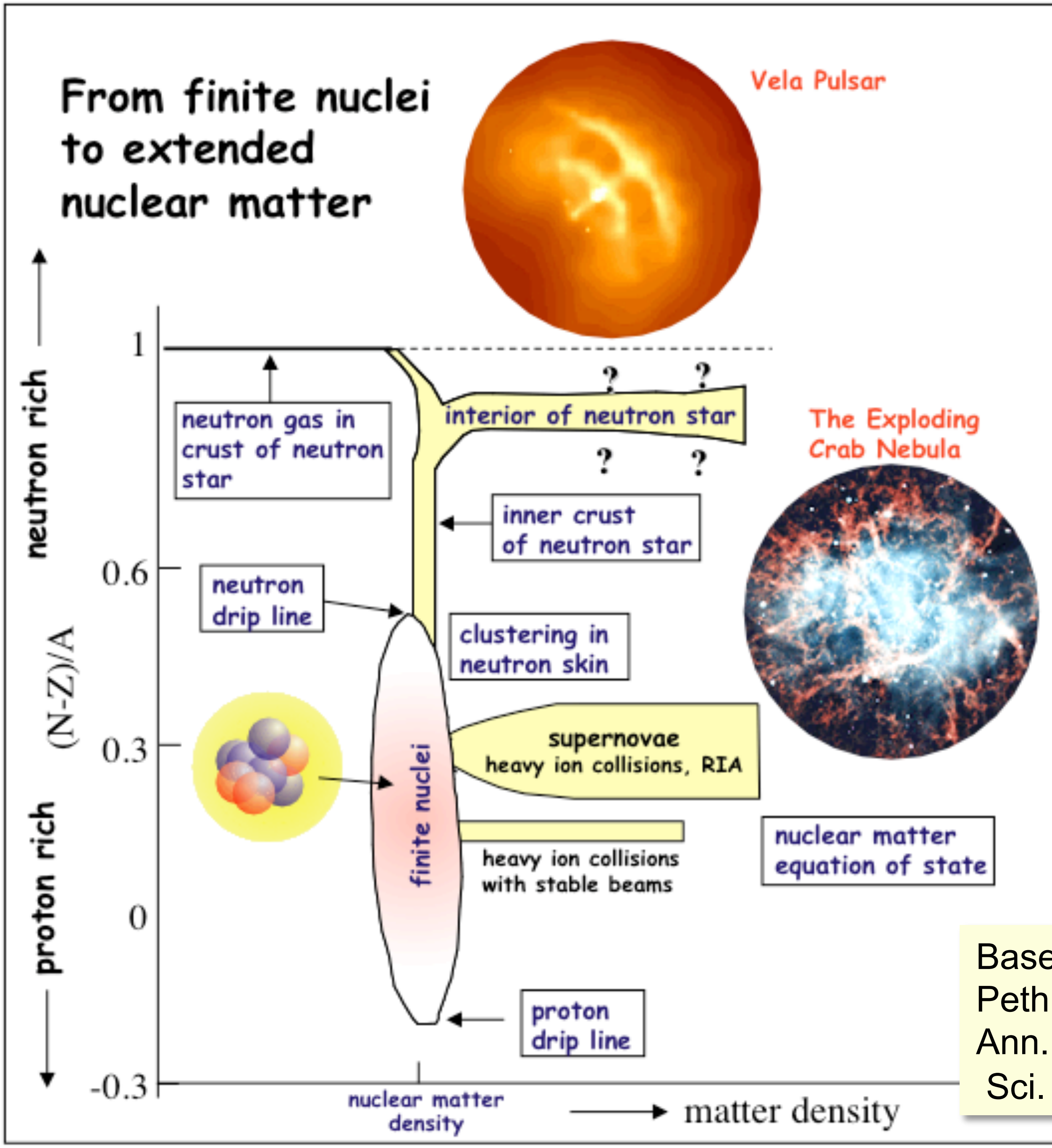
Theory of neutron-rich nuclei and nuclear radii

Witold Nazarewicz (with Paul-Gerhard Reinhard)

PREX Workshop, JLab, August 17-19, 2008

- Introduction to neutron-rich nuclei
- Radii, skins, and halos
- From finite to bulk
  - How to extrapolate from A=208 to A=∞?
- Correlation analysis and theoretical uncertainties
  - Which quantities correlate?
  - What is theoretical error bar on neutron skin?
- Perspectives

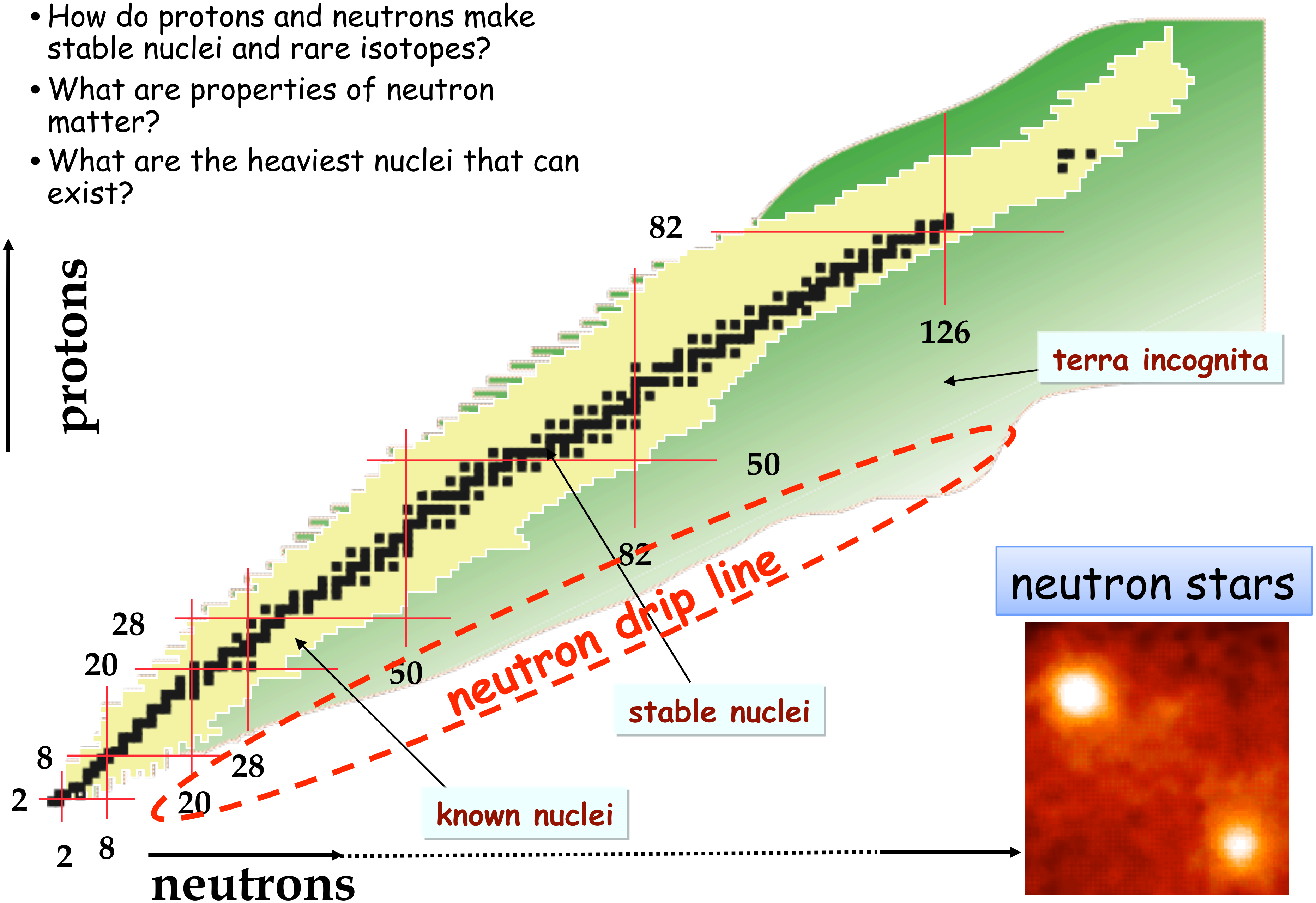
Introduction



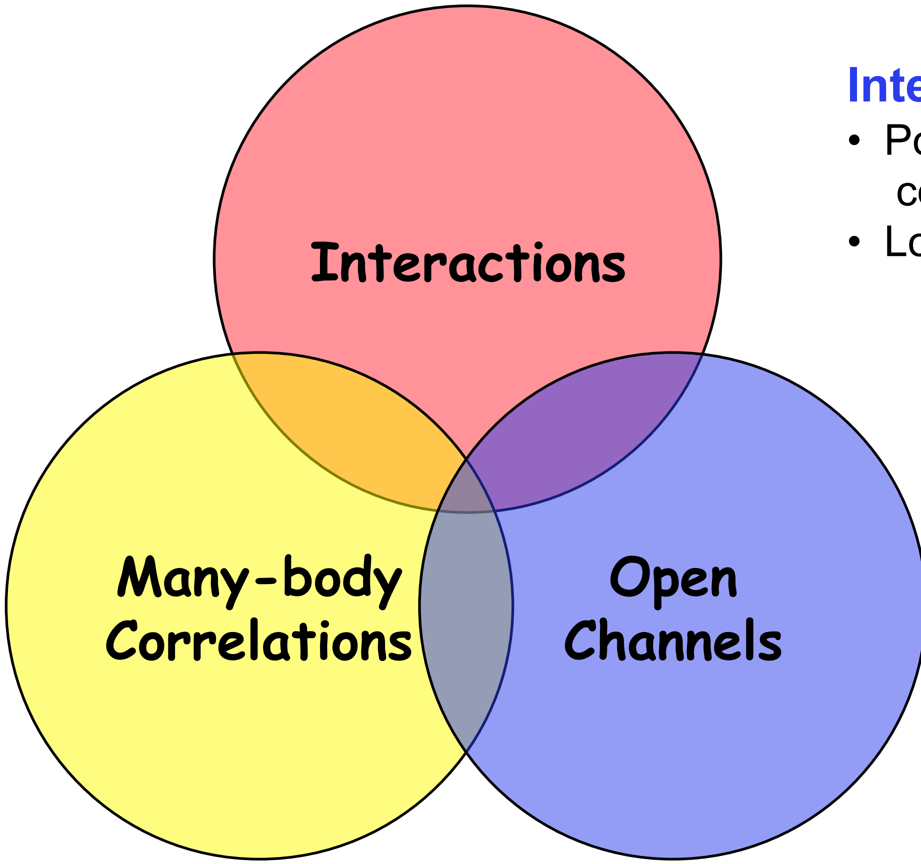
Based on
Pethick & Ravenhall
Ann. Rev. Nucl. Part. Sci. 45 (1995) 429

The Nuclear Landscape

- How do protons and neutrons make stable nuclei and rare isotopes?
- What are properties of neutron matter?
- What are the heaviest nuclei that can exist?



A remark: physics of neutron-rich nuclei is demanding



Interactions

- Poorly-known spin-isospin components come into play
- Long isotopic chains ***crucial***

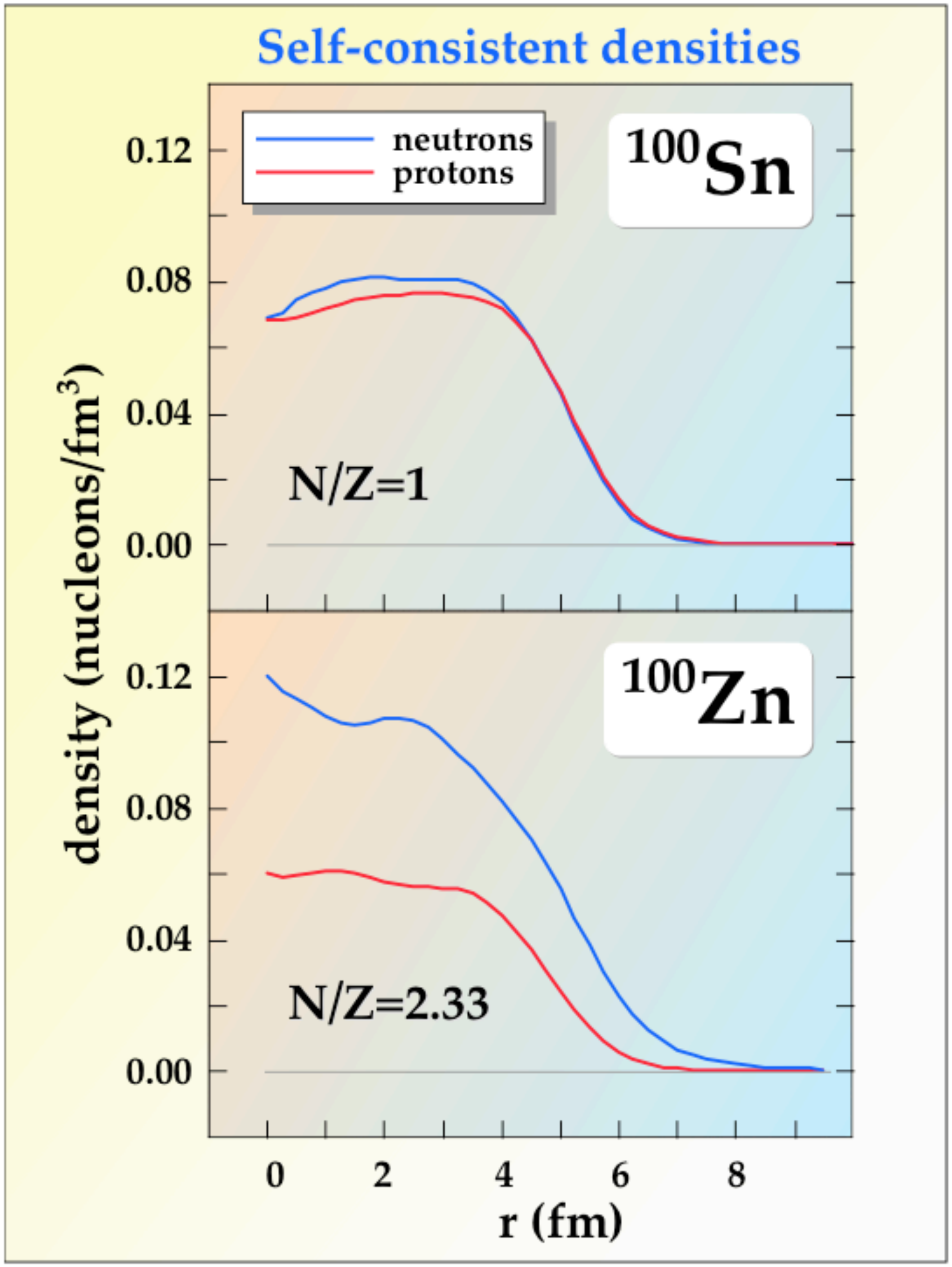
Configuration interaction

- Mean-field concept often ***questionable***
- Asymmetry of proton and neutron Fermi surfaces gives rise to new couplings
- New collective modes; polarization effects

Open channels

- Nuclei are ***open quantum systems***
- Exotic nuclei have low-energy decay thresholds
- Coupling to the continuum important
  - Virtual scattering
  - Unbound states
  - Impact on in-medium Interactions

Mean-Field Theory ⇒ Density Functional Theory



Nuclear DFT

- two fermi liquids
- self-bound
- superfluid

- mean-field ⇒ one-body densities
- zero-range ⇒ local densities
- finite-range ⇒ gradient terms
- particle-hole and pairing channels
- Has been extremely successful. A broken-symmetry generalized product state does surprisingly good job for nuclei.

Construction of the functional

Perlinska et al., Phys. Rev. C 69, 014316 (2004)

isoscalar (T=0) density $\left(\rho_0 = \rho_n + \rho_p\right)$

isovector (T=1) density $\left(\rho_1 = \rho_n - \rho_p\right)$

+isoscalar and isovector densities:
spin, current, spin-current tensor,
kinetic, and kinetic-spin
+ pairing densities

p-h density p-p density (pairing functional)

$$\mathcal{H}(r) = \frac{\hbar^2}{2m}\tau_0(r) + \sum_{t=0,1}\left(\chi_t(r) + \breve{\chi}_t(r)\right)$$

Most general second order expansion in densities and their derivatives

$$\begin{aligned}
\chi_0(r) &= C_0^\rho \rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^\tau \rho_0\tau_0 + C_0^{J0}J_0^2 + C_0^{J1}J_0^2 + C_0^{J2}\underline{J}_0^2 + C_0^{\nabla J}\rho_0\nabla\cdot J_0 \\
&+ C_0^s s_0^2 + C_0^{\Delta s}s_0\cdot\Delta s_0 + C_0^T s_0\cdot T_0 + C_0^j j_0^2 + C_0^{\nabla j}s_0\cdot(\nabla\times j_0) + C_0^{\nabla s}(\nabla\cdot s_0)^2 + C_0^F s_0\cdot F_0, \\
\chi_1(r) &= C_1^\rho \vec{\rho}^2 + C_1^{\Delta\rho}\vec{\rho}\circ\Delta\vec{\rho} + C_1^\tau \vec{\rho}\circ\vec{\tau} + C_1^{J0}\vec{J}^2 + C_1^{J1}\vec{J}^2 + C_1^{J2}\vec{\underline{J}}^2 + C_1^{\nabla J}\vec{\rho}\circ\nabla\cdot\vec{J} \\
&+ C_1^s \vec{s}^2 + C_1^{\Delta s}\vec{s}\cdot\circ\Delta\vec{s} + C_1^T\vec{s}\cdot\circ\vec{T} + C_1^j\vec{j}^2 + C_1^{\nabla j}\vec{s}\cdot\circ(\nabla\times\vec{j}) + C_1^{\nabla s}(\nabla\cdot\vec{s})^2 + C_1^F\vec{s}\cdot\circ\vec{F},
\end{aligned}$$

- Constrained by microscopic theory: ab-initio functionals
- Not all terms are equally important. Usually ~12 terms considered
- Some terms probe specific experimental data
- Pairing functional poorly determined. Usually 1-2 terms active.
- Becomes very simple in limiting cases (e.g., unitary limit)

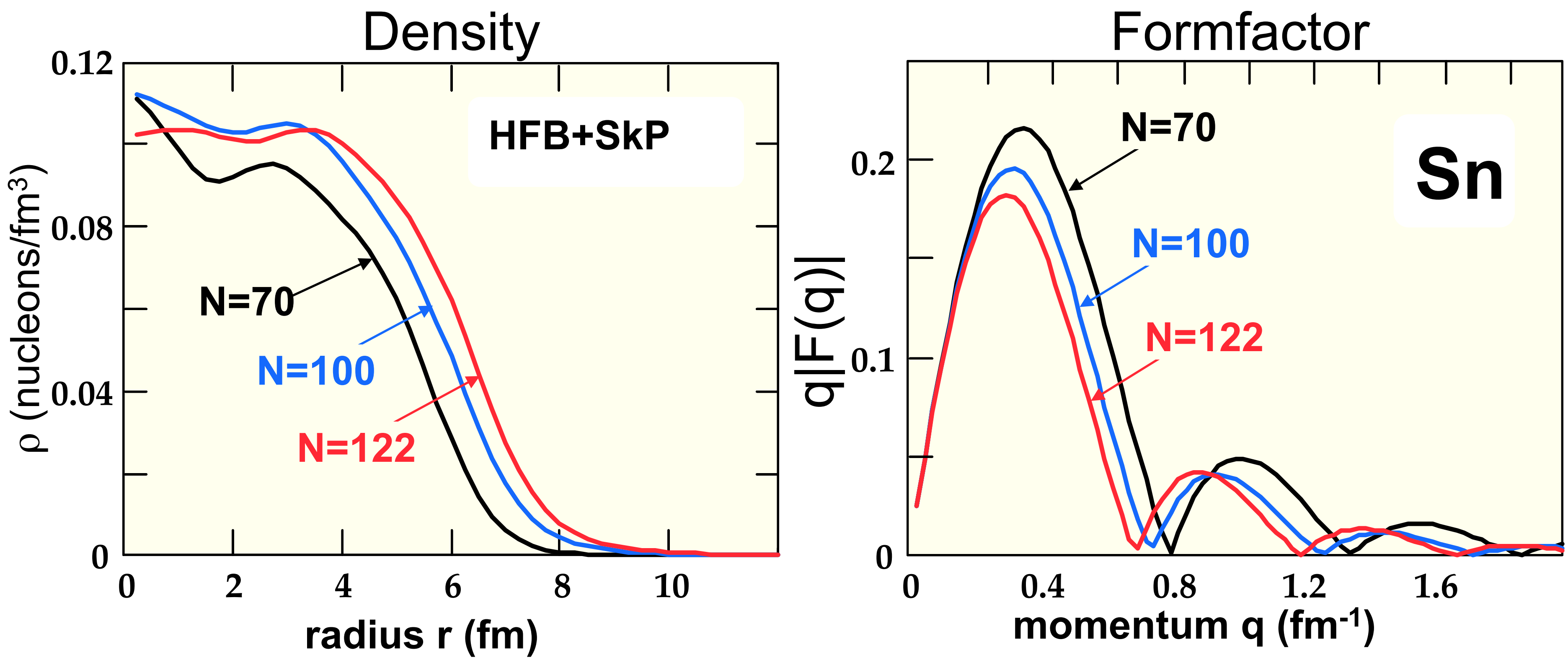
Universal Nuclear Energy Density Functional

http://unedf.org/

- Funded (on a competitive basis) by
  - Office of Science
  - ASCR
  - NNSA
- 15 institutions
- ~50 researchers
  - physics
  - computer science
  - applied mathematics
- foreign collaborators

…unprecedented theoretical effort !

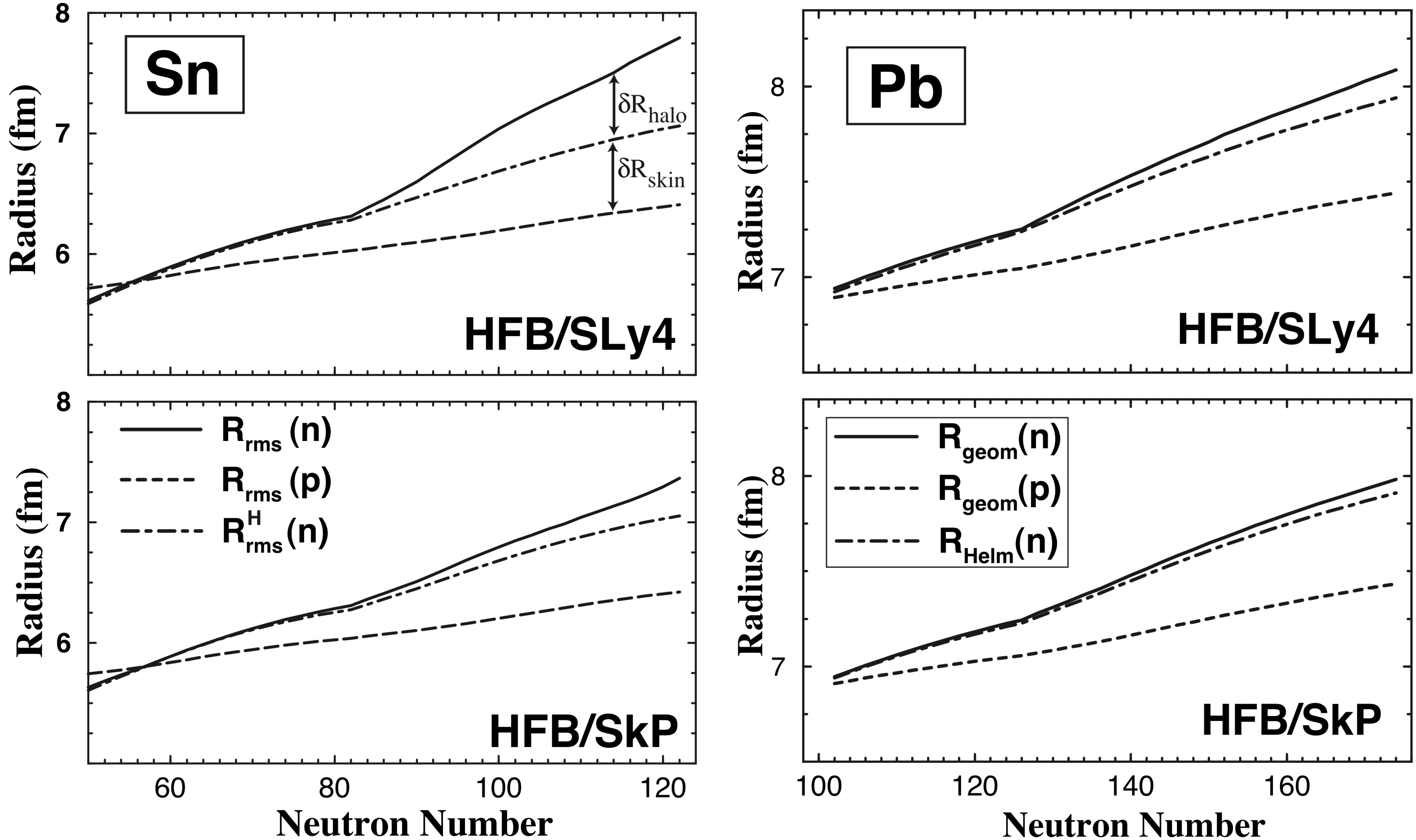
Radii, skins, halos…



First zero of F(q) ⇨ R_{diff}
First maximum of F(q) ⇨ surface thickness

$$R_{\rm rms}^{(\rm H)} = \sqrt{\frac{3}{5}\left(R_0^2 + 5\sigma^2\right)} \qquad R_{\rm geom} = \sqrt{\frac{5}{3}} R_{\rm rms}$$

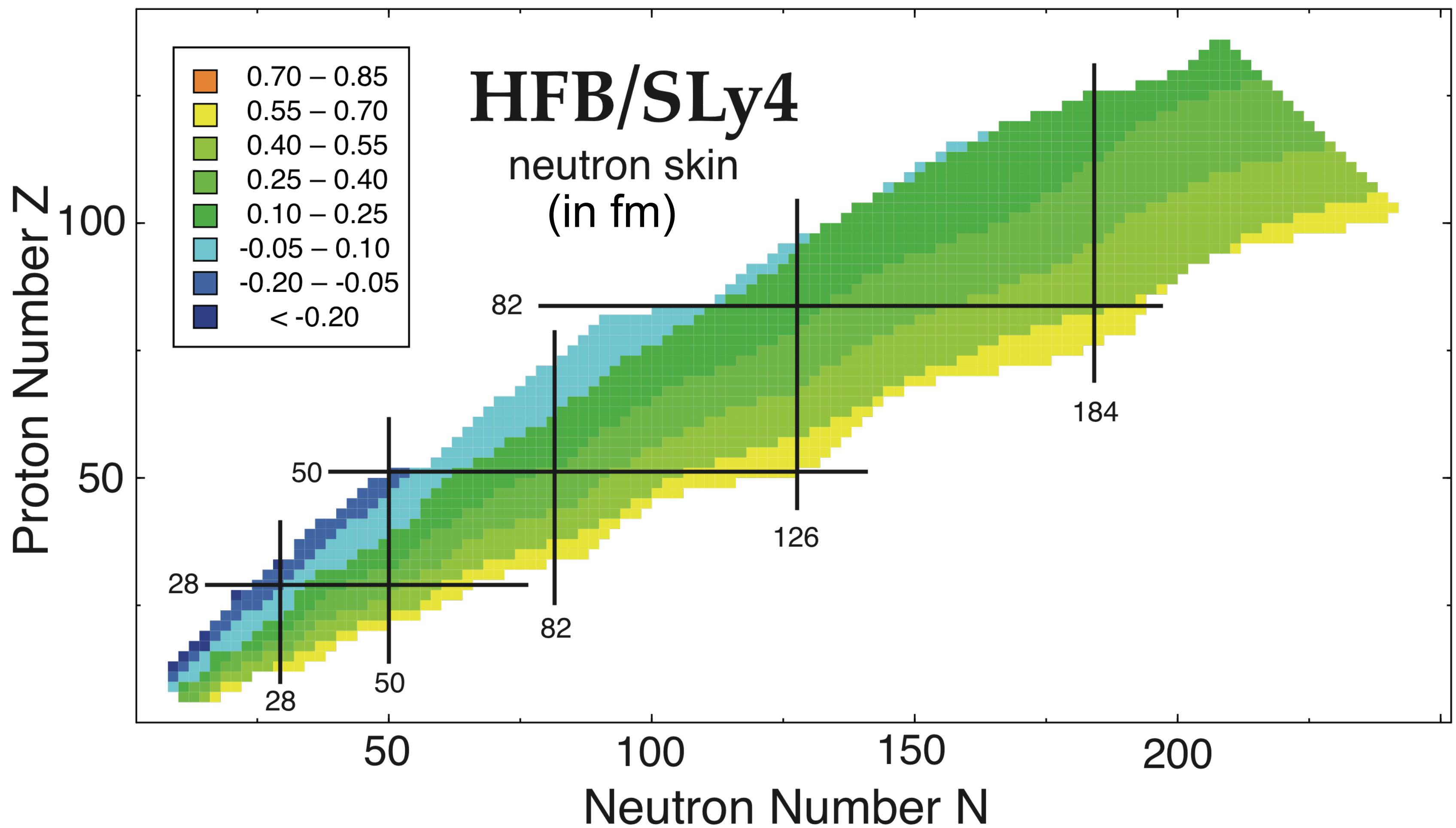
$$R_{\rm geom}(\rm p) \approx R_{\rm Helm}(\rm p)$$



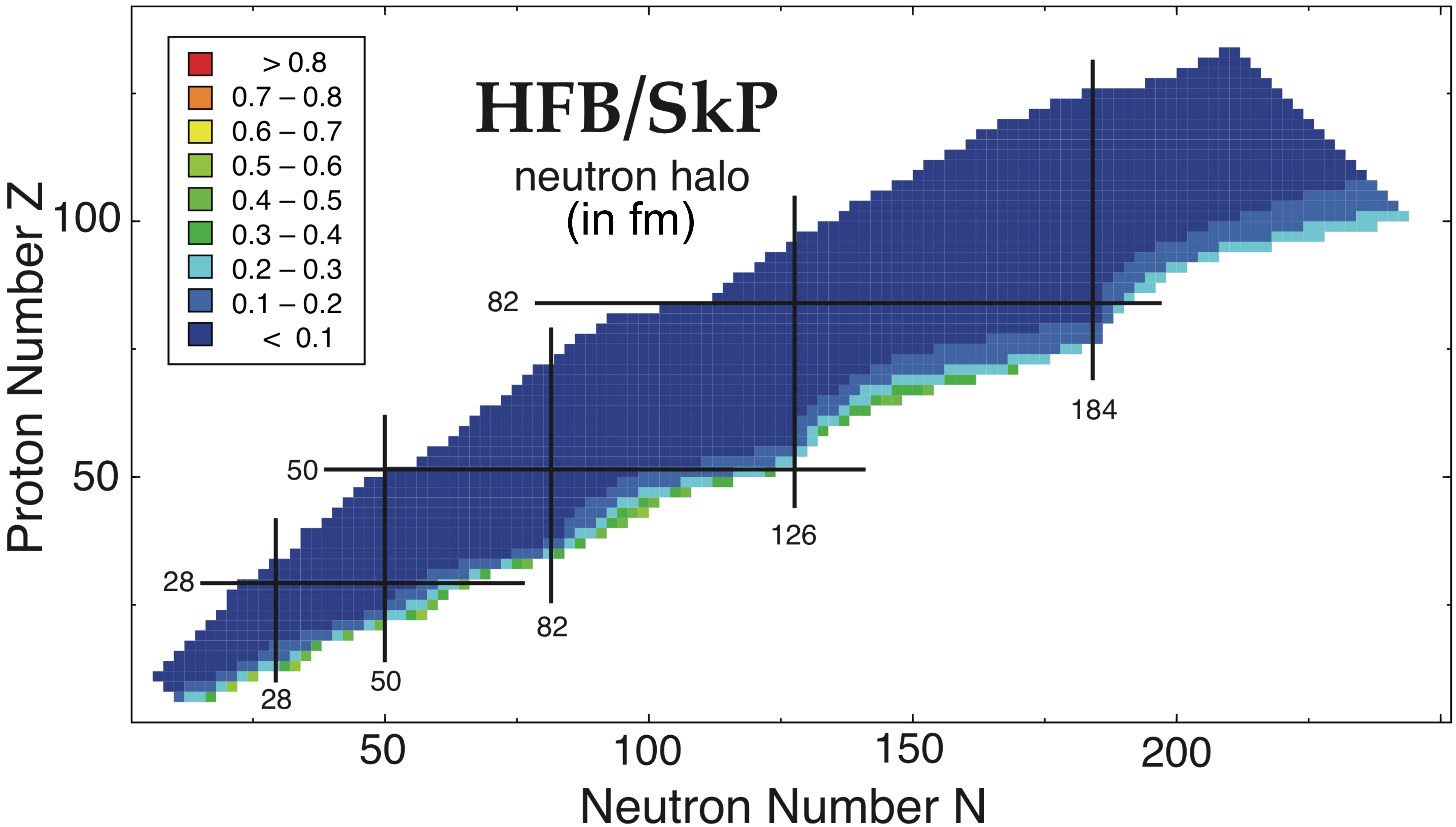
$$\delta R_{\rm halo} \equiv R_{\rm geom} - R_{\rm Helm}$$

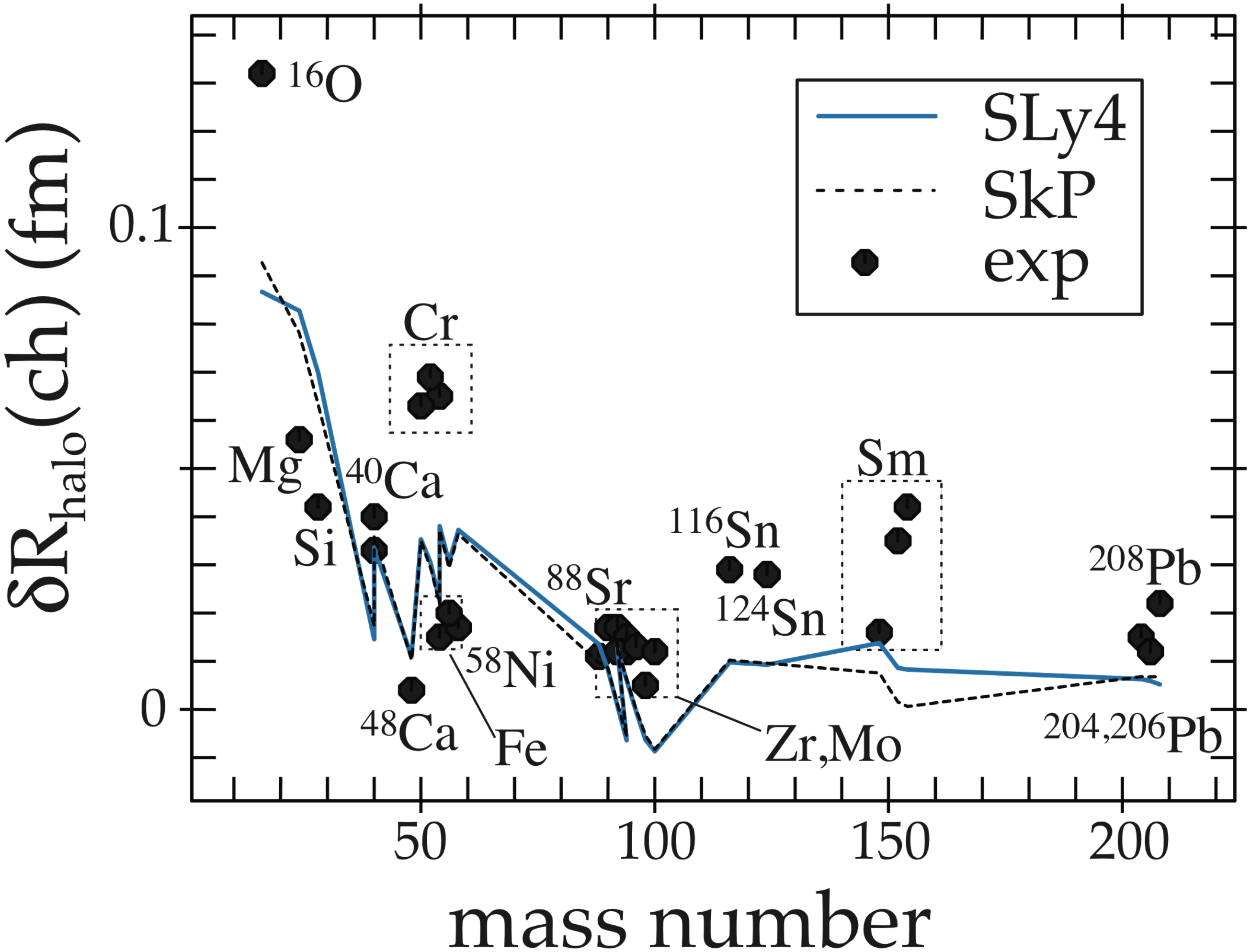
$$\delta R_{\rm skin} \equiv R_{\rm Helm}(n) - R_{\rm Helm}(p)$$

S. Mizutori et al., Phys. Rev. C61, 044326 (2000)



S. Mizutori et al., Phys. Rev. C61, 044326 (2000)





δR_halo(ch) (fm)
mass number
SLy4
SkP
exp
16O
Cr
Mg
40Ca
Si
58Ni
48Ca
Fe
88Sr
Zr,Mo
116Sn
124Sn
Sm
208Pb
204,206Pb
0.1
0
50
100
150
200

Finite size effects…

From Finite Nuclei to the Nuclear Liquid Drop

Leptodermous Expansion Based on the Self-consistent Theory

P.G. Reinhard, M. Bender, W.N., T. Vertse, Phys. Rev. C 73, 014309 (2006)

The parameters of the nuclear liquid drop model, such as the volume, surface, symmetry, and curvature constants, as well as bulk radii, are extracted from the non-relativistic and relativistic energy density functionals used in microscopic calculations for finite nuclei. The microscopic liquid drop energy, obtained self-consistently for a large sample of finite, spherical nuclei, has been expanded in terms of powers of $A^{-1/3}$ (or inverse nuclear radius) and the isospin excess (or neutron-to-proton asymmetry). In order to perform a reliable extrapolation in the inverse radius, **the calculations have been carried out for nuclei with huge numbers of nucleons**, of the order of 10^6.

The limitations of applying the leptodermous expansion for finite nuclei are discussed. **While the leading terms in the macroscopic energy expansion can be extracted very precisely, the higher-order, isospin-dependent terms are prone to large uncertainties due to finite-size effects**.

$$\mathcal{E}^{(\mathrm{smooth})} = \frac{E}{A} - \delta\mathcal{E}^{(\mathrm{shell})}$$

From HF or RMF

Shell correction estimated using the Green's function method

Liquid-Drop Expansion

$$\mathcal{E}^{(\mathrm{LDM})} = \mathcal{E}^{(\mathrm{smooth})}(A, I)$$

$$= a_{\mathrm{vol}} \quad + a_{\mathrm{surf}} A^{-1/3} + a_{\mathrm{sym}} I^2 \quad + a_{\mathrm{curv}} A^{-2/3} + a_{\mathrm{ssym}} I^2 A^{-1/3} + a^{(2)}_{\mathrm{sym}} I^4 \; .$$

$$I = \frac{N-Z}{N+Z}$$

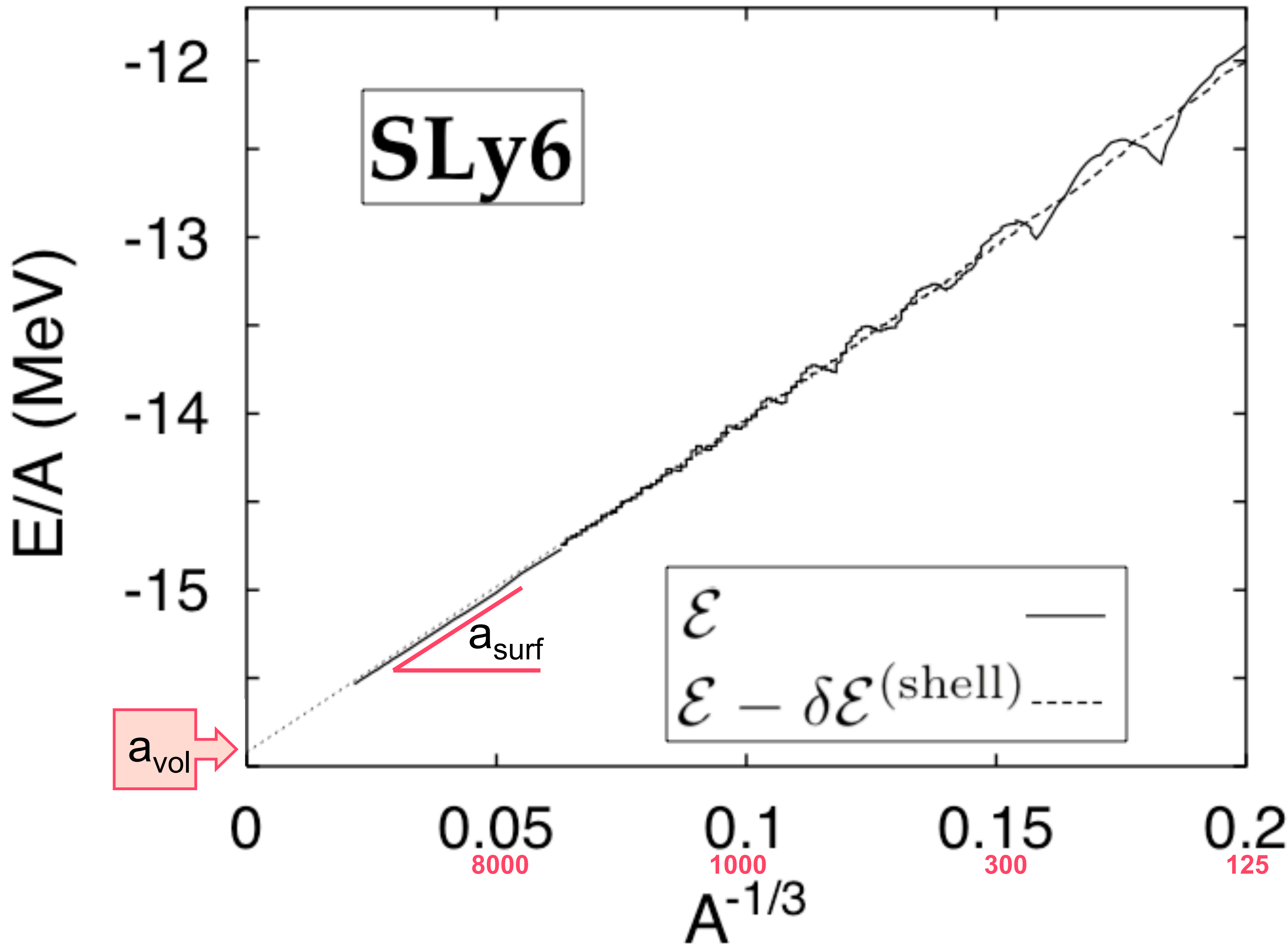
O(0)	O(1)	O(2)
a_{vol}	$+ a_{\mathrm{surf}} A^{-1/3}$	$+ a_{\mathrm{curv}} A^{-2/3}$
	$+ a_{\mathrm{sym}} I^2$	$+ a_{\mathrm{ssym}} I^2 A^{-1/3}$
		$+ a^{(2)}_{\mathrm{sym}} I^4$.

Droplet Model Expansion

Myers, Swiatecki 1974

$$\begin{aligned} \mathcal{E}^{(\mathrm{drop})} &= \mathcal{E}^{(\mathrm{drop})}(A, I, \epsilon, d) \\ &= a_{\mathrm{vol}} + a_{\mathrm{surf}} A^{-1/3} + \tilde{a}_{\mathrm{curv}} A^{-2/3} + 2a_{\mathrm{surf}} A^{-1/3}\epsilon + \tfrac{K}{2}\epsilon^2 \\ &\quad + a_{\mathrm{sym}} I^2 + \tilde{a}_{\mathrm{ssym}} A^{-1/3} f(I,d) - 3a'_{\mathrm{sym}} I^2 \epsilon \\ &\quad + \tilde{a}^{(2)}_{\mathrm{sym}} I^4 \end{aligned}$$

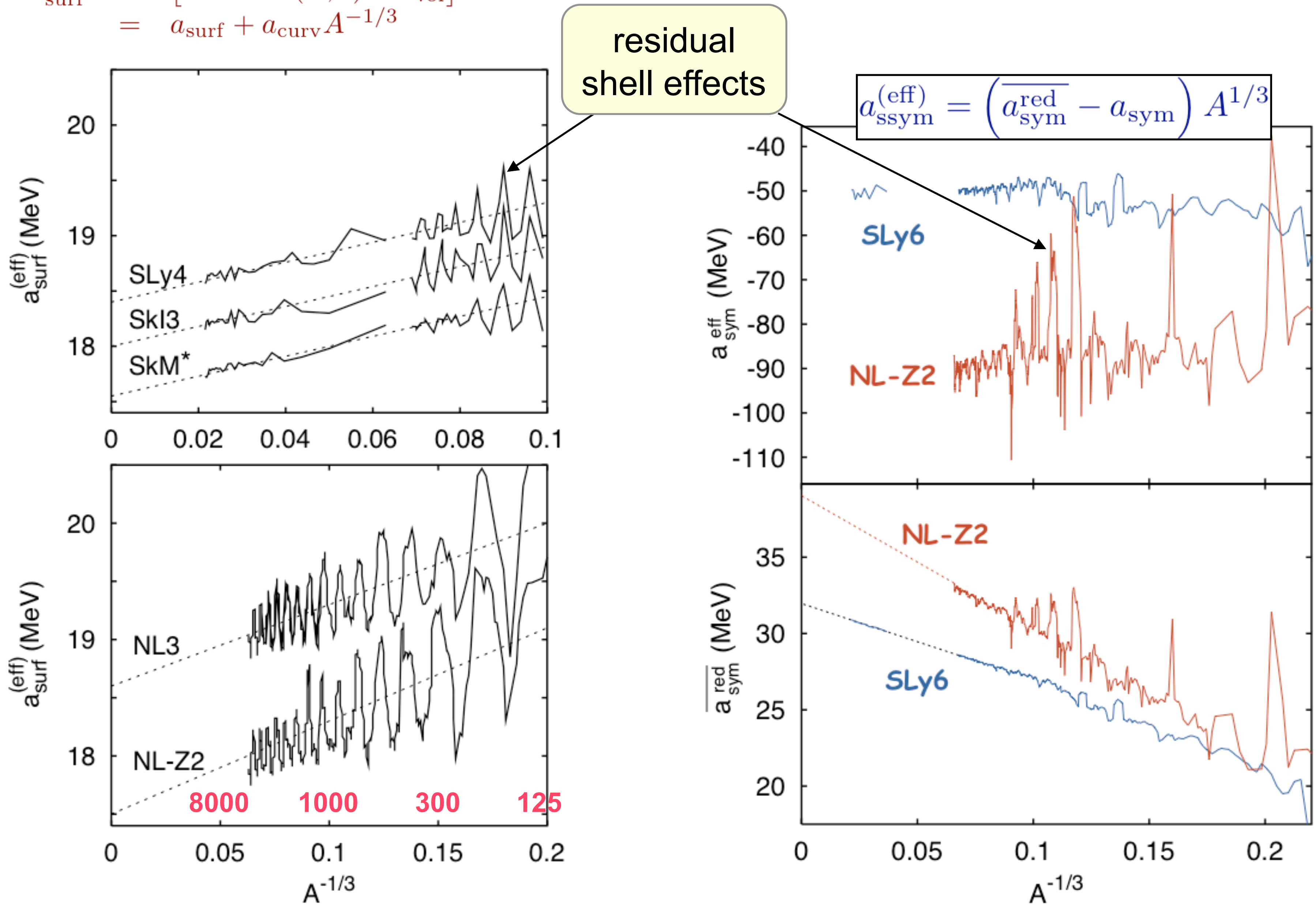
$$K \equiv 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{E}{A}\bigg|_{\rho=\rho_0} \qquad a'_{\mathrm{sym}} = \frac{\partial a_{\mathrm{sym}}}{\partial \rho}\bigg|_{\rho=\rho_0} \qquad a^{(2)}_{\mathrm{sym}} = \tilde{a}^{(2)}_{\mathrm{sym}} - \frac{9}{2}(a'_{\mathrm{sym}})^2 \frac{\rho_0^2}{K}$$



SLy6
E/A (MeV)
-12
-13
-14
-15
a_surf
a_vol
0
0.05
0.1
0.15
0.2
8000
1000
300
125
A^-1/3
ℰ
ℰ − δℰ^(shell)

$$
\begin{aligned}
a_{\rm surf}^{\rm (eff)} &= \left[\mathcal{E}^{\rm (smooth)}(A,0) - a_{\rm vol}\right] A^{1/3} \\
&= a_{\rm surf} + a_{\rm curv} A^{-1/3}
\end{aligned}
$$

$$
a_{\rm sym}^{\rm red}(A,I) = \frac{\mathcal{E}^{\rm (smooth)}(A,I) - \mathcal{E}^{\rm (smooth)}(A,0) + \frac{9(a'_{\rm sym})^2 \rho_0^2}{2K} I^4}{I^2}
$$

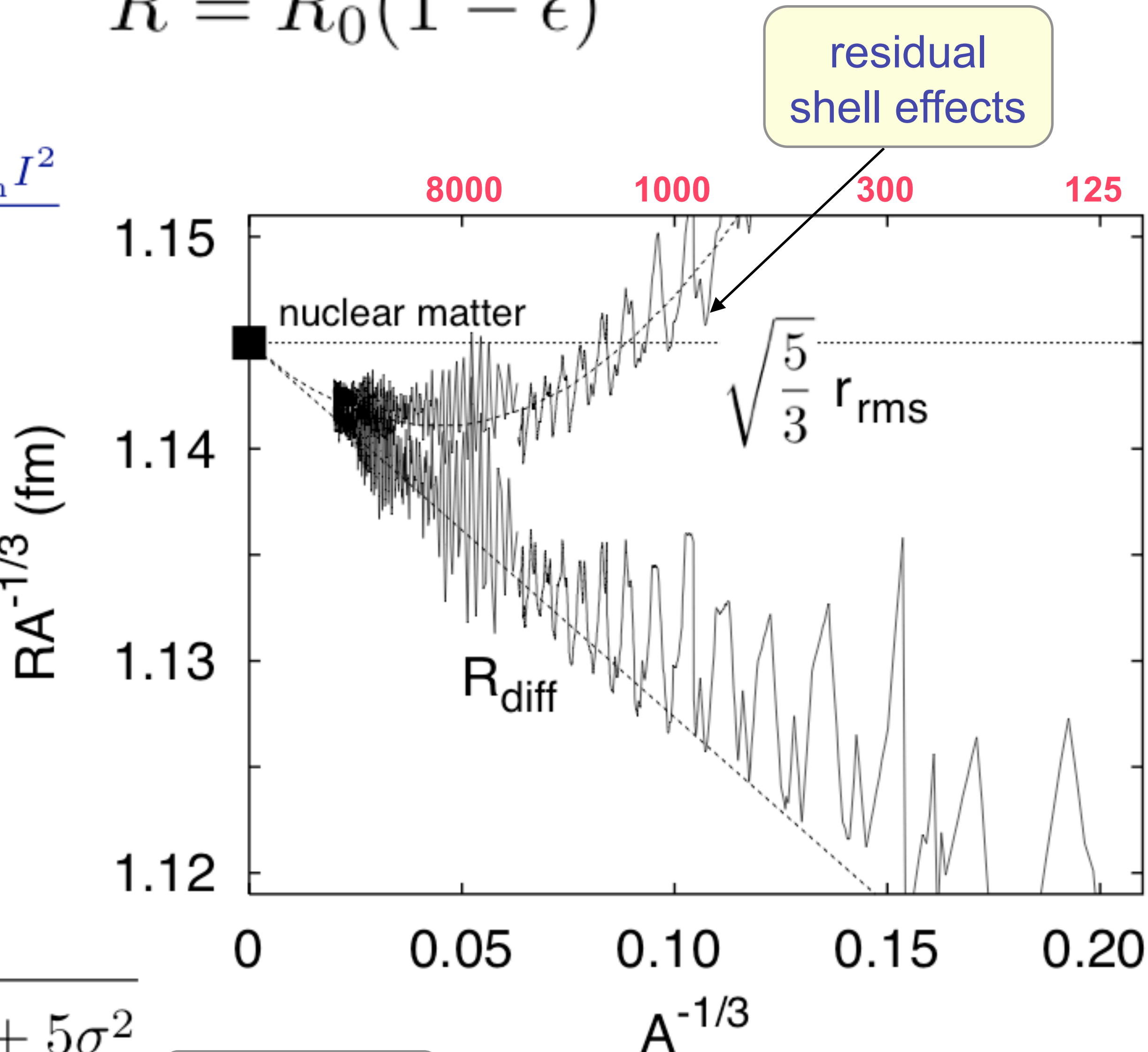


$$
a_{\rm ssym}^{\rm (eff)} = \left(\overline{a_{\rm sym}^{\rm red}} - a_{\rm sym}\right) A^{1/3}
$$

Macroscopic Droplet Model Radii

$$R = R_0(1-\epsilon)$$

$$\epsilon = -\frac{\rho-\rho_0}{3\rho_0} = \frac{-2a_{\rm surf}A^{-1/3}+3a'_{\rm sym}I^2}{K}$$



$$r_{\rm rms} = \sqrt{\frac{3}{5}}\sqrt{R_{\rm diff}^2 + 5\sigma^2}$$

around 1fm

Correlations, alignment, uncertainty…

Correlations between observables

(P.G. Reinhard and WN)

Consider an EDF described by coupling constants $\mathbf{p} = (p_1, ..., p_F)$

The optimum parameter set $\mathbf{p}_0 : \quad \chi^2(\mathbf{p}_0) = \chi^2_{\min} = \text{minimal.}$

$$\chi^2(\mathbf{p}) - \chi^2_{\min} \approx \sum_{i,j=1}^{F} (p_i - p_{i,0}) \mathcal{M}_{ij} (p_j - p_{j,0}), \quad \mathcal{M}_{ij} = \partial_{p_i} \partial_{p_j} \chi^2 \Big|_{\mathbf{p}_0}$$

Uncertainty in variable A:

$$\overline{\Delta A^2} = \sum_{ij} \partial_{p_i} A (\hat{M}^{-1})_{ij} \partial_{p_j} A, \quad \partial_{p_i} A = \partial_{p_i} A \Big|_{\mathbf{p}_0}$$

Correlation between variables A and B:

$$\overline{\Delta A \, \Delta B} = \sum_{ij} \partial_{p_i} A (\hat{M}^{-1})_{ij} \partial_{p_j} B$$

Alignment of variables A and B:

$$c_{AB} = \frac{\overline{\Delta A \, \Delta B}}{\sqrt{\overline{\Delta A^2} \; \overline{\Delta B^2}}}$$

=1: full alignment/correlation
=0: not aligned/statistically independent

P. Klüpfel et al, arXi:0804.3385

force	K	m^*/m	a_{sym}	a'_{sym}	κ	ρ_{eq}	E/A
SV-min	222	0.95	30.7	93	0.08	0.1610	-15.91
$\pm$	8	0.15	1.4	89	0.40	0.0013	0.06

sum-rule enhancement

Quantities of interest...

bulk equilibrium symmetry energy

$$a_{\rm sym} = \frac{\partial^2}{\partial I^2}\frac{E}{A}\Big|_{\rho=\rho_{\rm eq.}}, \quad I = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

symmetry energy at surface density

$$a_{\rm sym}({\rm surface}) = a_{\rm sym} - 0.08\frac{\partial}{\partial\rho}a_{\rm sym}$$

slope of binding energy of neutron matter

$$\frac{\partial}{\partial\rho}\frac{E_{\rm neut}}{A}\Big|_{\rho=0.1{\rm fm}^{-3}}$$

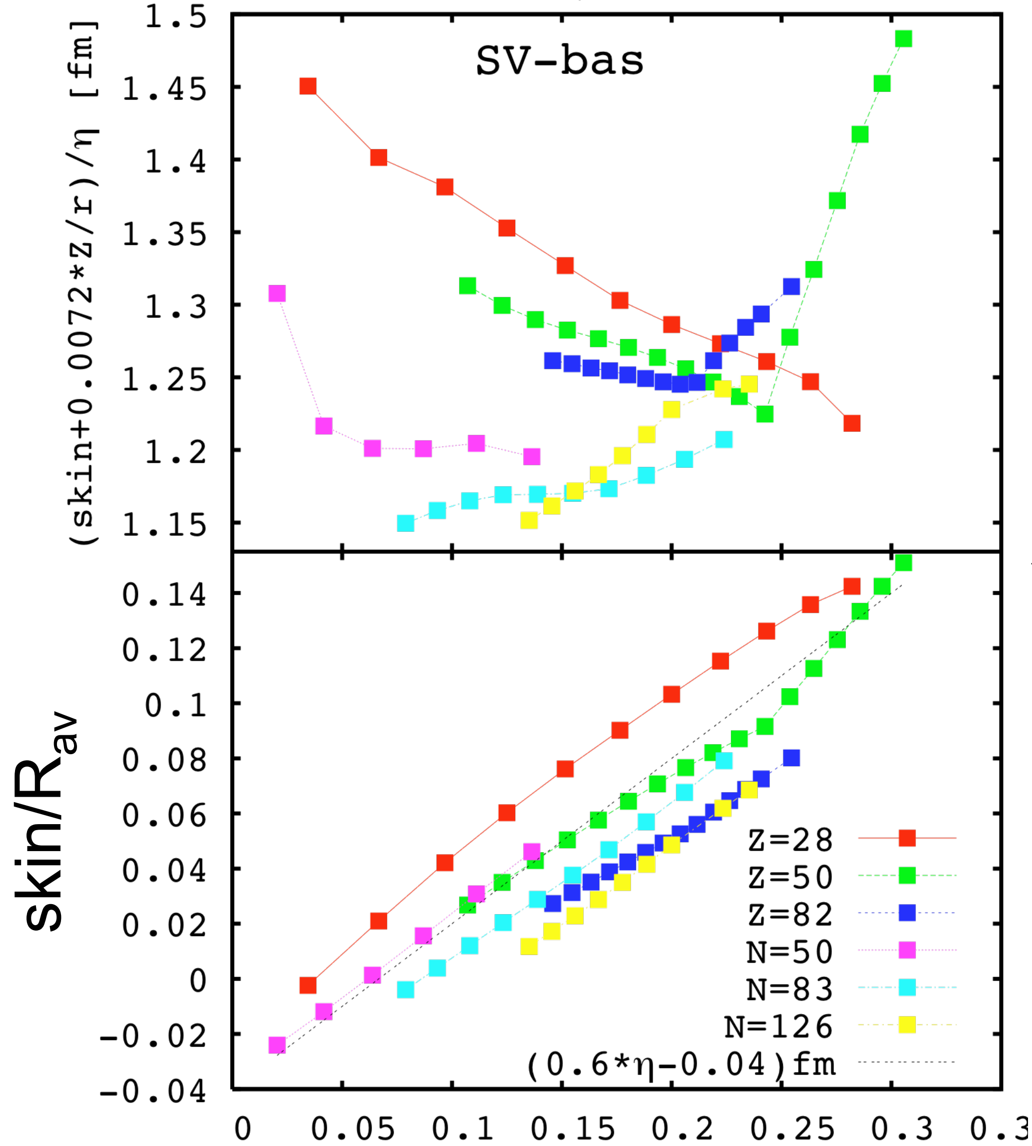
dipole polarizability

$$\alpha_D = \sum_{n\in{\rm RPA}}\frac{1}{E_n}|\langle\Phi_n|\hat{D}|\Phi_0\rangle|^2$$

rescaled polarizabiliy

$$a_{\rm sym}({\rm pol.dip.}) = \frac{Ar^2}{24\alpha_D}$$

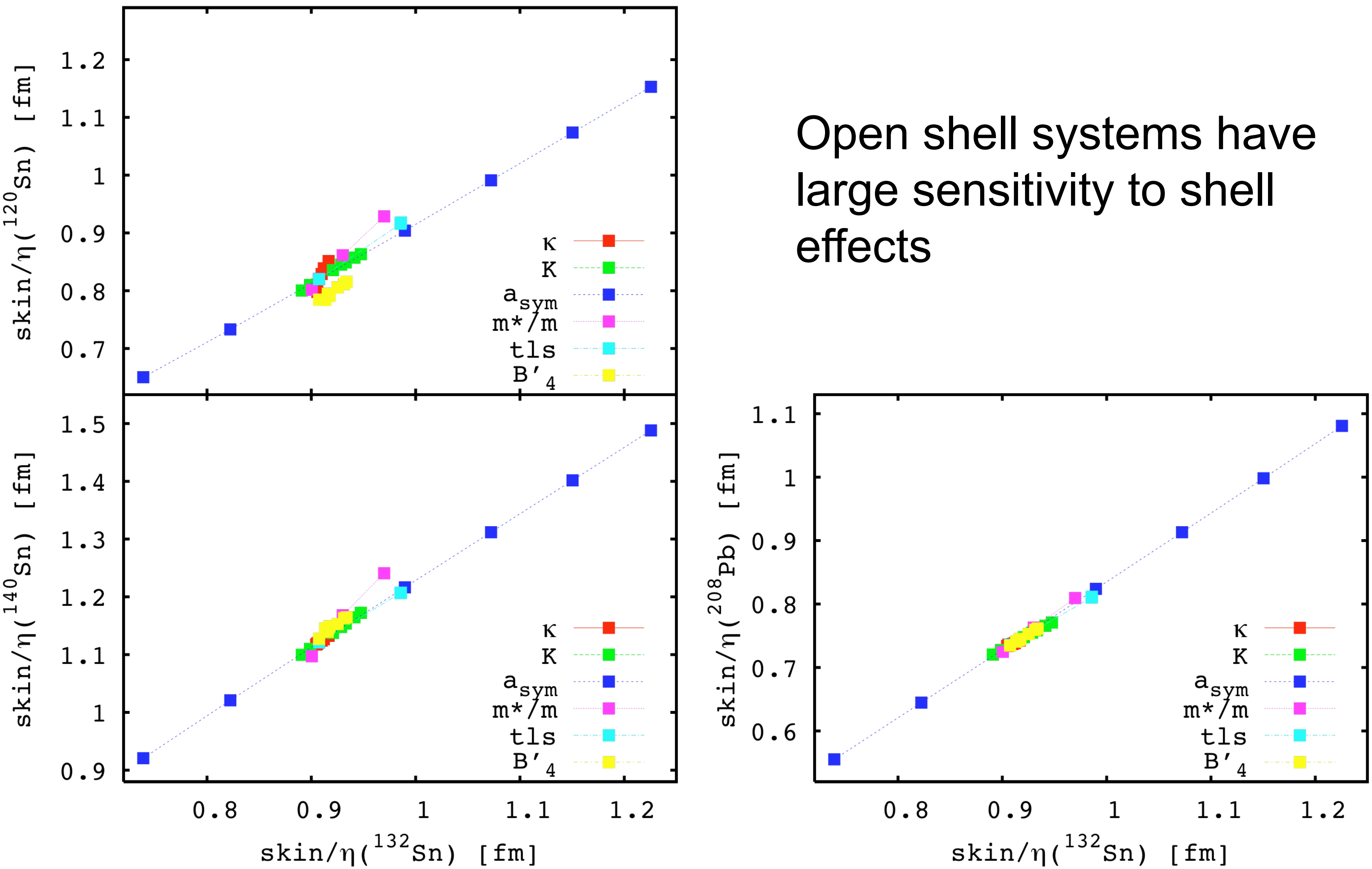
$$\frac{r_n - r_p + 0.0072\,Z/r_p}{\eta}$$



$$\eta = (N-Z)/(N+Z)$$

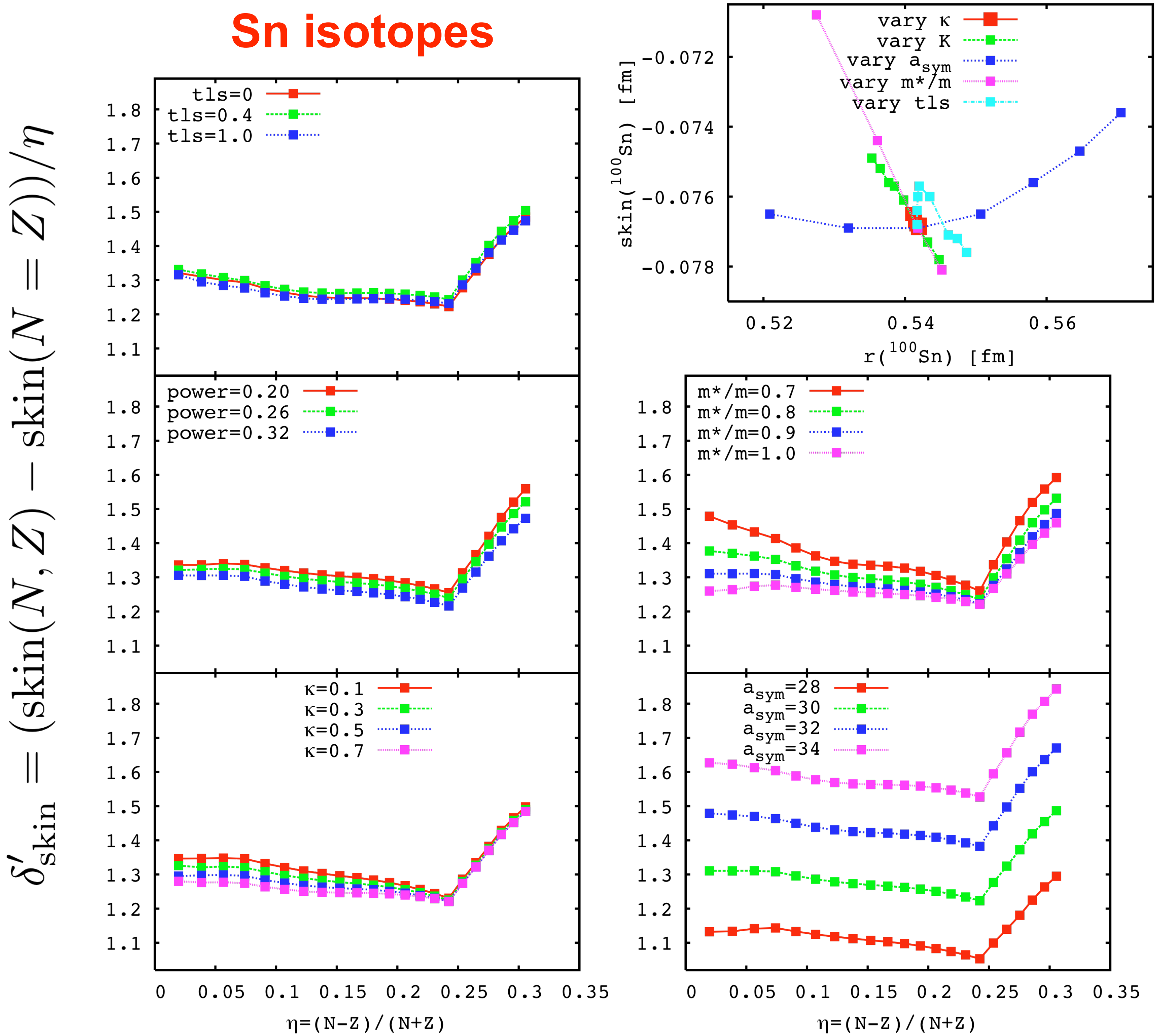
Shell effects can influence the skin!

$$\text{skin} = r_{rms}(n) - r_{rms}(p)$$

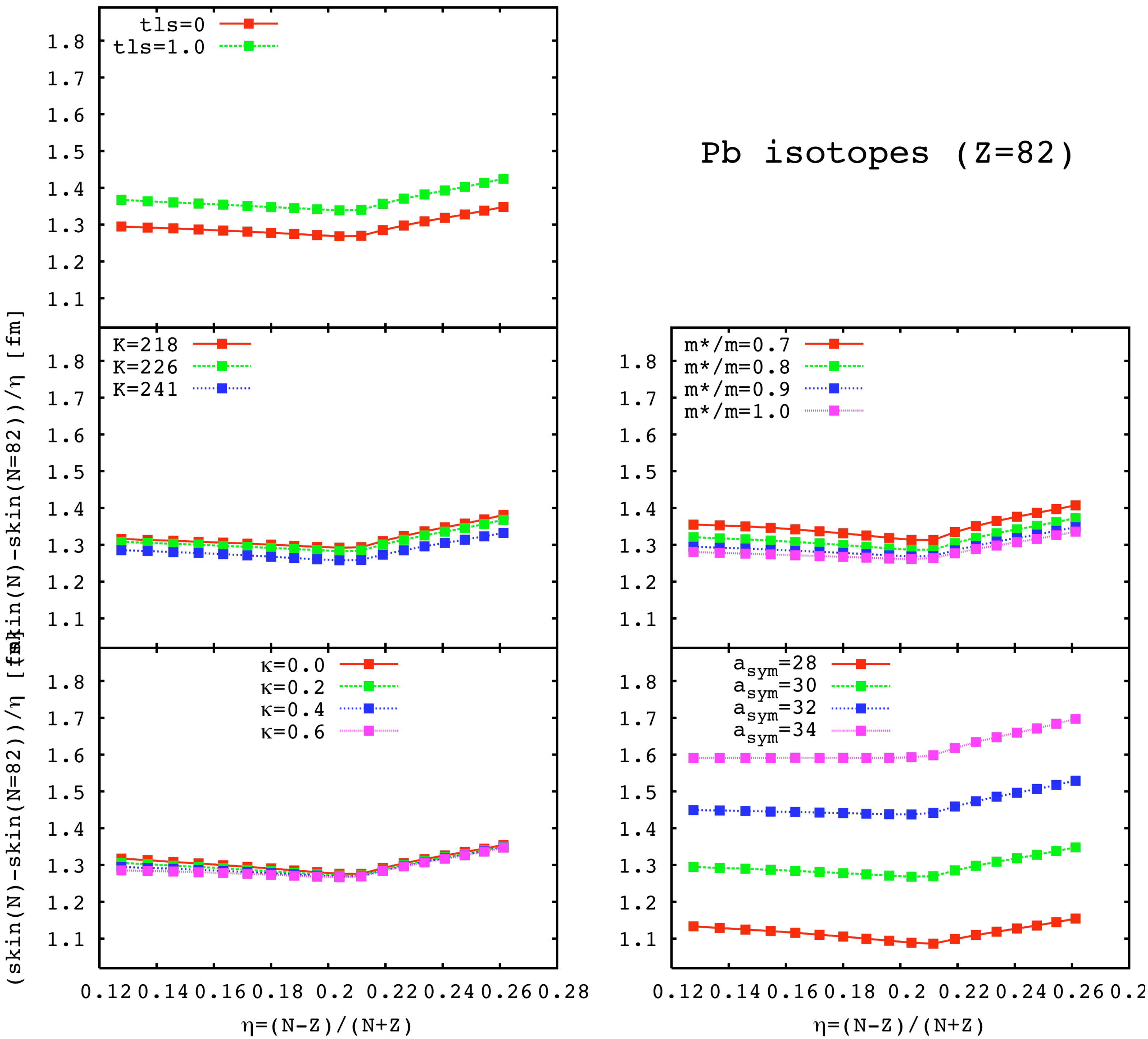


Open shell systems have large sensitivity to shell effects

Sn isotopes

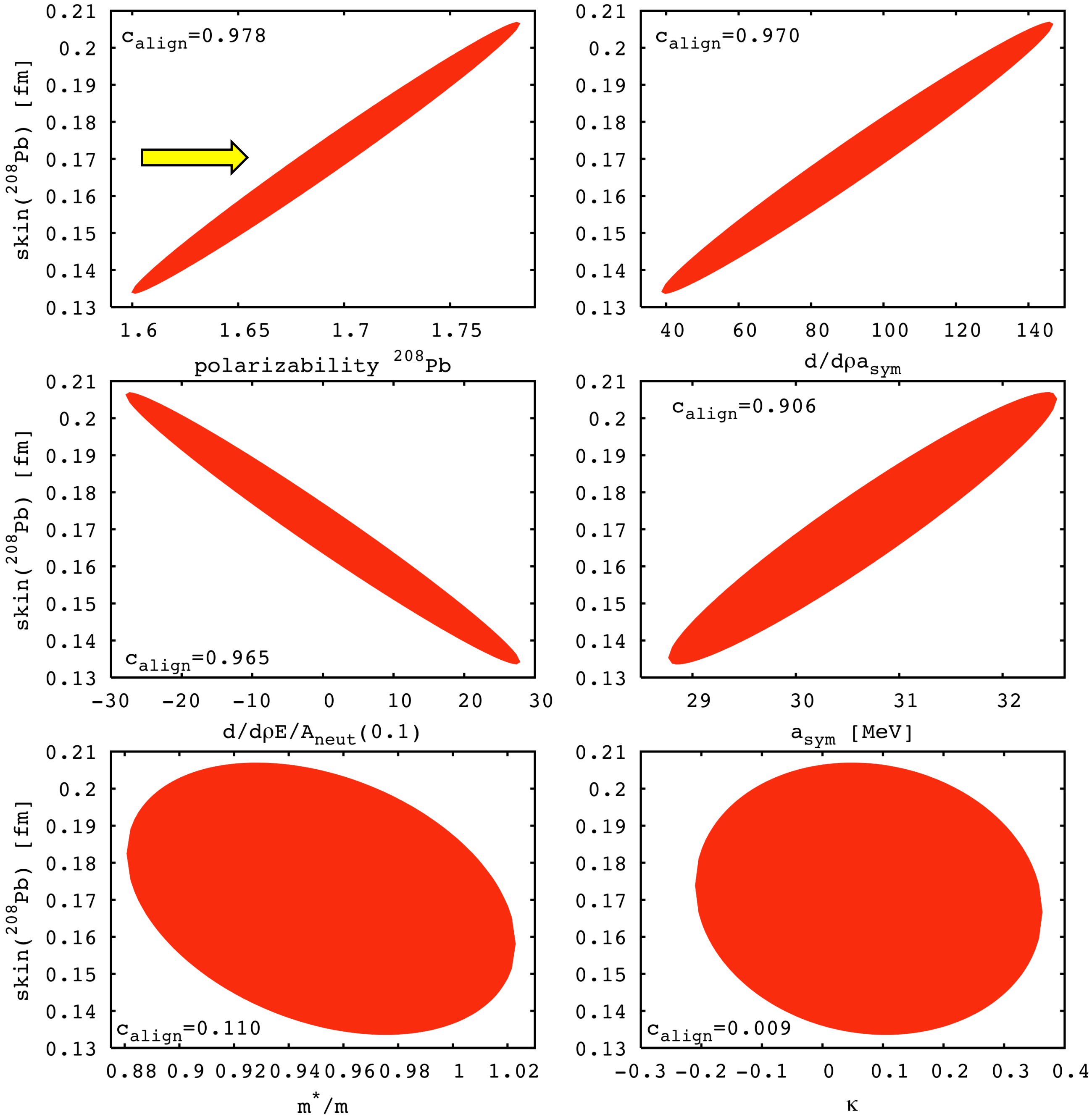


$\delta'_{\rm skin} = ({\rm skin}(N,Z) - {\rm skin}(N=Z))/\eta$
tls=0
tls=0.4
tls=1.0
power=0.20
power=0.26
power=0.32
κ=0.1
κ=0.3
κ=0.5
κ=0.7
η=(N-Z)/(N+Z)
vary κ
vary K
vary a_sym
vary m*/m
vary tls
skin(100Sn) [fm]
r(100Sn) [fm]
m*/m=0.7
m*/m=0.8
m*/m=0.9
m*/m=1.0
a_sym=28
a_sym=30
a_sym=32
a_sym=34

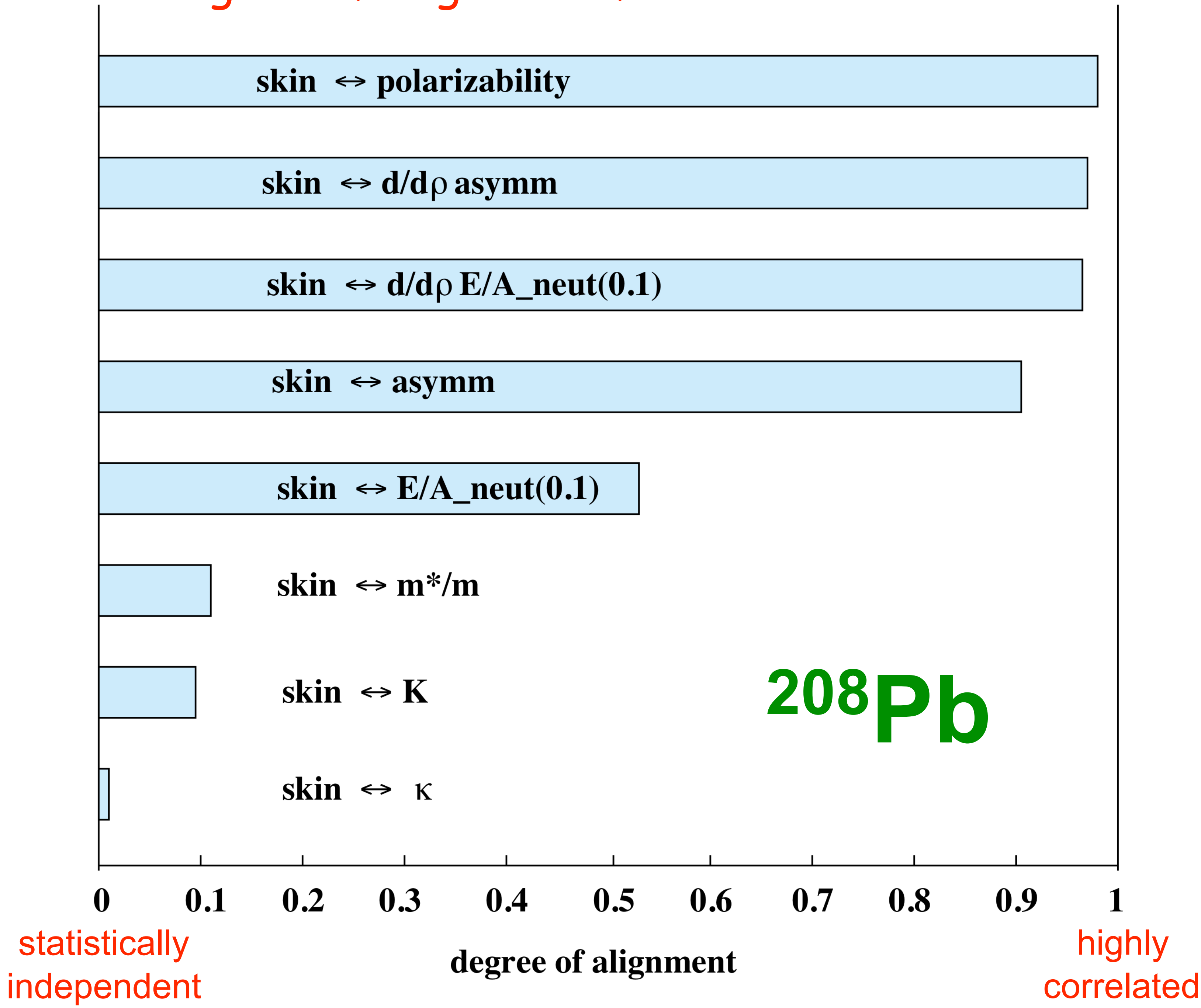


Pb isotopes (Z=82)
tls=0
tls=1.0
K=218
K=226
K=241
m*/m=0.7
m*/m=0.8
m*/m=0.9
m*/m=1.0
κ=0.0
κ=0.2
κ=0.4
κ=0.6
a_sym=28
a_sym=30
a_sym=32
a_sym=34
(skin(N)-skin(N=82))/η [fm]
η=(N-Z)/(N+Z)

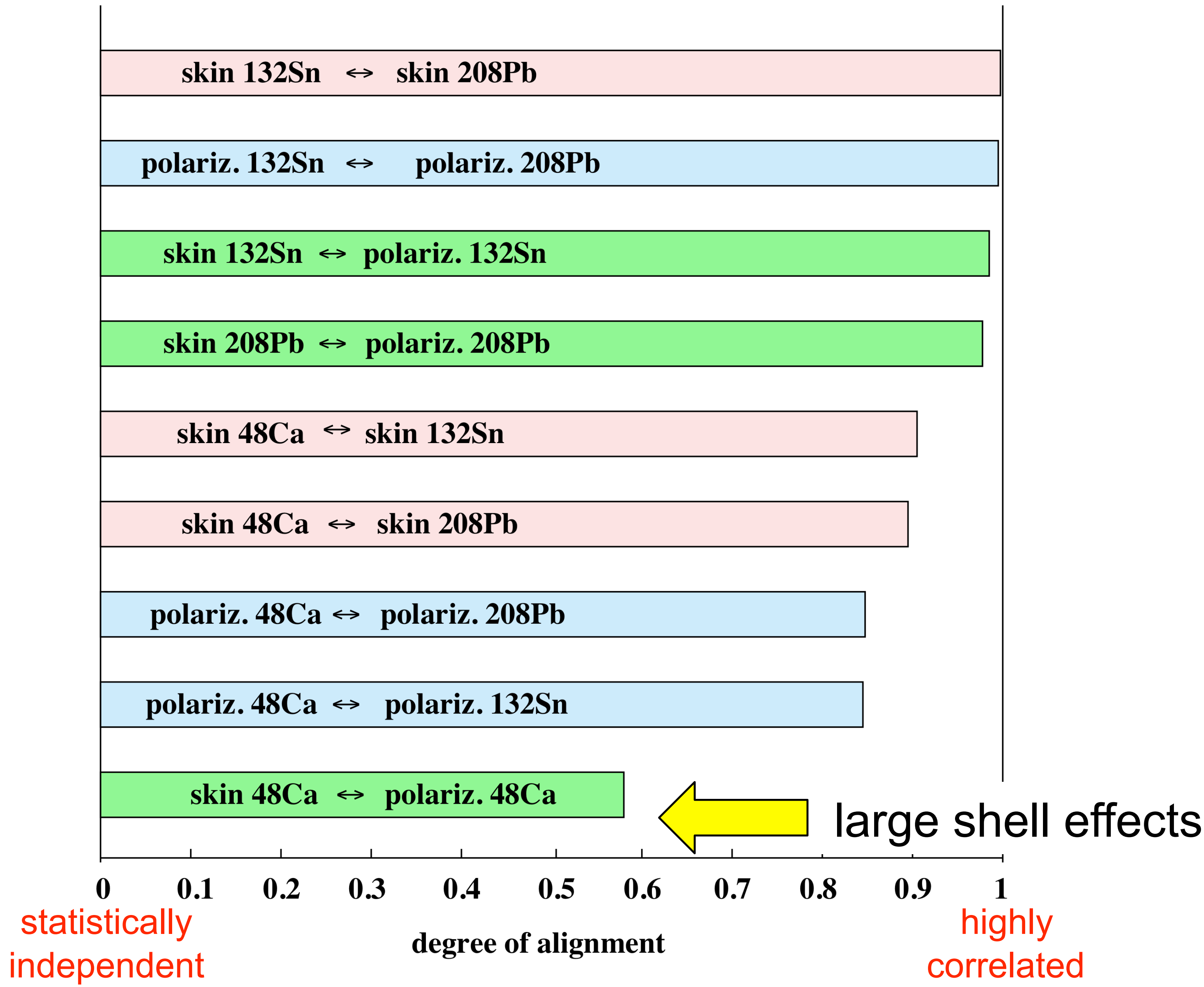
Correlation between observables



The degree of alignment for various observables



Alignmen between skin and polarizability for doubly magic nuclei



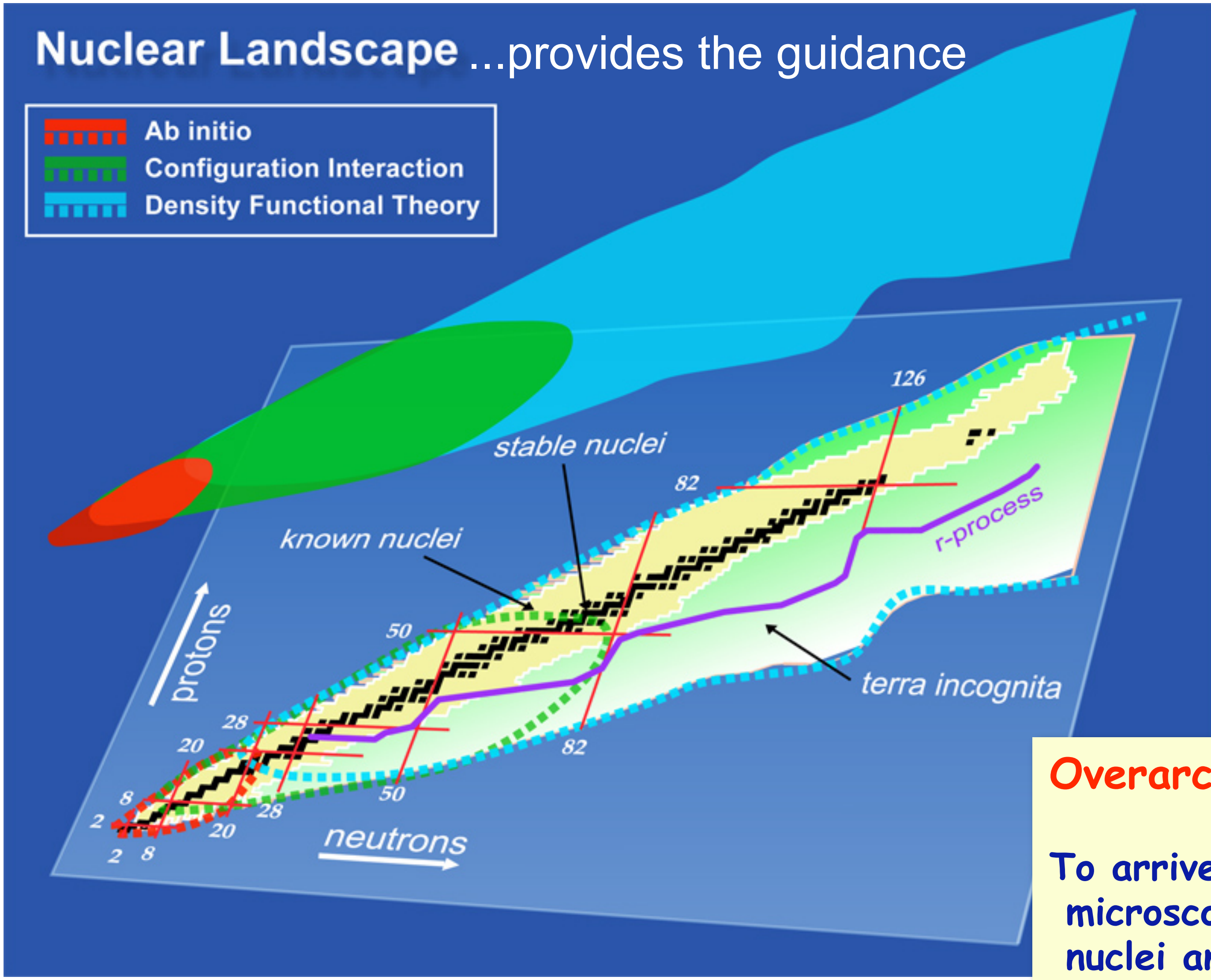
Summary

- For well bound systems, various definitions of skin are basically equivalent
- Skin of ^{208}Pb shows relatively weak (but not negligible) dependence on shell structure
- Strong correlation between skin and dipole polarizability but no correlation with the average GDR frequency
- Strong correlation between skin and slope of binding energy of neutron matter
- A fully free variation of EDF parameters yields an extrapolation uncertainty of 0.07 fm for the skin. If PREX measures it with this (or better) precision, we will learn a lot!

Thank You

Backup

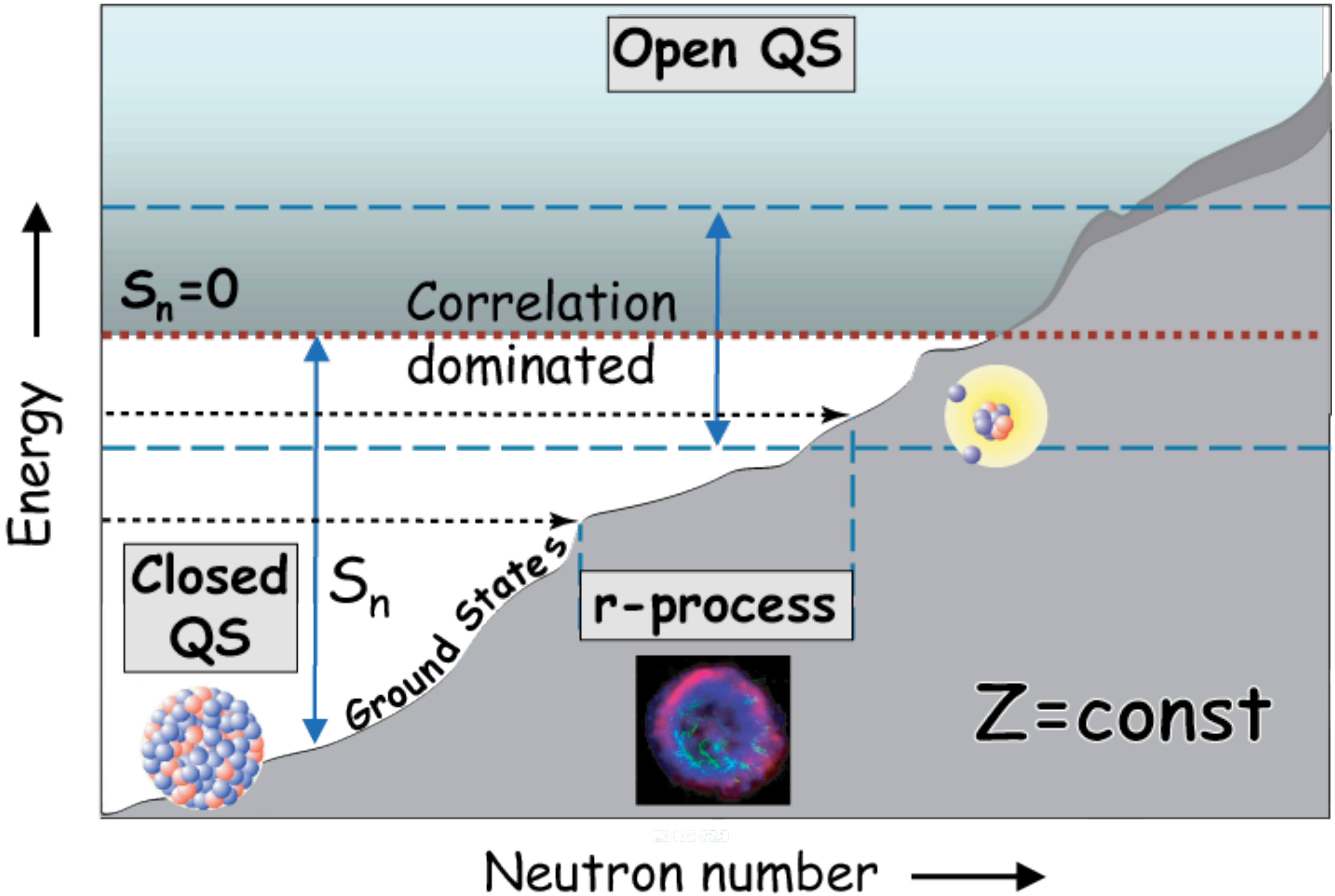
Roadmap for Theory of Nuclei



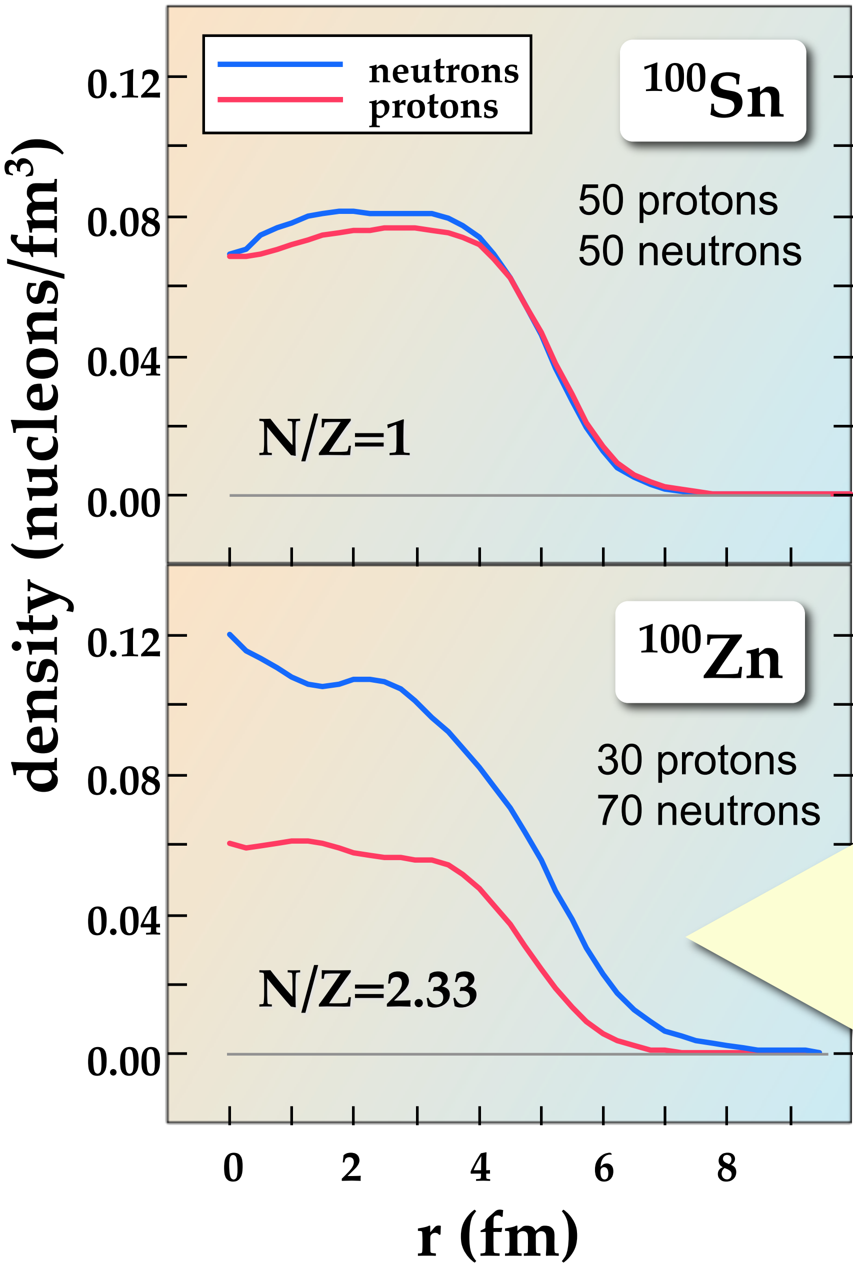
Overarching goal:

To arrive at a comprehensive microscopic description of all nuclei and low-energy reactions from the the basic interactions between the constituent nucleons

Prog. Part. Nucl. Phys. 59, 432 (2007)



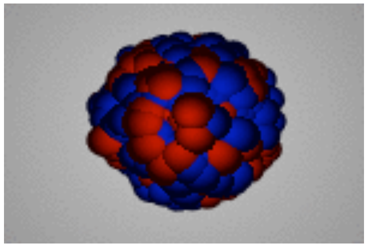
Neutron skins



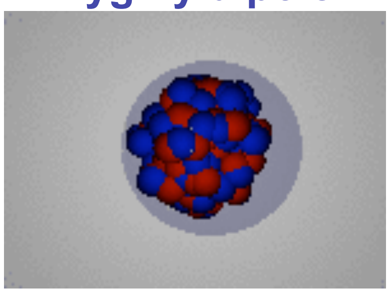
The only laboratory access to matter made essentially of pure neutrons

Neutron-rich matter and neutron skins

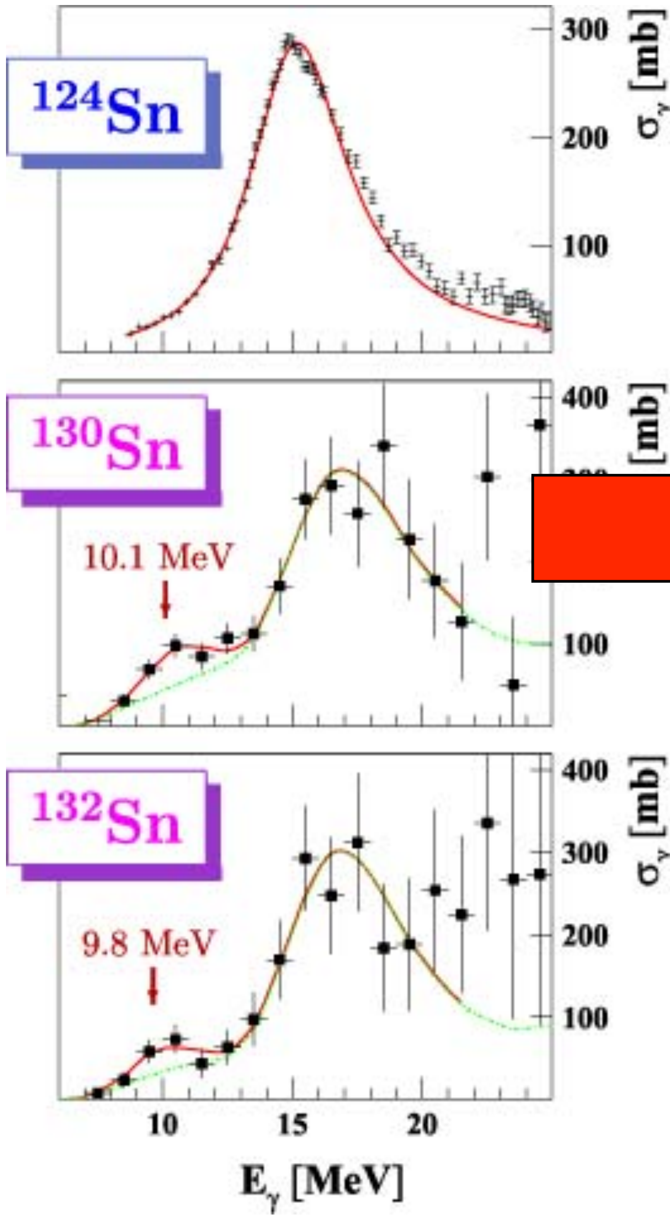
Giant dipole

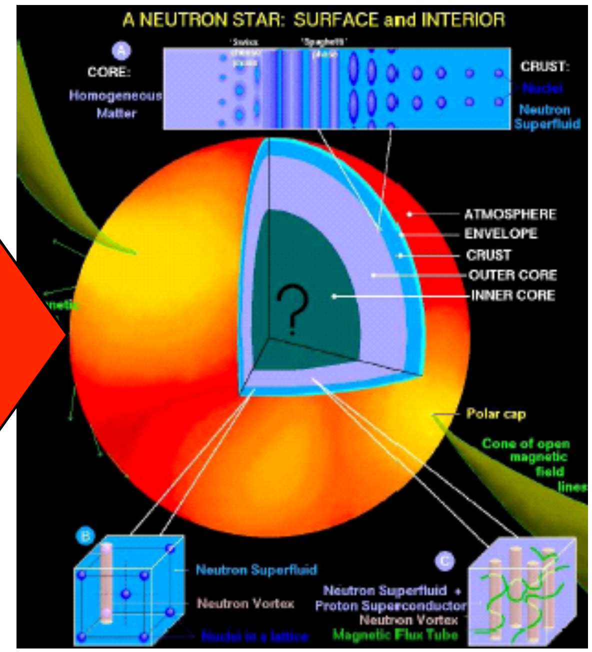


Pygmy dipole



size~10^{-14}m





Mass-radius relationship
of neutron stars

size~10^4m

LDM and Droplet Model Coefficients

	bulk properties					semi bulk	from finite nuclei		
Model	ρ_0	K	a_{vol}	a_{sym}	a'_{sym}	$a_{surf}^{(NM)}$	a_{surf}	a_{curv}	a_{ssym}
SkM*	0.1603	216.6	-15.752	30.04	95.25	17.70	17.6	9	-52
SkP	0.1625	201.0	-15.930	30.01	40.43	18.22	18.2	9.5	-45
BSk1	0.1572	231.4	-15.804	27.81	15.76	17.54	17.5	9.5	-36
BSk6	0.1575	229.2	-15.748	28.00	35.67		17.3	10	-33
SLy4	0.1596	230.1	-15.972	32.01	95.97		18.4	9	-54
SLy6	0.1590	230.0	-15.920	31.96	99.48	17.74	17.7	10	-51
SkI3	0.1577	258.1	-15.962	34.84	212.47		18.0	9	-75
SkI4	0.1601	247.9	-15.925	29.51	125.80		17.7	9	-34
SkO	0.1605	223.5	-15.835	31.98	163.50		17.3	9	-58
NL1	0.1518	211.3	-16.425	43.48	311.18		18.8	9	-110
NL3	0.1482	271.7	-16.242	37.40	269.16	18.5	18.6	7	-86
NL-Z	0.1509	173.0	-16.187	41.74	299.51		17.8	9	$<$ -125
NL-Z2	0.1510	172.4	-16.067	39.03	281.40	17.7	17.4	10	-90
LDM	0.153		-16.00	30.56			21.1		-48.6
LDM	0.1611	234.4	-16.24	32.65			18.6	12	
LDM	0.1417		-15.848	29.28			19.4		-38.4
LSD	0.1324		-15.492	28.82			17.0	3.9	-[illegible]9