TMDs at small-x: What is our current understanding?

Emil Avsar, Penn State University

Pre-DIS meeting 2011, 8-9 April, Newport News, VA

The terminology, jargon and problems at small x

"$k_\perp$ dependent gluon distribution", or "unintegrated gluon distribution" appear in many formalisms.

Intuitively, same meaning as "TMD" distribution.

Problem: what is exactly meant by these "TMD PDFs" is not so clear since explicit definitions not always provided.

Problem: When provided, sometimes conflicting definitions appear.

$k_\perp$factorization: Hard to find proofs in literature. For outsider not clear what is known, guessed, conjectured, hoped...

Some formulas have different forms, and the PDFs entering these can be different, even though that they are all referred to as "unintegrated PDFs".

A great deal needs to be checked so we can be sure about the physics done!

Formalisms dealing with small-x physics

Many approaches to deal with small x:

For additional formalisms, see earlier talks on TMDs

Here are some names:

Not exhaustive list!!!

Obviously BFKL

Balitsky's approach → Non-linear "Balitsky-Kovchegov" (BK) equation.

The Color Glass Condensate (CGC) based on McLerran-Venugopalan (MV) model. → JIMWLK evolution

k⊥-factorization of Catani, Ciafaloni and Hautmann (CCH).

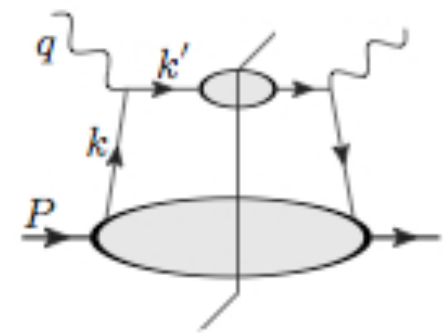
Catani, Ciafaloni, Fiorani and Marchesini (CCFM) formalism. ← Implemented in CASCADE MC (Jung)

The "dipole" formalism ← Close relation to BFKL. Widely used.

Applications of k⊥-factorized formula originating from Gribov, Levin and Ryskin (GLR).

Time will not permit me to go through all of these so I narrow my focus sharply. I will highlight some problems I have found in some of the assertions made.

Parton distributions from model theory



Intuitive ideas about parton densities can be made exact in a model field theory.

Super-renormalizable and non-gauge

Let $a^\dagger(k^+, k_\perp)$ and $a(k^+, k_\perp)$ be light-front creation and annihilation operators.

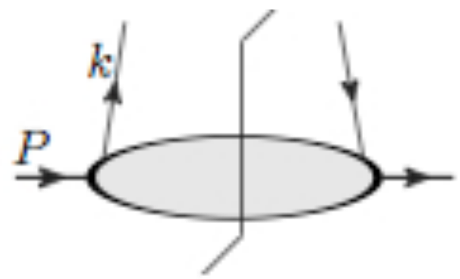
Intuitive definition of TMD PDF is that it is a number density:

$$f_\alpha(z, k_\perp) = \frac{1}{2z(2\pi)^3} \frac{\langle P | a^\dagger_{k,i,\alpha} a_{k,i,\alpha} | P \rangle}{\langle P | P \rangle}$$

$$k^+ = zP^+$$

Integrated PDF:

$$f_\alpha(z) = \frac{1}{2z} \int \frac{d^2k_\perp}{(2\pi)^3} \frac{\langle P | a^\dagger_{k,i,\alpha} a_{k,i,\alpha} | P \rangle}{\langle P | P \rangle}$$



Note that integral is over *all* momenta

Then in parton model, exactly true that: $f_\alpha(z) = \int d^2k_\perp f_\alpha(z, k_\perp)$

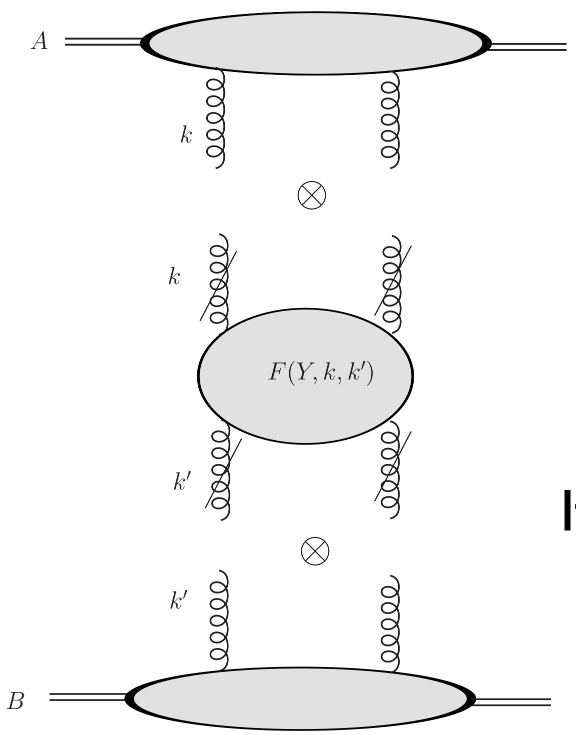
Thus the terminology "unintegrated density/distribution". Note, however, assumptions. If assumptions relaxed then none of these results can be taken as true anymore.

Gluon TMD density from BFKL

BFKL: Prototype of all small-x calculations.

Amplitude for scattering of objects *A* and *B* written as ($s/t \gg 1$)

$$\frac{\mathrm{Im}A(s,t)}{s} = \int \frac{d^2k_\perp}{k_\perp^2}\frac{d^2k'_\perp}{(k'_\perp - q_\perp)^2} I_A(k_\perp, q_\perp) I_B(k'_\perp, q_\perp) F(s, k_\perp, k'_\perp, q_\perp)$$



"Impact factors"

BFKL "Green's function",
"gg scattering amplitude",
"gg absorptive part"

If one defines $\mathcal{F}(s, k_\perp) = \int \frac{d^2k'_\perp}{k'^2_\perp} I_B(k'_\perp) F(s, k_\perp, k'_\perp)$

At leading log exact parameter here not so important. But one needs more care at NLL

then $\frac{\mathrm{Im}A(s,0)}{s} = \int \frac{d^2k_\perp}{k_\perp^2} I_A(k_\perp)\mathcal{F}(s, k_\perp)$

Common to call $\mathcal{F}$ "unintegrated gluon distribution".

More on small-x gluon distribution, and the dipole picture

The BFKL result for scattering amplitude is of "$k_\perp$-factorized" form.

It is then commonly asserted that "integrated" gluon density given by

$$G(x, Q^2) = \mathcal{C} \int^{Q^2} d^2k_\perp \mathcal{F}(x, k_\perp).$$

Asserted to be density of gluons with k^+ between xP^+ and $(x+dx)P^+$

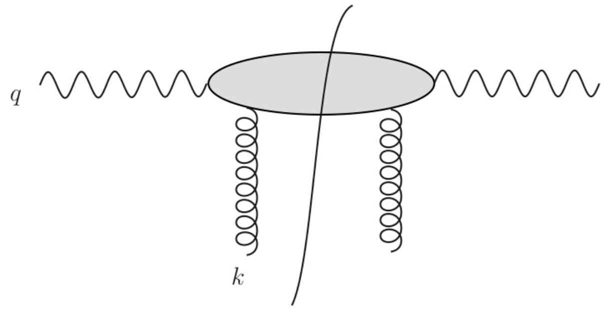
Not always universal prefactor

Note this dependence on both sides
Are they same? Actually not!

There is an issue with the meaning of this parameter, come back to this.

At this point not at all clear to what definition these objects correspond to.

A popular way to rewrite the BFKL result is via the so-called "dipole" formalism. This emerges when impact factor in DIS written as



$$I_{\gamma^*}(k_\perp) = \int d^2r_\perp |\psi(r_\perp)|^2 (1 - e^{ik_\perp r_\perp})$$

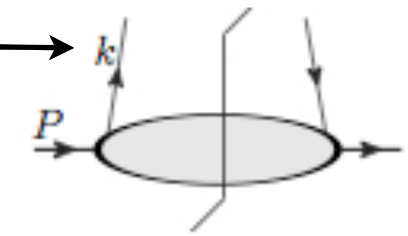
"Wave-function" of photon

In this picture the photon splits into a qq pair (dipole) "long" before interaction.
This dipole then interacts with hadron. This interaction coded in $\mathcal{F}$ above.

The meaning of the parameters in TMD PDF

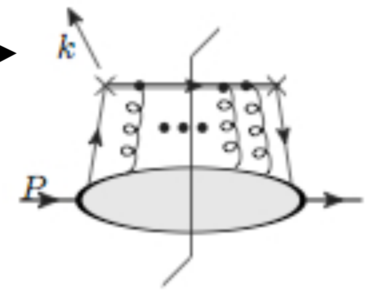
In parton model, PDF is number density. In that case the “integrated” density depends on z while “unintegrated” density on z and $k_\perp$

These are literally momentum variables of parton →



Turning on gauge interactions ⟶ arbitrarily many gluon exchanges ⟶ TMDs with eikonal Wilson lines in operator. def.

Now k is sum of exchanged momentum in intermediate state →



Then for TMDs, rapidity divergences appear if light like Wilson lines are used. These have to be cut-off by a parameter ζ

y_n used in Collins’ talk

Thus $f_{TMD} = f_{TMD}(z, k_\perp; \zeta)$ For collinear (“integrated”) distribution no rapidity divergence, thus no ζ dependence.

For renormalizable theory like QCD, also dependence on ren. scale μ

Thus generally $f_{TMD}(z, k_\perp; \zeta, \mu)$ and $G = G(z; \mu)$

These extra parameters needed in QCD also complicates relation between G and F

What is the meaning of x in $\mathcal{F}(x, k_\perp)$?

In derivation of BFKL type factorized formula, or in dipole model, k^+ actually neglected. Thus no dependence on k^+ in $\mathcal{F}(x, k_\perp)$

Then what is x? Answer is that it is actually the rapidity cut-off, and not the z from parton model.

If associate $\mathcal{F}(x, k_\perp)$ with TMD distributions, then it would mean

$$f_{TMD}(z = 0, k_\perp; \zeta = x, \mu)$$

But then what is the meaning of the relation $G(x, Q^2) = \mathcal{C} \int^{Q^2} d^2k_\perp \mathcal{F}(x, k_\perp).$?

It would be $G(z = x, \mu) = \int^{\mu^2} d^2k_\perp f_{TMD}(z = 0, k_\perp; \zeta = x, \mu)$

Conceptually this does not make much sense, and if not careful things can go wrong.
I will give one example of this.

Also, being careful with the parameters makes a difference, I will give an example of this too.

Dipole scattering, Wilson lines and connection to "BFKL factorization"

In small-x literature the "gluon distribution" also written using Wilson lines:

Balitsky's treatment of $\gamma^*\gamma^*$ scattering leads to

$$A(s,t) = i\frac{s}{2}\sum_i e_i^2 \int \frac{d^2k_\perp}{4\pi^2} I_A(k_\perp, q_\perp) \int d^2x_\perp e^{-ik_\perp\cdot x_\perp} \left\langle \mathrm{Tr}\{U(x_\perp)U^\dagger(0_\perp)\}\right\rangle$$

where $$U(x_\perp) = P\exp\left(-ig\int_{-\infty}^{\infty} d\lambda\, n_1 \cdot A^a(x_\perp + \lambda n)t_F^a\right)$$

n_1: vector along direction of motion of dipole

Similarly in dipole model one models interaction via same Wilson lines. In CGC same formula taken for DIS.

To avoid rapidity divergence, Wilson line taken off light cone by ζ

Evolution eq wrt ζ $\longrightarrow$ BK equation for $\mathcal{N}_\zeta(x_\perp, y_\perp) \equiv 1 - \frac{1}{N_c}\langle \mathrm{Tr}\{U_\zeta(x_\perp)U_\zeta^\dagger(y_\perp)\rangle$

$$\equiv 1 - \mathcal{S}_\zeta(x_\perp, y_\perp)$$

$\mathcal{N}_\zeta$: Dipole "scattering amplitude"

Dipole formalism and an application: Inclusive gluon production, and problems

A myriad of "gluon distributions" appear in dipole formalism:

$$\phi(\zeta, k_\perp) = \mathcal{C}_\phi \int \frac{d^2 x_\perp}{x_\perp^2} e^{-ik_\perp \cdot x_\perp} \mathcal{N}_\zeta(x_\perp)$$

$$\varphi(\zeta, k_\perp) = \mathcal{C}_\varphi \, k_\perp^2 \int d^2 x_\perp e^{-ik_\perp \cdot x_\perp} \mathcal{N}_\zeta(x_\perp)$$

$$\mathcal{F}(\zeta, k_\perp) = \mathcal{C}_\mathcal{F} \int d^2 x_\perp e^{-ik_\perp \cdot x_\perp} \mathcal{S}_\zeta(x_\perp)$$

I have not been able to pin down universal prefactors so let me leave them unspecified

These appear in different formulas in phenomenological applications and I here give one example. Which is "real" gluon distribution...? Note that again none has z dependence...

Kovchegov and Tuchin (hep-ph/0111362) studied single inclusive gluon production in DIS on "classical" nucleus. Using dipole formalism they arrive at a formula which is then "identified" with GLR formula:

$$\frac{d\sigma}{d^2 k_\perp dy} = \frac{2\alpha_s}{C_F k_\perp^2} \int d^2 q_\perp \frac{f_1(x_1, q_\perp^2) \, f_2(x_2, |k_\perp - q_\perp|^2)}{q_\perp^2 (k_\perp - q_\perp)^2}$$

Now, what is then f here?

Continuation from previous slide

In GLR, f was "defined" as:

$$f(x, q_\perp^2) = \frac{dxG(x, q_\perp^2)}{d\ln q_\perp^2} \qquad \text{(A)}$$

Using thus GLR, KT identified $f(x, q_\perp^2)$ with $\varphi(x, q_\perp^2)$ with prefactor $\mathcal{C}_\varphi = \frac{N_c}{(2\pi)^4 \alpha_s}$

Comments and problems:

To begin with note again the issue with the meaning of x in these formulas.

Given the classical formulation of the nucleus it was possible to directly calculate G, with the definition that it is a number density in longitudinal momentum.

Note: This is classical calculation so problems of QCD not appear

Using same formalism one could also calculate $\varphi(x, q_\perp^2)$

Then using (A) above the results could be compared.

Yet the results of the two different calculations do not agree...

Continuing...

Thus the integral of "unintegrated" distribution did not agree with integrated one...

The identification of $\varphi(x, q_\perp^2)$ with GLR formula is still being used, however.

Thus one needs to be extra careful here. Moreover, GLR formula came with different forms...

Formula used in pp and AA collision where TMD factorization has not been established. In fact there are explicit counter examples that it fails in pp...

Collins, Qiu 0705.2141
Rogers, Mulders 1001.2977

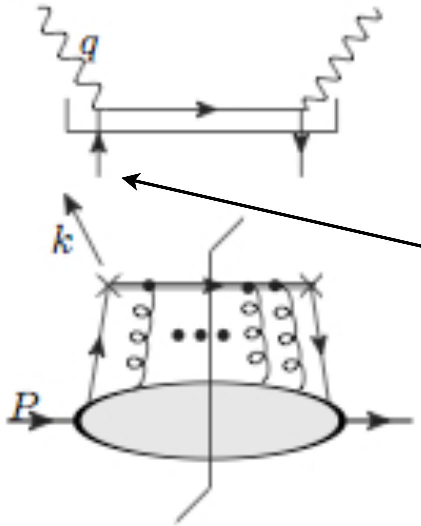
Additional warning: $\frac{d\sigma}{d^2k_\perp dy}$ will give y dependence of produced particle.

Leading small-x formalism extremely poor for this, however, because of large kinematical approximations.

Naive application dangerous

Importance of kinematics and possible implications

Remember again picture of factorization:



k enters in impact factor too

In formula

$$\sigma = \int \frac{d^2 k_\perp}{k_\perp^2} I(k_\perp) \mathcal{F}(x_{bj}, k_\perp)$$

kinematical approximations as noted earlier

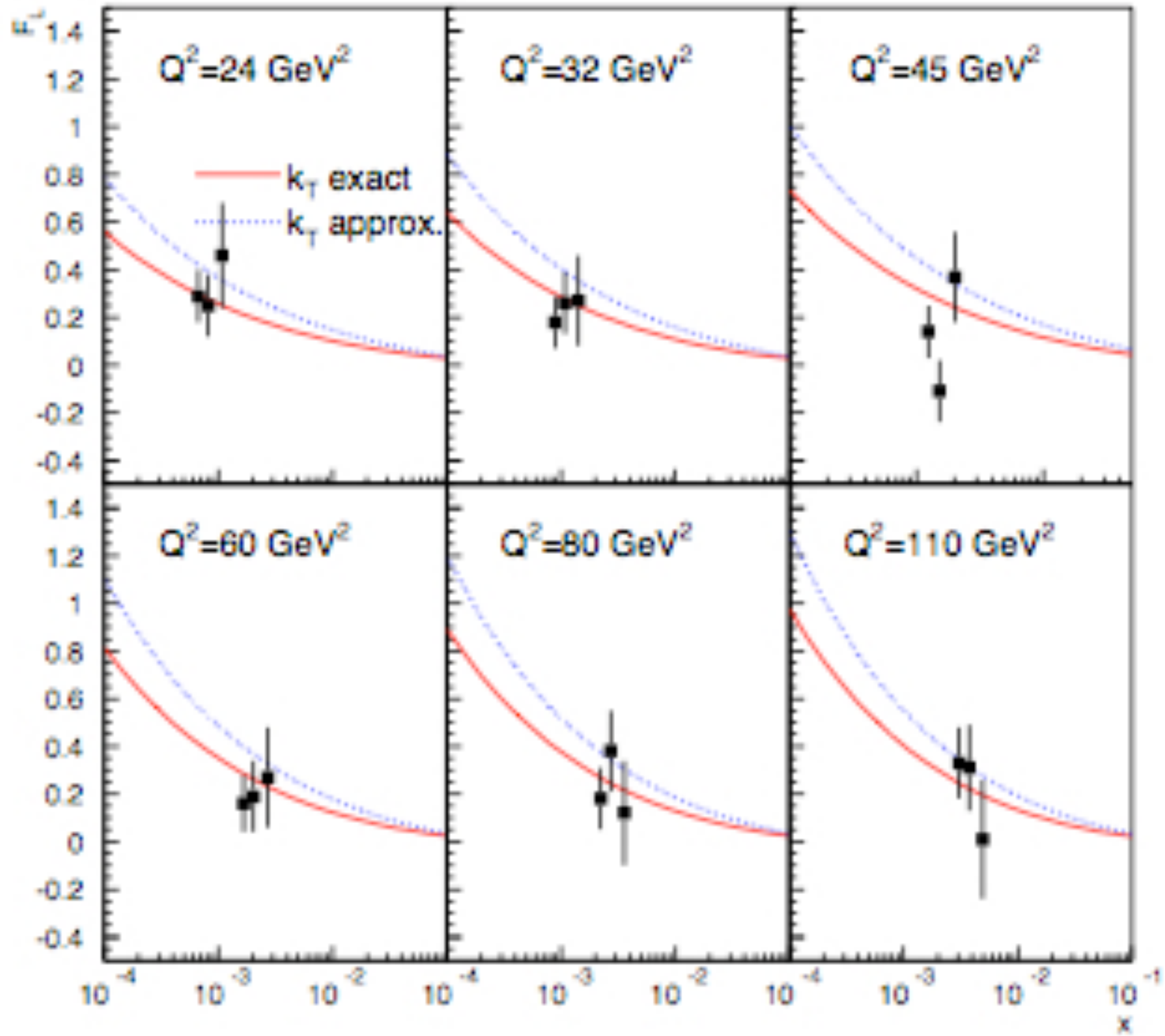
Then in impact factor x_{bj} used instead of true momentum of parton

Can make a big difference!

Exact kinematics shifts x to higher values: Important for non-linear physics, especially an issue at low Q^2

Example of application for FL

Golec-Biernat, Stasto: 0905.1321



Actually effect for F2 is bigger...

When looking for saturation in data, we must be careful of these effects.

Question is whether we are?