Improving the kinematics in BK/BFKL to resum the dominant part of higher orders

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QCD Evolution Workshop: from collinear to non collinear case
Jefferson Lab, April 8, 2011

Motivation

Gluon saturation in hadronic or nuclear wave-functions at small x (a.k.a. Color Glass Condensate) is progressively becoming a mature topic:

- There are strong experimental evidence for it in d-A collisions at RHIC, and less strong ones in DIS at HERA.
- Numerical solutions of the LO BK equation with running coupling provide a very good description of these data.
- The BK equation is now available at NLO accuracy.
- CGC is becoming the standard framework for initial conditions in heavy ion collisions.

Motivation

Hence, one should now try to push gluon saturation phenomenology from LO with running coupling to full NLO.

In the high energy QCD evolution (BK or JIMWLK), the nonlinearity of gluon saturation plays effectively the role of a dynamical boundary condition, and the evolution itself is driven by the BFKL kernel.

Hence, most of the issues met in developing BFKL phenomenology at NLO are also there for gluon saturation.

Outline

1. Remarks about the BFKL equation at NLO
2. LO and NLO BK equations
    - Generalities about the (N)LO BK equations
    - BK evolution in k^+
    - BK evolution in k^-
3. Resummation via double ordering of gluons

Solution of the BFKL equation

Mellin representation of the unintegrated gluon distribution $f(x, \mathbf{k})$:

$$f(x, \mathbf{k}) = \int \frac{d\gamma}{2\pi i} \int \frac{d\omega}{2\pi i} \, x^{-\omega} \left(\frac{\mathbf{k}^2}{\Lambda^2} \right)^{-\gamma} \tilde{f}(\omega, \gamma)$$

$f(x, \mathbf{k})$ is a solution of the BFKL equation (neglecting running coupling effects) if the only contribution to the ω integration comes from a pole of $\tilde{f}(\omega, \gamma)$ at the eigenvalue of the BFKL kernel:

$$\omega = \bar{\alpha}\, \chi_0(\gamma) + \bar{\alpha}^2\, \chi_1(\gamma) + \mathcal{O}(\bar{\alpha}^3)\,,$$

$$\text{where} \quad \bar{\alpha} = \frac{N_c}{\pi}\, \alpha_s$$

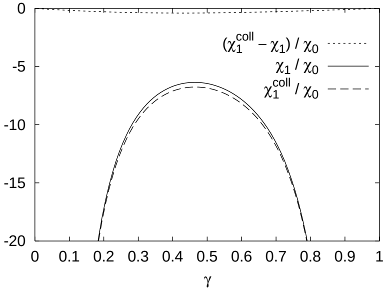
$$\text{and} \quad \chi_0(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma) = \frac{1}{\gamma} + \frac{1}{1-\gamma} + \cdots$$

NLO BFKL eigenvalue

Problem: the NLO BFKL eigenvalue $\chi_1(\gamma)$ is very large.

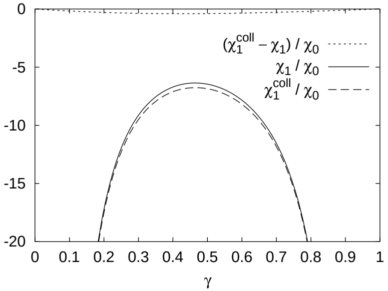
Fadin, Lipatov (1998); Camici, Ciafaloni (1998)

$\Rightarrow$ Apparent breakdown of perturbation theory.



Plot from Salam: hep-ph/9910492

NLO BFKL eigenvalue



Remark: the NLO BFKL eigenvalue $\chi_1(\gamma)$ is dominated by contributions of collinear and anti-collinear double and triple poles:

$$\chi_1^{coll}(\gamma) = \frac{A_1(0)}{\gamma^2} + \frac{A_1(0) - b}{(1-\gamma)^2} - \frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3}$$

$$\text{with} \quad A_1(0) = -\frac{11}{12} - \frac{N_f}{6N_c^3} \quad \text{and} \quad b = \frac{11}{12} - \frac{N_f}{6N_c}$$

Origin of NLO BFKL corrections: non-eikonal gluon emission

In Mellin space, the (reversed) DGLAP evolution writes

$$\gamma = \bar{\alpha}\left(\frac{1}{\omega} + A_1(\omega)\right) + \mathcal{O}\left(\frac{\bar{\alpha}^2}{\omega}\right)$$

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Hence: $$\omega = \frac{\bar{\alpha}}{\gamma}\Big(1 + \omega A_1(\omega)\Big) + \mathcal{O}\left(\frac{\bar{\alpha}^2}{\gamma}\right)$$

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Hence: $$\omega = \left[\frac{\bar{\alpha}}{\gamma} + A_1(0)\frac{\bar{\alpha}^2}{\gamma^2} + \mathcal{O}\left(\frac{\bar{\alpha}^3}{\gamma^3}\right)\right] + \mathcal{O}\left(\frac{\bar{\alpha}^2}{\gamma}\right)$$

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The reversed LO DGLAP evolution induces the $A_1(0)/\gamma^2$ in $\chi_1(\gamma)$, and $\gamma^{-(n+1)}$ terms in the N^{n}LO BKL eigenvalue.

The usual DGLAP evolution has a similar effect, up to $\gamma \mapsto 1-\gamma$.

Origin of NLO BFKL corrections: running coupling prescription

Depending on the prescription to choose the scale of the running coupling in front of the LO BFKL kernel, one gets a different NLO BFKL kernel:

Non-optimal prescriptions induce double poles terms in $\gamma = 0$ and/or 1 in the NLO eigenvalue, and more generally $\gamma^{-(n+1)}$ and/or $(1-\gamma)^{-(n+1)}$ terms in the N^{n}LO eigenvalue.

The optimal choice is roughly to take the coupling always at the hardest available $\mathbf{k_t}$.

Origin of NLO BFKL corrections: kinematics and choice of evolution variable

The triple poles in $\chi_1(\gamma)$ at $\gamma = 0$ and 1 are due to some too crude kinematical approximations usually performed in the derivation of the LO BFKL equation.

Those approximations actually induce $\gamma^{-(2n+1)}$ and $(1-\gamma)^{-(2n+1)}$ terms in the N^{n}LO eigenvalue.

The respective coefficients of these singularities at $\gamma = 0$ or 1 depend on the precise choice of evolution variable: rapidity, momentum fraction, . . .

Collinear resummation of NLO BFKL

Finally, the physical origin of the pathologically large contributions to the NLO BFKL kernel is understood, and increasingly large contributions are expected at every N^{n}LO order.

$\Rightarrow$ Need to resum these large contributions before using the NLO BFKL kernel.

Salam (1998)
Ciafaloni, Colferai (1998)
Ciafaloni, Colferai, Salam, (Staśto) (1999-2007)
Altarelli, Ball, Forte (1999-2008)

High energy factorization

cf. Ian Balitsky, previous talk.

Convenient picture of high energy scattering:

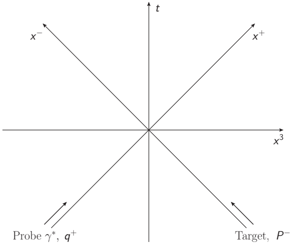
- Represent the target by a random semiclassical gluon field shockwave.
- Write the projectile wave-function on a Fock basis.
- Each Fock parton of the projectile scatter eikonally on the classical field, via a Wilson line.

Need to regularize the Wilson lines (rapidity divergence) and the projectile wave function (soft divergence).

$\Rightarrow$ Separation between the gluons belonging to the projectile or the target via a cut-off along a longitudinal or time direction.

Invariance of S matrix wrt the cut-off value $\Rightarrow$ BK equation for the dipole operators.

Example: F_L in DIS

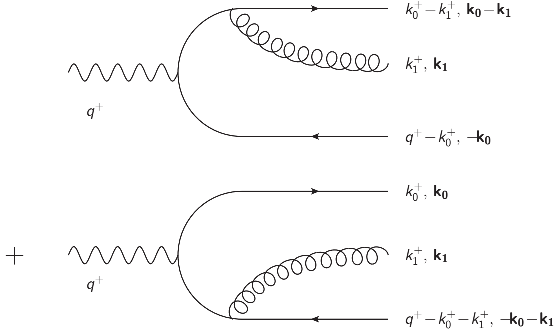


Scattering of a longitudinal photon of virtuality Q^2 on a target.

- Virtual photon: q^+, $q^- = -\frac{Q^2}{2q^+}$, and $\mathbf{q} = 0$.
- Target: $\mathcal{P}^+ = \frac{M^2}{2\mathcal{P}^-}$, $\mathcal{P}^-$, and $\mathbf{P} = 0$.

Bjorken x: $x = \frac{Q^2}{2q^+\mathcal{P}^-}$.

$q\bar{q}g$ component of the longitudinal γ^*



Notation: longitudinal momentum fraction: $z_i = \dfrac{k_i^+}{q^+}$.

$q\bar{q}g$ component of the longitudinal γ^*

$$\left|\gamma^*\right\rangle\Big|_{q\bar{q}g} = \sum_{q.n.} e\, e_f\, g\, T^a \frac{Q}{\sqrt{2}} \int_0^1 \frac{\mathrm{d}z_0}{\sqrt{z_0(1-z_0)}} \int \frac{\mathrm{d}^2\mathbf{k_0}}{(2\pi)^3} \frac{1}{Q^2 + \frac{\mathbf{k_0}^2}{z_0(1-z_0)}}$$

$$\int^1 \frac{\mathrm{d}z_1}{z_1} \int \frac{\mathrm{d}^2\mathbf{k_1}}{(2\pi)^3} \frac{\epsilon_\lambda^* \cdot \mathbf{k_1}}{z_1} \left\{ \frac{\left|q(z_0-z_1, \mathbf{k_0}-\mathbf{k_1})\bar{q}(1-z_0, -\mathbf{k_0})g(z_1, \mathbf{k_1})\right\rangle}{Q^2 + \frac{(\mathbf{k_0}-\mathbf{k_1})^2}{z_0-z_1} + \frac{\mathbf{k_0}^2}{1-z_0} + \frac{\mathbf{k_1}^2}{z_1}} \right.$$

$$\left. - \frac{\left|q(z_0, \mathbf{k_0})\bar{q}(1-z_0-z_1, -\mathbf{k_0}-\mathbf{k_1})g(z_1, \mathbf{k_1})\right\rangle}{Q^2 + \frac{(\mathbf{k_0})^2}{z_0} + \frac{(\mathbf{k_0}+\mathbf{k_1})^2}{1-z_0-z_1} + \frac{\mathbf{k_1}^2}{z_1}} \right\}$$

k^+ ordering

In most of the literature:

Leading logarithms $\Leftrightarrow$ strong ordering in z_i:

$$z_0,\, 1 - z_0 \gg z_1 \gg z_2 \gg \cdots$$

And one also assumes that

$$Q^2 \simeq \mathbf{k_0}^2 \simeq \mathbf{k_1}^2 \simeq \mathbf{k_2}^2 \simeq \cdots$$

in order to simplify the energy denominators as

$$Q^2 + \frac{(\mathbf{k_0} - \mathbf{k_1})^2}{z_0 - z_1} + \frac{\mathbf{k_0}^2}{1 - z_0} + \frac{\mathbf{k_1}^2}{z_1} \simeq \frac{\mathbf{k_1}^2}{z_1}$$

Problem: it's wrong when $\mathbf{k_1}^2$ is smaller enough than $\mathbf{k_0}^2$.

k^+ ordered $q\bar{q}g$ component of γ^*

$$\left|\gamma^*\right\rangle\Big|_{q\bar{q}g} = \sum_{q.n.} e\, e_f\, g\, T^a \frac{Q}{\sqrt{2}} \int_0^1 \frac{\mathrm{d}z_0}{\sqrt{z_0(1-z_0)}} \int \frac{\mathrm{d}^2\mathbf{k_0}}{(2\pi)^3} \frac{1}{Q^2 + \frac{\mathbf{k_0}^2}{z_0(1-z_0)}}$$
$$\int_{z_{cut}}^{z_0(1-z_0)} \frac{\mathrm{d}z_1}{z_1} \int \frac{\mathrm{d}^2\mathbf{k_1}}{(2\pi)^3} \frac{\epsilon_\lambda^* \cdot \mathbf{k_1}}{\mathbf{k_1}^2} \Big\{ \big|q(z_0, \mathbf{k_0}-\mathbf{k_1})\bar{q}(1-z_0, -\mathbf{k_0})g(z_1, \mathbf{k_1})\big\rangle$$
$$- \big|q(z_0, \mathbf{k_0})\bar{q}(1-z_0, -\mathbf{k_0}-\mathbf{k_1})g(z_1, \mathbf{k_1})\big\rangle \Big\}$$

And by consistency, only gluons with $k^+ < z_{cut}\, q^+$ are included in the Wilson lines.

k^+ ordered NLO BK

The original calculation of NLO BK, with rigid cut-off in k^+ is based on a different framework, but similar approximations.
Balitsky, Chirilli (2007)
One can calculate the eigenvalue $\chi_1(\gamma)$ of its linearized version:

- It contains no triple pole at $\gamma = 0 \Rightarrow$ good behavior in the reversed DGLAP regime where

$$\mathbf{k_0}^2 \ll \mathbf{k_1}^2 \ll \mathbf{k_2}^2 \ll \cdots$$

- There is a triple pole at $\gamma = 1 \Rightarrow$ huge NLO corrections in the DGLAP regime where

$$\mathbf{k_0}^2 \gg \mathbf{k_1}^2 \gg \mathbf{k_2}^2 \gg \cdots$$

This is due to the too strong kinematical approximation of the energy denominators.

Conformal dipole version of NLO BK

NLO BK equation with a (quasi-)conformal regularization of dipole operators:
Balitsky, Chirilli (2009)

The eigenvalue of the kernel contains triple poles both at $\gamma = 0$ and $\gamma = 1$:

- reminiscent of the raw NLO BFKL result.
- breakdown of high energy perturbation theory both in the DGLAP and the reversed DGLAP regimes.

$\Rightarrow$ None of the two available versions of NLO BK seems to be suitable for phenomenology.

Another possibility

Instead of imposing strict ordering of the gluons in k^+, let's impose strict ordering in $k^- = \frac{\mathbf{k}^2}{2z\, q^+}$:

$$\frac{z_0}{\mathbf{k_0}^2}, \frac{1-z_0}{\mathbf{k_0}^2} \gg \frac{z_1}{\mathbf{k_1}^2} \gg \frac{z_2}{\mathbf{k_2}^2} \gg \cdots$$

And let's still assumes that

$$Q^2 \simeq \mathbf{k_0}^2 \simeq \mathbf{k_1}^2 \simeq \mathbf{k_2}^2 \simeq \cdots$$

in order to keep the simplification of the energy denominators

$$Q^2 + \frac{(\mathbf{k_0}-\mathbf{k_1})^2}{z_0-z_1} + \frac{\mathbf{k_0}^2}{1-z_0} + \frac{\mathbf{k_1}^2}{z_1} \simeq \frac{\mathbf{k_1}^2}{z_1}$$

It's ok except in some domain where $\mathbf{k_1}^2$ is greater enough than $\mathbf{k_0}^2$.

k^- ordered $q\bar{q}g$ component of γ^*

$$\left|\gamma^*\right\rangle\Big|_{q\bar{q}g} = \sum_{q.n.} e\, e_f\, g\, T^a \frac{Q}{\sqrt{2}} \int_0^1 \frac{\mathrm{d}z_0}{\sqrt{z_0(1-z_0)}} \int \frac{\mathrm{d}^2\mathbf{k_0}}{(2\pi)^3} \frac{1}{Q^2 + \frac{\mathbf{k_0}^2}{z_0(1-z_0)}}$$

$$\int \frac{\mathrm{d}z_1}{z_1} \int \frac{\mathrm{d}^2\mathbf{k_1}}{(2\pi)^3} \frac{\epsilon_\lambda^* \cdot \mathbf{k_1}}{\mathbf{k_1}^2} \Big\{ \left|q(z_0, \mathbf{k_0}-\mathbf{k_1})\bar{q}(1-z_0, -\mathbf{k_0})g(z_1, \mathbf{k_1})\right\rangle$$

$$-\left|q(z_0, \mathbf{k_0})\bar{q}(1-z_0, -\mathbf{k_0}-\mathbf{k_1})g(z_1, \mathbf{k_1})\right\rangle \Big\}$$

Formally same expression as previously. By consistency, let's change variable as $\hat{z}_1 = \frac{z_1}{\mathbf{k_1}^2}$ and impose the cut-off on $\hat{z}_1$ instead of z_1.

k^- ordered $q\bar{q}g$ component of γ^*

$$\left|\gamma^*\right\rangle\Big|_{q\bar{q}g} = \sum_{q.n.} e\, e_f\, g\, T^a \frac{Q}{\sqrt{2}} \int_0^1 \frac{\mathrm{d}z_0}{\sqrt{z_0(1-z_0)}} \int \frac{\mathrm{d}^2\mathbf{k_0}}{(2\pi)^3} \frac{1}{Q^2 + \frac{\mathbf{k_0}^2}{z_0(1-z_0)}}$$
$$\int_{\hat{z}_{cut}}^{z_0(1-z_0)/\mathbf{k_0}^2} \frac{\mathrm{d}\hat{z}_1}{\hat{z}_1} \int \frac{\mathrm{d}^2\mathbf{k_1}}{(2\pi)^3} \frac{\epsilon_\lambda^* \cdot \mathbf{k_1}}{\mathbf{k_1}^2}$$
$$\Big\{ \left| q(z_0, \mathbf{k_0}-\mathbf{k_1}) \bar{q}(1-z_0, -\mathbf{k_0}) g(\hat{z}_1, \mathbf{k_1}) \right\rangle$$
$$- \left| q(z_0, \mathbf{k_0}) \bar{q}(1-z_0, -\mathbf{k_0}-\mathbf{k_1}) g(\hat{z}_1, \mathbf{k_1}) \right\rangle \Big\}$$

From this, one deduces the LO BK equation for Wilson lines operators with a rigid cut-off in k^-.

Comparison of k^- and k^+ ordering

The BK equation is formally identical at LO in both cases, up to the definition of the evolution variable. However, it should differ at NLO.

For the momentum space representation of the linearized BK equation:

Different evolution variable $\Rightarrow$ different Mellin variables: (ω, γ) for the k^+ ordered equation and $(\omega, \hat{\gamma} = \gamma - \omega)$ for the k^- ordered equation.

The former gives $\omega = \bar{\alpha}\chi_0(\gamma) + \bar{\alpha}^2\chi_1^{(+)}(\gamma) + \mathcal{O}(\bar{\alpha}^3)$, and the latter

$$
\begin{aligned}
\omega &= \bar{\alpha}\chi_0(\hat{\gamma}) + \bar{\alpha}^2\chi_1^{(-)}(\hat{\gamma}) + \mathcal{O}(\bar{\alpha}^3) \\
&= \bar{\alpha}\chi_0(\gamma) - \omega\bar{\alpha}\chi_0'(\gamma) + \bar{\alpha}^2\chi_1^{(-)}(\gamma) + \mathcal{O}(\bar{\alpha}^3) + \mathcal{O}(\bar{\alpha}^2\omega) \\
&= \bar{\alpha}\chi_0(\gamma) + \bar{\alpha}^2\left[\chi_1^{(-)}(\gamma) - \chi_0(\gamma)\chi_0'(\gamma)\right] + \mathcal{O}(\bar{\alpha}^3)
\end{aligned}
$$

Comparison of k^- and k^+ ordering

Hence, the NLO BK kernel eigenvalue in the k^- ordered case has to be

$$\chi_1^{(-)}(\gamma) = \chi_1^{(+)}(\gamma) + \chi_0(\gamma)\chi_0'(\gamma)\,.$$

The new term cancels the triple pole at $\gamma = 1$, but add one one at $\gamma = 0$.

$\Rightarrow$ k^- ordering leads to an optimal behavior of the high energy perturbation theory in the DGLAP regime, but a bad one in the reversed DGLAP regime.

$\Rightarrow$ Better starting point for NLO gluon saturation phenomenology than any of the two available NLO BK equations.

Double ordering

In practice, the triple pole at $\gamma = 0$ also gives a quite large correction, so that we should try to get rid it also, by a better treatment of kinematics.

- The usual approximation for the energy denominators actually requires simultaneously the k^+ and the k^- ordering of the gluons to be valid.
- Outside this regime, the true expression of the energy denominators leads to less logarithms: contributes to the evolution only at higher orders.

$\Rightarrow$ We should impose simultaneously k^+ and k^- ordering of the gluons, already at LO.

Resummation

2 ways to ensure the simultaneous ordering in k^+ and k^-:

- Construct a composite evolution variable, implementing both ordering simultanously.
  $\longrightarrow$ Presumably possible but cumbersome.
- Use either k^+ or k^- as evolution variable (preferably k^-), and impose ordering of along the other by a kinematical constraint in the kernel.

BFKL case:
Andersson, Gustafson, Kharraziha, Samuelsson (1996)
Kwieciński, Martin, Sutton (1996)

BK case:
Motyka, Staśto (2009)
Berger, Staśto (2010)

Conclusion: status of the large NLO BK terms

- Kinematical corrections associated with the Mellin triple poles: Can be resummed via a modified kernel à la Motyka-Staśto. But I would recommend to use k^- as evolution variable in BK, and then impose the kinematical constraint on k^+.
- Corrections associated with the running coupling scale: exactly resummed by Balitsky's running coupling prescription. Balitsky (2006)
- Contributions of non-eikonal gluon emission: still to be done for BK.

$\rightarrow$ And by the way, what about the DIS impact factors?