AdS/CFT: Scattering amplitudes from weak to strong coupling

Jochen Bartels, Hamburg University

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- Introduction
- Weak coupling
- Strong coupling
- Conclusions

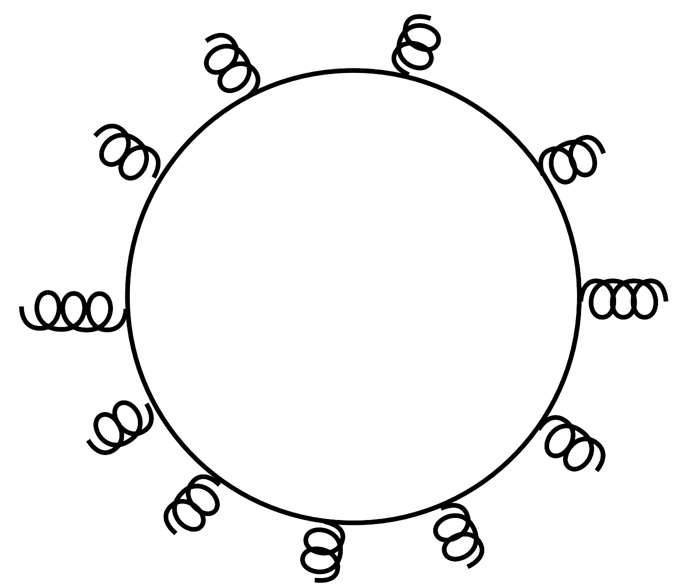
Collaborations:
JB., L.Lipatov, A.SabioVera, A.Prygarin
J.B., J.Kotanski, V.Schomerus

Introduction

Ultimate Goal: N=4 SYM could be solvable
(could play the role of 'harmonic oscillator in QFT').

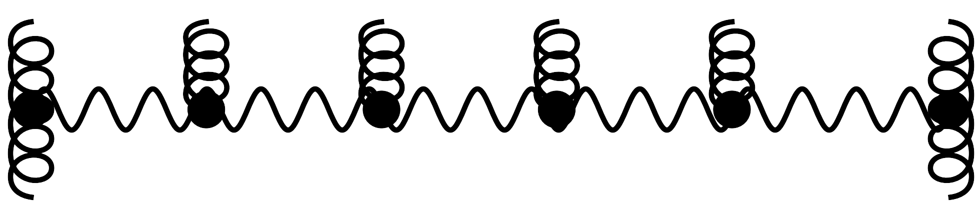
Main progess: anomalous dimensions for dilatation

More recent aim:
computation of scattering amplitudes
at weak and at strong coupling



Special interest:
multi-leg scattering at high energies (Regge limit)

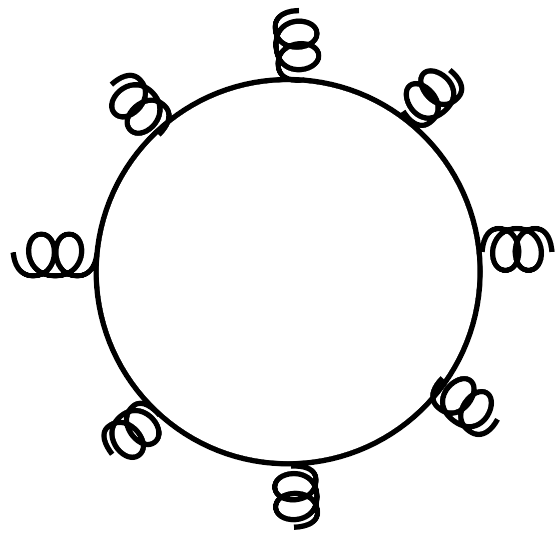
Why Regge limit:
personal interest in Regge limit
experience shows that Regge limit plays
special role
integrability in the Regge limit

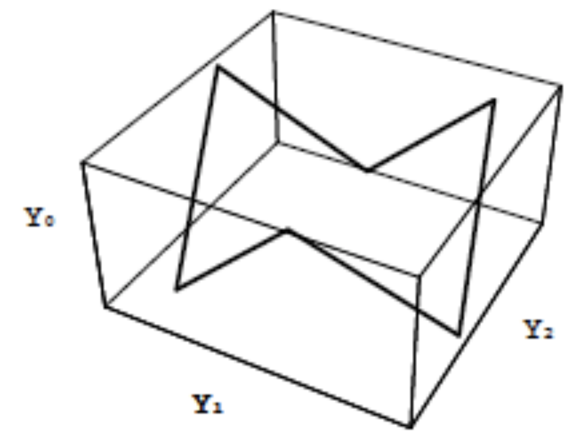


s large

Framework:
correspondence

N=4 SYM	$\rightleftarrows$	AdS_5 x S_5
$g,\ N_c,\ \lambda = g^2 N_c$		$g^2 N_c = \dfrac{R^4}{l_s^4}$





weak coupling:
perturbation theory

strong coupling:
semiclassical approximation,
mimimal area of polygon

Weak coupling

In recent years enormous activity.
Very helpful: two loop calculations

Del Duca et al.
Goncharov et al.

BDS conjecture for MHV amplitudes:

Bern et al.

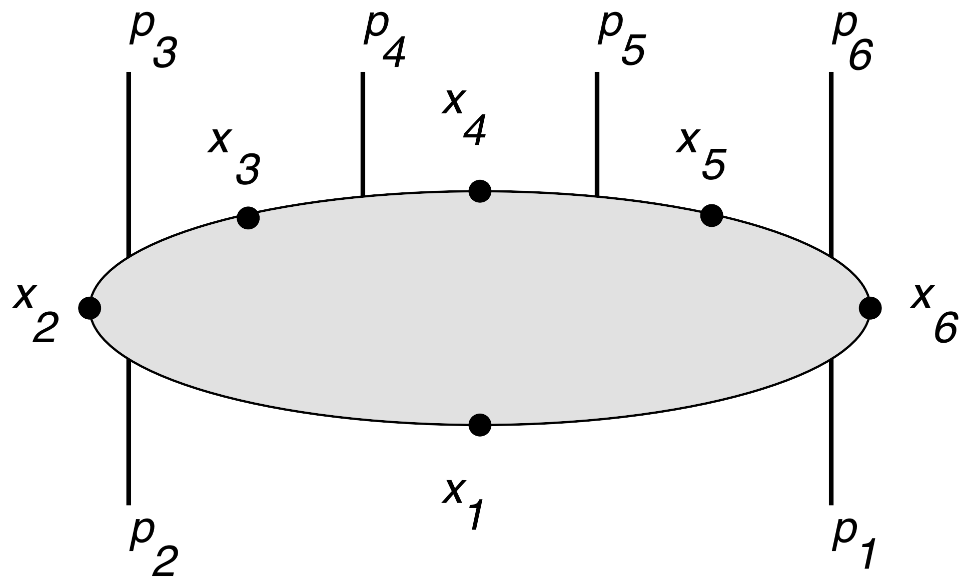
$$tr(T^{a_1}...T^{a_n}) + noncycl.perm, \quad A_n = A_n^{tree} \cdot M_n(\epsilon)$$

$$\ln M_n = \sum_l a^l \Big[\Big(f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \Big) + C^{(l)} + E_n^{(l)}[\epsilon] \Big]$$

$$a = \frac{N_c \alpha}{2\pi}(4\pi e^{-\gamma})^{\epsilon}, \; d = 4 - 2\epsilon$$

Present understanding:
formula correct for n=4 and n=5
Corrections needed for n $\geq$ 6: remainder function $R^{(n)}$

Dual conformal symmetry:



$$x_{i+1} - x_i = p_i$$

Conformal transformations in dual space.

In unphysical region (all invariants negative):

$$M_n \sim \exp(\ln M_n^{BDS} + R^{(n)})$$

Correspondence: amplitude - Wilson loop (color non-singlet!).

Remainder function depends upon anharmonic ratios, e.g. for n=6:

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, u_2 = \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{41}^2}, u_3 = \frac{x_{26}^2 x_{35}^2}{x_{25}^2 x_{36}^2} = \frac{s_2 s}{s_{345} s_{456}}$$

(instead of 8 s,t invariants)

Goal: find remainder function $R^{(6)}$

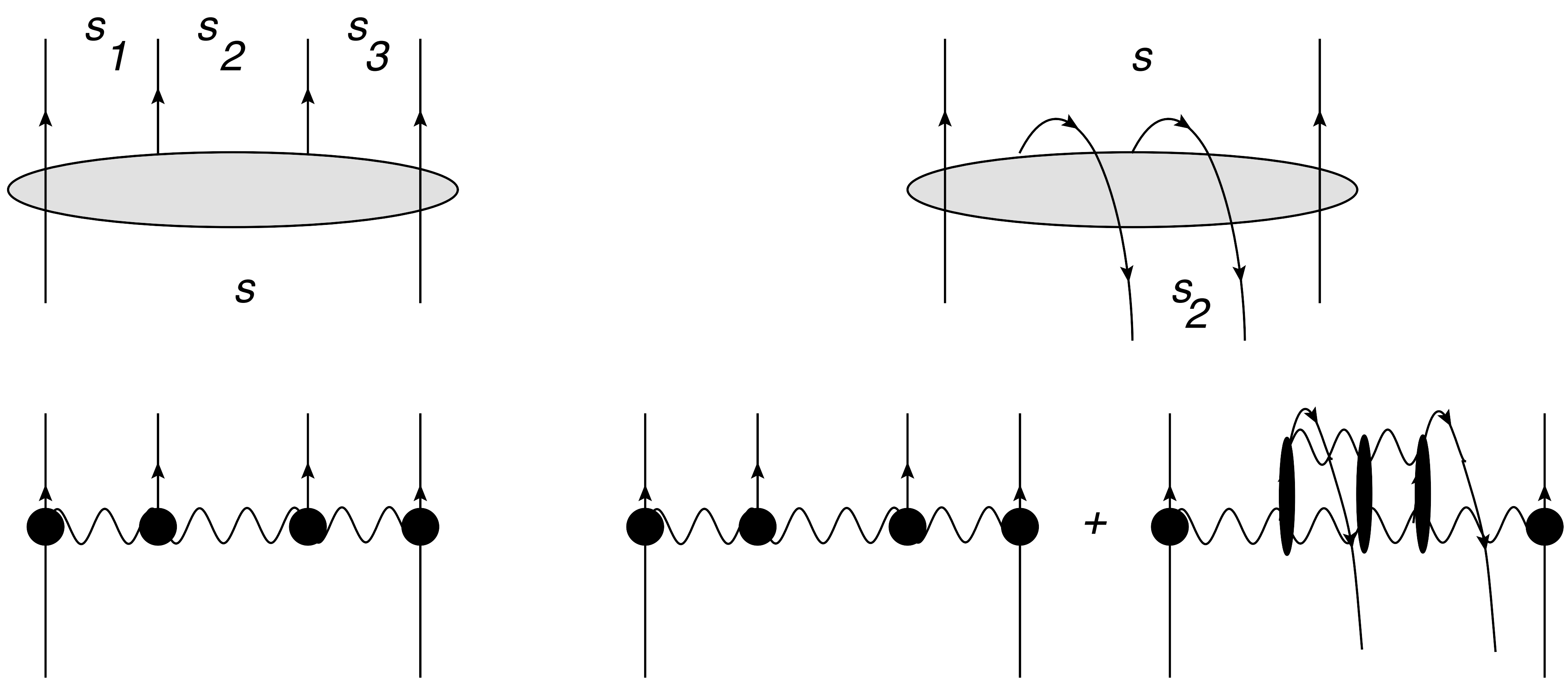
What do we know about $R^{(n)}$:

- vanishes for n=4,5
  (BDS correct; no unharmonic ratios)

- exact two loop results (Del Duca; Goncharov)

- n=6:
  Comparison with leading log calculations in QCD
  BDS contains Regge - pole model (gluon) : Veneziano amplitude

- missing piece: Regge cut, visible only in special physical region (known since 1979)



all energies positive

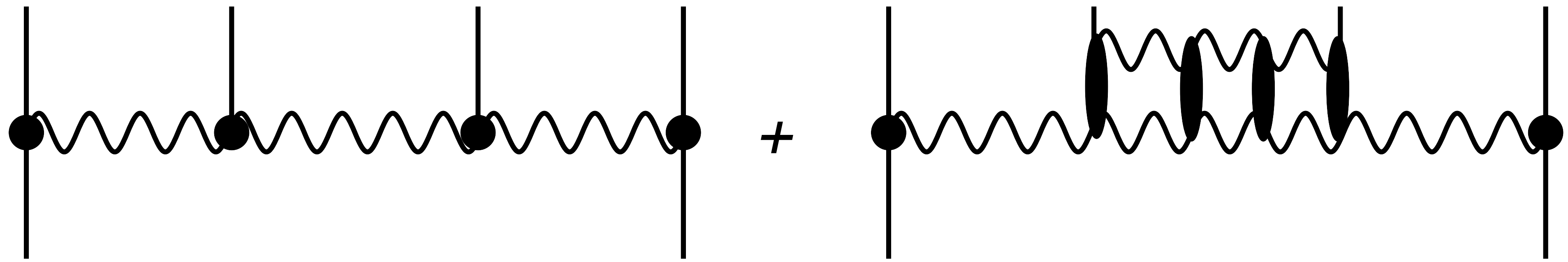
$u_3 = 1$

mixed region: $s > 0,\ s_2 > 0;\ \ s_{123} < 0,\ s_{234} < 0$

$u_3 = e^{-2\pi i}$

Most important: integrability

Lipatov

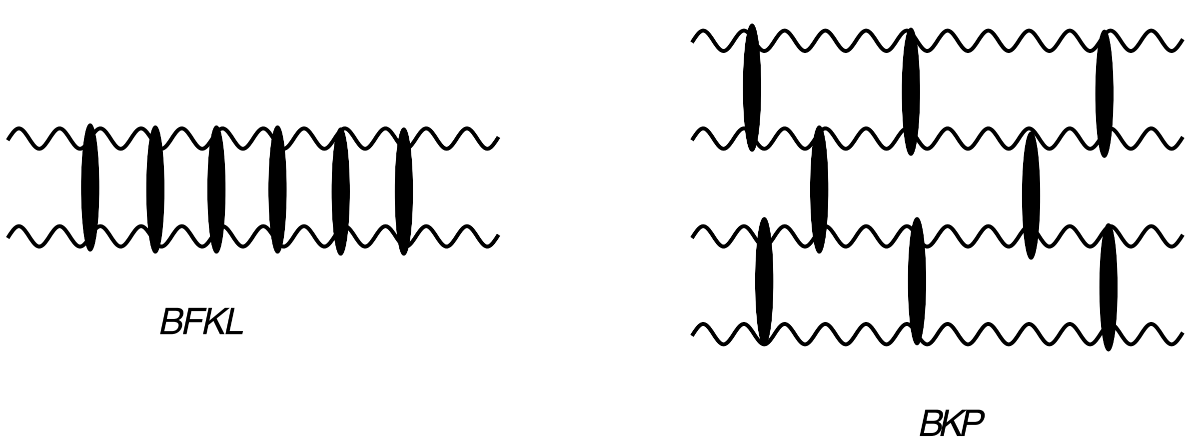


$$\Delta T_{2\to 4} \sim s_2^{-E^{(2)}}$$

$E^{(2)}$ is the lowest eigenvalue of the BFKL-octet Hamiltonian $H^{(2)}$: integrable

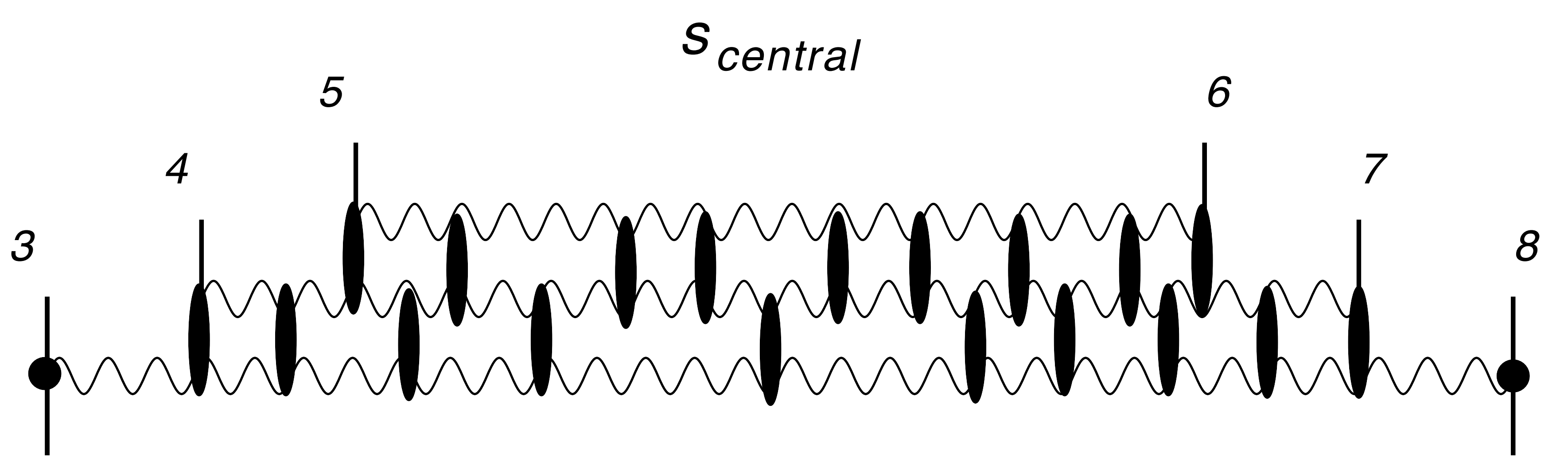
History of integrability in Regge limit:

large-N_c color singlet BFKL, BKP Hamiltonian $\sim$ closed spin chain: integrable



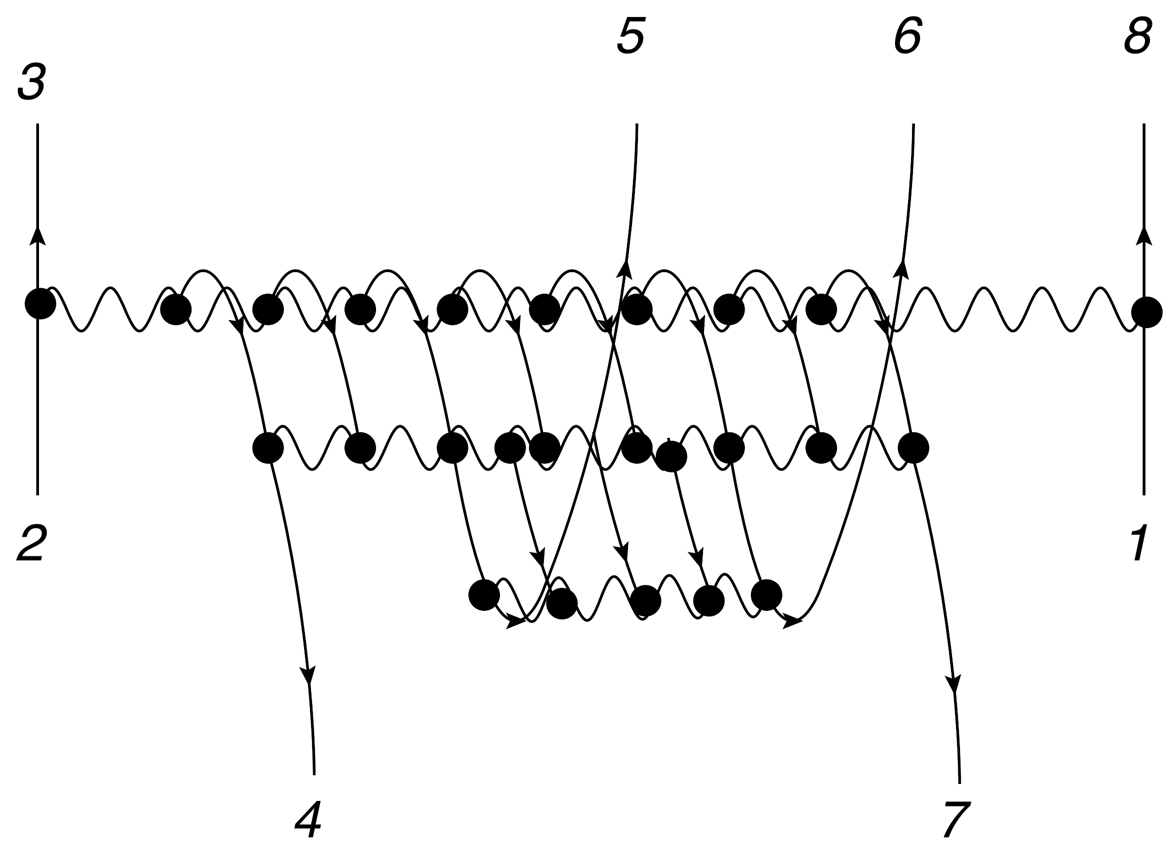
New: color octet BFKL, BKP Hamiltonian $\sim$ open spin chain: integrable

More particles: $T_{2\to 2n}$



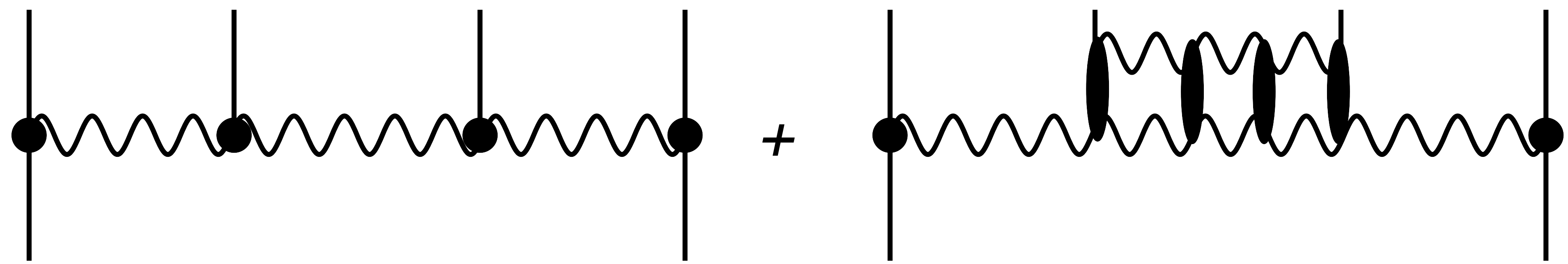
$E^{(3)}, \; H^{(3)}$

'mixed' region:



Should all be contained in remainder function!

$$\mathrm{Amp}' = \mathrm{Amp}^{BDS}_{2\to4}\ \ (1 + i\Delta_{2\to4})$$



$$\Delta_{2\to4} \ = \ \frac{a}{2}\sum_{n=-\infty}^{n=\infty}(-1)^n\int\frac{d\nu}{\nu^2+\frac{n^2}{4}}\left(\frac{q_2^*p_4^*}{p_5^*q_1^*}\right)^{i\nu-\frac{n}{2}}\ (s_2^{\omega(\nu,n)}-1)\left(\frac{q_2p_4}{p_5q_1}\right)$$

$$\omega(\nu,n) \ = \ 4a\mathcal{R}\left(2\psi(1)-\psi(1+i\nu+\frac{n}{2})-\psi(1+i\nu-\frac{n}{2})\right) \ .$$

Leading: $\mathrm{n}=1,\nu=0$: $\omega(0,1)=\frac{\lambda}{\pi^2}(2\ln 2-1)$

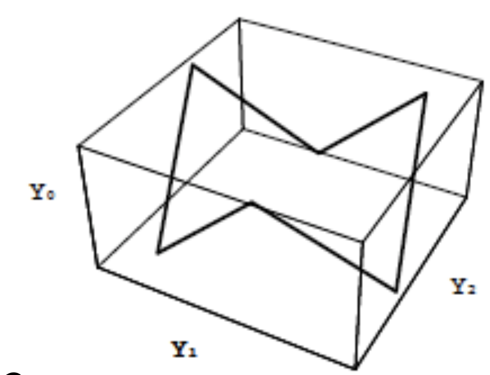
What about exponentiation?

Strong coupling

Correspondence: scattering amplitude is given by minimal area A.

$$\mathrm{Amp} \sim \langle W \rangle \sim \exp\left[-\frac{\sqrt{\lambda}}{2\pi} A\right] = \exp\left[-\frac{\sqrt{\lambda}}{2\pi}(A_{\mathrm{div}} + A_{\mathrm{BDS}} - R)\right]$$

Contours (lightlike polygons) of the area are determined by kinematics, i.e. by the values of the unharmonic ratios.



For n=6 we have three ratios: $u_1, \ u_2, \ u_3$

solve Euler-Lagrange equations
classical integrable system ~ use auxiliary quantum system
Y-Equations
area related to free energy of this system

Alday, Maldacena,Sever,Vieira;
Alday,Gaiotto,Maldacena

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Goal: solve Y-equations

$$
\begin{aligned}
\log Y_2(\theta) &= -m\sqrt{2}\cosh(\theta - i\phi) - 2\int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log(1+Y_2(\theta')) \\
&\quad - \int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta'))) \\
\log Y_{2\pm 1}(\theta) &= -m\cosh(\theta - i\phi) \pm C - \int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log(1+Y_2(\theta')) \\
&\quad - \int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta'))) \ .
\end{aligned}
$$

$$
\begin{aligned}
u_1 &= \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \left(1 + \frac{1}{Y_2(\theta = -i\pi/4)}\right)^{-1} \\
u_2 &= \frac{x_{24}^2 x_{15}^2}{x_{25}^2 x_{14}^2} = \left(1 + \frac{1}{Y_2(\theta = i\pi/4)}\right)^{-1} \\
u_3 &= \frac{x_{35}^2 x_{26}^2}{x_{36}^2 x_{25}^2} = \left(1 + \frac{1}{Y_2(\theta = -3i\pi/4)}\right)^{-1}
\end{aligned}
$$

with $m = m(u_1, u_2, u_3),\ \phi = \phi(u_1, u_2, u_3),\ C = C(u_1, u_2, u_3)$

Closer look: Regge limit provides great simplification

Special values of anharmonic ratios: $\mathrm{u}_1 \to 0, \; u_2 \to 0, \; u_3 \to 1 \; (e^{-2\pi i})$
allow to disregard the inhomogeneous terms of the Y-equations.

After continuation in u_3 :
solution becomes analytic.

Main results for the Regge limit:

- remainder function vanishes (up to constant) before analytic continuation
- after analytic continuation a new term appears which has Regge behavior

Strong coupling:

$$(1-u_3)\sim 1/s_2 \qquad \tilde{u}_{1,2}=\frac{u_{1,2}}{1-u_3}$$

$$\mathrm{Amp}' \sim \langle W'\rangle \sim \exp\left[-\frac{\sqrt{\lambda}}{2\pi}A'\right]=\exp\left[-\frac{\sqrt{\lambda}}{2\pi}(A'_{\mathrm{div}}+A'_{\mathrm{BDS}}-R')\right]$$

$$e^{\frac{\sqrt{\lambda}}{2\pi}R'} \sim e^{-i\frac{\pi}{2}\frac{\sqrt{\lambda}}{4\pi}\ln(\tilde{u}_1\tilde{u}_2)}\left((1-u_3)\sqrt{\tilde{u}_1\tilde{u}_2}\right)^{\frac{\sqrt{\lambda}}{2\pi}e_2}\left(\frac{\tilde{u}_1}{\tilde{u}_2}\right)^{-\frac{\sqrt{\lambda}}{\sqrt{2}\pi}}$$

$$e_2 = \left(\sqrt{2}+\frac{1}{2}\log(3+2\sqrt{2})\right)$$

Weak coupling:

$$\mathrm{Amp}'=\mathrm{Amp}^{BDS}_{2\to 4}\ (1+i\Delta_{2\to 4})$$

$$\Delta_{2\to 4} = \frac{a}{2}\sum_{n=0}^{n=\infty}(-1)^n\int\frac{d\nu}{\nu^2+\frac{n^2}{4}}((1-u_3)^{-\omega(\nu,n)}-1)|w|^{2i\nu}\cosh nC$$

$$w \approx \sqrt{\frac{\tilde{u}_1}{\tilde{u}_2}} \quad \text{and} \quad \cosh C = \frac{1-u_1-u_2-u_3}{2\sqrt{u_1u_2u_3}} \quad \text{and} \quad \omega(0,1) = -E_2 = \frac{\lambda}{\pi^2}(2\ln 2-1)$$

Possible interpretation: $\nu_{saddle}=i\frac{\sqrt{\lambda}}{\sqrt{2}\pi},\ -\omega(\nu_{saddle},0)=\frac{\sqrt{\lambda}}{2\pi}e_2$

Conclusions

Scattering amplitudes within AdS/CFT:

Weak coupling:

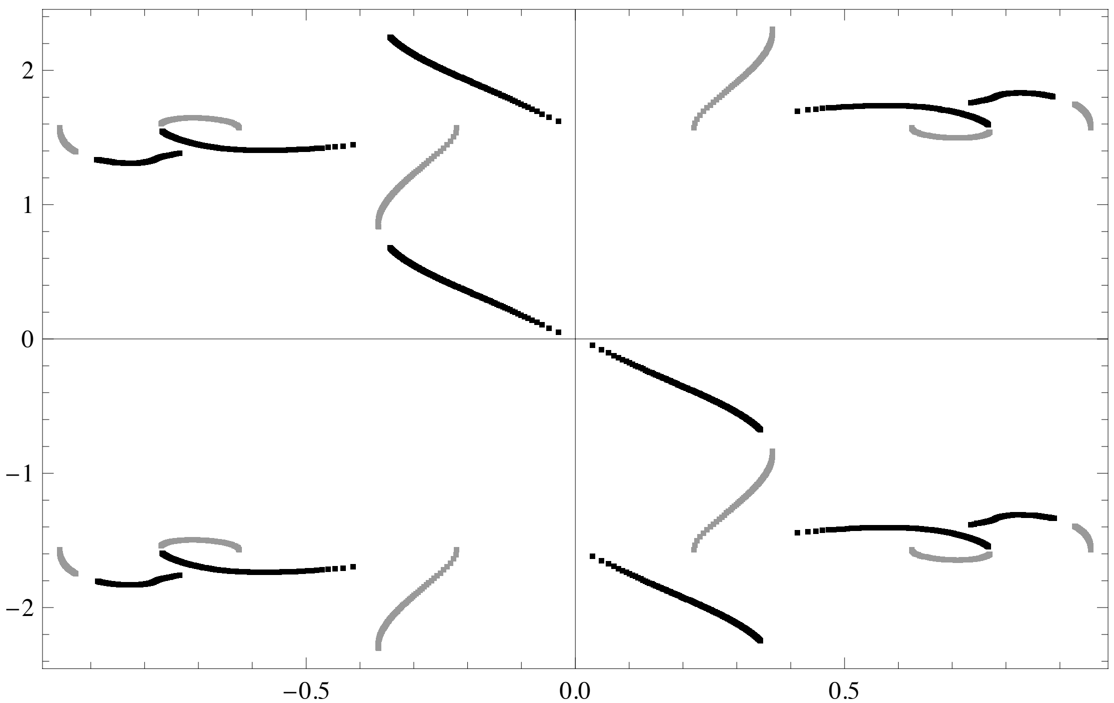
- identify corrections to BDS: remainder function
- Regge limit helps: analytic structure, integrability

Strong coupling (leading order):

- Set of nonlinear equations - Y-equations -
  determine the remainder function for $n \geq 6$
- First attempt to solve these equations ($n=6$):
  Regge limit simplifies solving the equations (analytic solutions)
- new piece after analytic continuation: excited state in TBA
- matches the structure at weak coupling (?)

Next steps:

- generalization to more legs (larger n): more excited TBA states
- connection with collinear limit (OPE expansion)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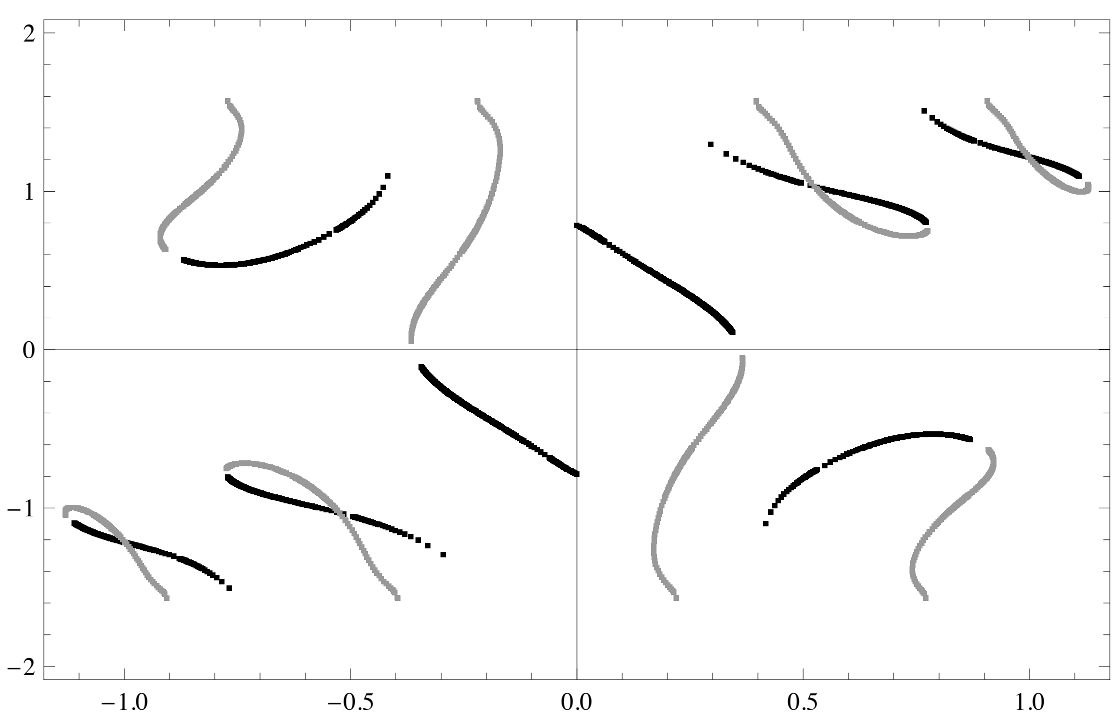
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