An Introduction to Neutron Stars

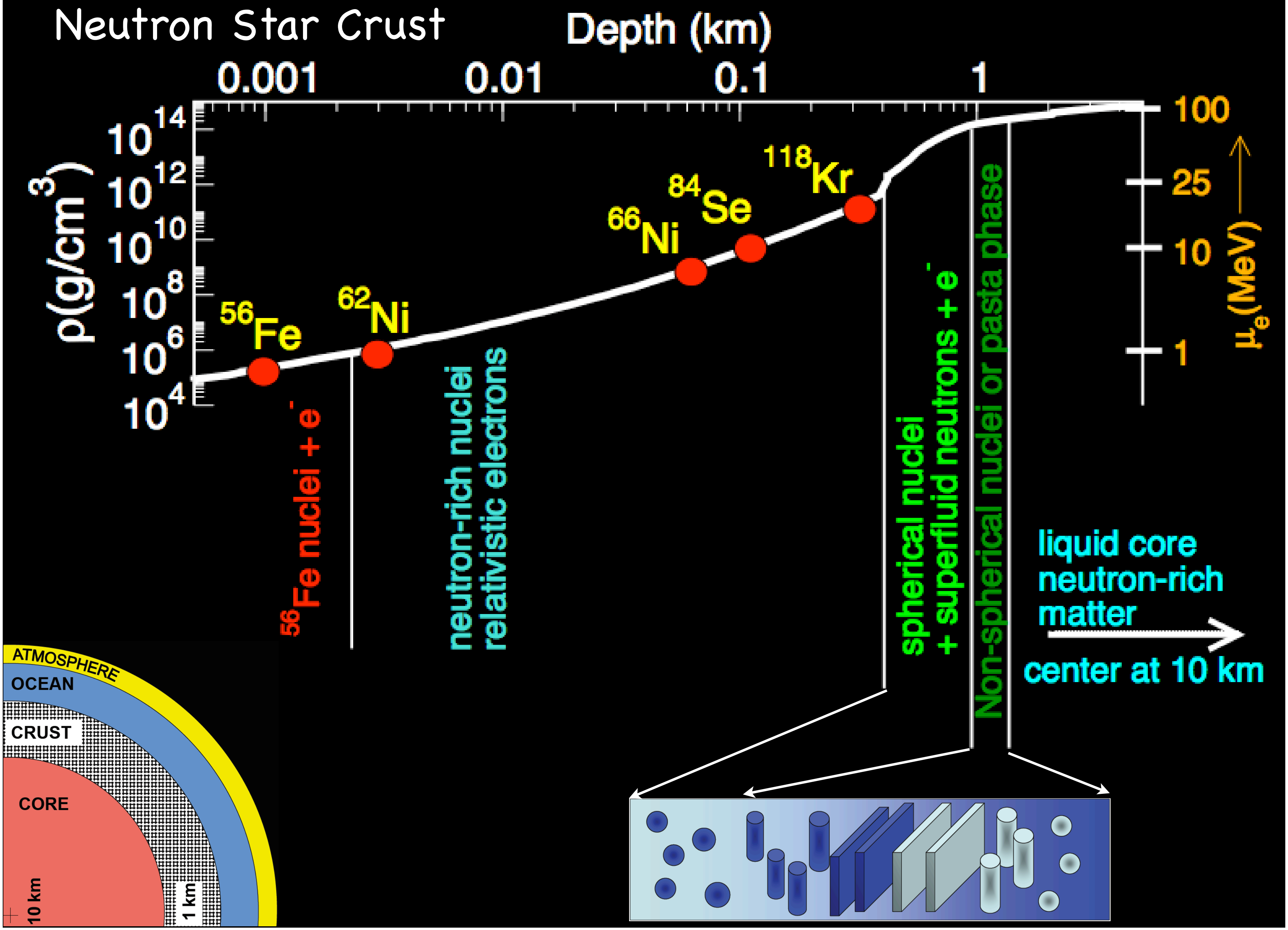
A nuclear theory perspective

Sanjay Reddy
Theoretical Division
Los Alamos National Lab

Compressing matter: Liberating degrees of freedom

Density	Energy	Phenomena
10^3 - 10^6 g/cm^3	Electron Chemical Pot. μ_e= 10 keV– MeV	Ionization
10^6 - 10^{11} g/cm^3	Electron Chemical Pot. μ_e= 1–25 MeV	Neutron-rich Nuclei
10^{11} - 10^{14} g/cm^3	Neutron Chemical Pot. μ_n= 1–30 MeV	Neutron-drip
10^{14} - 10^{15} g/cm^3	Neutron Chemical Pot. μ_n=30–1000 MeV	Nuclear matter Hyperons or Quarks ?

Neutron Star Crust



Building a Neutron Star

Hydrostatic Structure Equation :

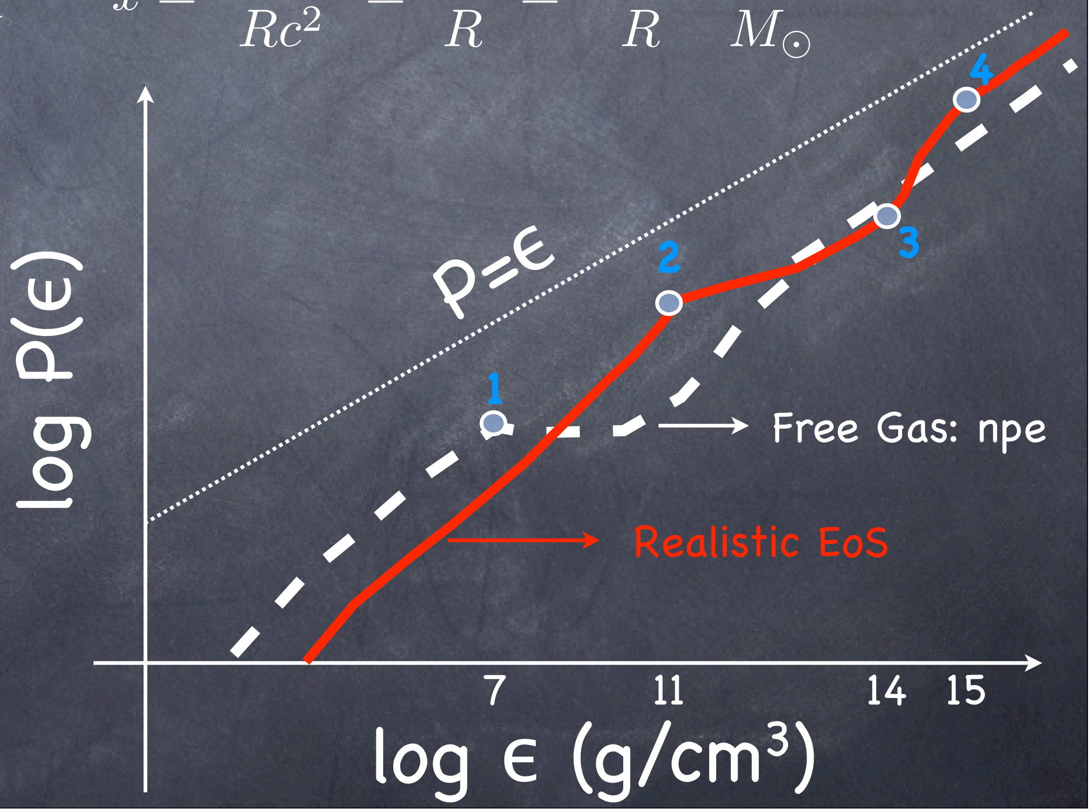
$$\frac{dP}{dr} = -\frac{\tilde{G}\ M(r)(\epsilon(r)+P(r))}{r^2c^2}\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)$$

$$\tilde{G} = \frac{G}{1-\frac{2GM(r)}{rc^2}}$$

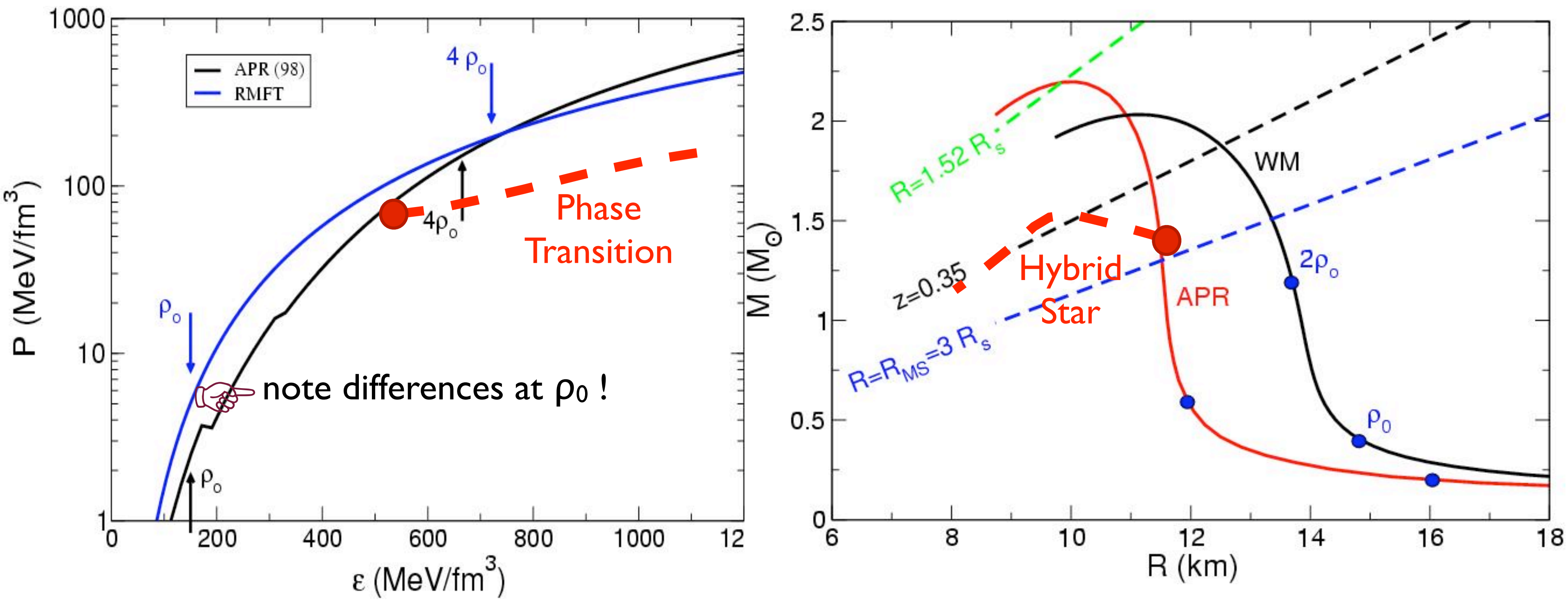
$$P_{\mathrm{central}} \simeq \frac{x}{1-3\ x}\,\epsilon_{\mathrm{central}} \qquad x = \frac{2GM}{Rc^2} = \frac{R_S}{R} = \frac{3\ \mathrm{km}}{R}\,\frac{M}{M_\odot}$$

Equation of State:

1. **Neutron Threshold**
2. **Neutron Drip**
3. **Nuclear Matter**
4. **Possible phase transition**



Mass-Radius Curve



note differences at ρ0 !

Phase Transition

Hybrid Star

Mass-Radius relation is “unique” to the underlying EoS.

- Soft EoS: low maximum mass and small radii
- Stiff EoS: high maximum mass and large radii

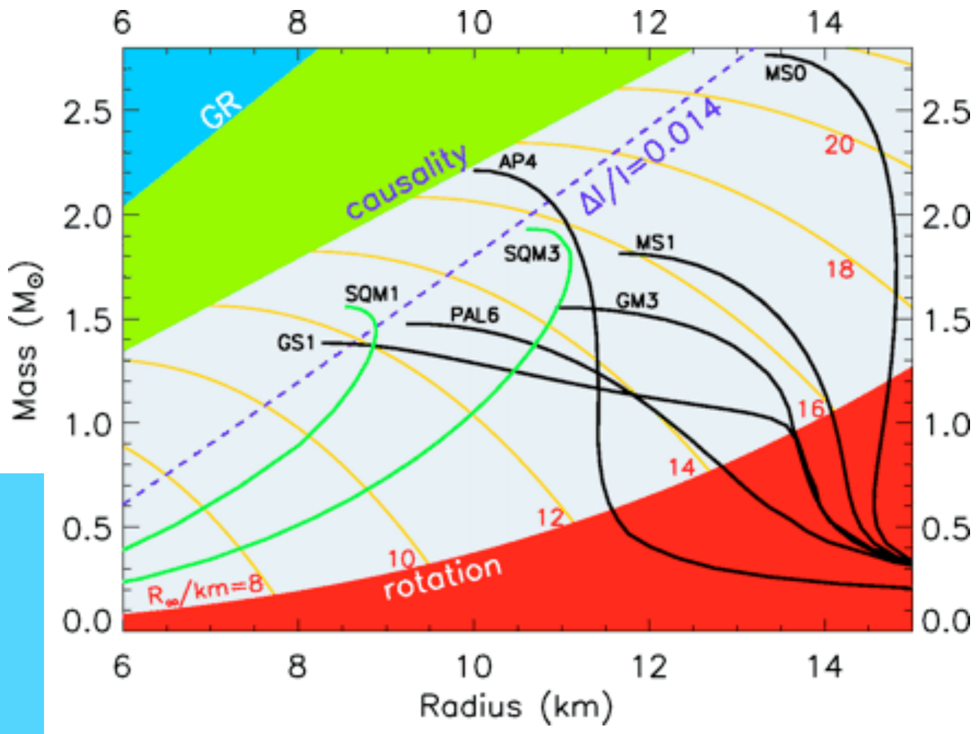
The compact object zoo:

Three Classes:

(1)Nucleon Stars

(2)Hybrid Stars

(3)Strange Quark Stars



Lattimer & Prakash, Science 2004

Predictions for the radii of nucleon stars vary and depends sensitively on the pressure of neutron-rich matter at ρ=1-2 ρ₀

Hybrid stars typically have smaller radii and smaller maximum masses - but difficult to quantify.

Strange stars can be arbitrarily small. Very speculative but has been difficult to conclusively rule them out.

Neutron Star Observations

- Pulsar timing. Can accurately measure the neutron star masses in compact binaries.
- Surface photon luminosity. Can provide information about radius and thermal evolution.
- X-ray bursts & Superbursts. Can provide simultaneous information mass, radius and internal temperature.
- Supernova neutrinos. Can directly probe the dense core.
- Giant-flare QPOs on magnetars. Can probe the crust thickness and radius - if they are shear modes.
- Gravity waves. Can measure the quadrapole moment in rotating stars and oscillation modes in newly born neutron stars and binary mergers.

Pulsar Timing:

Potentially observable relativistic effects

- Periastron advance (c.f. perihelion advance of Mercury)

$$\dot{\omega} \sim 40\,\mathrm{deg\,yr^{-1}} \left(\frac{P_b}{\mathrm{hr}}\right)^{-5/3} \left(\frac{1}{1-e^2}\right) \left(\frac{m_p+m_c}{\mathrm{M}_\odot}\right)^{2/3}$$

- Orbital decay (gravitational-wave emission)

$$\dot{P}_b \sim -4\times10^{-12} \left(\frac{P_b}{\mathrm{hr}}\right)^{-5/3} \frac{(1+\frac{73}{24}e^2+\frac{37}{96}e^4)}{(1-e^2)^{7/2}} \frac{m_p m_c}{(m_p+m_c)^{1/3}}$$

- Gravitational redshift and time dilation

$$\gamma \sim 2.4\,\mathrm{ms} \left(\frac{P_b}{\mathrm{hr}}\right)^{1/3} e\frac{m_c(m_p+2m_c)}{(m_p+m_c)^{4/3}}$$

- Shapiro delay (light bending by companion)

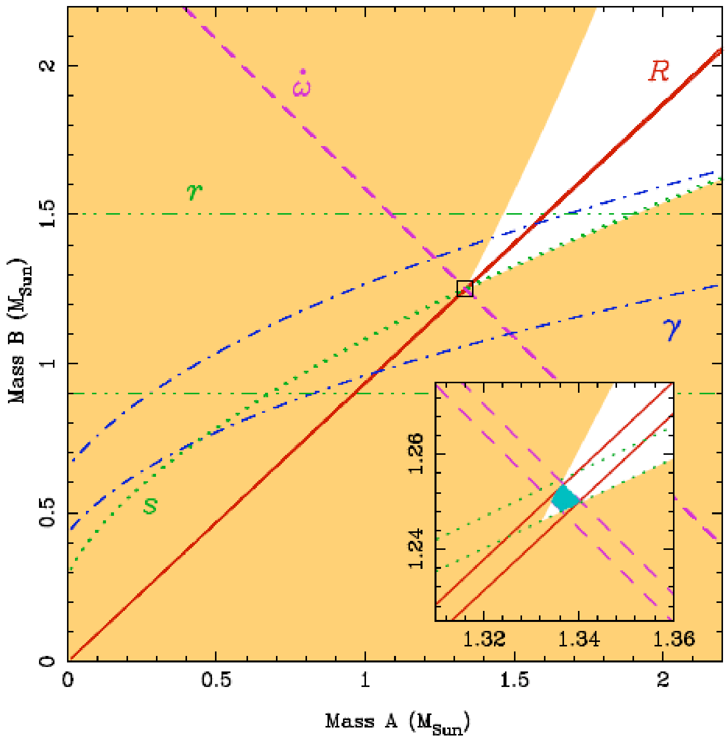
$$r \sim 5\mu\mathrm{s} \times m_c$$

$$s = \sin i \sim 0.9 \left(\frac{P_b}{\mathrm{hr}}\right)^{-2/3} x \frac{(m_p+m_c)^{2/3}}{m_c}$$

- Geodetic precession (roughly one/third of $\dot{\omega}$)

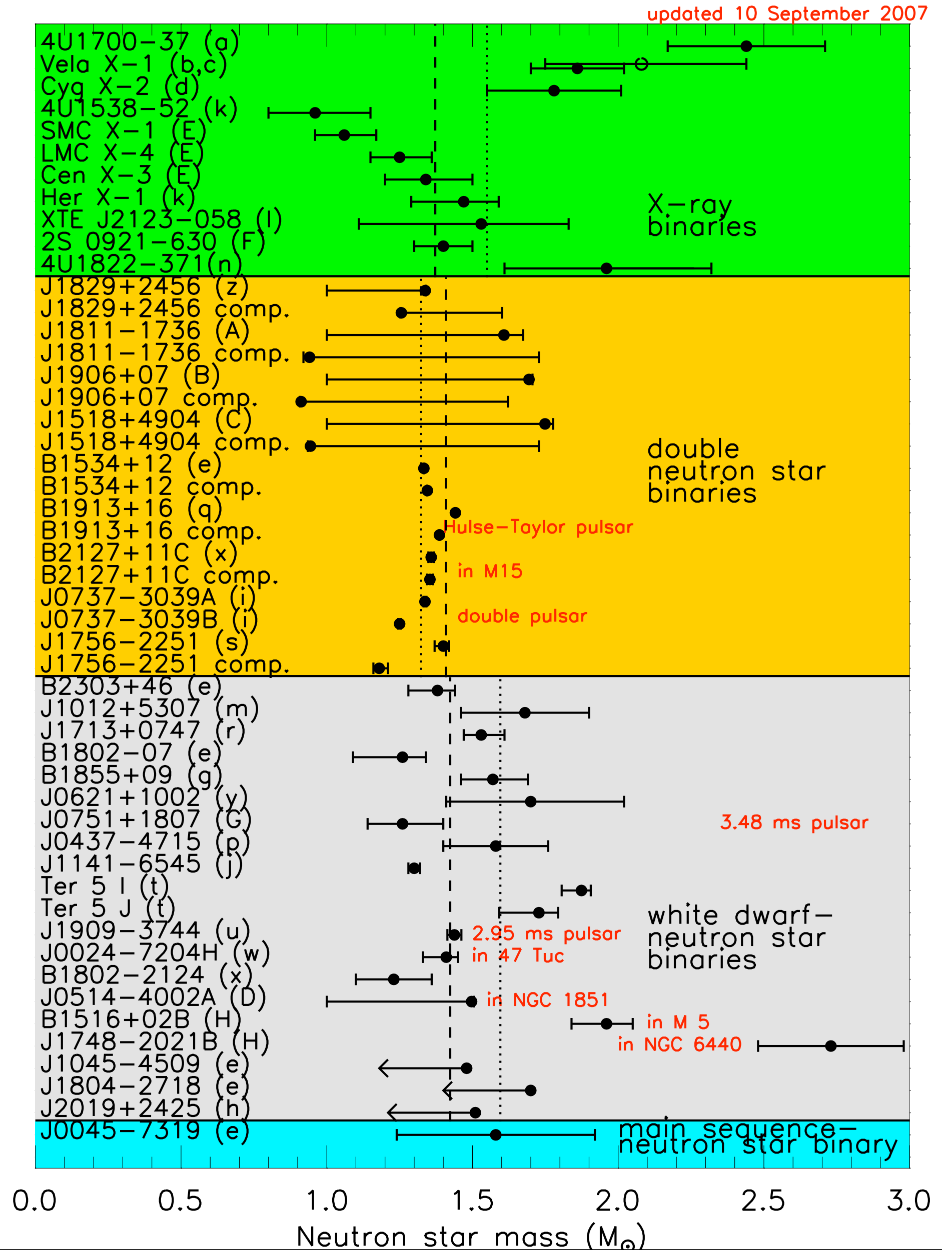
$$P_{\mathrm{geod}} \sim 54\,\mathrm{yr} \left(\frac{P_b}{\mathrm{hr}}\right)^{5/3} \frac{(M)^{4/3}}{m_c(4m_p+3m_c)} (1-e^2)$$

Mass–mass diagram for the double pulsar



Discovery of J0737–3039A (Burgay et al. 2003)

Neutron Star Masses



What is the origin of the clustering ?

Is Vela X-1 the heaviest NS ?

Is J1748-2021B for real ?

Massive neutron stars provide very useful constraints on the high density EoS.

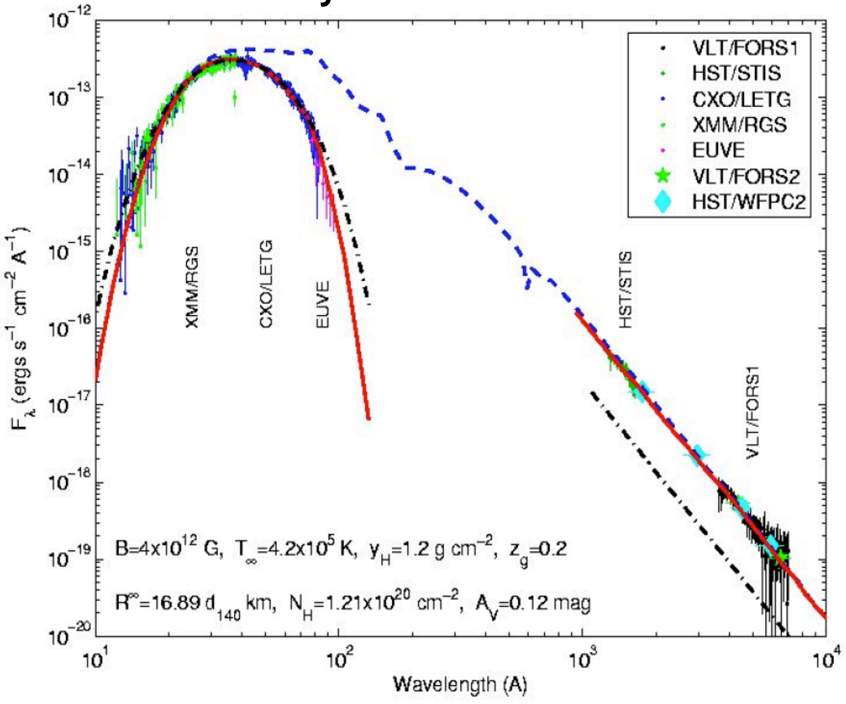
Figure courtesy: J. Lattimer

Radius

If NS radiates as a black body the observed flux :

$$\begin{aligned} F_{\rm BB} &= 4\pi \, \frac{R_\infty^2}{d^2} \, \sigma_{SB} \, T_\infty^4 \\ R_\infty &= \frac{R}{\sqrt{1 - R_S/R}} \\ T_\infty &= \sqrt{1 - R_S/R} \quad T \end{aligned}$$

RXJ185635-3754

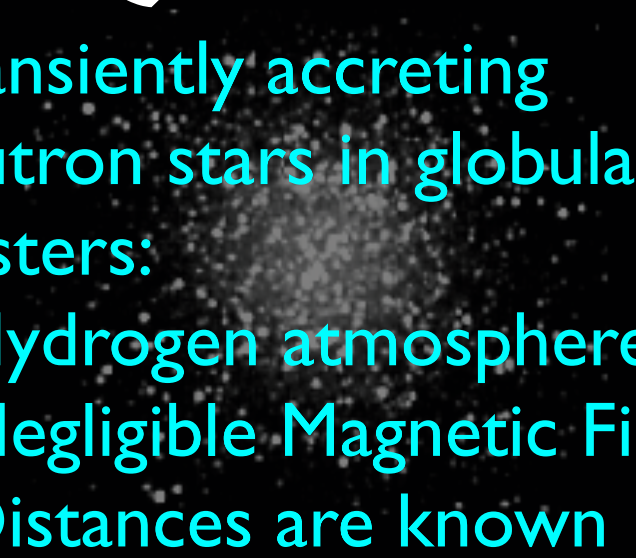


NS spectra depend on:

1. Composition of atmosphere
2. Magnetic fields

Difficult to model accurately

Quiescent NSs in LMXB



Transiently accreting neutron stars in globular clusters:

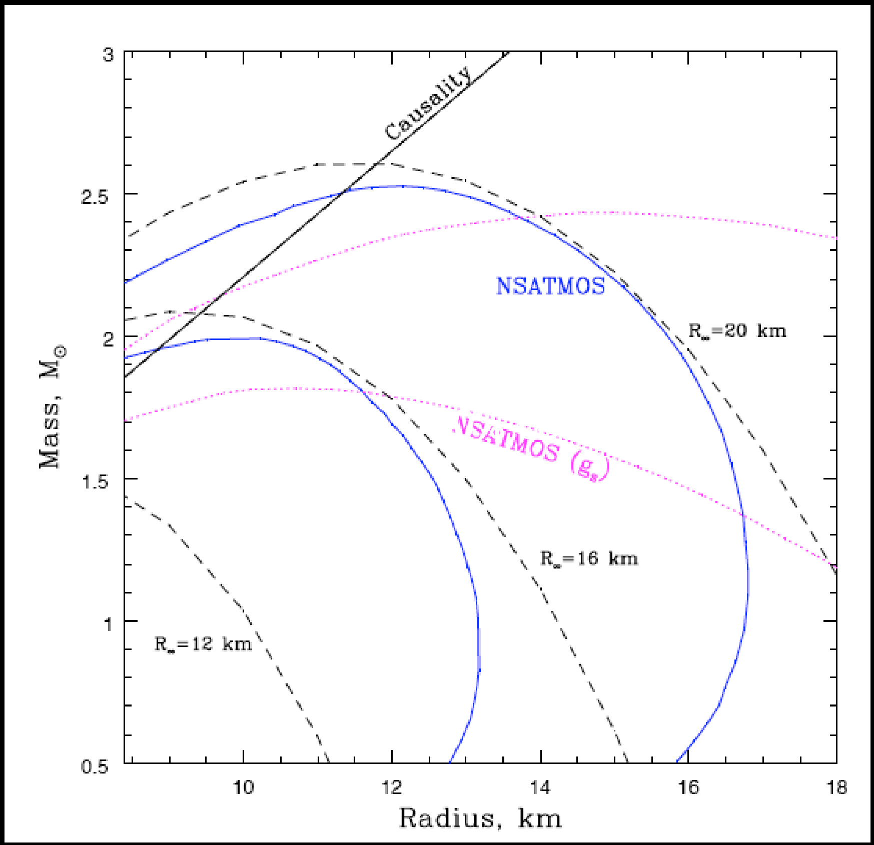
1. Hydrogen atmosphere
2. Negligible Magnetic Fields
3. Distances are known

Rutledge et al. (2004)

NS	R∞	Ref.
ω Cen	13.6± 0.3	Gendre et al. (2002)
M13	12.6± 0.4	Gendre et al. (2002)
X7*	14.5+1.6-1.4	Heinke et al. (2005)
M28	14.5+6.8-3.9	Becker et al. (2003)

Radius of qLMXB X7 in 47 Tuc:

Rybicki et al. (2005)

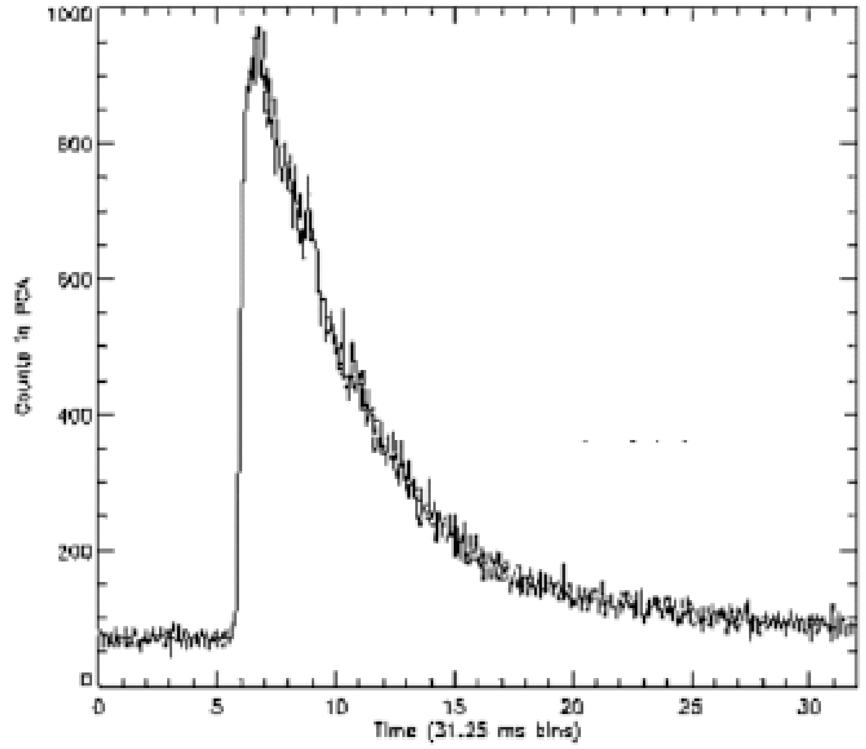


3 more found in GC : NGC 6304

Mass and Radius from X-ray Bursts:

Unstable burning of accreted material produces x-ray bursts

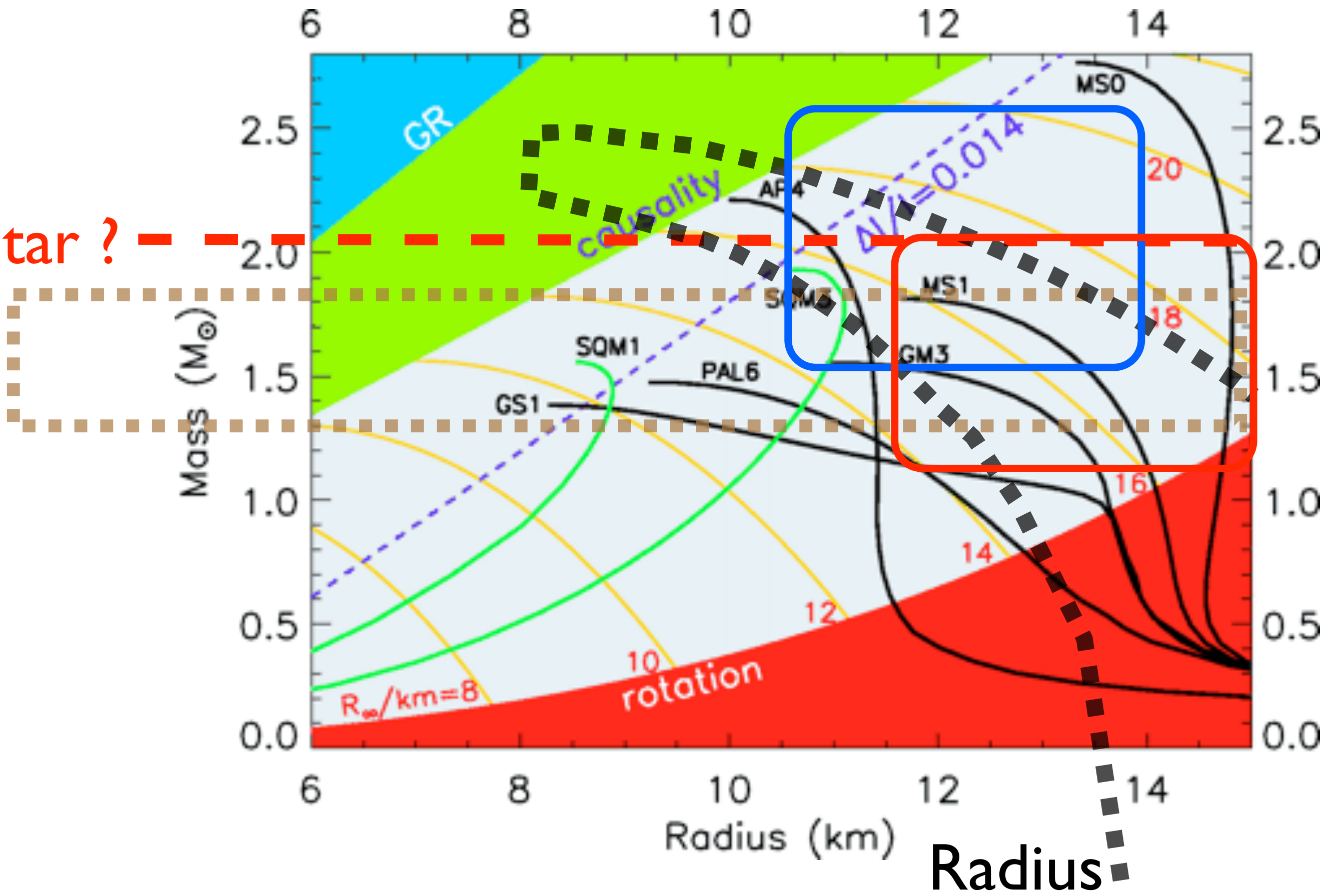
- Most common cosmic explosion in the universe.
- Light curve powered by nuclear reactions (rp process).
- Features in the light curve are be sensitive to Mass and Radius.



From Mass-Radius back to EoS

Massive neutron star ?

Not even wrong



Radius

Simultaneous Mass & Radius measurement

+ Temperature information

Important Correlation:

Large radii favor rapid cooling of neutron stars

Generic Trends

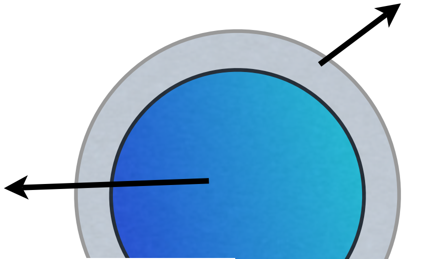
- A stiff low-density equation of state has a larger proton fraction.

$$
\begin{aligned}
E(\rho, x_p) &= E_o(\rho, x_p) + E_{\rm sym}\ \delta^2 + \cdots \\
P(\rho, x_p) &= P(\rho, \frac{1}{2}) + \rho\ E_{\rm sym}\ \left[(1-2x_p)^2\ \frac{\rho\ E'_{\rm sym}}{E_{\rm sym}} + x_p(1-2x_p) \right] \\
x_p &\simeq 0.04 \left[\frac{E_{\rm sym}(\rho_o)}{28\ {\rm MeV}} \right]^3 \left[1 + \frac{E'_{\rm sym}}{E_{\rm sym}(\rho_o)}\ (\rho - \rho_o) \right]
\end{aligned}
$$

Steiner, Lattimer, Prakash & Ellis (2005)

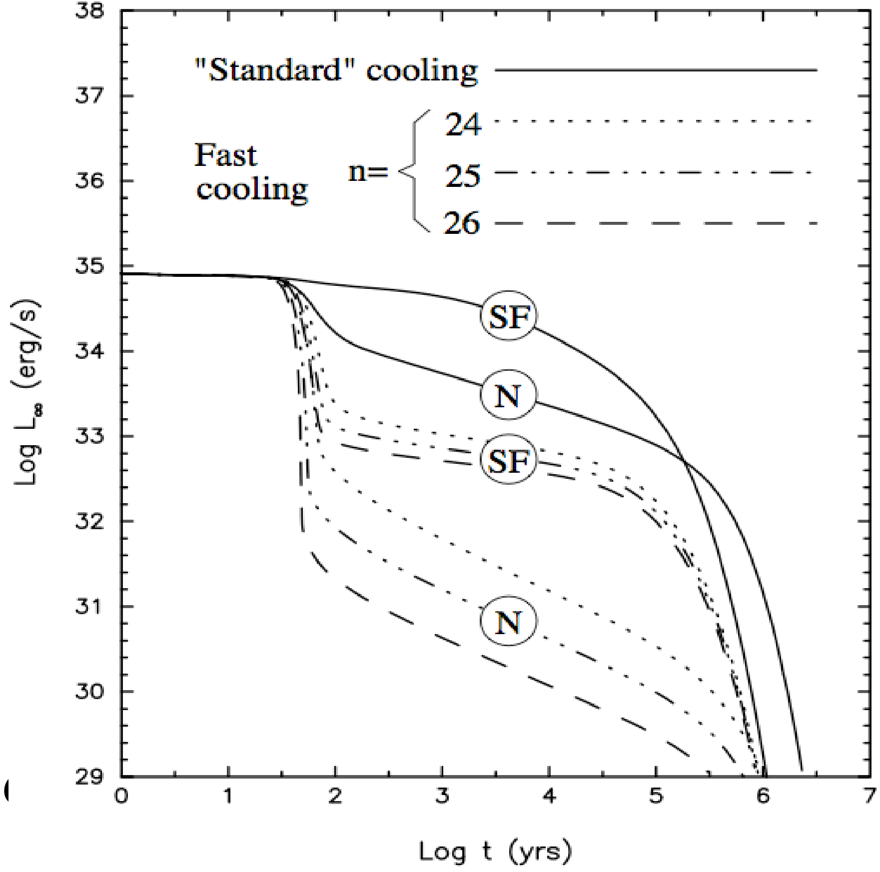
- A large radius (> 12 km) for a canonical neutron star of mass ~ 1.4 M☼ would favor $x_p > 0.1$ in the core.

Neutron Star Cooling



Crust cools by conduction

Isothermal core cools by neutrino emission



face photon emission dominates

ate time t > 10^6 yrs

e

$+ \bar{\nu}_e$

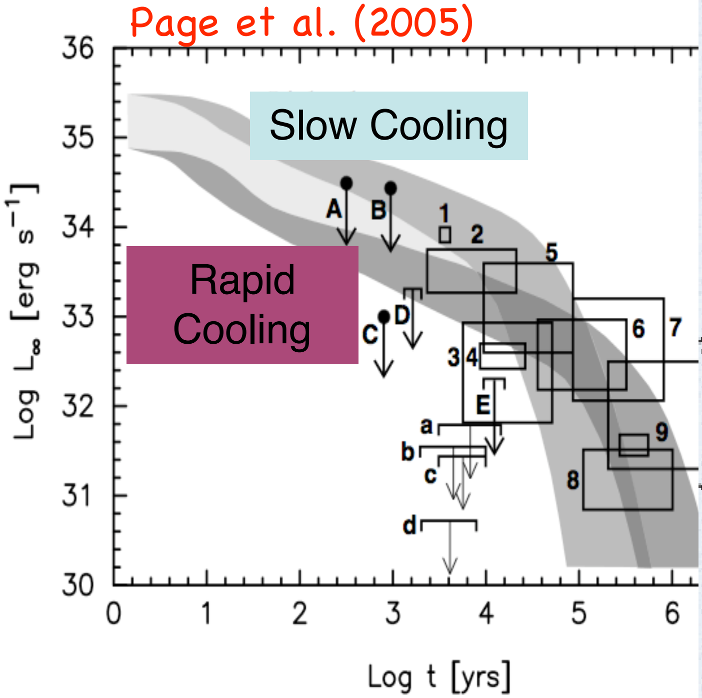
$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{25}\ T_9^6\ \frac{\text{ergs}}{\text{cm}^3\ \text{s}}$$

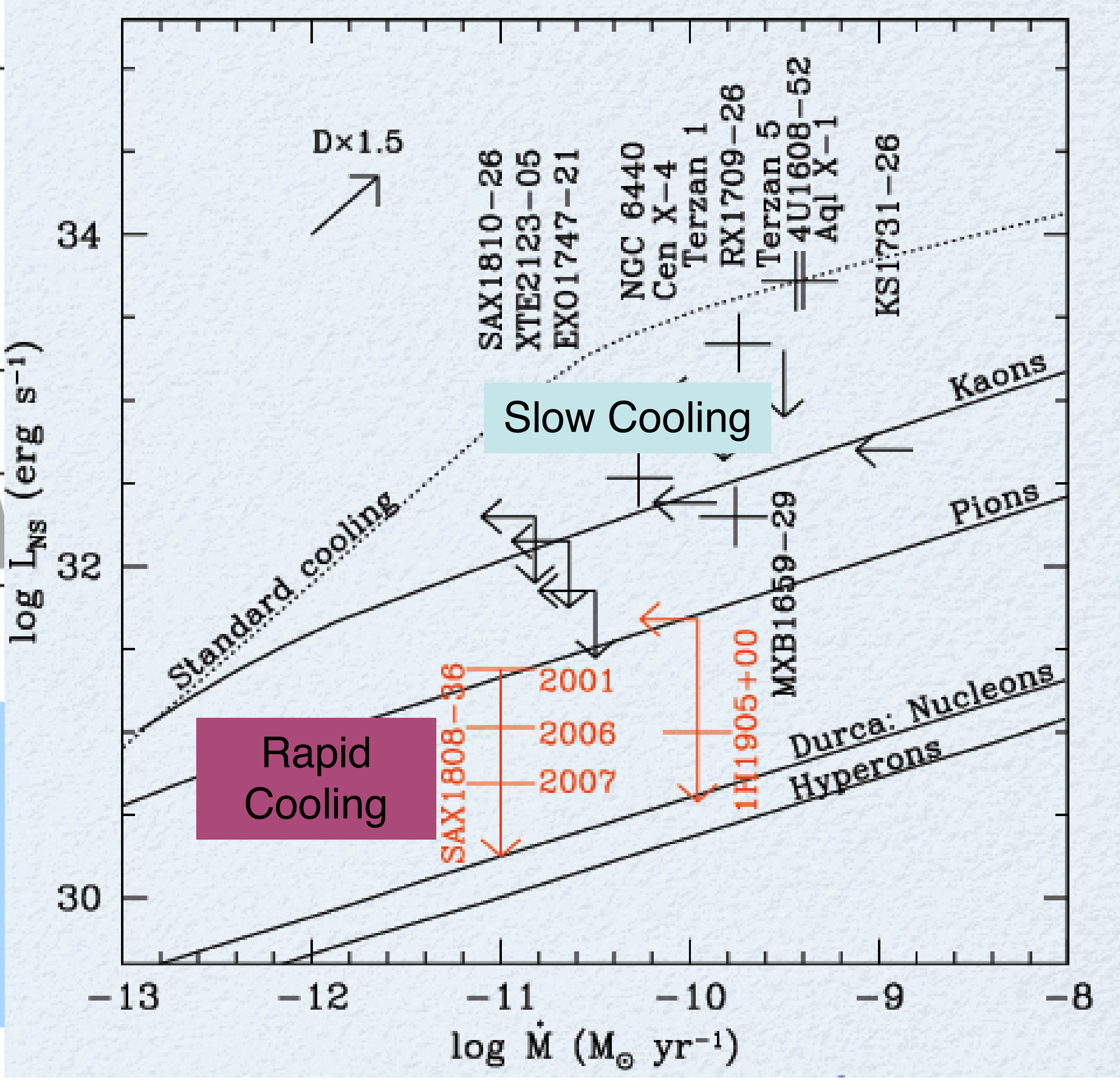
Fast: Direct URCA

$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{22}\ T_9^8\ \frac{\text{ergs}}{\text{cm}^3\ \text{s}}$$

Slow: Modified URCA

Neutron Star Cooling

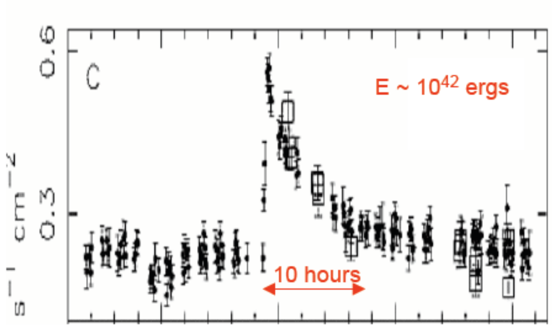

Page et al. (2005)



Quiescent emission after periods of bursting in accreting neutron stars (SXRT)

Rutledge (2004) Yakovlev (2006) Heinke (2007)

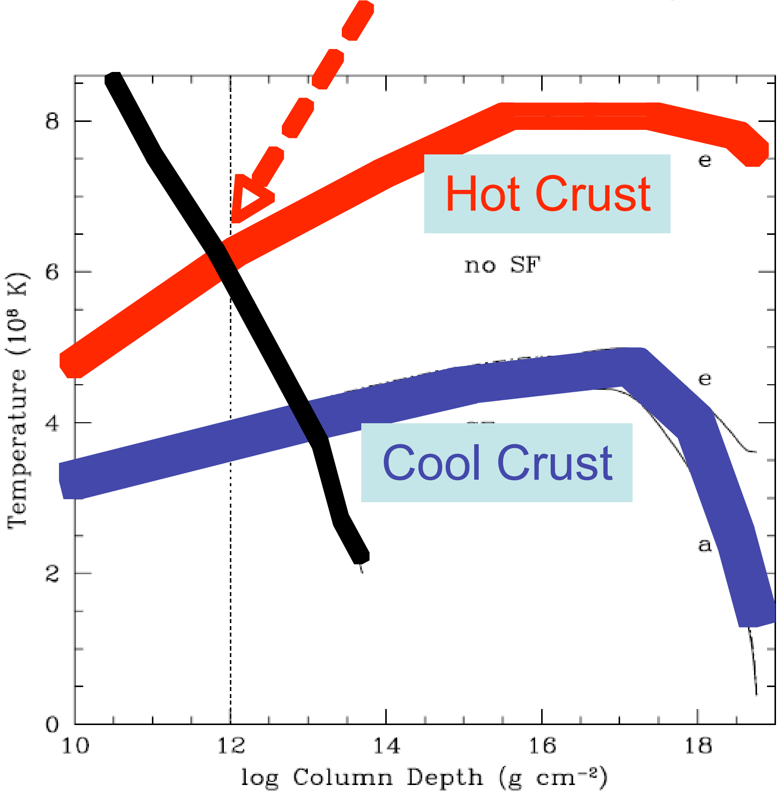
Superbursts as neutron star thermometers



Superbursts are longer duration (hours) bursts with *recurrence times days-years*.
Likely to be ignition of carbon poor ashes produced during XRB activity.

Woosley & Taam (1976), Cumming & Bildsten (2001)
Strohmayer & Brown (2002)

Carbon-ignition at column depth ~10^{12}g/cm^2

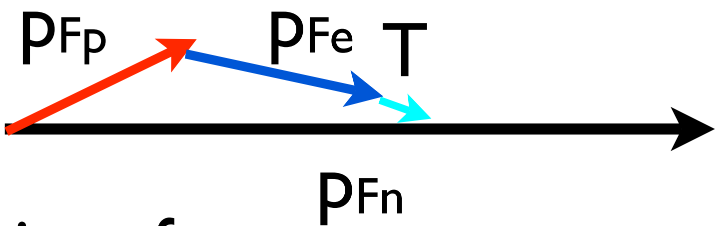


Ignition depth is very sensitive to the thermal profile of the neutron star crust.

Brown (2005) Cumming (2006)

Correlation between Cooling and EoS

Kinematics:



Single neutron decay at the Fermi surface cannot conserve momentum if $x_p \sim (p_{Fp} / p_{Fn})^3 < 0.12\text{-}14$

Dynamics:

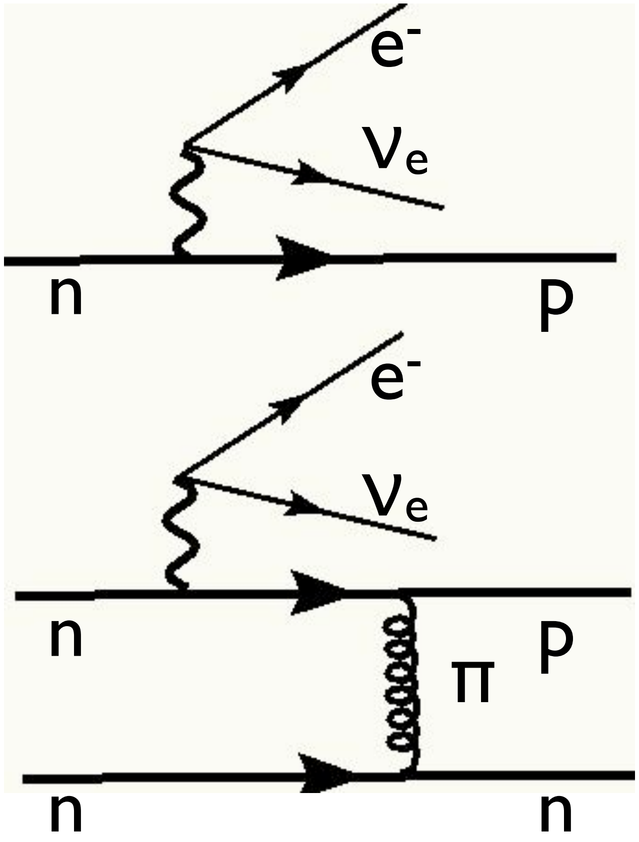
- Nuclear symmetry energy can favor large proton fraction and allow direct-urca.
- Nucleon-nucleon interactions induce two and many-body processes that produce neutrinos by the modified URCA reaction.

Neutrino Emissivity & Fluctuations

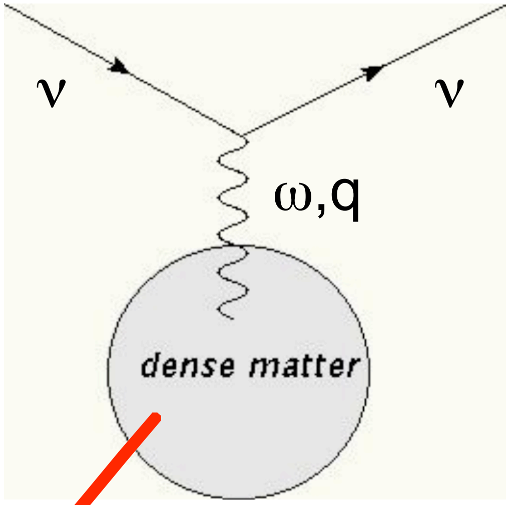
Neutrinos couple to nucleon density and spin, Density and spin fluctuations lead to neutrino emission.

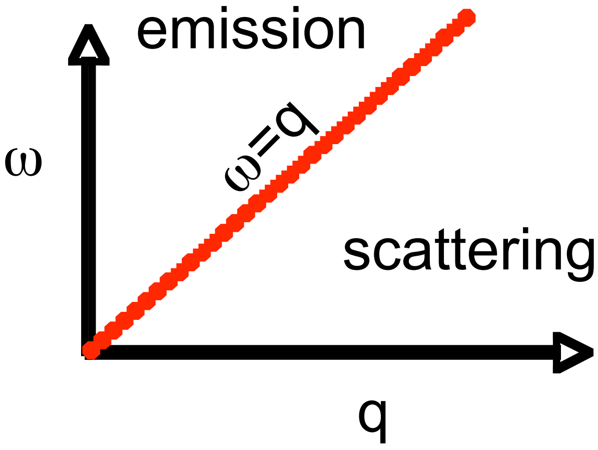
Single -particle reactions are fast. Need unstable particles

Multi-particle reactions are slow - typically of the Bremsstrahlung type.



Neutrino Rates





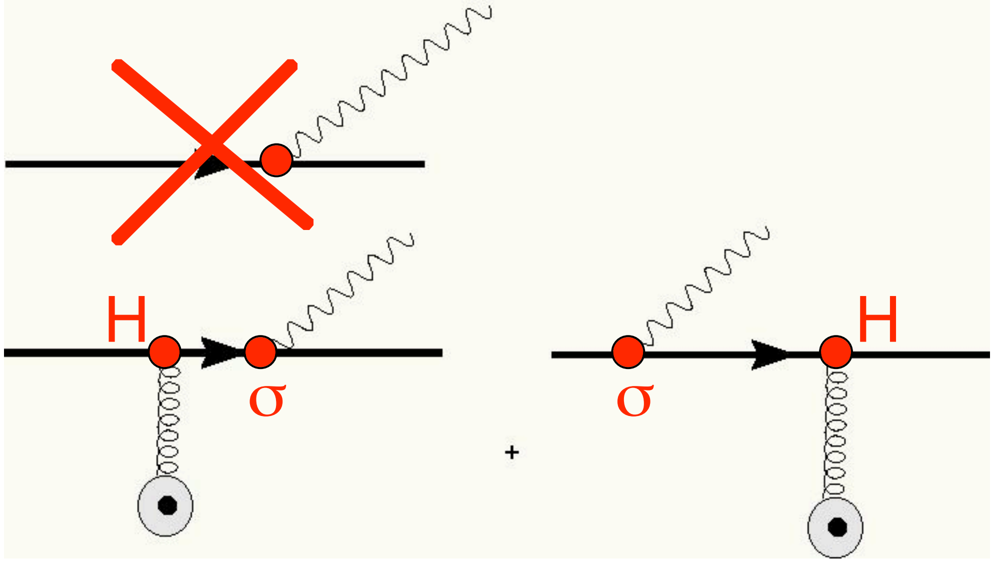
$$\frac{d^2\sigma}{V\, d\cos\theta\, dE'} \approx G_F^2 \frac{E}{E'} \,\mathrm{Im}\left[L_{\mu\nu}(k,k+q)\,\Pi^{\mu\nu}(q)\right]$$

$$L_{\mu\nu}(k,k+q) = \mathrm{Tr}\,[\; l_\mu(k)\; l_\nu(k+q)\;]$$

$$\Pi^{\mu\nu}(\omega,\vec{q}) = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{Tr}\,[\; j^\mu(p)\;\; j^\nu(p+q)\;]$$

Dynamic structure factor: spectrum of density, spin and current fluctuations.

Subtle nuclear aspects of neutrino emission:



Kinematically forbidden

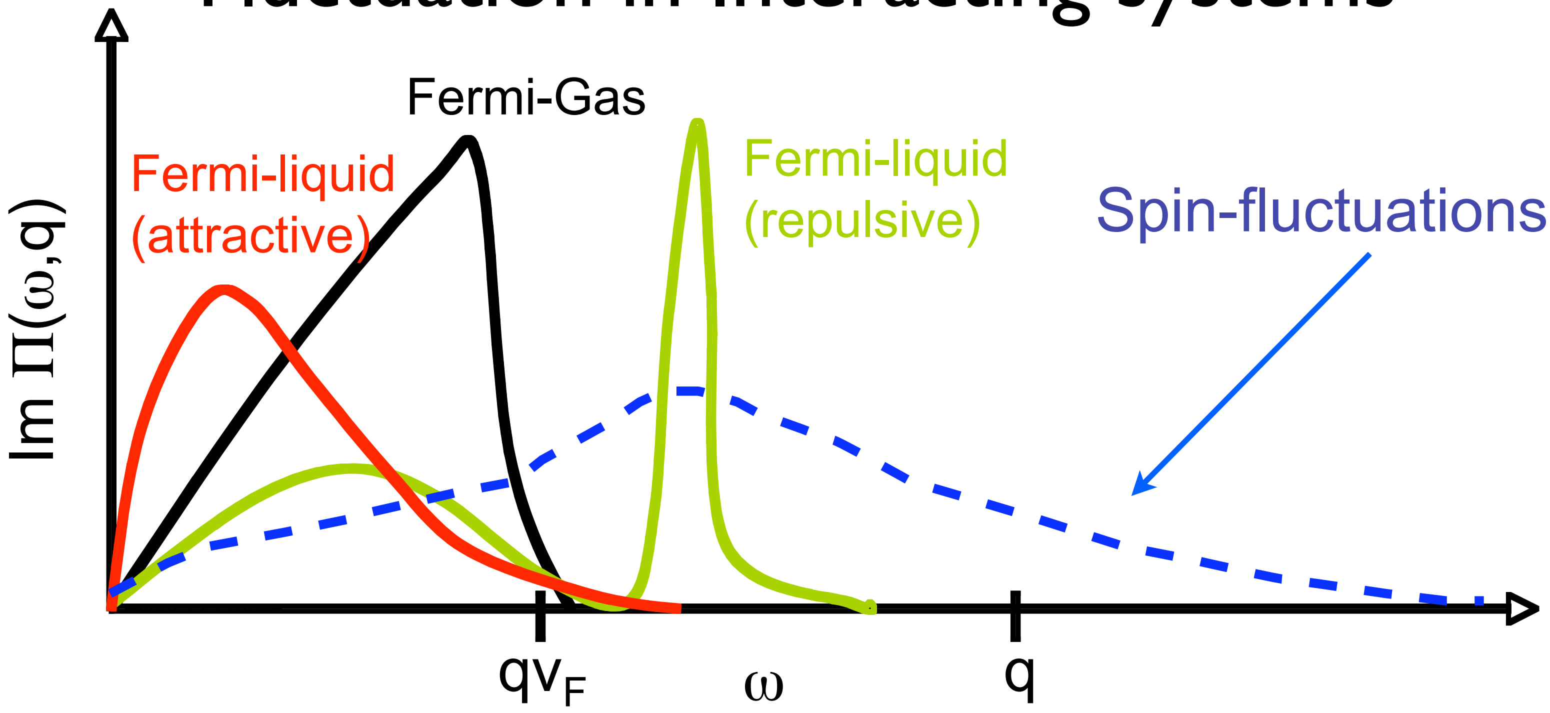
Need acceleration for radiation

$$\frac{1}{\frac{p\cdot q}{m}+\frac{q^2}{2m}+\omega}+\frac{1}{\frac{p\cdot q}{m}-\frac{q^2}{2m}-\omega}\approx\frac{1}{\omega}\frac{p}{m}\frac{q}{\omega}$$

Radiation without acceleration set q=0:

$$\frac{H\,\sigma}{\omega}-\frac{\sigma\,H}{\omega}\approx\frac{1}{\omega}[H,\sigma]$$

Fluctuation in Interacting systems



Spectrum of fluctuations of spin and spin-isospin can be strongly modified by non-central interactions.

In the long-wave length limit these are related to the EoS through the susceptibilities and energy-weighted sum-rules.

Phase Transitions

Nuclear transitions:

Driving force: Large Fermi surface enhances attractive fluctuations.

1. Neutron superfluidity and proton superconductivity
2. Pion condensation or spin-isospin ordering
3. Ferromagnetism or spin-ordering

Strange transitions:

Driving force: Dense charge neutral nuclear matter is neutron-rich. Too many down quarks.

1. Hyperon matter
2. Kaon condensation
3. Quark matter

Pairing in neutron matter

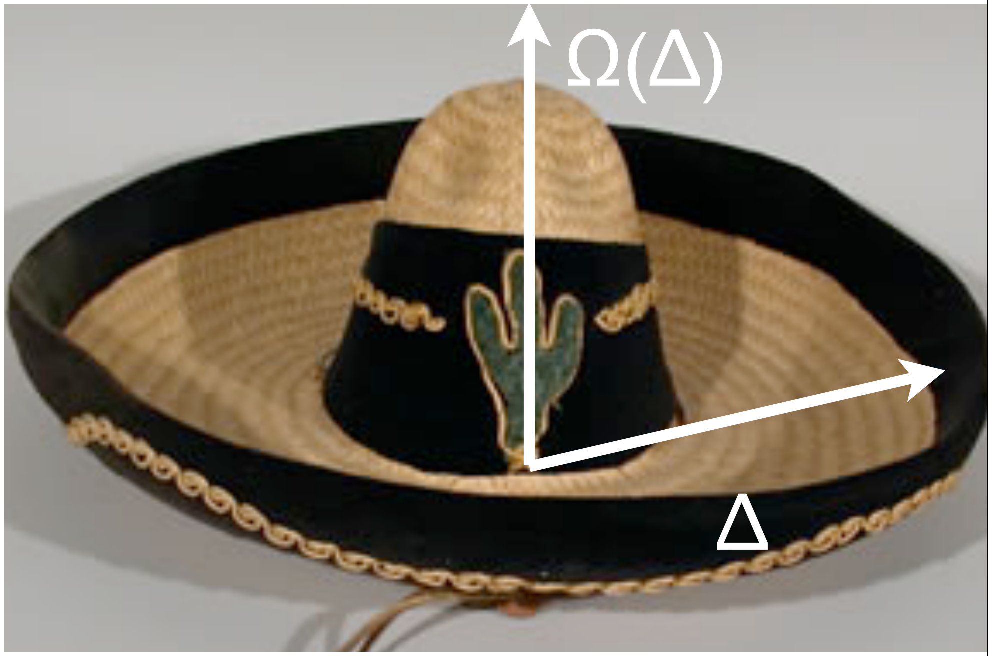
Attractive interactions destabilize the Fermi surface:

$$H = \sum_{k,s=\uparrow,\downarrow} (\frac{k^2}{2m} - \mu)\; a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

Cooper-pair condensation results in superfluidity and superconductivity:

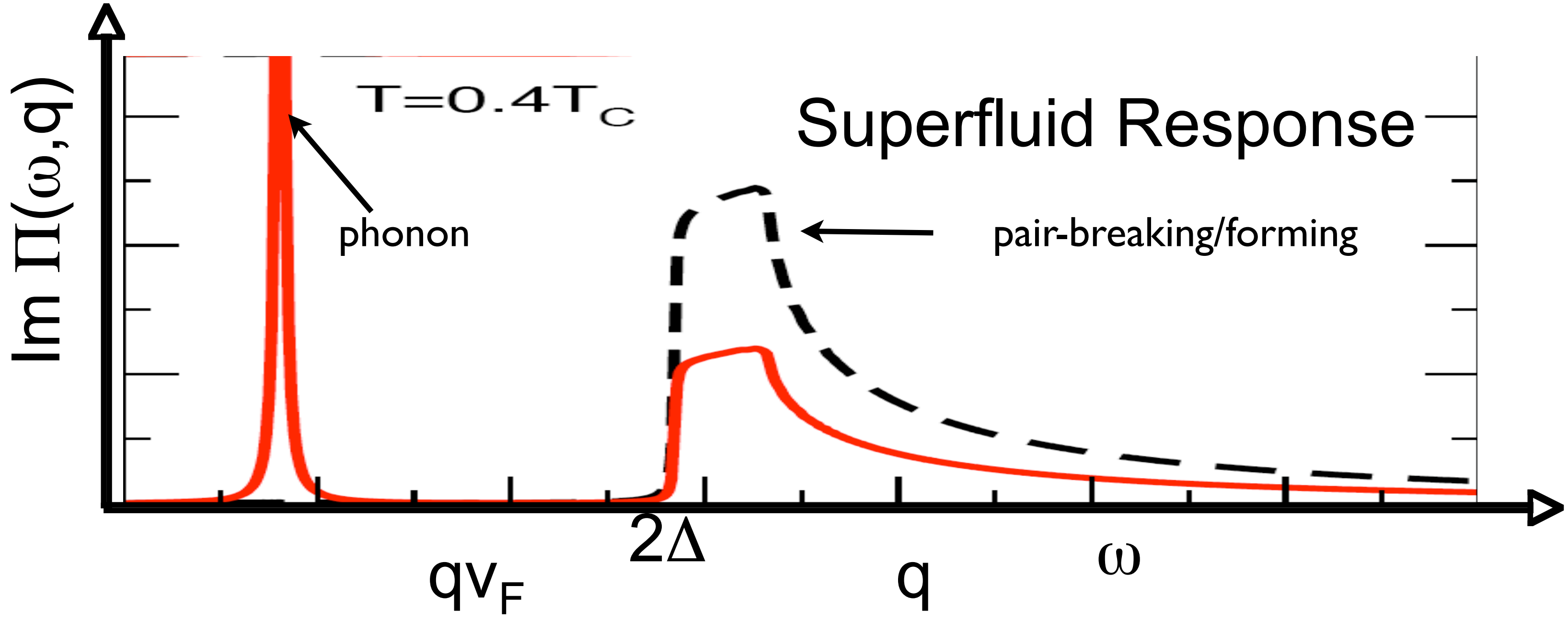
- •Energy-gap for fermions
- •New collective excitations (Goldstone modes)



$$E(p) = \sqrt{(\frac{p^2}{2m} - \mu)^2 + \Delta^2}$$

$$\omega_{\text{phonon}} = v_s\; q$$

Can suppress and enhance fluctuations



- Neutrino emission and specific heat are exponentially suppressed for T << Δ.
- For T ~ Δ, pair breaking/forming fluctuations can enhance neutrino cooling.

Greatly complicates interpretation of cooling data

Other Nuclear Transitions

There is more than one type of "symmetry energy":

$$E[\rho, \delta_\tau, \delta_\sigma, \delta_{\sigma\tau}] = E_0 + \boxed{E_{\rm sym}\delta_\tau^2} + \underset{\substack{\uparrow \\ \text{spin polarization}}}{E_\sigma \delta_\sigma^2} + \underset{\substack{\uparrow \\ \text{spin-isospin polarization}}}{E_{\sigma\tau}\delta_{\sigma\tau}^2} + \cdots$$

$$\delta_\sigma = \frac{\rho_\uparrow - \rho_\downarrow}{\rho} \qquad \delta_{\sigma\tau} = \frac{\rho_{n\uparrow} - \rho_{p\downarrow}}{\rho}$$

In nuclei, these terms are probed by spin and isospin dependent excitations.

In matter, they are encoded in the response functions:

$$\delta\rho_i(q,\omega) = \Pi(q,\omega)\ \phi(q,\omega)$$

Static Susceptibility :

$\Pi_\rho = \dfrac{\Pi_\rho^{\rm FG}}{1+F_0}$	$\Pi_\tau = \dfrac{\Pi_\rho^{\rm FG}}{1+F_0'}$
$\Pi_\sigma = \dfrac{\Pi_\rho^{\rm FG}}{1+G_0}$	$\Pi_{\sigma\tau} = \dfrac{\Pi_\rho^{\rm FG}}{1+G_0'}$

Transitions to matter with strangeness

1. Hyperons:

$\mu_n > M_\Lambda$

$\mu_n + \mu_e > M_{\Sigma^-}$

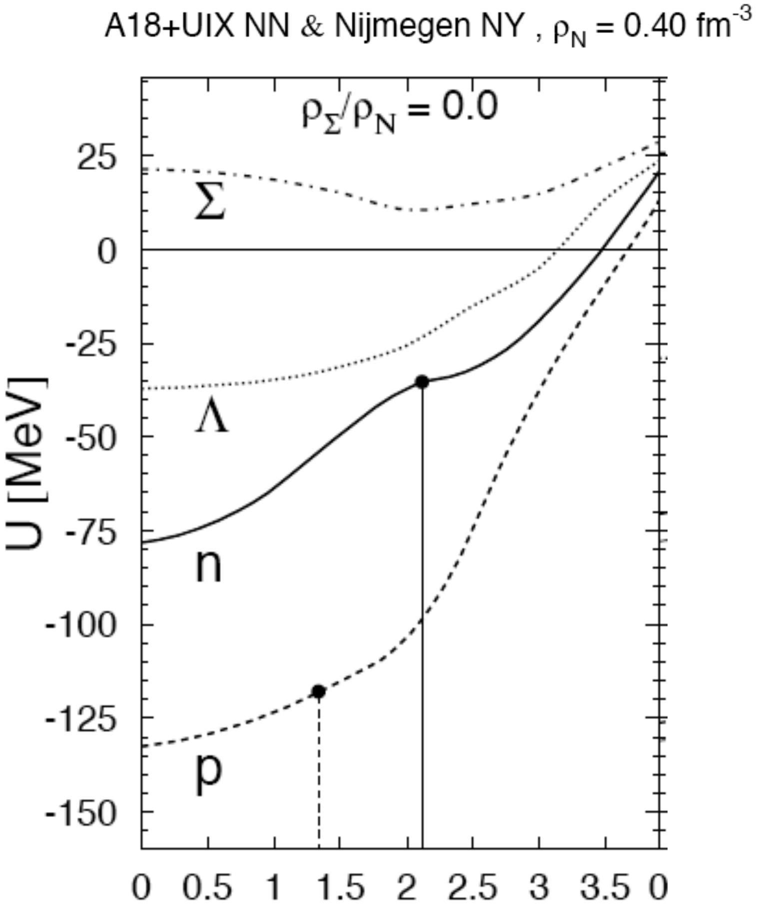
$\mu_n \sim 1400\text{-}1500$ MeV

in the center of 2 $M_\odot$

neutron star.

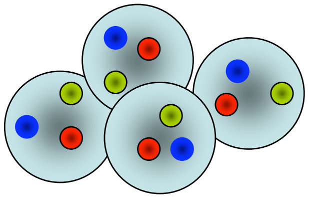
Need strong repulsive nucleon-hyperon interactions to prevent Λ and Σ^-

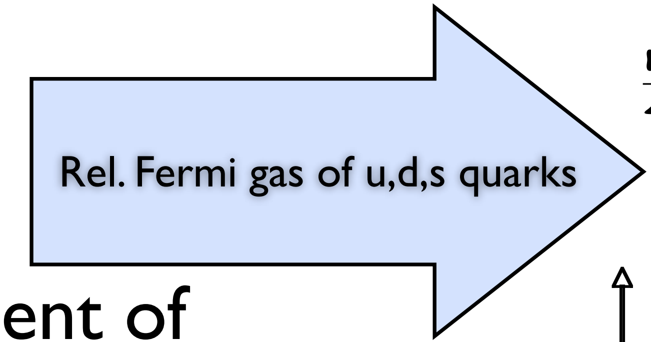
If they appear hyperons soften the EoS

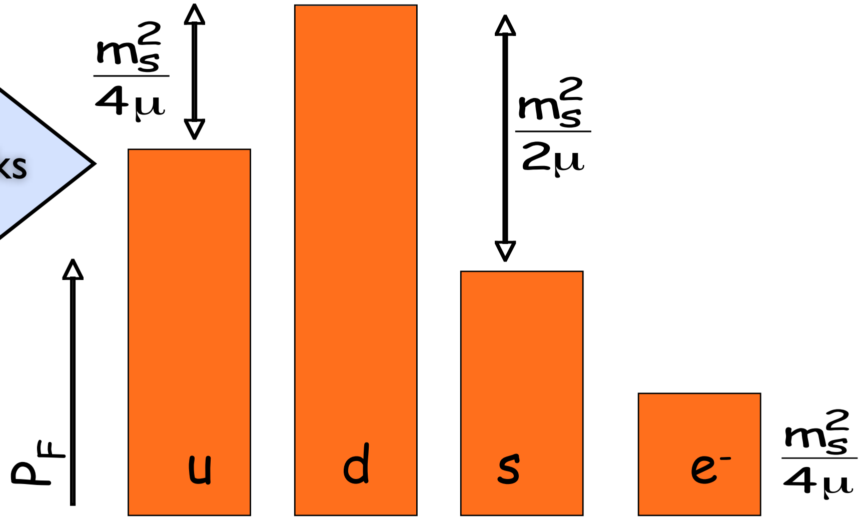


Baldo, Burgio & Schulze (2003)

Quark Matter





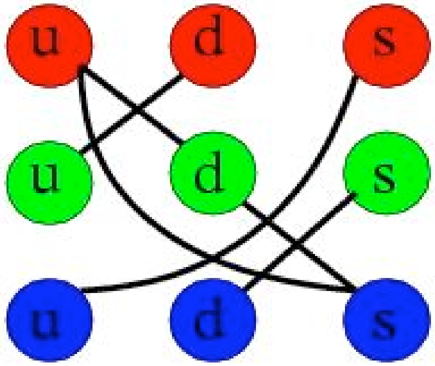


De-confinement of
quarks from nucleons

Attraction leads to pairing and color superconductivity

Complex Phase
Structure when

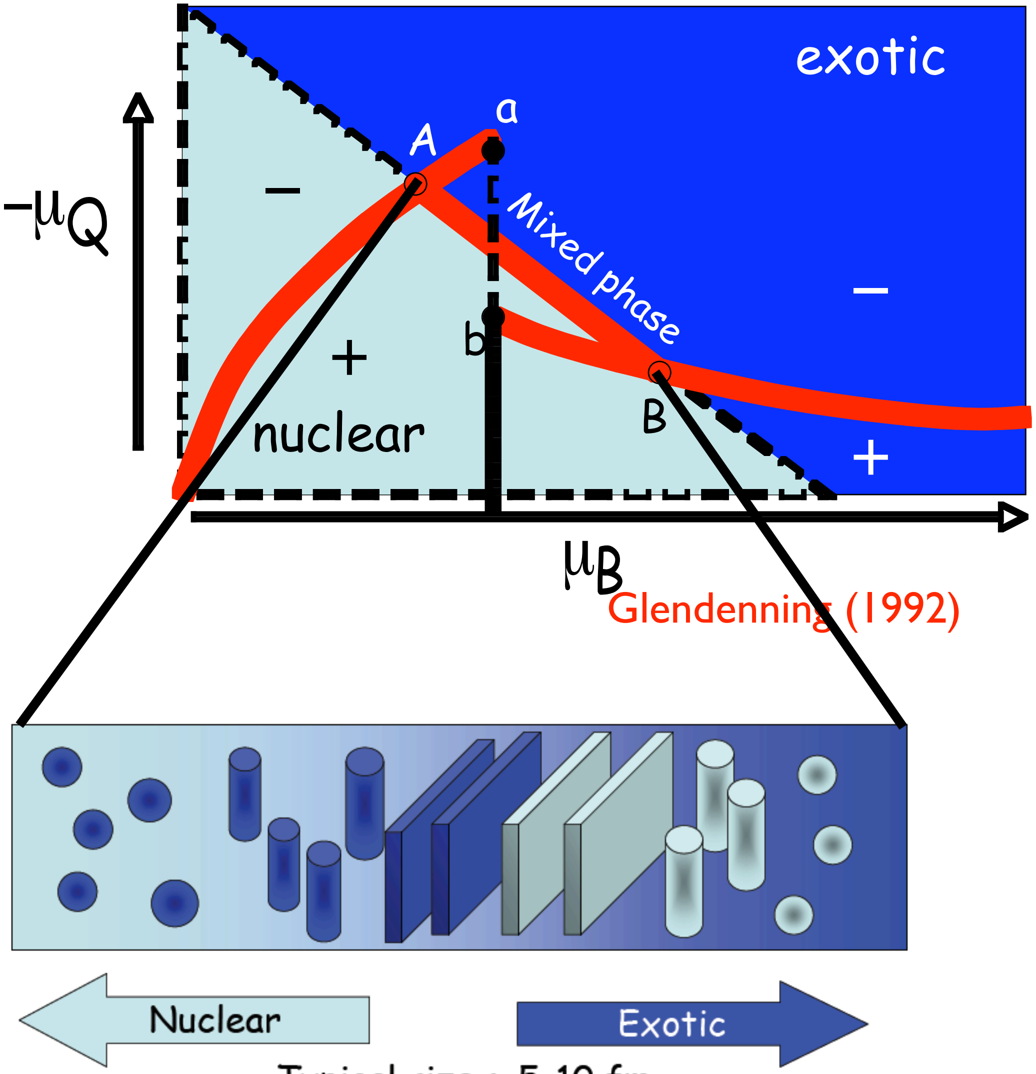
$$\Delta \simeq \frac{m_s^2}{4\mu}$$



Except at asymptotic density, EoS and
neutrino processes difficult to calculate

Solid or Liquid ?

- First-order transitions can lead to an extended mixed solid phase in the core.
- Its existence depends on the competition between surface tension and nuclear symmetry energy.



Glendenning (1992)

Typical size ~ 5-10 fm

Elevator Pitch

- Pressure of neutron-rich matter around saturation density important for the radius.
- Soft nuclear EoS makes compact and proton poor neutron stars.
- Very soft EoSs predicted by strong first-order transitions are disfavored by high mass neutron stars.
- An important correlation exists between mass, radius and temperature. Observation + theory can probe this.
- Its difficult to reconcile a rapid cooling neutron star with mass ~ 1.4 $M_{\odot}$ and radius < 12 km.
- A slow cooler with large radius would require pairing or other correlations to suppress direct-URCA.