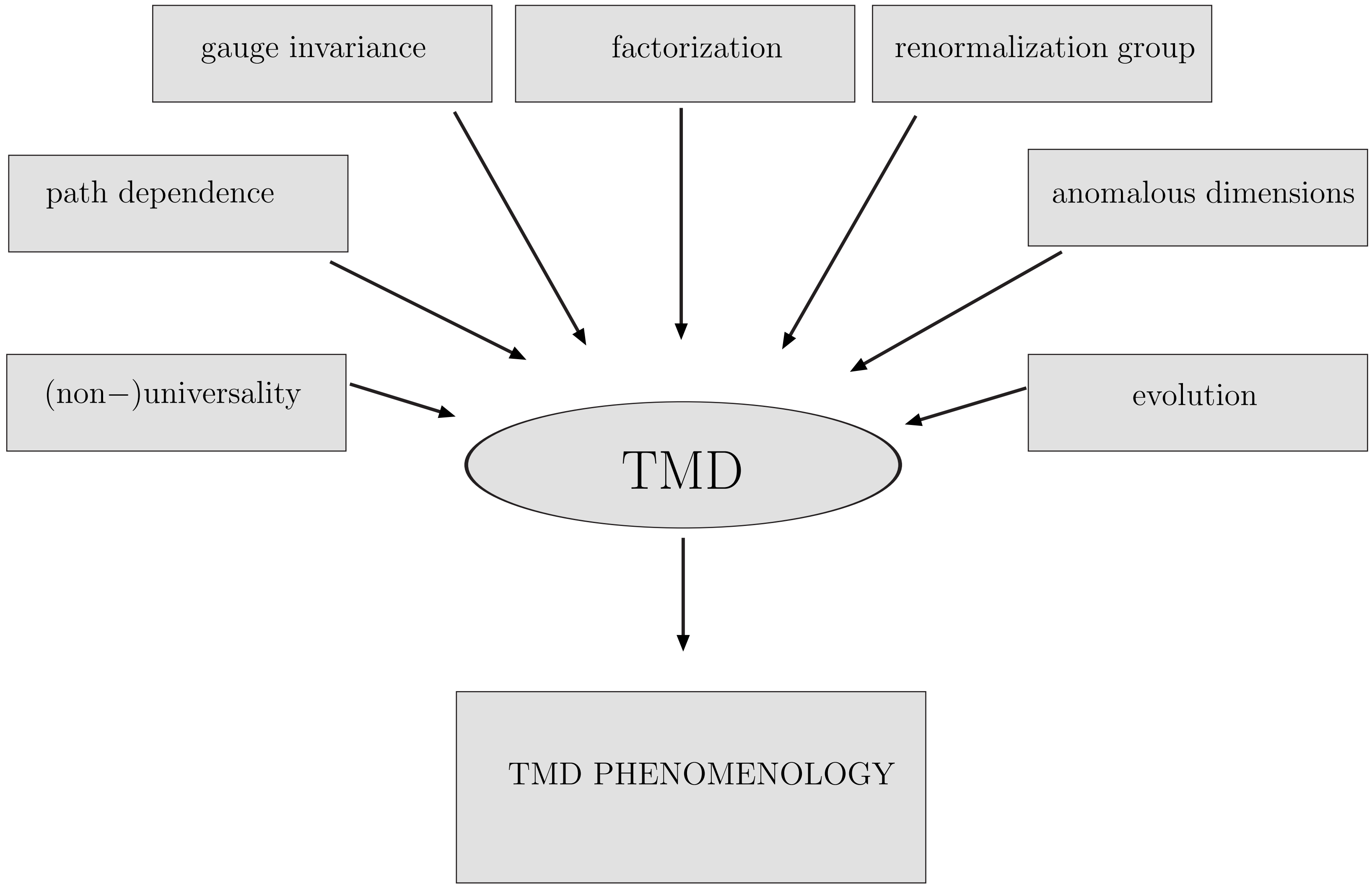
Rapidity evolution:

$$\theta \frac{\partial}{\partial \theta} \mathcal{F}_{[\mathrm{An}]} (x, \mathbf{b}_\perp; \mu, \theta) = [K_{\mathrm{n}}(\mu, \mathbf{b}_\perp) + G_{\mathrm{n}}(\mu, \theta)] \, \mathcal{F}_{[\mathrm{An}]} (x, \mathbf{b}_\perp; \mu, \theta)$$

Small-x evolution?

$\rightarrow$ see talks by Balitsky, Jalilian-Marian, Dominguez, Collins, Qiu, Pirnay, Yuan, Chirilli, Avsar...

TMD ROADMAP


gauge invariance
factorization
renormalization group
path dependence
anomalous dimensions
(non−)universality
evolution
TMD
TMD PHENOMENOLOGY