TMD Factorization and Evolution for TMD Correlation Functions

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In collaboration with Ted C. Rogers

Based on:
arXiv: 1101.5057
and Foundations of Perturbative QCD, J.C. Collins
http://www.cambridge.org/uk/catalogue/catalogue.asp?isbn=9780521855334

PreDIS 2011 - JLab

Outline

- Collinear and TMD factorization
- A unified treatment that includes evolution
- New TMD definitions
- Evolution for TMDs
- Conclusions and Outlook

Factorization in QCD

QCD gains its predictive power through factorization

Consider Drell-Yan process: $P_1 + P_2 \to l\bar{l}(Q^2) + X$

$$\frac{d\sigma_{P_1 P_2 \to ll'(Q^2)+X}(s, Q^2)}{dQ^2} = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_{i/P_1}(x_1, \mu^2)\, f_{j/P_2}(x_2, \mu^2)\, \mathcal{H}_{ij}\left(\frac{Q^2}{x_1 x_2 s}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Collins, Soper and Sterman (1985,1988)

PDFs: Long distance dynamics, non-perturbative, universal, evolution through DGLAP

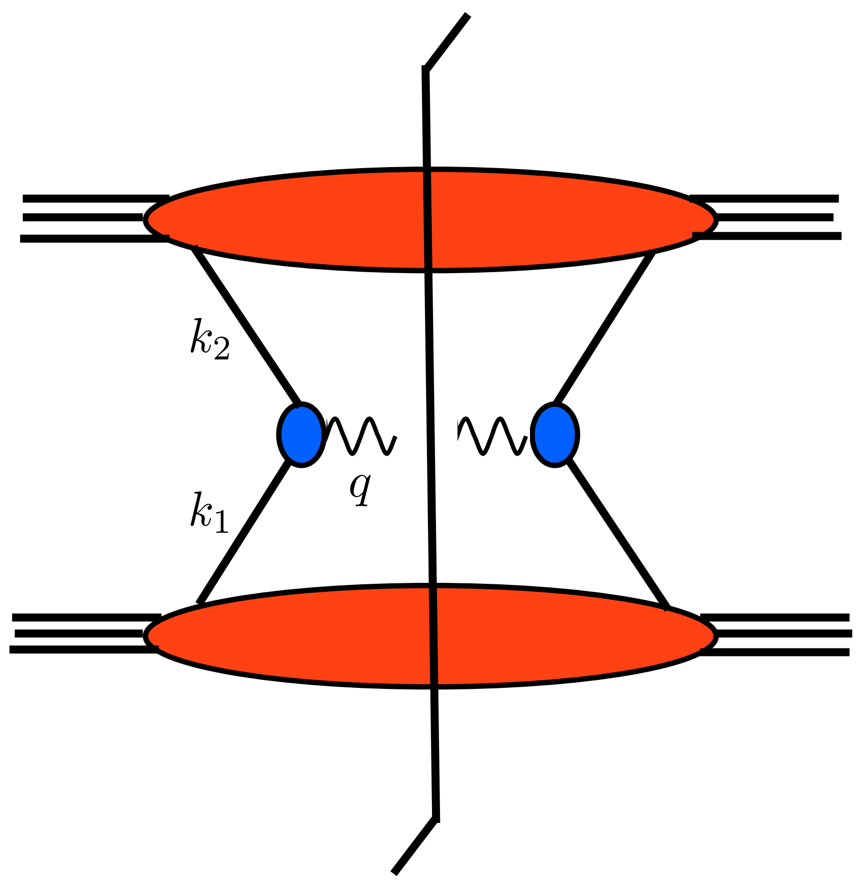
Hard scattering function: short distance dynamics, pQCD calculations

Not complete description → transverse momentum of incoming partons are important

examples: DY, SIDIS, hadron production at e^+e^- collisions...

TMD Factorization

Consider Drell-Yan process



We want to get $\frac{d\sigma}{dq_T}$ **for all** q_T

Two Common Approaches

A) Typical implementation of CSS formalism

- Parametrize the non-perturbative parts.

- Global fit to several experiments (example: Tevatron data).

  Landry et al (2003); Konychev and Nadolsky (2006)

CSS Formalism: Typical Implementation

Collins-Soper-Sterman formalism for DY gives

(1985)

$$
\begin{aligned}
d\sigma \quad \sim \quad & \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\mathbf{q}_T} \\
& \int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*, \mu_b^2, \mu_b, g(\mu_b))\, f_{j/P_1}(\hat{x}_1, \mu_b) \\
& \int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{f/j}(x_2/\hat{x}_2, b_*, \mu_b^2, \mu_b, g(\mu_b))\, f_{j/P_2}(\hat{x}_2, \mu_b) \\
& \exp\left[\int_{1/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \left\{ \mathcal{A}(\alpha_s(\mu')) \ln \frac{Q^2}{\mu'^2} + \mathcal{B}(\alpha_s(\mu')) \right\} \right] \\
& \exp\left[- g_K(b) \ln \frac{Q^2}{Q_0^2} - g_1(x_1, b) - g_2(x_2, b) \right] \\
& \qquad\qquad + \text{Large}\, q_T \text{ term}
\end{aligned}
$$

Where is the TMD PDF ?

CSS Formalism: Typical Implementations

- Underlying presence of individual TMD PDFs is hidden.
  - TMD PDF → hadron structure.
  - T-Odd effects (Sivers ...) require TMD Correlation functions.

- Explicit soft factors → different pieces entangled.
- Different processes → start over → new fits.

} Universal ? Predictive ?

A unified treatment is needed to relate different experiments!

Want: Analogous to collinear factorization

Two Common Approaches

A) Typical implementation of CSS formalism

- Parametrize the non-perturbative parts.
- Global fit to several experiments (example: Tevatron data).

  Landry et al (2003); Konychev and Nadolsky (2006)

B) Use gaussian parametrization

- Assume x and k_T behaviors factorize.
- Fixed scale, no evolution.
- Ok for fixed, small scales but redo the fits for each experiment and for each scale.

  Schweitzer, Teckentrup and Metz (2010)

Main Philosophy and the Goal

Extend collinear factorization methodology to TMD factorization.

- Repository of well defined TMD fits with evolution for use in phenomenology.

https://projects.hepforge.org/tmd

- Unified treatment → use existing fits to build a global fit.

- Connection between operator definitions and phenomenology.

Relation to Generalized Parton Model??

Correct QCD Formula

$$\zeta_F = 2M_P^2 x^2 e^{2(y_p - y_s)}$$

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu} \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} \tilde{F}_{f/P_1}(x_1,\mathbf{k}_{1T},\mu,\zeta_F)\tilde{F}_{\bar{f}/P_2}(x_2,\mathbf{k}_{2T},\mu,\zeta_F)\delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)$$

$+$ corrections

Hard Part

PDFs

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2)|^{\mu\nu} \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1,\mathbf{k}_{1T})F_{\bar{f}/P_2}(x_2,\mathbf{k}_{2T})\delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)$$

Generalized Parton Model

Definitions of TMD Correlation Functions

- Dictated by requirements of factorization.
- Identified with operator definitions - universality/non-universality properties clear.
- Deal with all divergences.

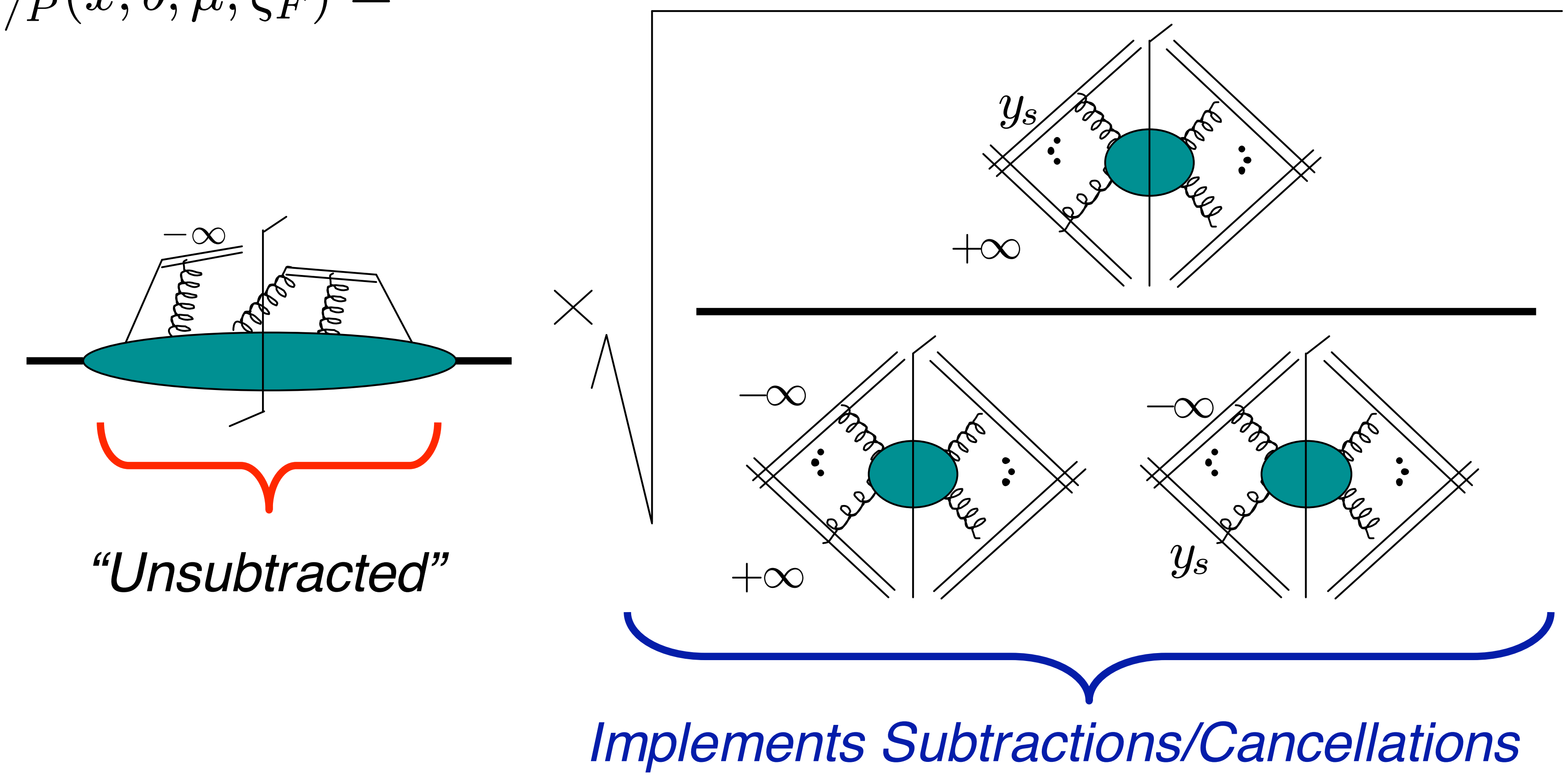
Consistently defined TMD correlation functions

- Have evolution equations associated with them individually.
- Be analogous to generalized parton model picture.

See talk by J. Collins for the new, consistent definitions of TMD parton densities.

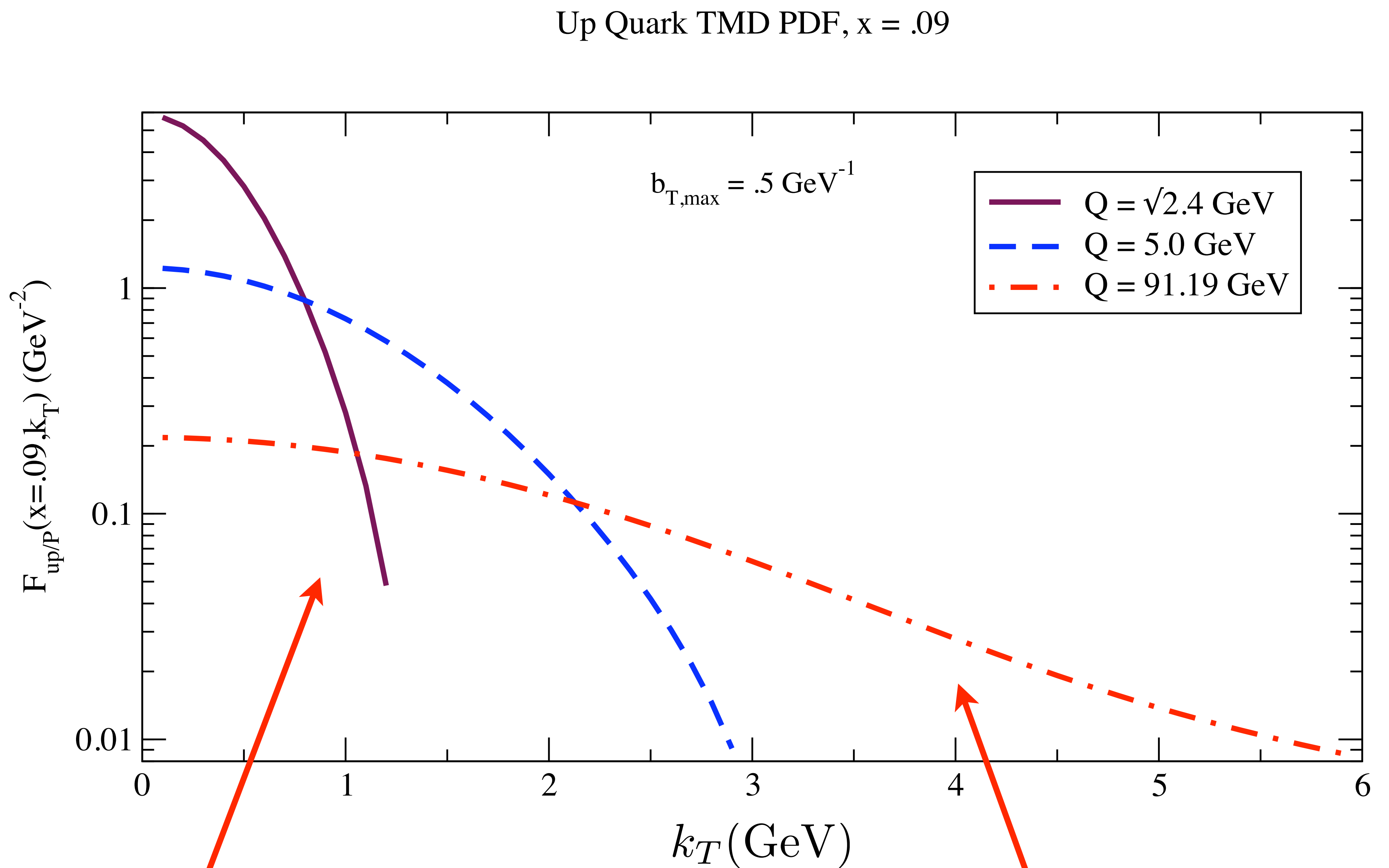
New TMD Definitions

$F_{f/P}(x, b; \mu; \zeta_F) =$



From ***Foundations of Perturbative QCD***, J. Collins
(See talk by J. Collins and also J. Collins, ***TMD 2010 Trento Workshop*** talk)

Some Results



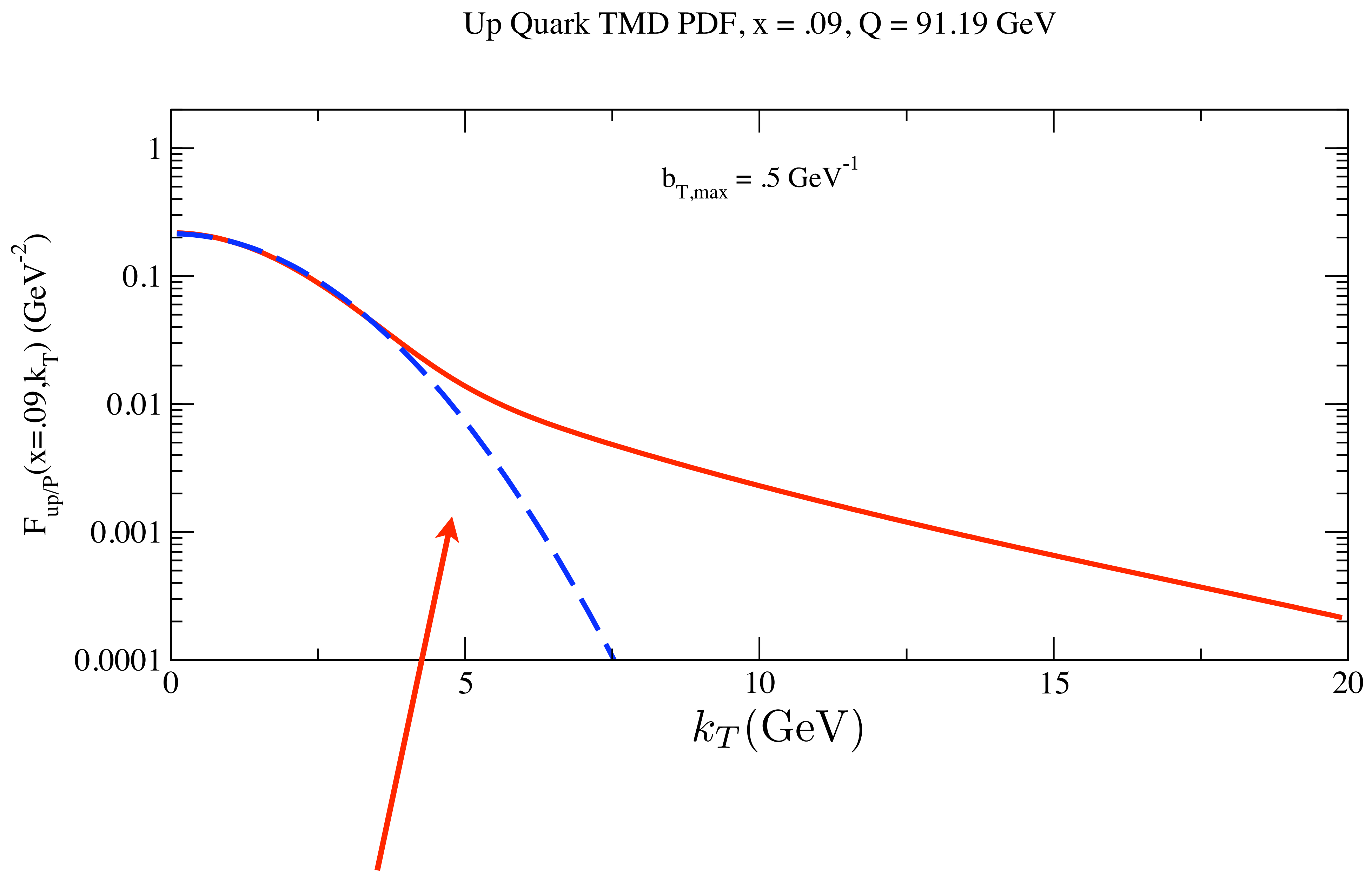
JLab Energies
matches STM fit

Schweitzer, Teckentrup and Metz (2010) (STM)

Tevatron Energies
matches BNLY fit

Brock, Landry, Nadolsky, Yuan (2003) (BLNY)

Some Results



gaussian fit does not capture the effects of evolution quite well

Evolution for TMDs

Energy evolution from Collins-Soper equation

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu) \quad \text{with} \quad \tilde{K}(b_T, \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Renormalization group equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \text{and} \quad \frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln\mu} = -\gamma_F(g(\mu), \zeta/\mu^2)$$

energy evolution for γ_F :

$$\gamma_F(g(\mu); \zeta_F/\mu^2) = \gamma_F(g(\mu); 1) - \frac{1}{2}\gamma_K(g(\mu)) \ln \frac{\zeta_F}{\mu^2}$$

Evolution for TMDs

Small b_T → collinear factorization formalism

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_T, \zeta_F, \mu, g(\mu)) \, f_{j/P}(\hat{x}, \mu) + \mathcal{O}((\lambda_{\rm QCD} b_T)^a)$$

At large b_T → perturbative description breaks down → scale dependence through evolution

Matching for large and small bT

$$\mathbf{b}_*(\mathbf{b}_T) = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins and Soper (1982)

- Use collinear factorization treatment for small b_T.
- Implement matching procedure.
- Apply evolution equations.

Evolved TMDs

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \zeta_F, \mu, g(\mu))\, f_{j/P}(\hat{x}, \mu)$$

$$\times \exp\left\{ \ln\frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'), 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

$$\tilde{C}_{j'/j}(x, b_T, \mu, \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{2\pi} \left\{ 2\left[\ln\left(\frac{2}{\mu b_T}\right) - \gamma_E \right] \left[\left(\frac{2}{1-x}\right)_+ - 1 - x \right] + 1 - x \right.$$

$$\left. + \delta(1-x) \left[-\frac{1}{2}[\ln(b_T^2\mu^2) - 2(\ln 2 - \gamma_E)]^2 - [\ln(b_T^2\mu^2) - 2(\ln 2 - \gamma_E)] \ln\left(\frac{\zeta_F}{\mu^2}\right) \right] \right\} + \mathcal{O}(\alpha_s^2)$$

Evolved TMDs

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \zeta_F, \mu, g(\mu))\, f_{j/P}(\hat{x}, \mu)$$

$$\times \exp\left\{ \ln\frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'), 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

$\mu_b = \dfrac{C_1}{b_*(b_T)}$ **with** $C_1 = 2e^{-\gamma_E}$

Evolved TMDs

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \zeta_F, \mu, g(\mu))\, f_{j/P}(\hat{x}, \mu)$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'), 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

$$\mu_b = \frac{C_1}{b_*(b_T)} \quad \text{with} \quad C_1 = 2e^{-\gamma_E}$$

nonperturbative b_T behavior in $\tilde{F}_{f/P}$

nonperturbative b_T behavior in $\tilde{K}$

Determination of the Non-perturbative Parts

Using CSS formalism for the full cross section, fits to DY Tevatron data

$$\exp\left\{ -\left[g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(100 x_1 x_2) \right] b_T^2 \right\}$$

Brock,Landry,Nadolsky,Yuan (2003) (BLNY):

Assuming flavor independence and symmetric role of PDFs
we use for one specific TMD

$$\exp\left\{ -\left[\frac{g_2}{2} \ln\frac{Q}{2Q_0} + g_1 \left(\frac{1}{2} + g_3 \ln\left(10 \frac{x x_0}{x_0 + x} \right) \right) \right] b_T^2 \right\}$$

with $g_1 = 0.21$ GeV^2, $g_2 = 0.68$ GeV^2 and $g_3 = -0.6$ GeV^2, using $Q_0 = 1.6$ GeV for $b_{\rm max} = 0.5$ GeV^{-1}.

- For large Q and small x: reduces to BLNY fit for DY
- For $x_0 = 0.02$: matches the STM fit for SIDIS at $x = 0.09$ and $Q = \sqrt{2.4}$ GeV

Conclusions and Outlook

- TMD correlation functions with evolution based on definitions by J. Collins.
- Combined previous fits (BNLY and STM) which apply at different scales.
- Use TMDs in actual calculations: DY and SIDIS (work in progress).
- Improve fits, include higher order.
- Evolution for polarization dependent TMDs.
- Gluon distribution.
- Quantify factorization breaking effects.

See DIS talk by Ted Rogers.

Stay tuned for new and improved results
at

https://projects.hepforge.org/tmd