NN and 3N interactions in neutron-rich nuclei

Achim Schwenk

TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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Outline

Resolution dependence of nuclear forces

Low-momentum NN and 3N interactions

Perturbative nuclear matter

Neutron matter and 3N interactions

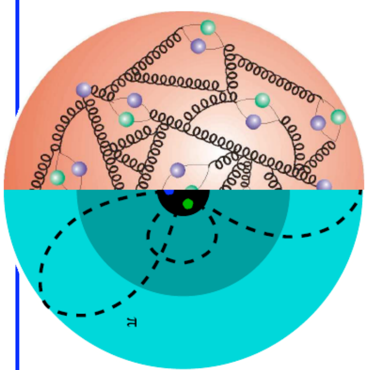
Neutron-rich nuclei and 3N interactions

Low-density matter

Λ / Resolution dependence of nuclear interactions

with high-energy probes:
quarks+gluons

cf. scale/scheme dependence
of parton distribution functions



Lattice QCD

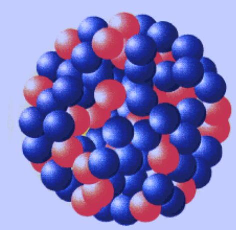
Effective theory for NN, many-N interactions,
operators depend on resolution scale Λ

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

$\Lambda_{\rm chiral}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory

nucleons interacting via pion exchanges + contact interactions



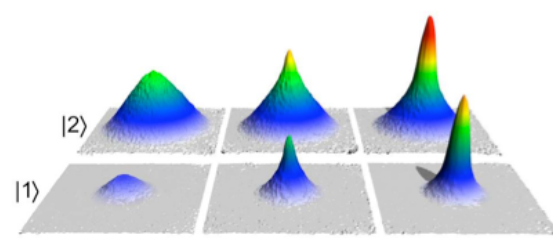
typical Fermi momenta in nuclei $\sim m_\pi$

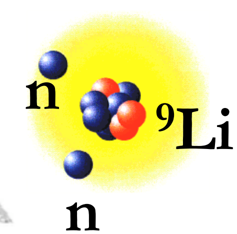
$\Lambda_{\rm pionless}$

$Q << m_\pi$=140 MeV: pion not resolved

pionless effective field theory

large scattering lengths + corrections



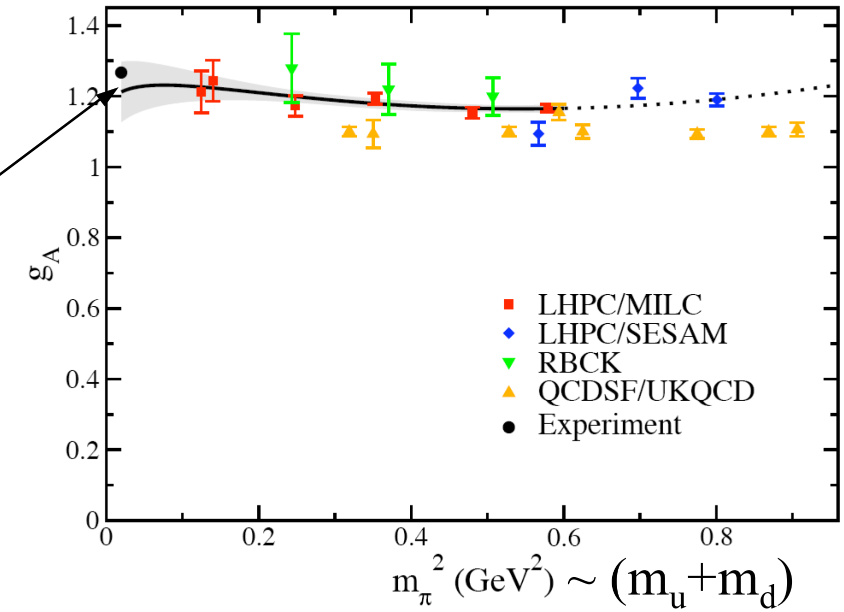


applicable to loosely-bound, dilute systems, reactions at astro energies

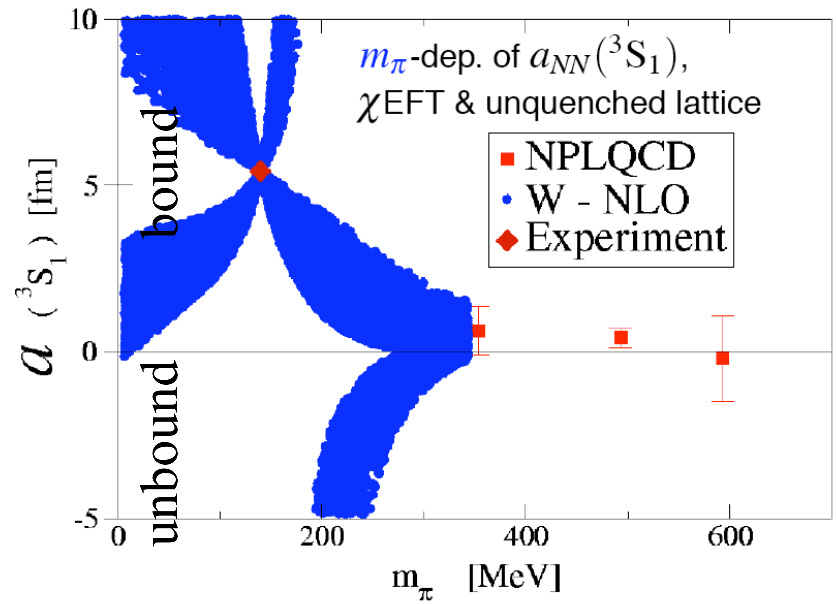
Lattice QCD and nuclear forces

pion-NN coupling g_a from full QCD
Edwards et al. (2006)

chiral EFT extrapolation to physical pion mass agrees with experiment



NN scattering lengths from full QCD, dependence on quark masses Beane et al. (2006)



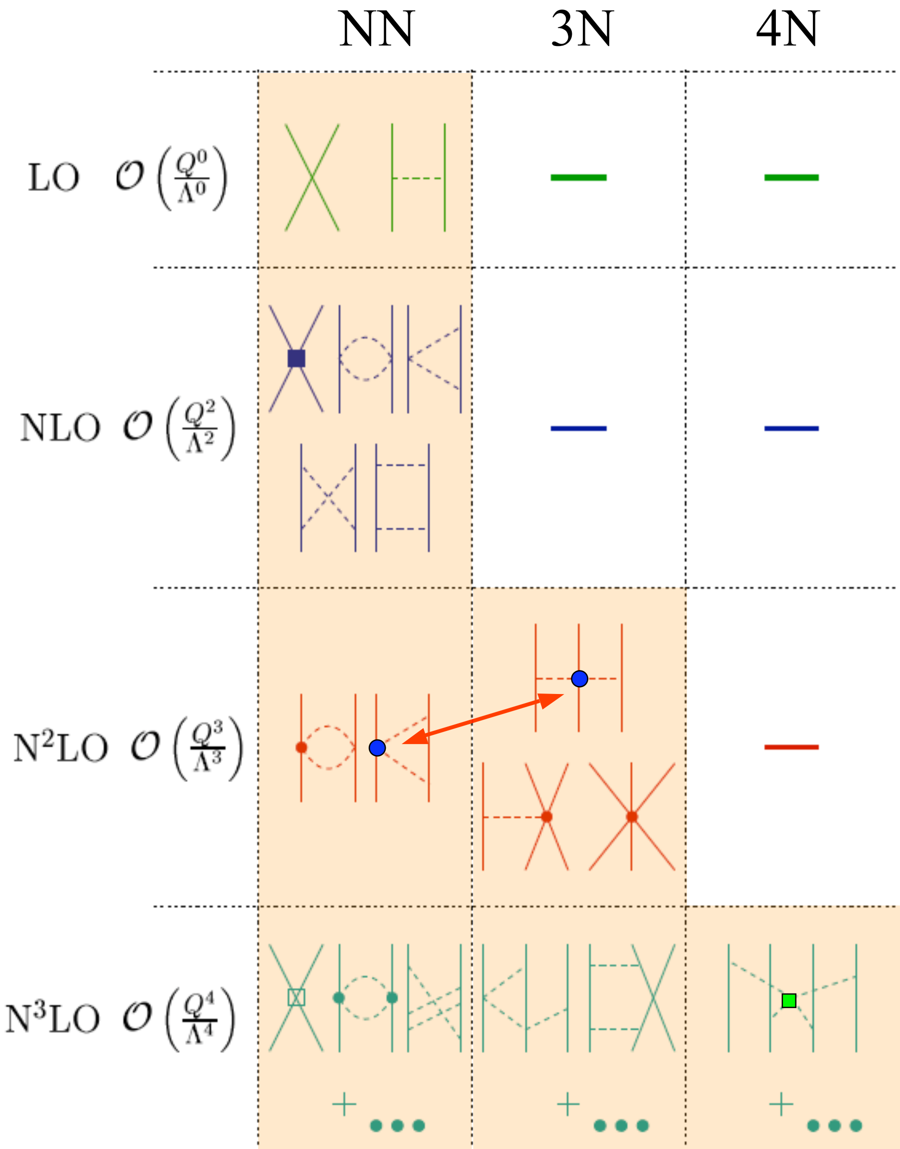
First coherent effort to connect nuclear forces to QCD, requires chiral EFT

Constrain low-energy couplings (long-range only?)

Constrain experimentally difficult observables: 3-neutron properties

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b



limited resolution at low energies, can expand in powers Q/Λ_b

details of short-distance physics not resolved

capture in few short-range couplings, fit to experiment once

include long-range physics explicitly, pions for chiral EFT

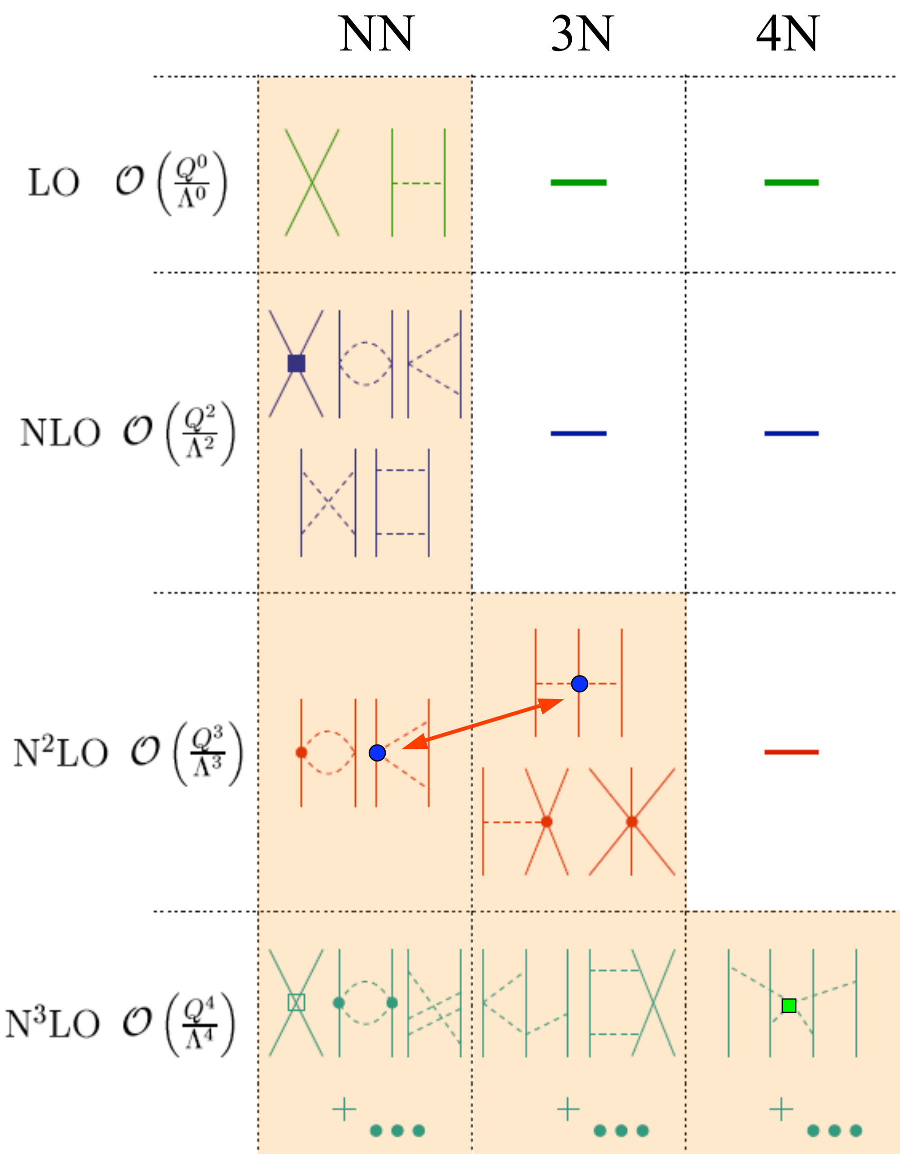
systematic: can work to desired accuracy and obtain error estimates

can connect to lattice QCD

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b



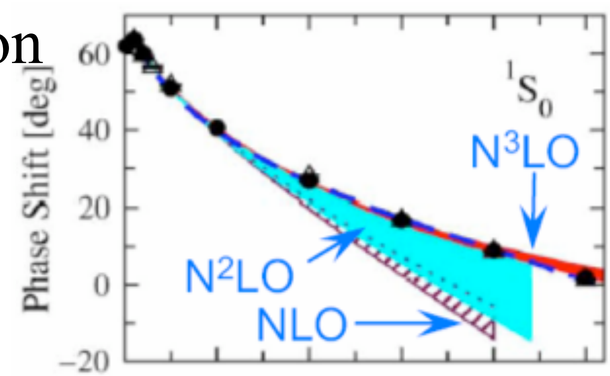
explains pheno hierarchy:
NN > 3N > 4N > …

NN-3N, πN, ππ, electro-weak,…
consistency

3N,4N: 2 new couplings to N^3LO

resolution/Λ-dependent couplings:
infinitely many N…LO potentials

error estimates from truncation order
and Λ variation

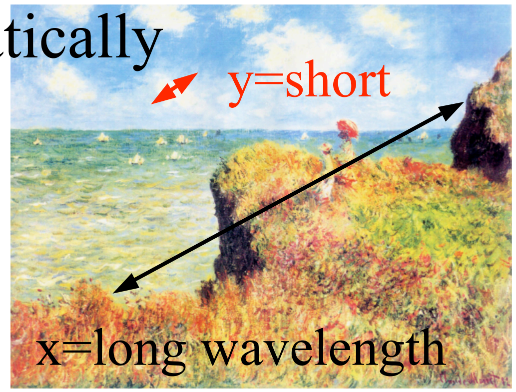


Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Renormalization group (RG) for nuclear forces

integrate out high-momentum modes that are not resolved
and incorporate their effects in couplings of effective theory

schematically



$$Z = \int dx \int dy\, e^{-S(x,y)} = \int dx \int dy\, e^{-a(x^2+y^2)-b(x^2+y^2)^2}$$

$$= \int dx\, e^{-S_{\rm eff}(x)} = \int dx\, e^{-a'x^2-b'x^4-c'x^6+\ldots}$$

separate into slow and fast modes $\phi(\omega,k) = \begin{cases} \phi_<(\omega,k) & \text{for } \omega, k < \Lambda \\ \phi_>(\omega,k) & \text{else} \end{cases}$
and integrate out fast modes

$$Z = \int \prod d\phi_<(\omega,k)\, e^{-S_{\rm free}[\phi_<]} \int \prod d\phi_>(\omega,k)\, e^{-S_{\rm free}[\phi_>]-S_{\rm int}[\phi_<,\phi_>]}$$

$$= \int \prod d\phi_<(\omega,k)\, e^{-S_{\rm eff}[\phi_<]}$$

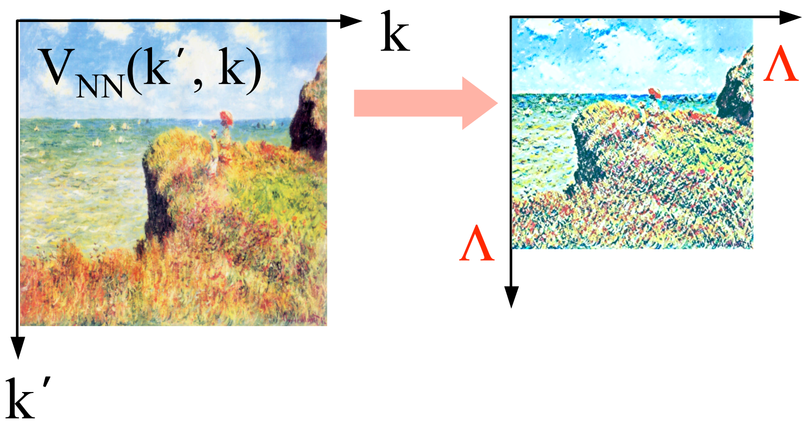
resolution/Λ-dependent couplings that follow RG equations

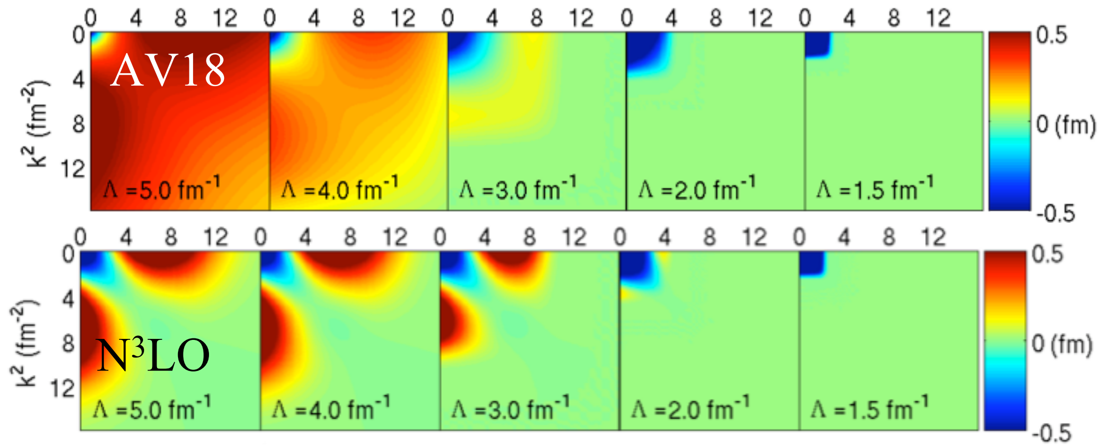
applied to pionless EFT $C_0(\Lambda) = \dfrac{4\pi}{m} \dfrac{1}{\frac{1}{a_s} - \frac{2}{\pi}\Lambda}$

for weak scattering $C_0(\Lambda) \simeq \dfrac{4\pi a_s}{m}$ vs. for strong interactions $C_0(\Lambda) = -\dfrac{2\pi^2}{m\Lambda}$

Low-momentum interactions from the Renormalization Group

evolve to lower resolution/cutoffs by integrating out high-momenta,
can be carried out exactly for NN interactions Bogner, Kuo, AS (2003)





implemented by RG equations
or unitary transformation

$$\frac{d}{d\Lambda} V^{\Lambda}_{\text{low}\,k}(k', k) = \frac{2}{\pi} \frac{V^{\Lambda}_{\text{low}\,k}(k', \Lambda) T^{\Lambda}(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

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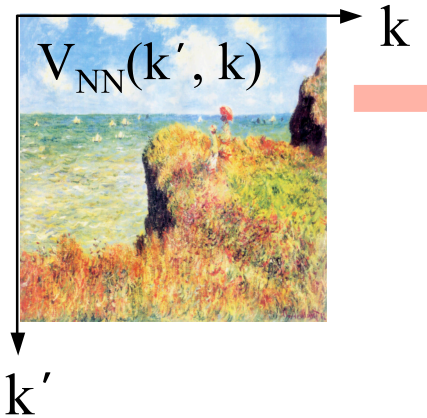
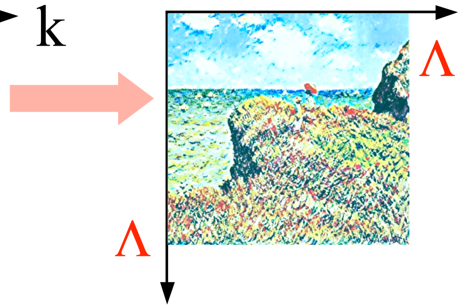
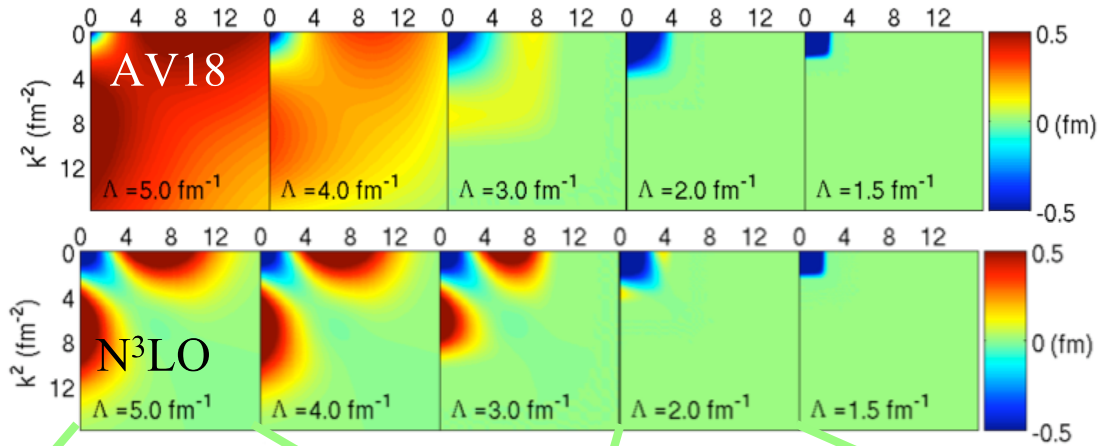


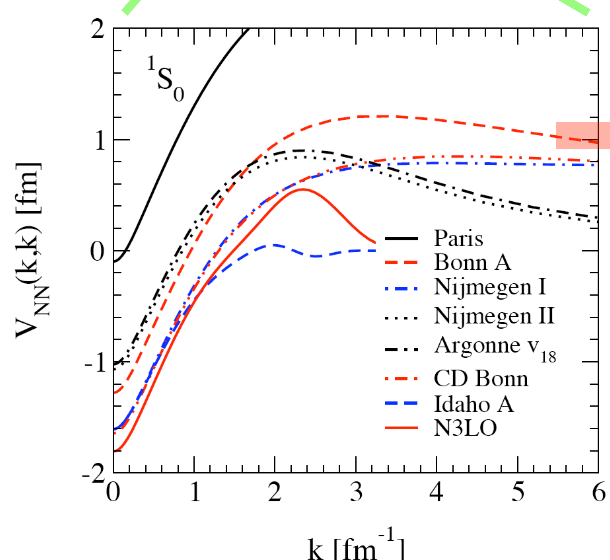


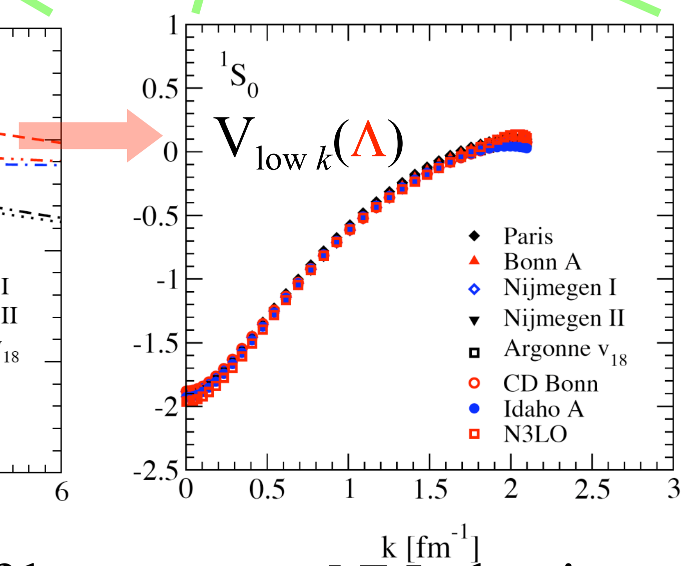


implemented by RG equations
or unitary transformation

≈ universal interaction
for low momenta

evolution to $V_{low\ k}(\Lambda)$
decouples high momenta



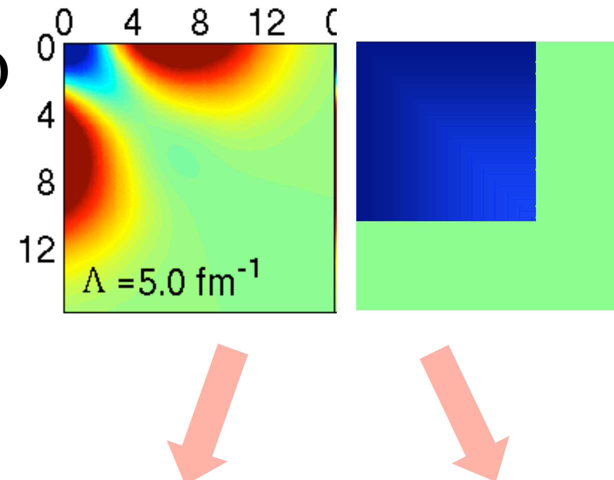


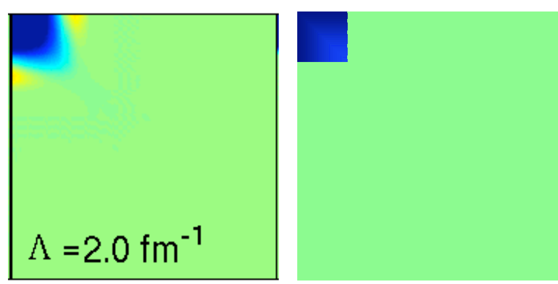
method to vary resolution scale without loss of low-energy NN physics

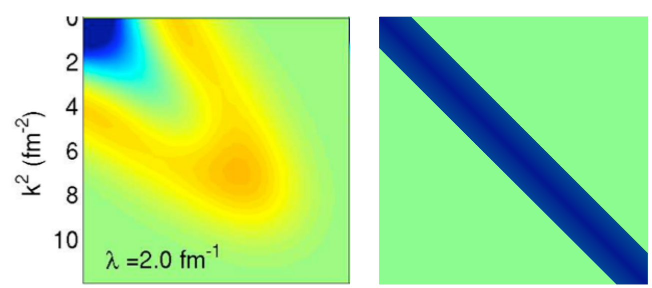
Different schemes: RG and Similarity RG (SRG)

all interactions are low-energy NN equivalent up to truncation errors and include same long-range physics

start from chiral EFT, here N^3LO







sharp or smooth cutoff $V_{\text{low }k}(\Lambda)$

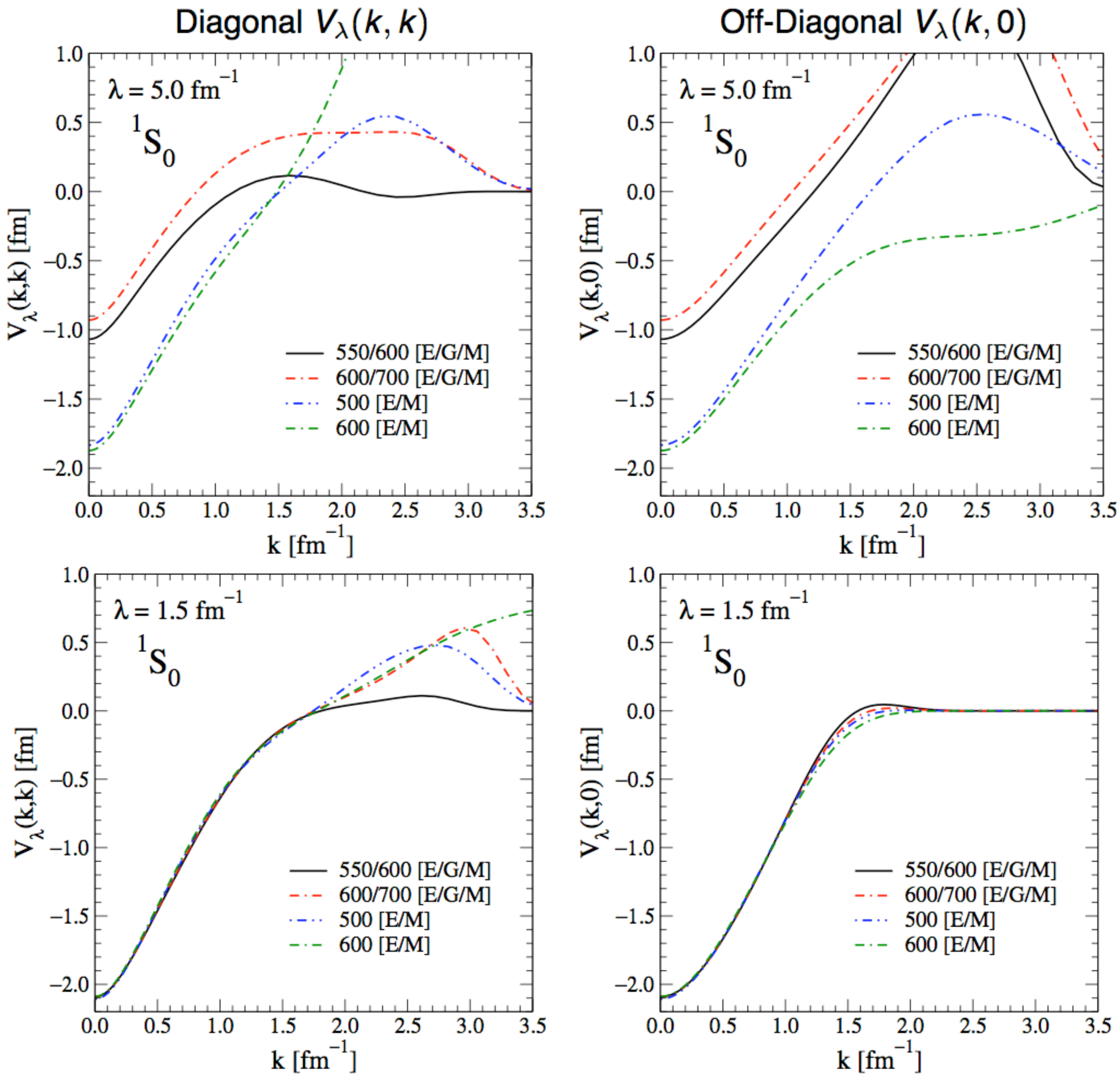
smooth cutoff $V_{\text{SRG}}(\lambda)$ evolves towards band diagonal Bogner et al. (2007)

RG generates all short-range operators so that low-energy NN is reproduced

Chiral EFT and RG

find ≈ universality from different N^3LO potentials

weakens off-diag coupling



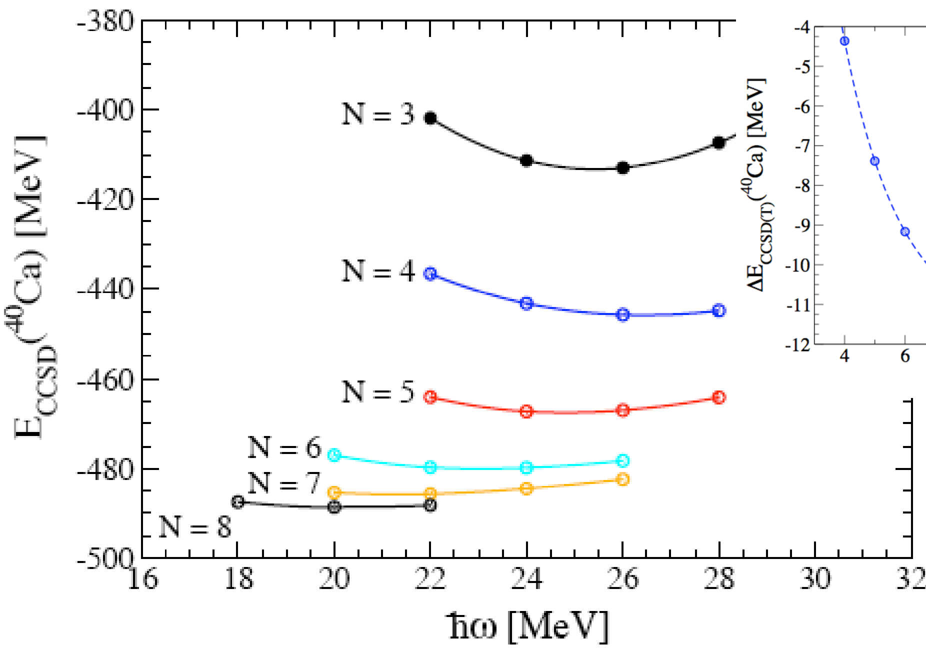
Pushing the limits

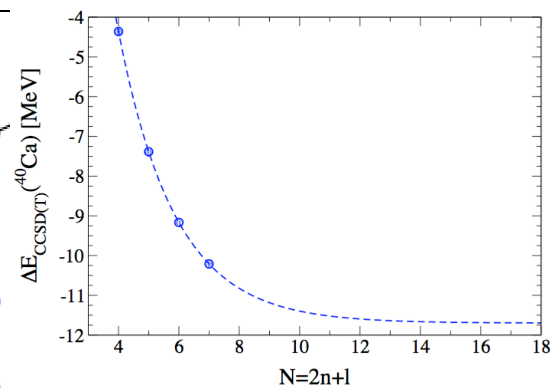
lower cutoffs need smaller basis: direct convergence

Coupled-cluster theory based on $V_{\text{low }k}(\Lambda)$ Hagen et al. (2007)

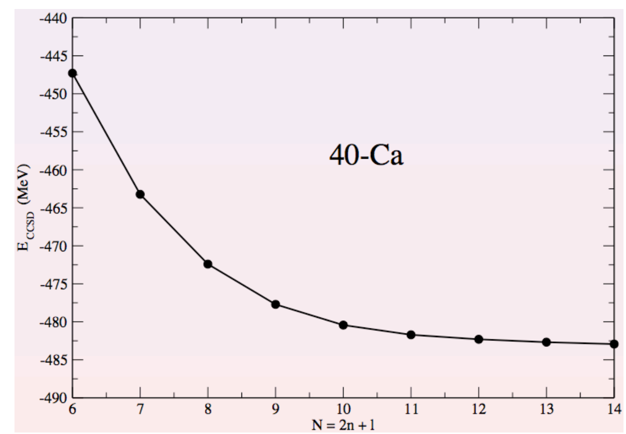
meets and sets benchmarks: within 10 keV of exact FY for ^{4}He

accurate for ^{16}O and ^{40}Ca





	^{4}He	^{16}O	^{40}Ca
E_0	-11.8	-60.2	-347.5
ΔE_{CCSD}	-17.1	-82.6	-143.7
$\Delta E_{\text{CCSD(T)}}$	-0.3	-5.4	-11.7
$E_{\text{CCSD(T)}}$	-29.2	-148.2	-502.9
exact (FY)	-29.19(5)		



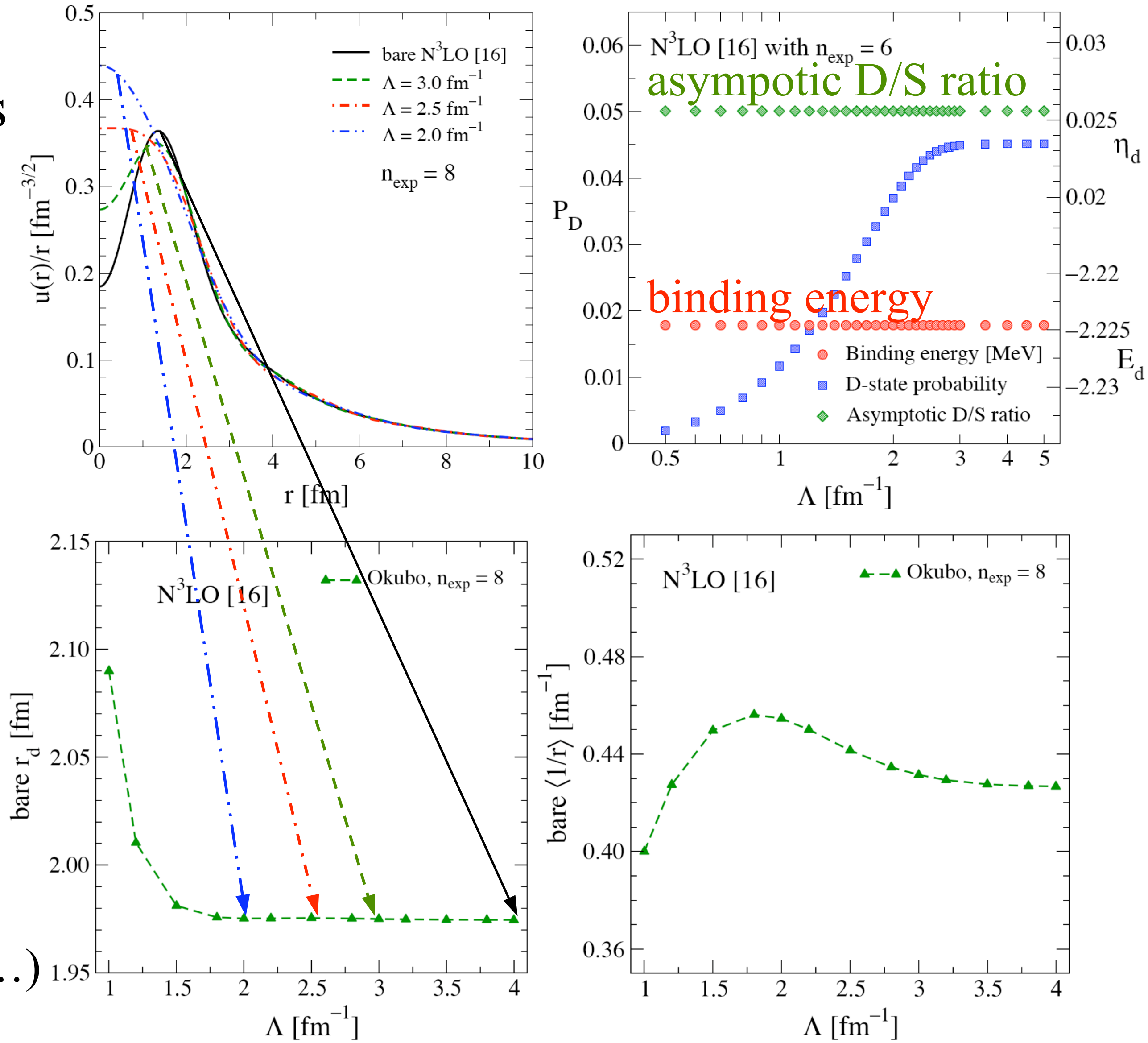
spherical CC code Hagen et al. (2008)

^{208}Pb radii will be possible!

Correlations

RG preserves long-range parts of interactions, deuteron observables, with dramatically different wave functions/correlations

short-range correlations depend on resolution scale, cf. parton df



weak renormalization of long-range operators r, Q, 1/r,…

short-range operators always have correction terms (exchange currents,…)

Three-nucleon interactions: a frontier

3N interactions crucial for:

binding energies

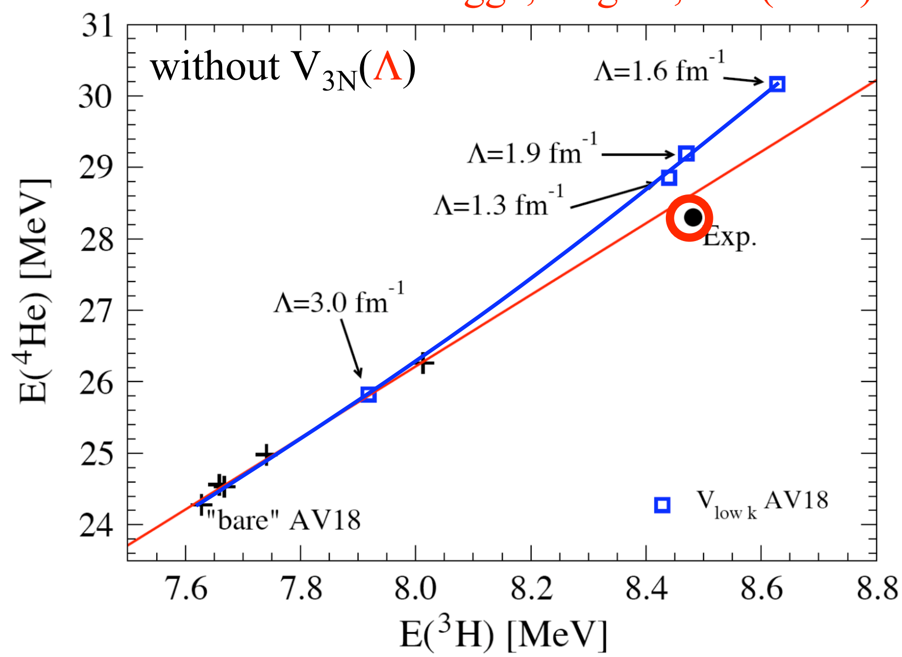
cutoff dependence explains Tjon line,
3N required by renormalization

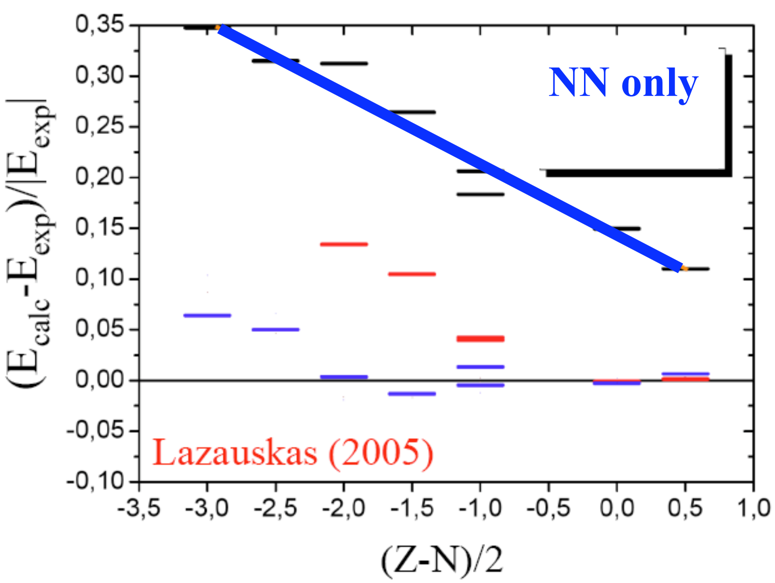
spin-orbit effects

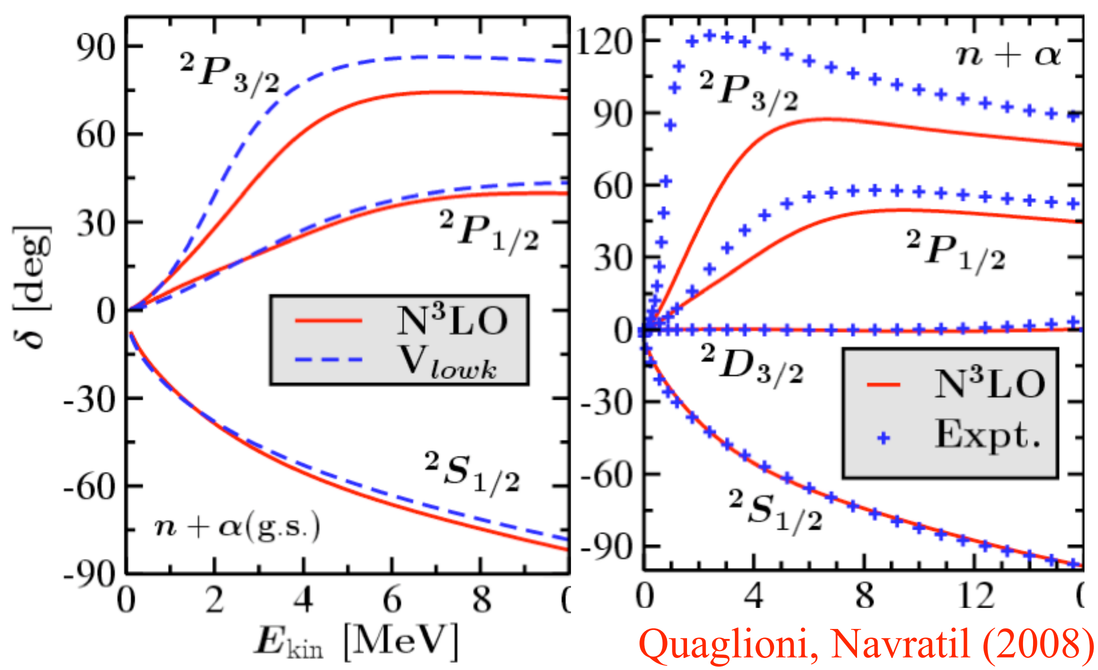
neutron/proton-rich systems

dense matter in astrophysics

Nogga, Bogner, AS (2004)





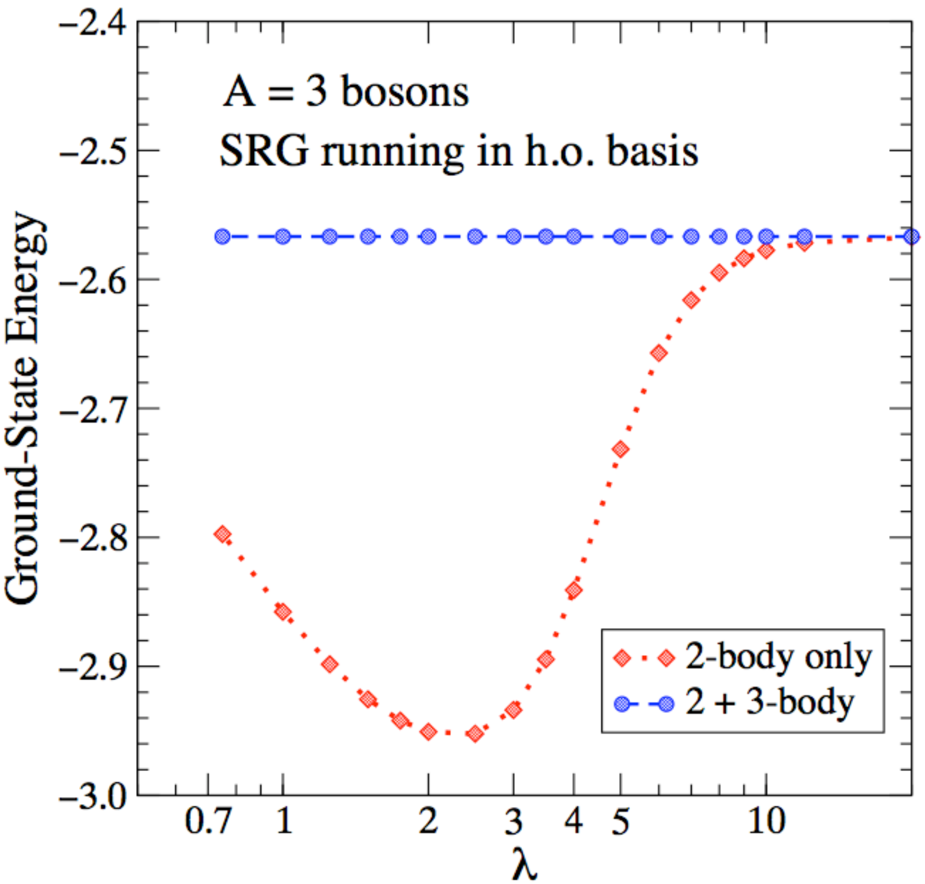


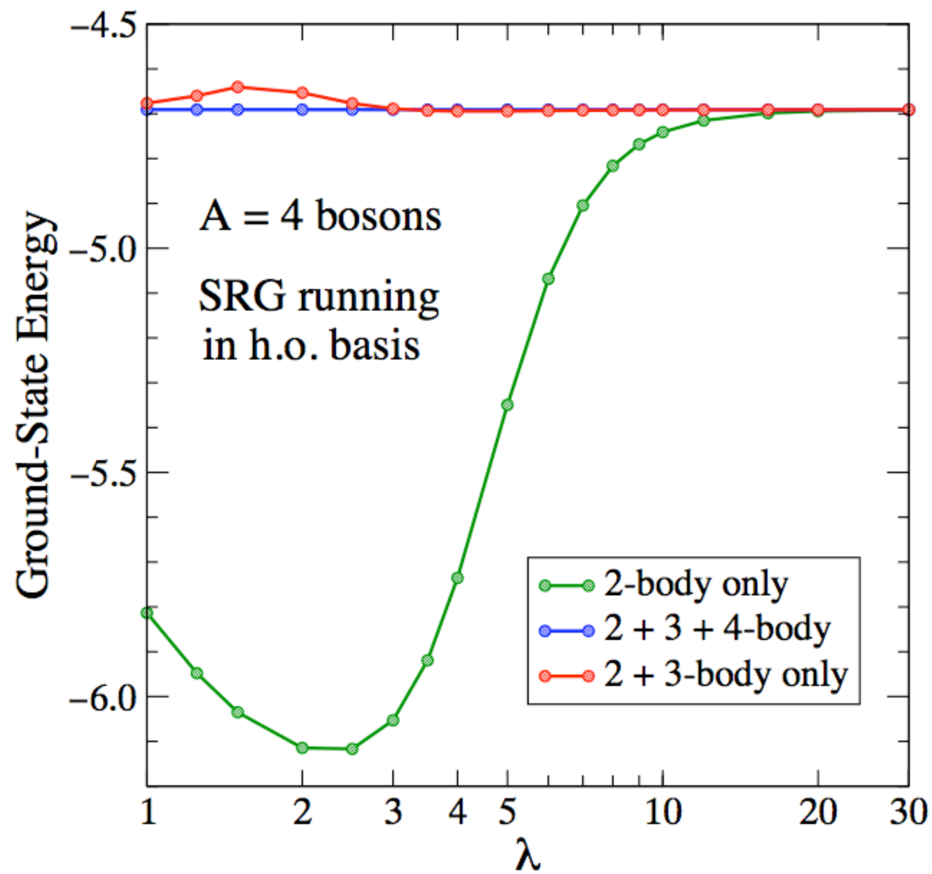
Quaglioni, Navratil (2008)

Towards evolving 3N interactions

SRG evolution for 1d systems with T_{rel} in Jacobi HO basis

Jurgenson et al. (OSU), in prep.





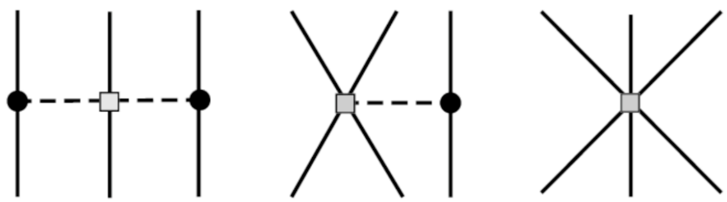
gearing up to evolve chiral 3N interactions

for now: use chiral EFT is complete basis

Low-momentum 3N interactions

from leading N^2LO chiral EFT ~ $(Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)

long (2π) intermed. (π) short-range



c_1,c_3,c_4 terms $D(\Lambda)$ term $E(\Lambda)$ term

$$= \frac{g_A^2}{8F_\pi^4}\frac{(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_3\cdot\vec{q}_3)}{[\vec{q}_1{}^2+M_\pi^2][\vec{q}_3{}^2+M_\pi^2]}\left\{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\left(-4c_1M_\pi^2+2c_3\,\vec{q}_1\cdot\vec{q}_3\right)+c_4\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3]\cdot\boldsymbol{\tau}_2\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2\right\} + \text{perm.}$$

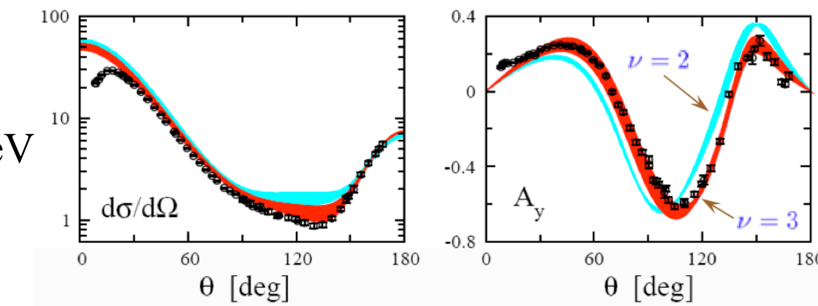
$$= -\frac{g_A D}{8F_\pi}\frac{(\vec{\sigma}_3\cdot\vec{q}_3)}{\vec{q}_3{}^2+M_\pi^2}(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\,(\vec{\sigma}_2\cdot\vec{q}_3)\; + \text{perm.}$$

$$= \frac{1}{2}E\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\; + \text{perm.}$$

c_i from πN, consistent with NN $c_1 = -0.9^{+0.2}_{-0.5}\,,\; c_3 = -4.7^{+1.2}_{-1.0}\,,\; c_4 = 3.5^{+0.5}_{-0.2}$

Meissner (2007)

generally improves 3N scattering pd @ 65MeV



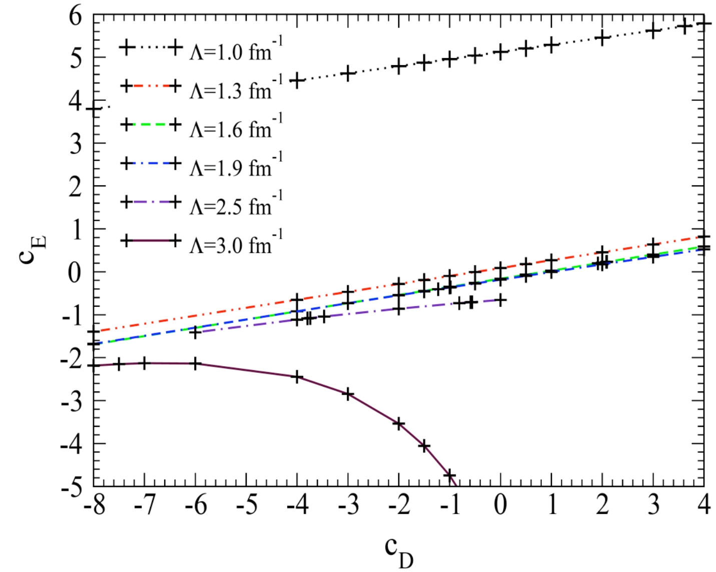
c_3,c_4 important for structure, large uncertainties at present

chiral EFT is complete basis → 3N up to truncation errors

D term could be fixed by tritium beta decay

Low-momentum 3N fits

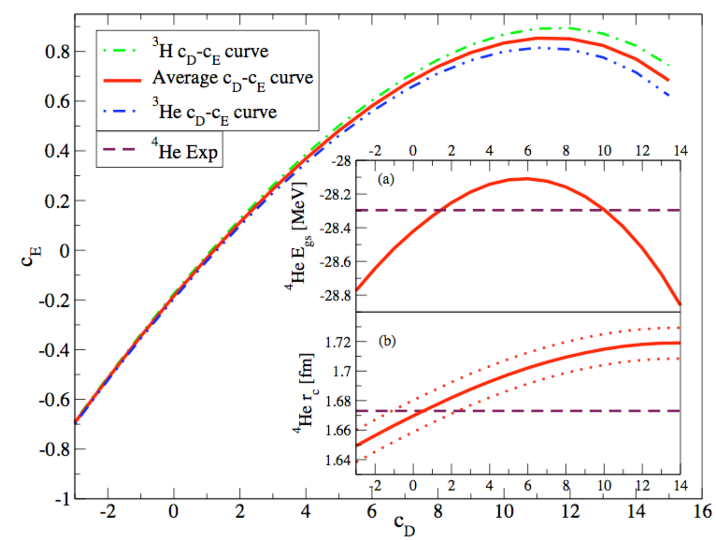
fit D,E couplings to A=3,4 binding energies
for range of cutoffs



linear dependences in fits to triton binding

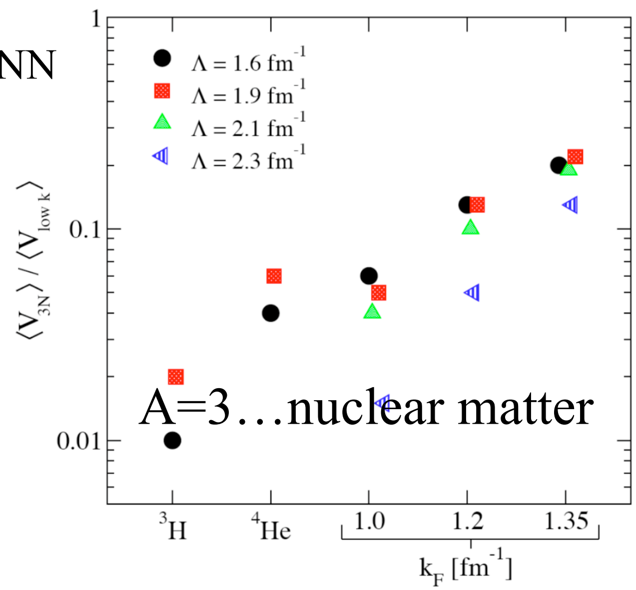
3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$
Nogga, Bogner, AS (2004)

nonperturbative at larger cutoffs
cf. chiral EFT $\Lambda \approx 3$ fm^{-1}



Navratil et al. (2007)

3N exp. values natural
$\sim(Q/\Lambda)^3\ V_{NN} \sim 0.1\ V_{NN}$

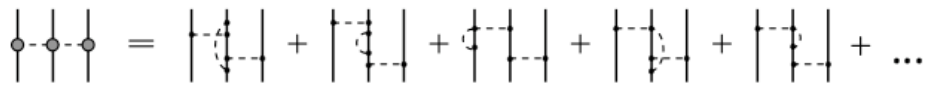


Subleading chiral EFT 3N interactions

parameter-free $N^3LO \sim (Q/\Lambda)^4$ Status from Epelbaum (2007)

- 1/m-corrections to 1 insertion from $\mathcal{L}^{(2)}_{1/m} =$  $+ \mathcal{O}(\pi^3)$
  — rich operator structure (includes spin-orbit interactions)
- 1-loop diagrams with all vertices from $\mathcal{L}^{(0)}_{\text{eff}}$

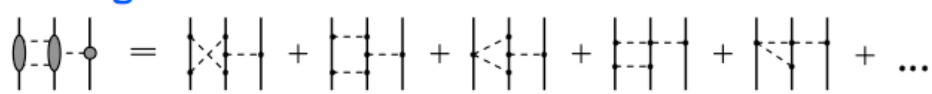
2π – exchange



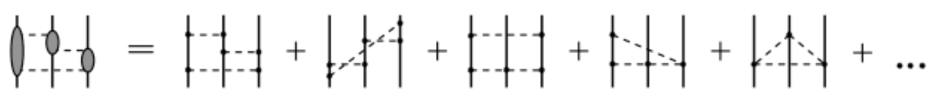
The calculated corrections simply shift the LECs c_i as follows:

$$\delta c_1 = \frac{g_A^2 M_\pi}{64\pi F_\pi^2} \sim 0.13\ \text{GeV}^{-1} \qquad \delta c_3 = \frac{3 g_A^4 M_\pi}{16\pi F_\pi^2} \sim 2.5\ \text{GeV}^{-1} \qquad \delta c_4 = -\frac{g_A^4 M_\pi}{16\pi F_\pi^2} \sim -0.85\ \text{GeV}^{-1}$$

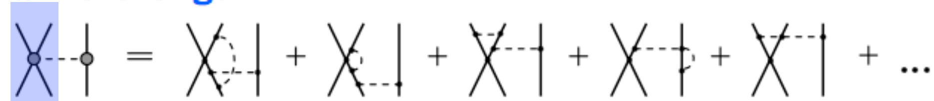
2π-1π – exchange



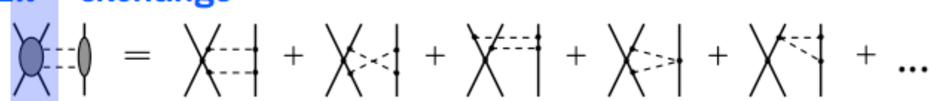
ring diagrams



contact-1π – exchange



contact-2π – exchange



N^3LO contributions to 3N interactions decrease c_i constants, cf. at N^2LO

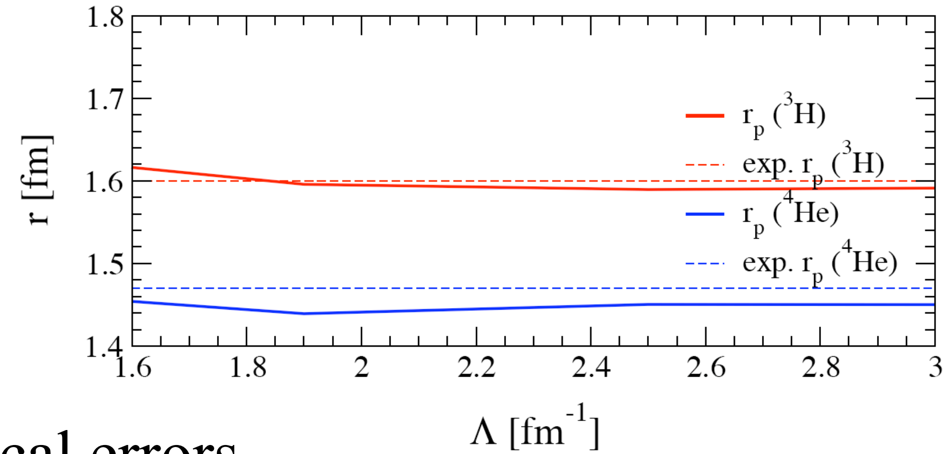
$$c_1 = -0.9^{+0.2}_{-0.5}$$

$$c_3 = -4.7^{+1.2}_{-1.0}\,, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

Theoretical uncertainties

Cutoff variation estimates errors due to neglected parts in H(Λ)

Radii of light nuclei approximately cutoff-independent, agree with exp.



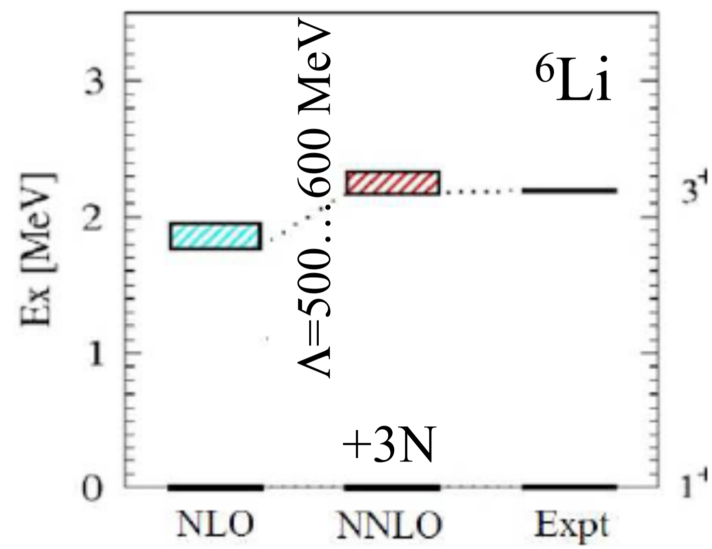
Can provide lower limits on theoretical errors

uncertainties of matrix elements needed in fundamental symmetry tests

isospin-symmetry breaking corrections
V_{ud}=0.97416(13) (14/18)theo.

neutrinoless double-beta decay

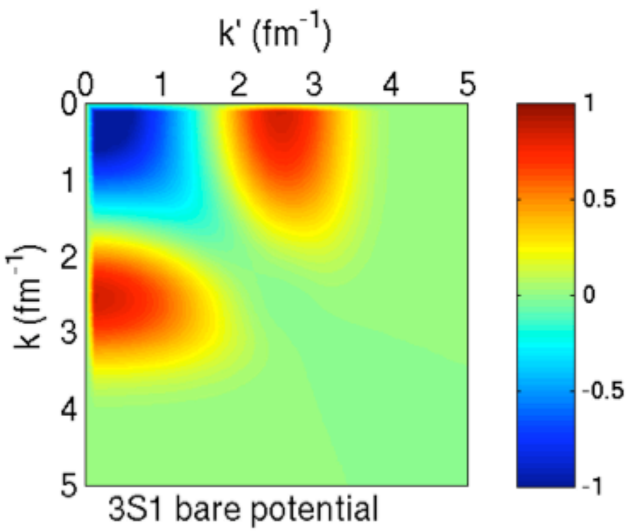
EDMs ……

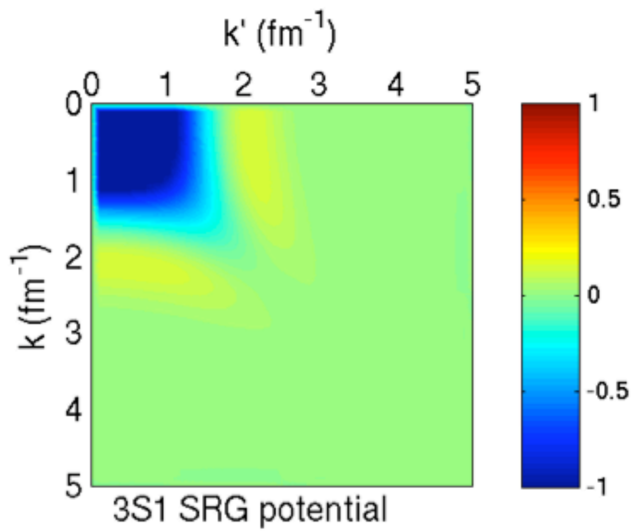


from A. Nogga

Advantages of low-momentum interactions for nuclei

conventional G matrix approach does not solve off-diagonal coupling, renders Bethe-Brueckner-Goldstone expansion necessarily nonperturb.

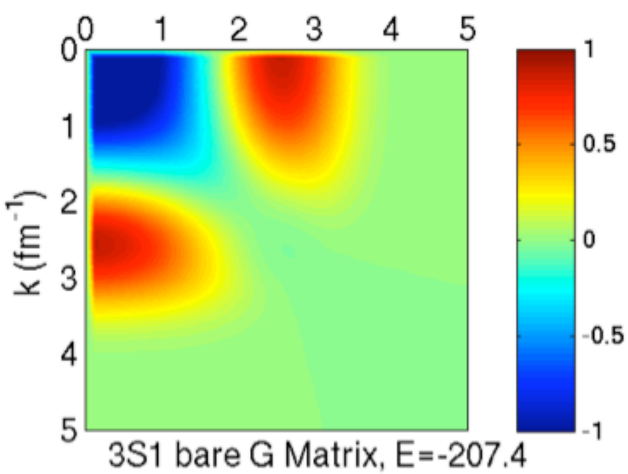


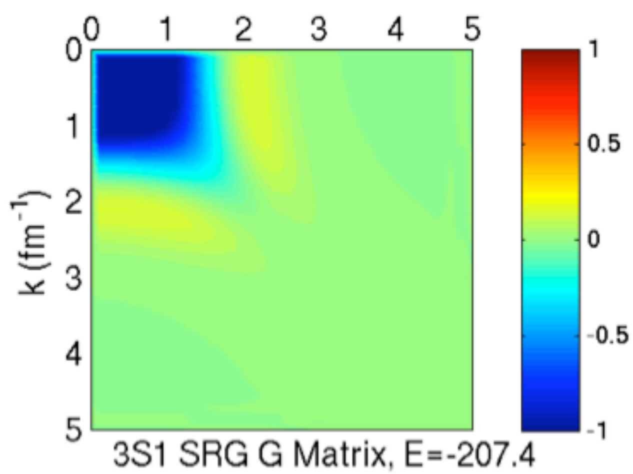


chiral N^{3}LO potential

SRG evolved

G matrix

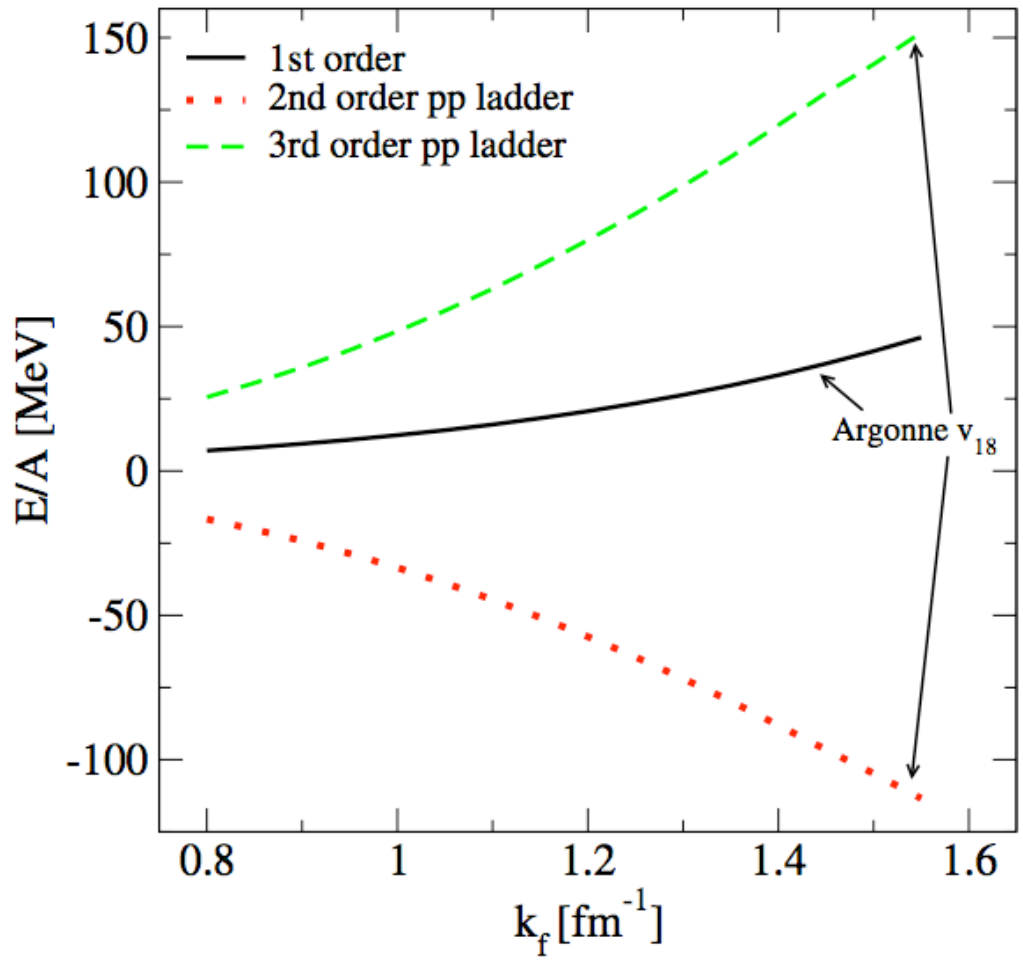




Possibility of perturbative nuclear matter with NN and 3N

start from chiral EFT to given order, soften with RG

nuclear matter converged at ≈ 2nd order,
motivated by Weinberg eigenvalue analysis



Possibility of perturbative nuclear matter with NN and 3N

start from chiral EFT to given order, soften with RG

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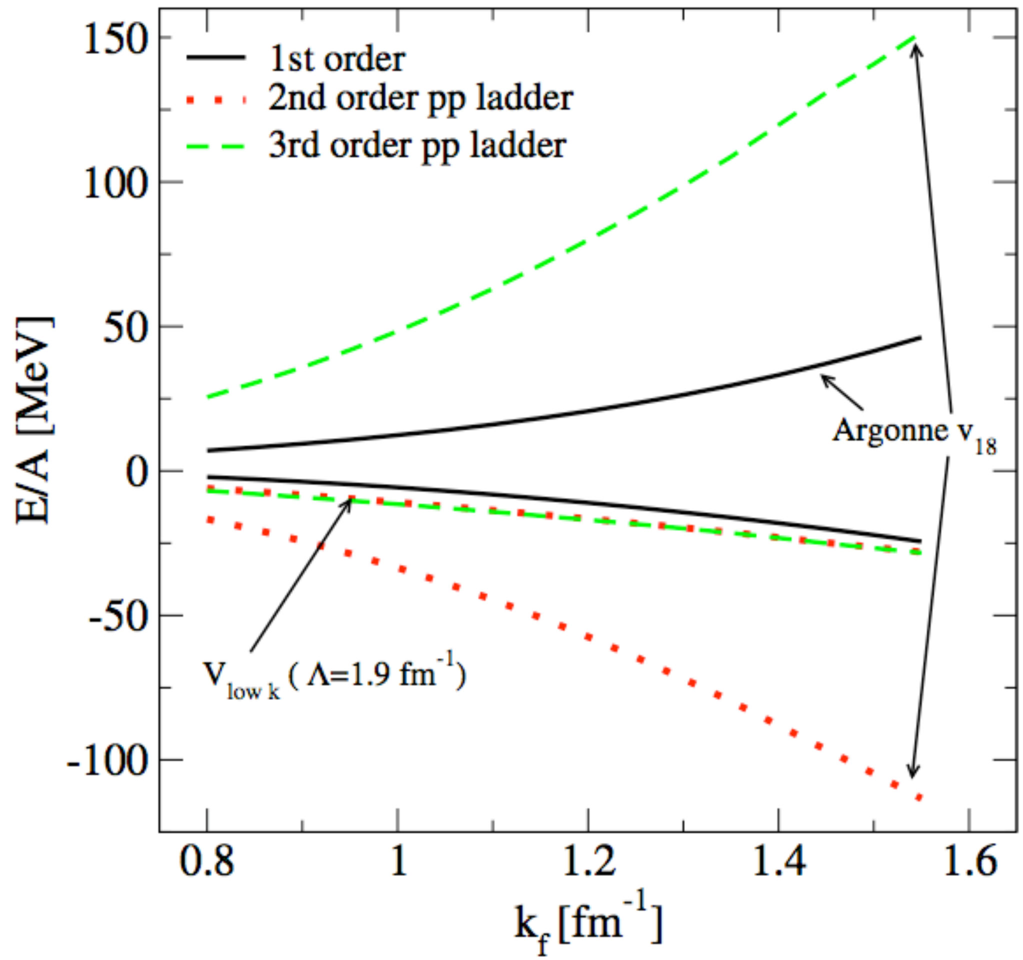

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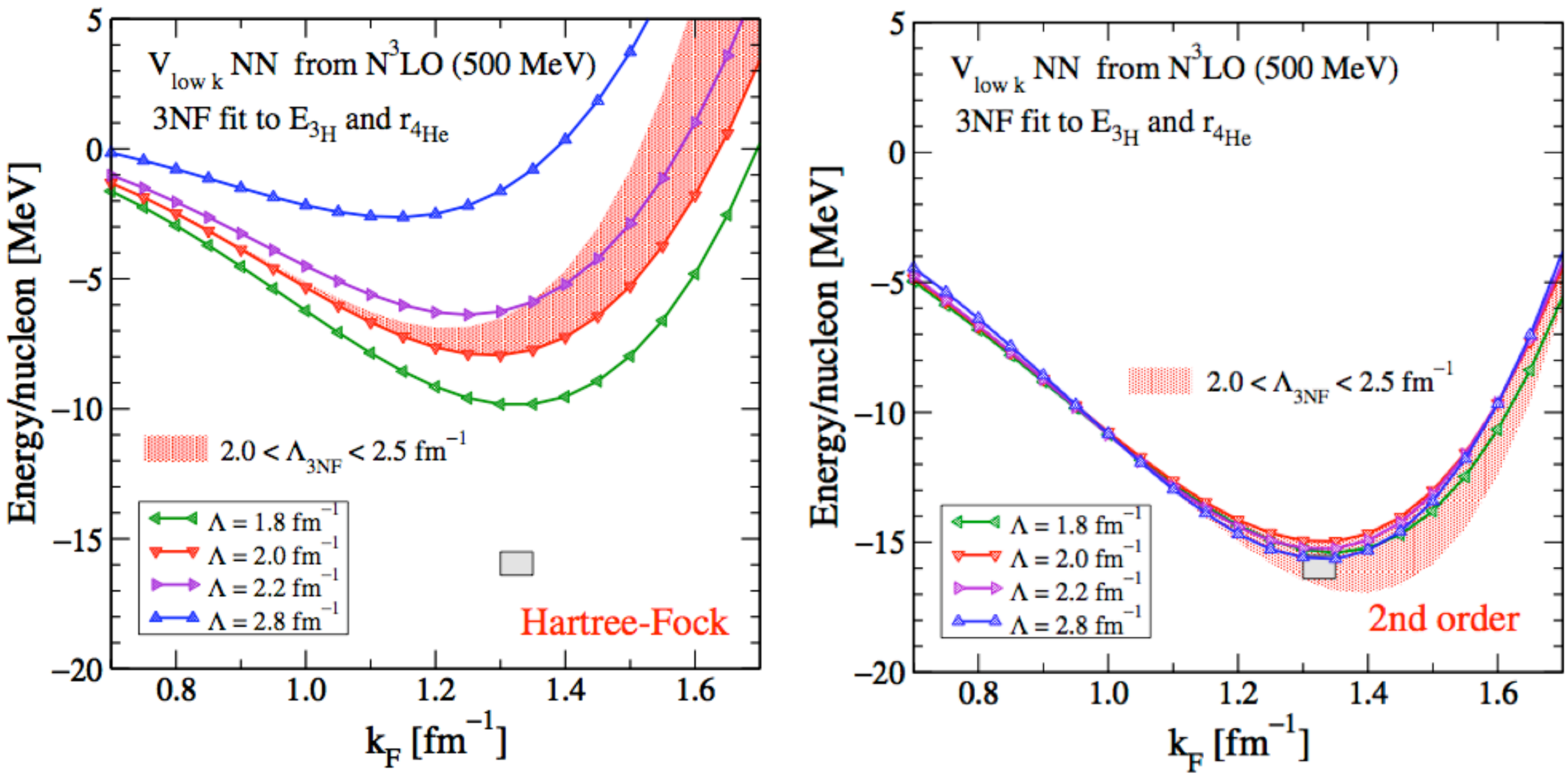
Bogner, AS, Furnstahl, Nogga (2005) + improvements, in prep.



provides guidance to UNEDF http://unedf.org

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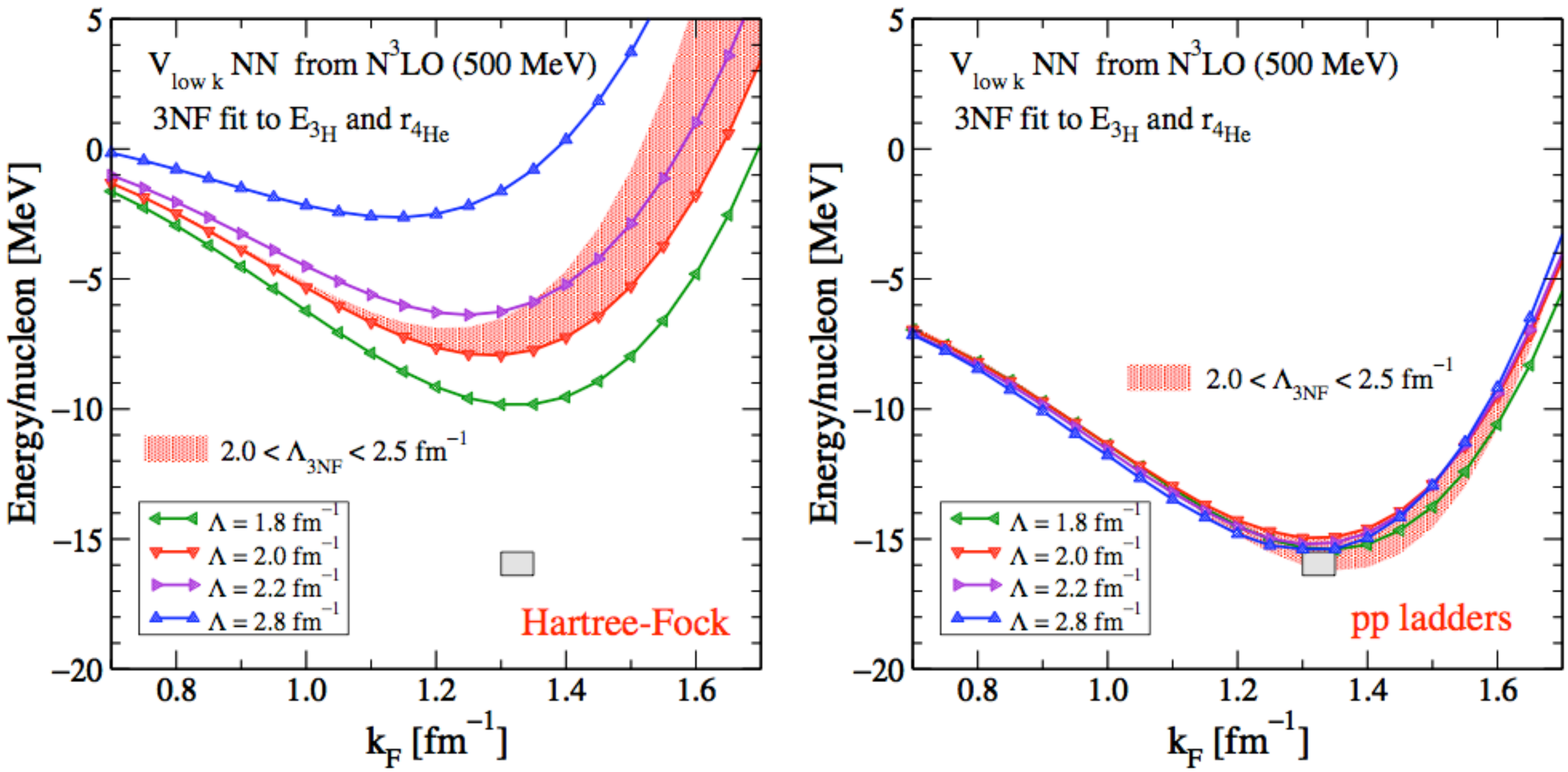

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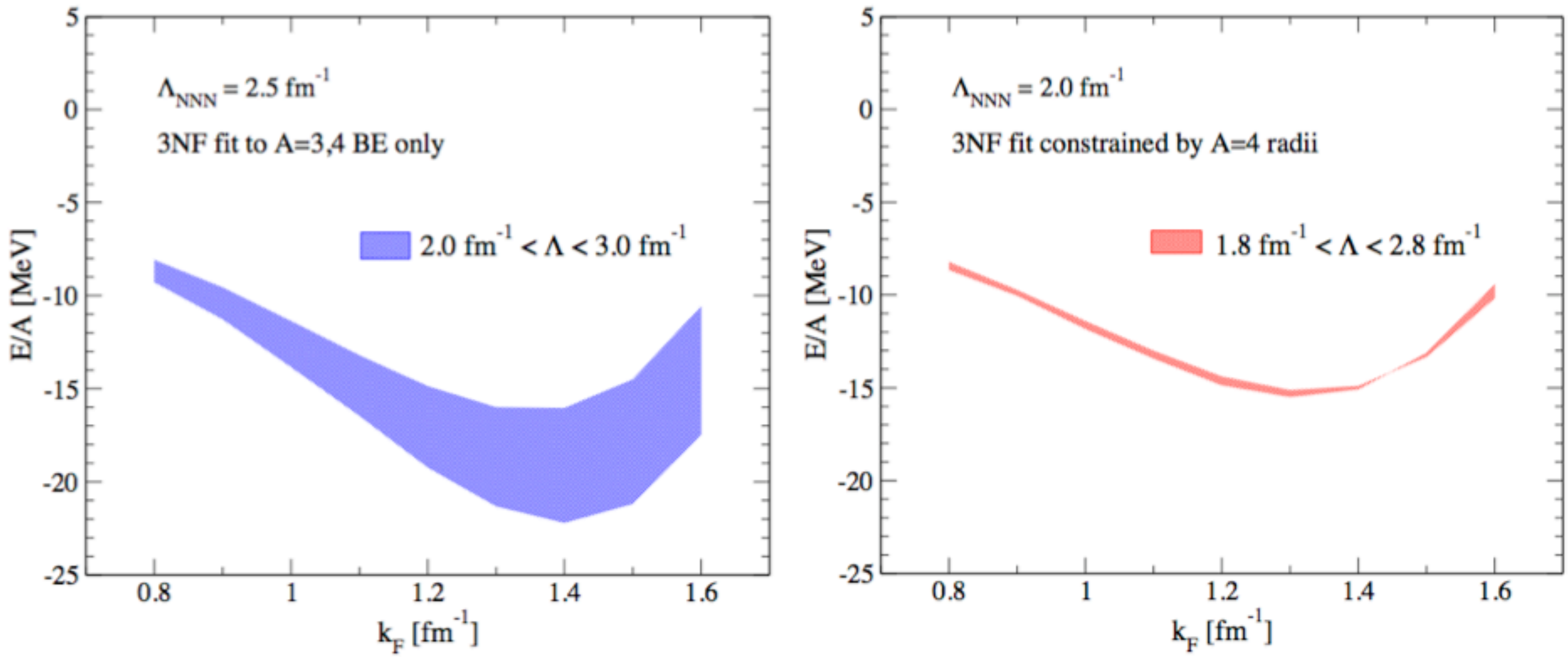
Nuclear matter with NN and 3N

comparison of 3N fits to ^{3}H, ^{4}He binding energies vs. ^{3}H be, ^{4}He radius

Bogner et al., in prep.

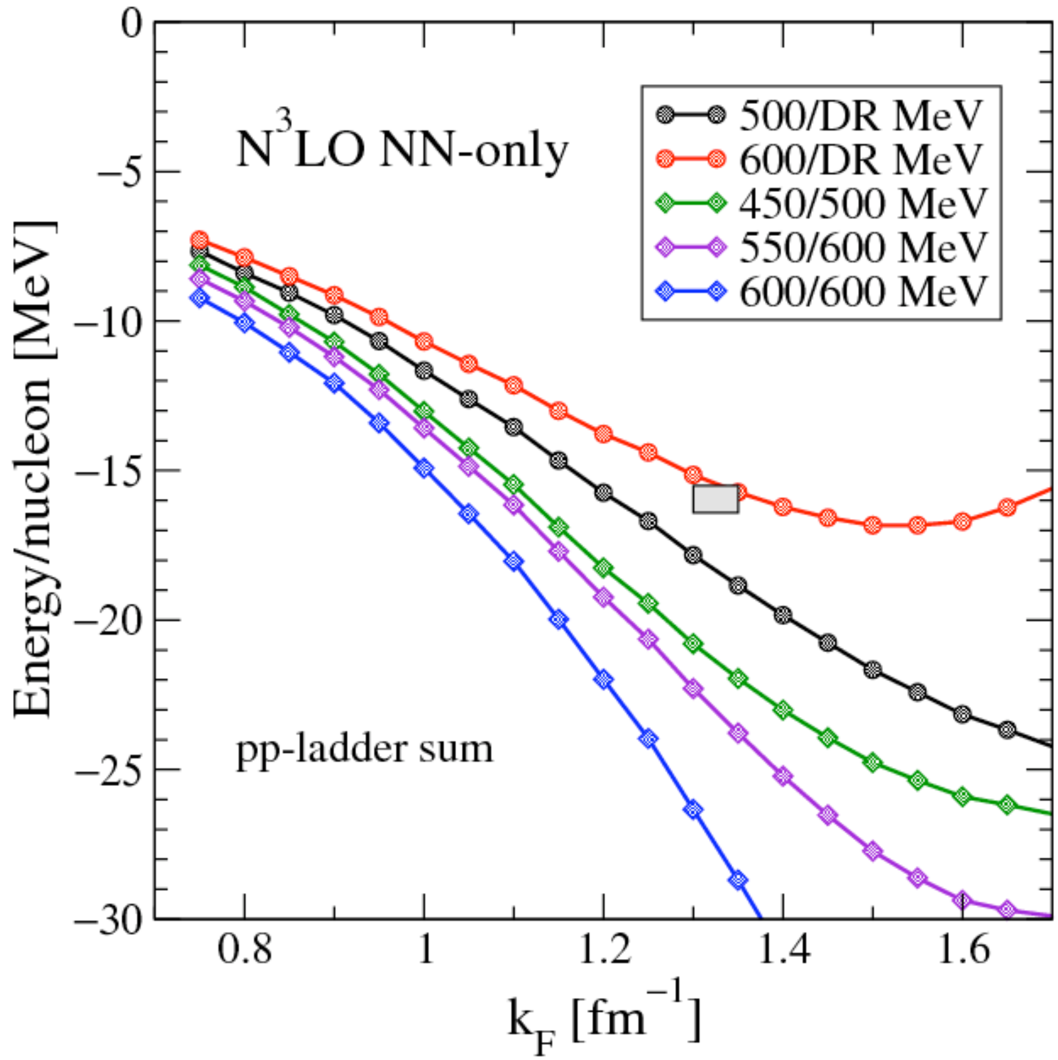


radius constraint improves cutoff dependence

Nuclear matter with "bare" N^3LO

3N contributions not expected to be small for N^3LO interactions

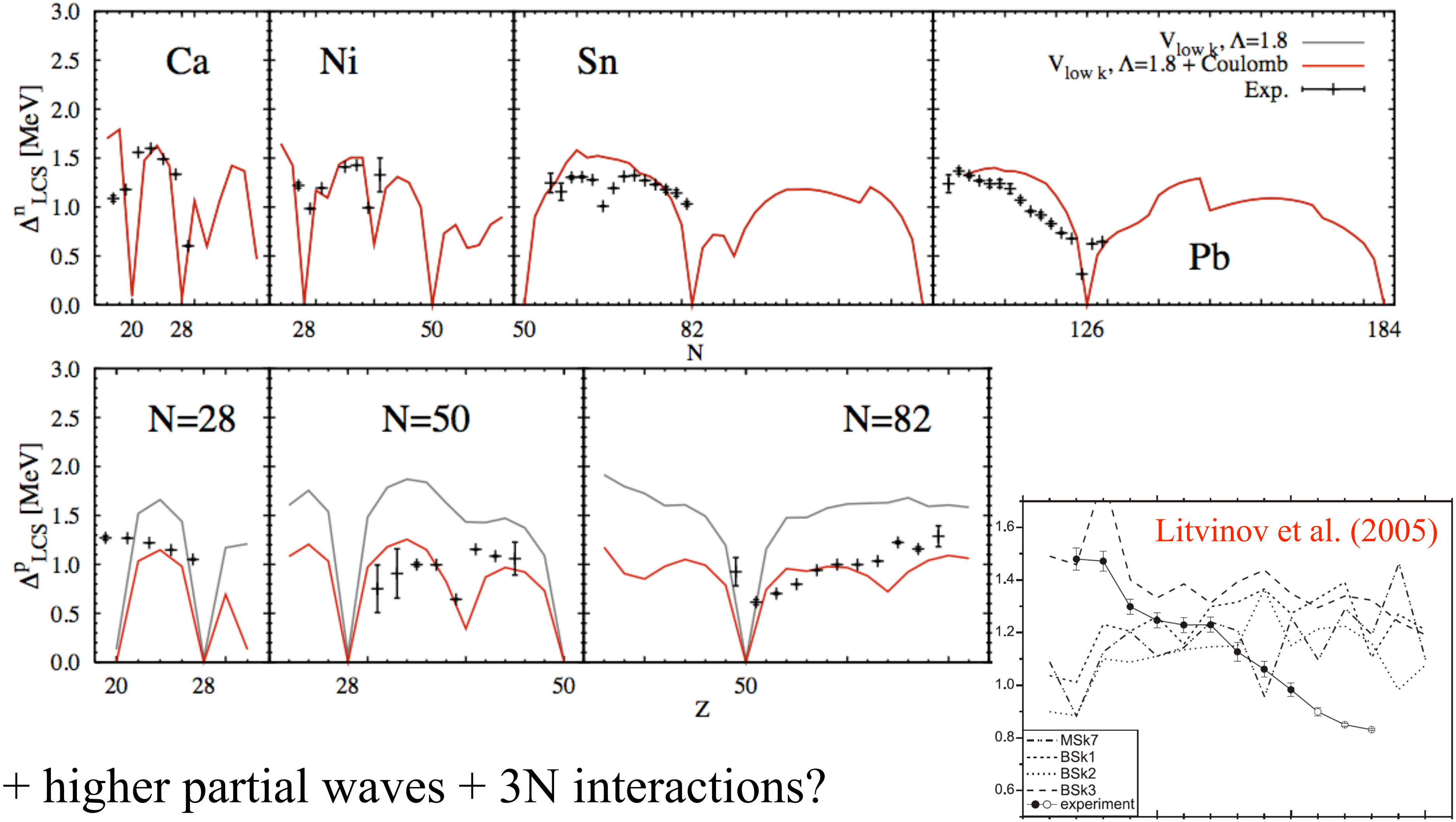
nuclear matter from different N^3LO potentials without 3N



Pairing gaps

first microscopic pairing functional from low-momentum interactions

Lesinski, Duguet, arXiv:0711.4386 and in prep.

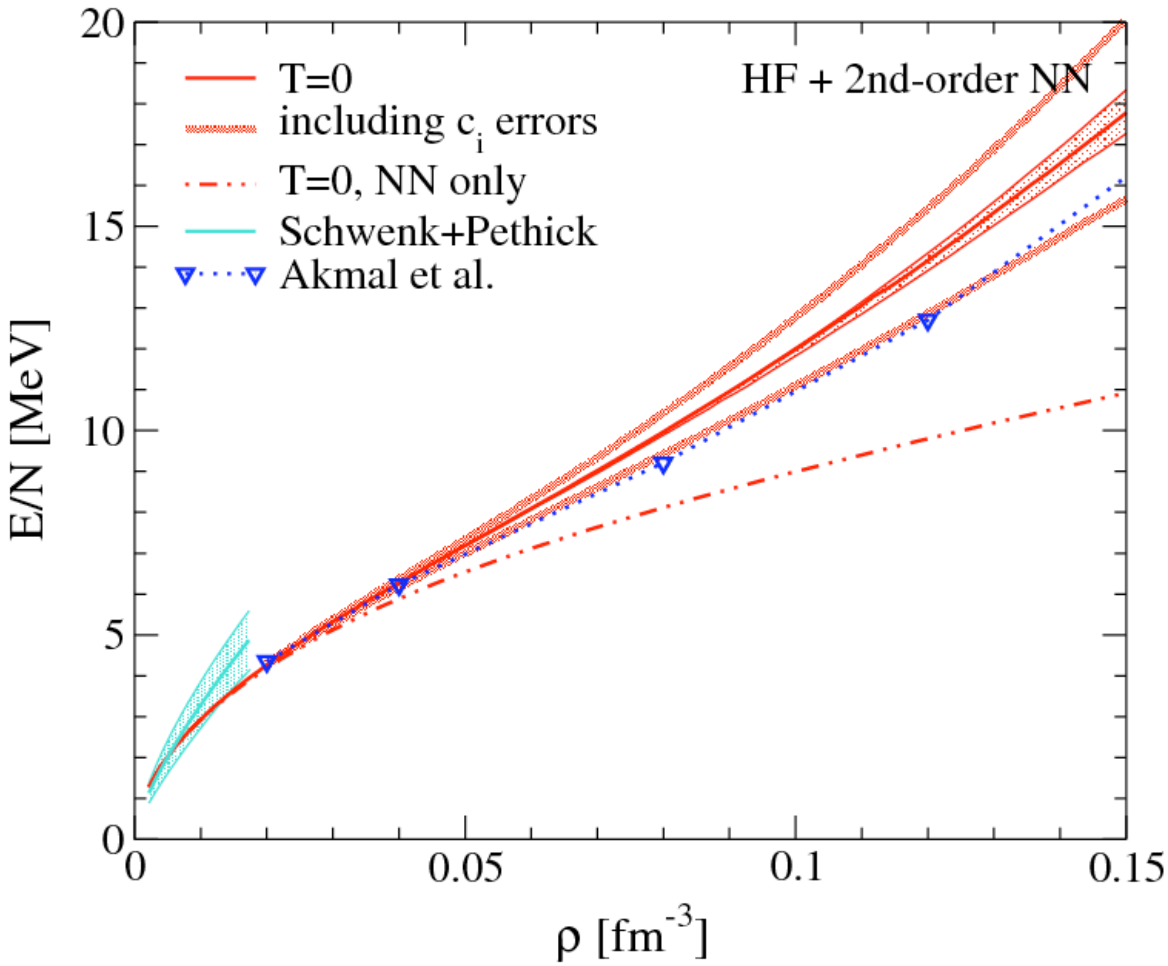


N - const. Z

+ higher partial waves + 3N interactions?

+ induced interactions in neutron-rich nuclei?

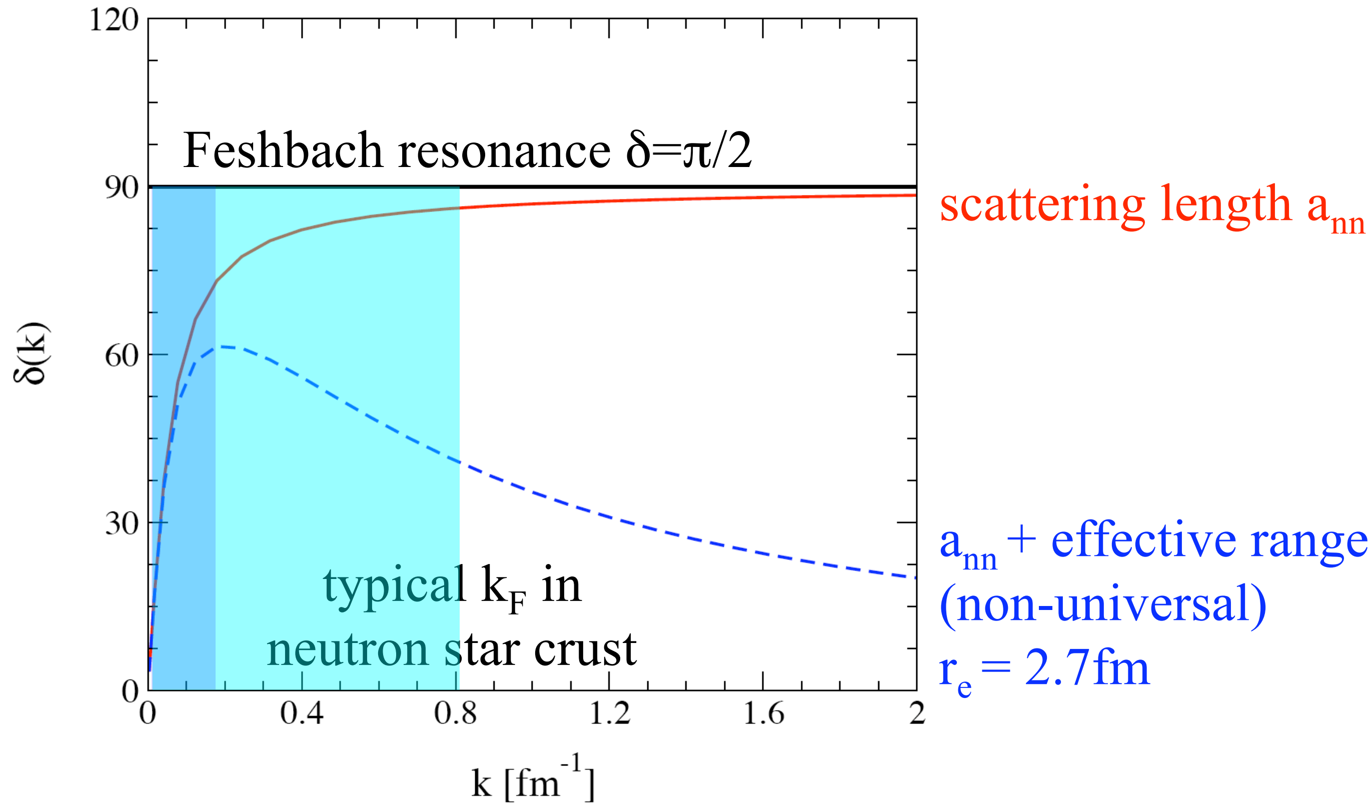
Neutron matter from NN and 3N



low densities from large scattering length and effective range

AS, Pethick (2005)

Neutron matter and non-universal corrections



phase shifts characterize strength of interaction

effective range important, weakens interactions at higher momenta

idea: large-N expansion, N = number of particles/resonantly-int. pairs

AS, Pethick (2005)

Neutron matter and non-universal corrections

di-Fermion EFT for large scattering length and large effective range

Weinberg (1963), Kaplan (1997),…

$$\mathcal{L} \;=\; \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2} \right) \psi - d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4} - \Delta \right) d \;-\; g \left(d^\dagger \psi\psi + d\,\psi^\dagger\psi^\dagger \right)$$

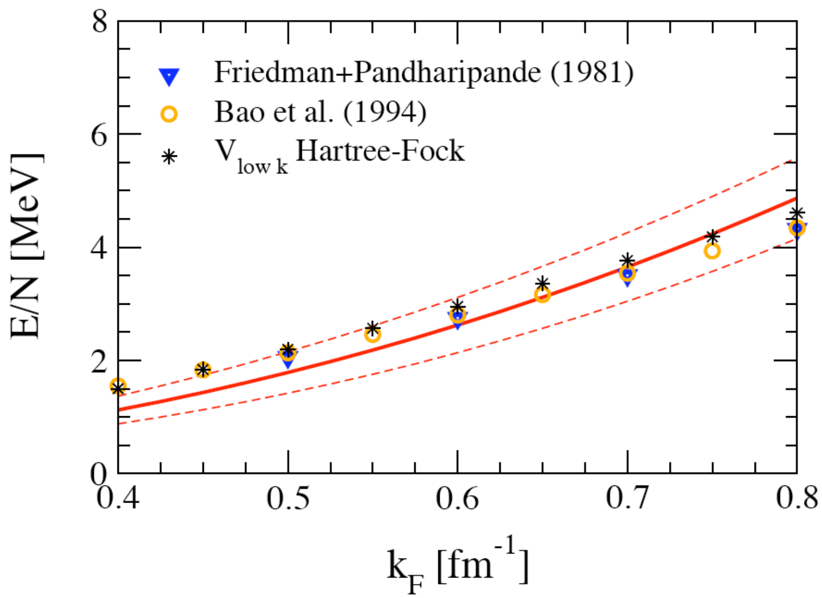
leading-order neutron matter E/N for $k_{\mathrm{F}} r_{\mathrm{e}} \lesssim 2$ or $\rho < 0.02\,\mathrm{fm}^{-3}$

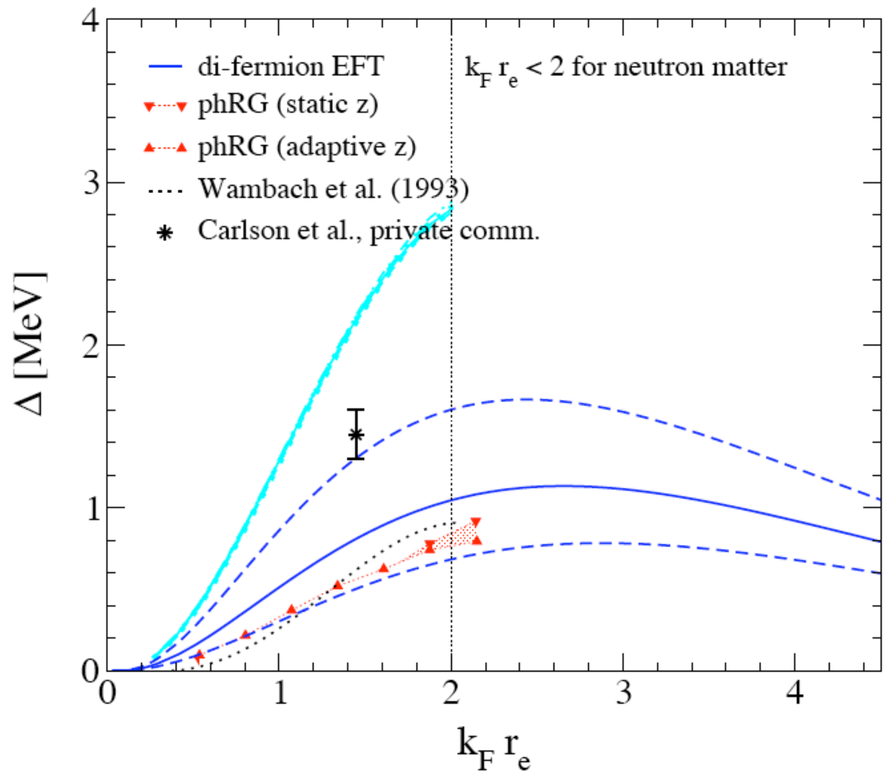
AS, Pethick (2005)

next-to-leading-order superfluid pairing gap

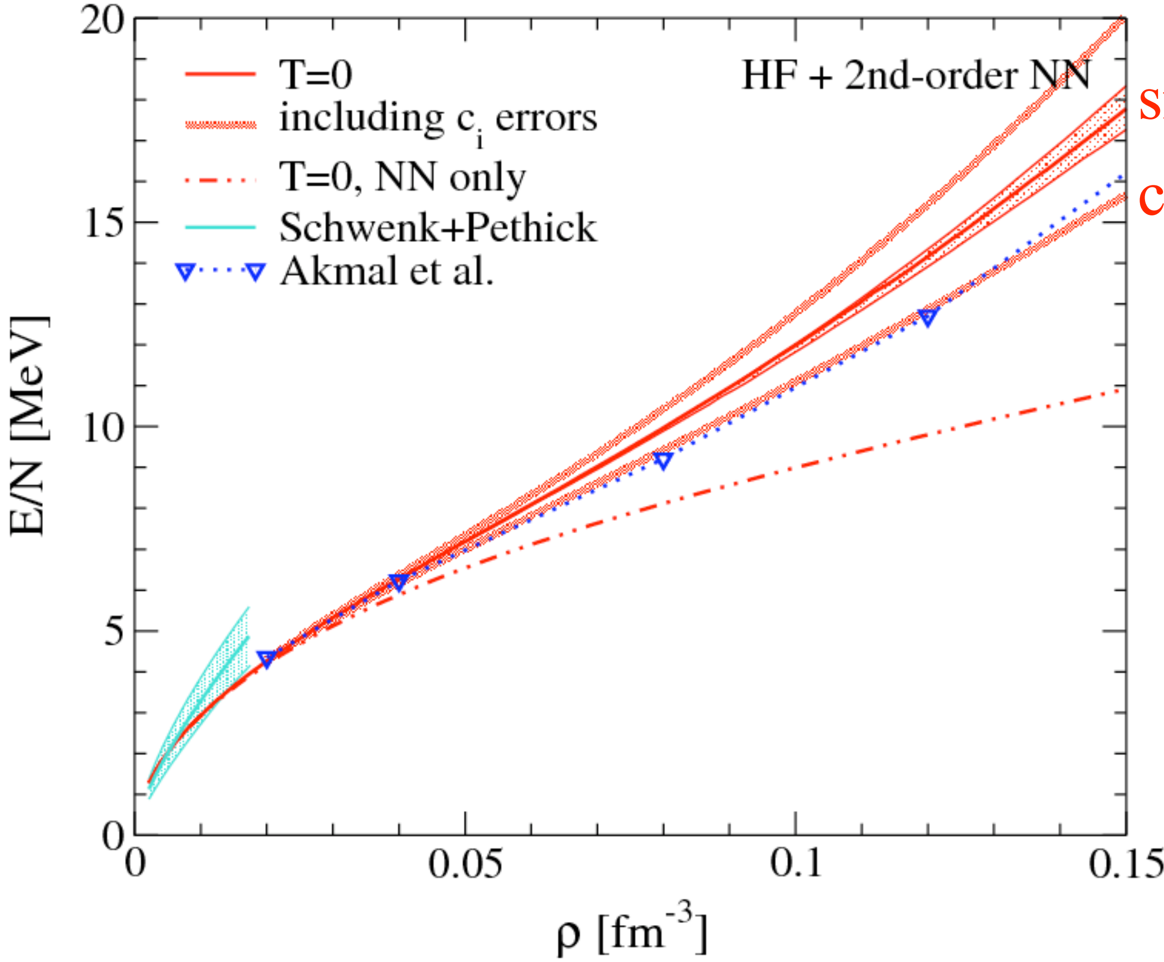
Reuter, AS, in prep.

microscopic calculations
within EFT error bands





Neutron matter from NN and 3N



small cutoff variation

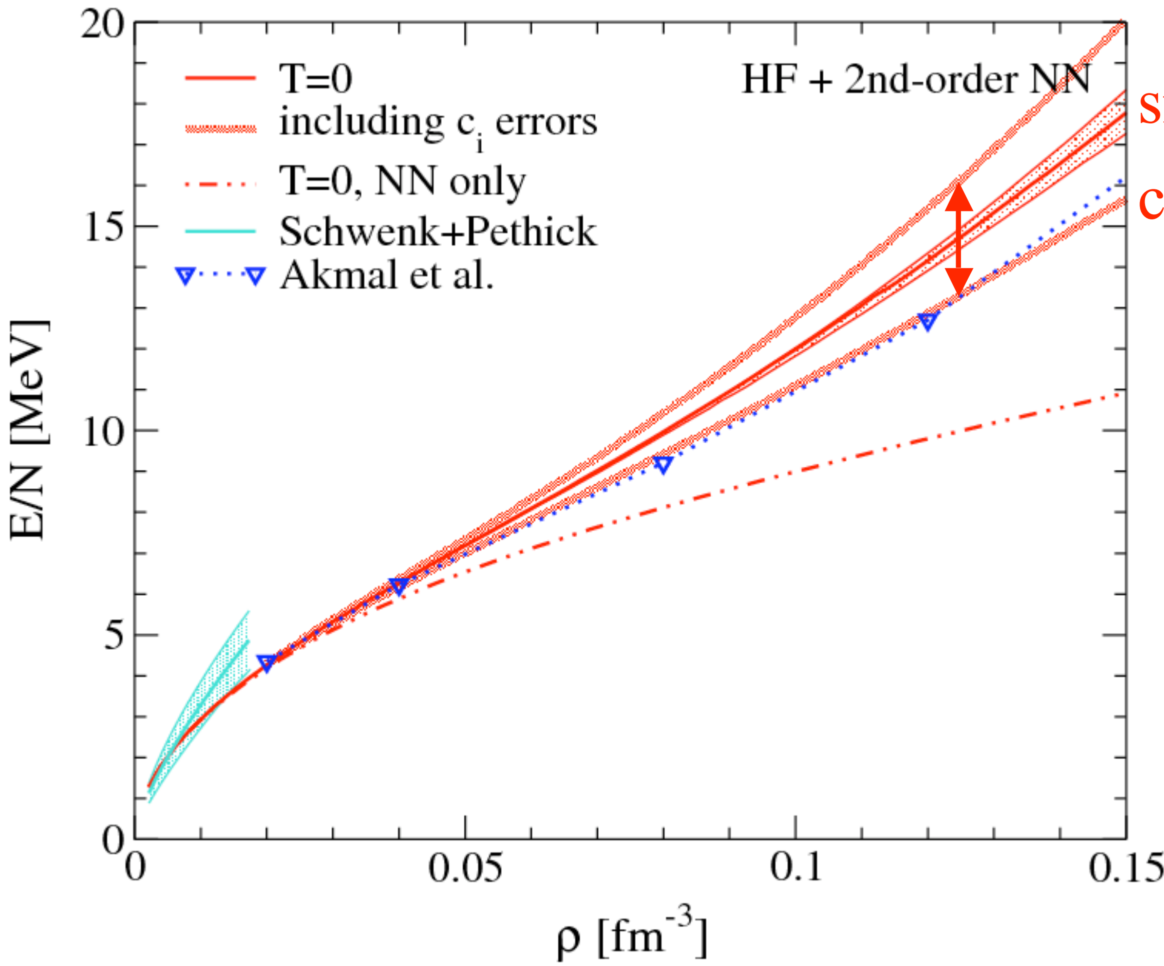
c_3 uncertainties

Tolos, Friman, AS (2007)

uncertainties from c_i overwhelm errors due to cutoff variation, mainly c_3 for neutron matter

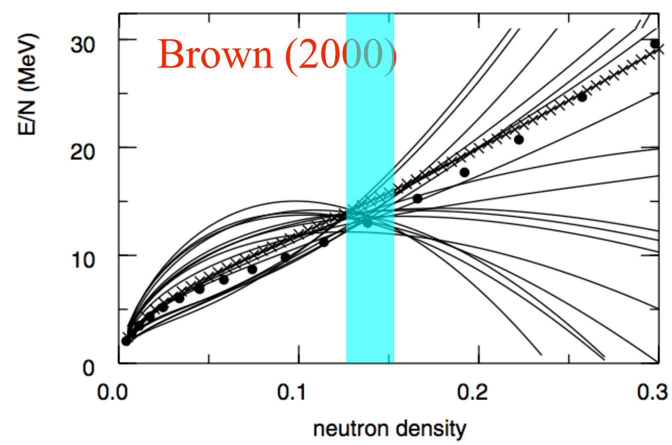
lower c_3 (Δ dominated): less repulsion, similar to results of AV18+UIX

Neutron matter from NN and 3N



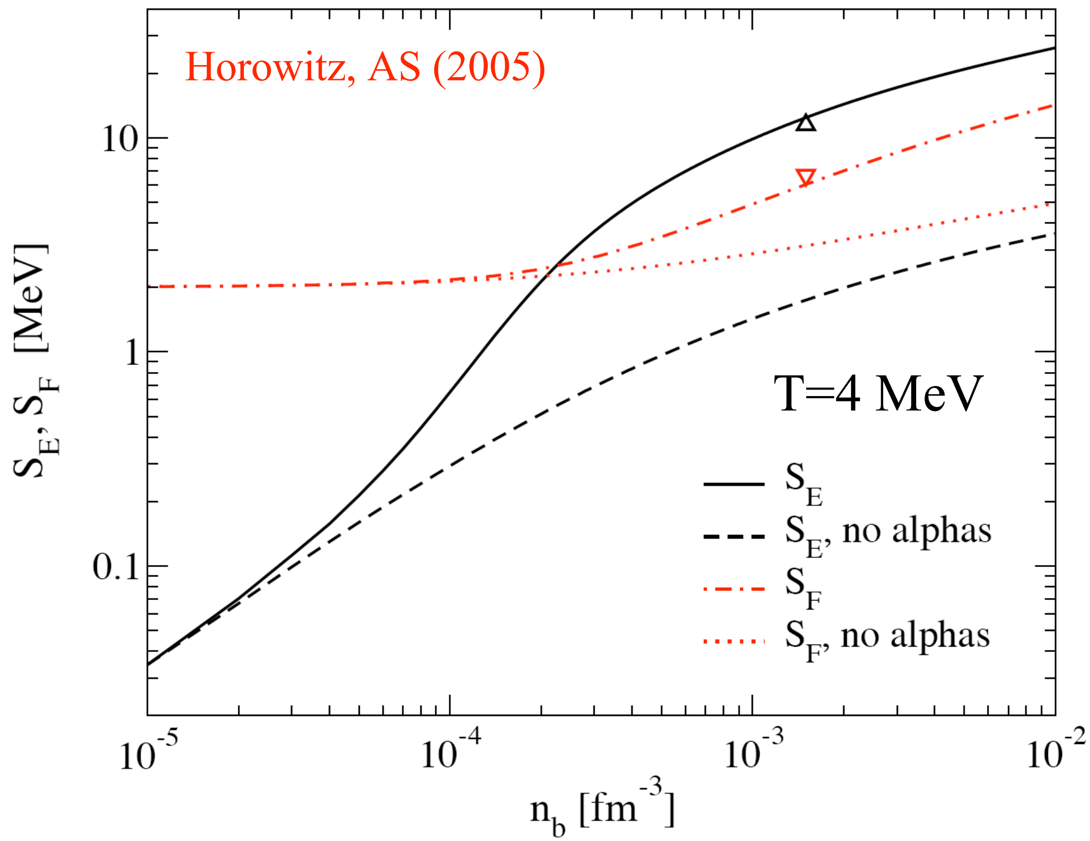
small cutoff variation

c_3 uncertainties



correlation of
low-momentum 3N interaction c_3 part
with symmetry energy Baroni et al., in prep.

Symmetry energy at low densities from virial expansion

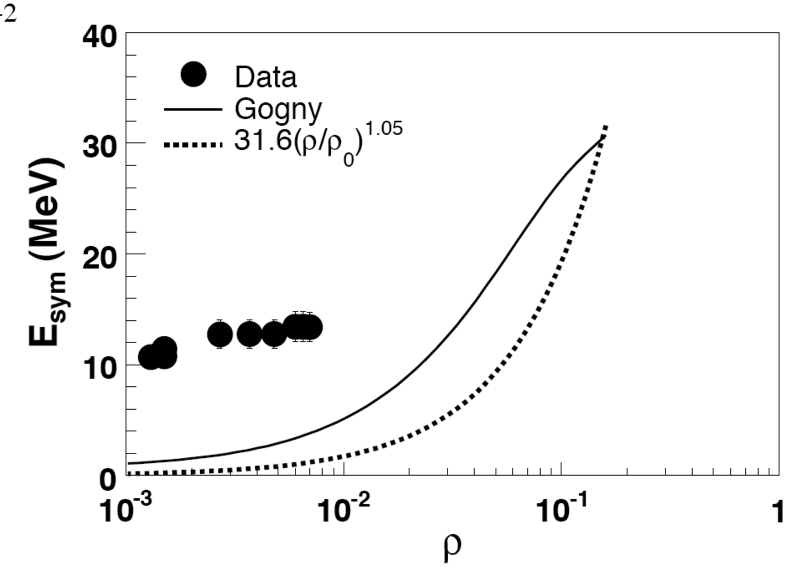


$$S_E = \frac{1}{8} \frac{\partial^2}{\partial {Y_p}^2} \frac{E}{A} \bigg|_{Y_p=1/2}$$

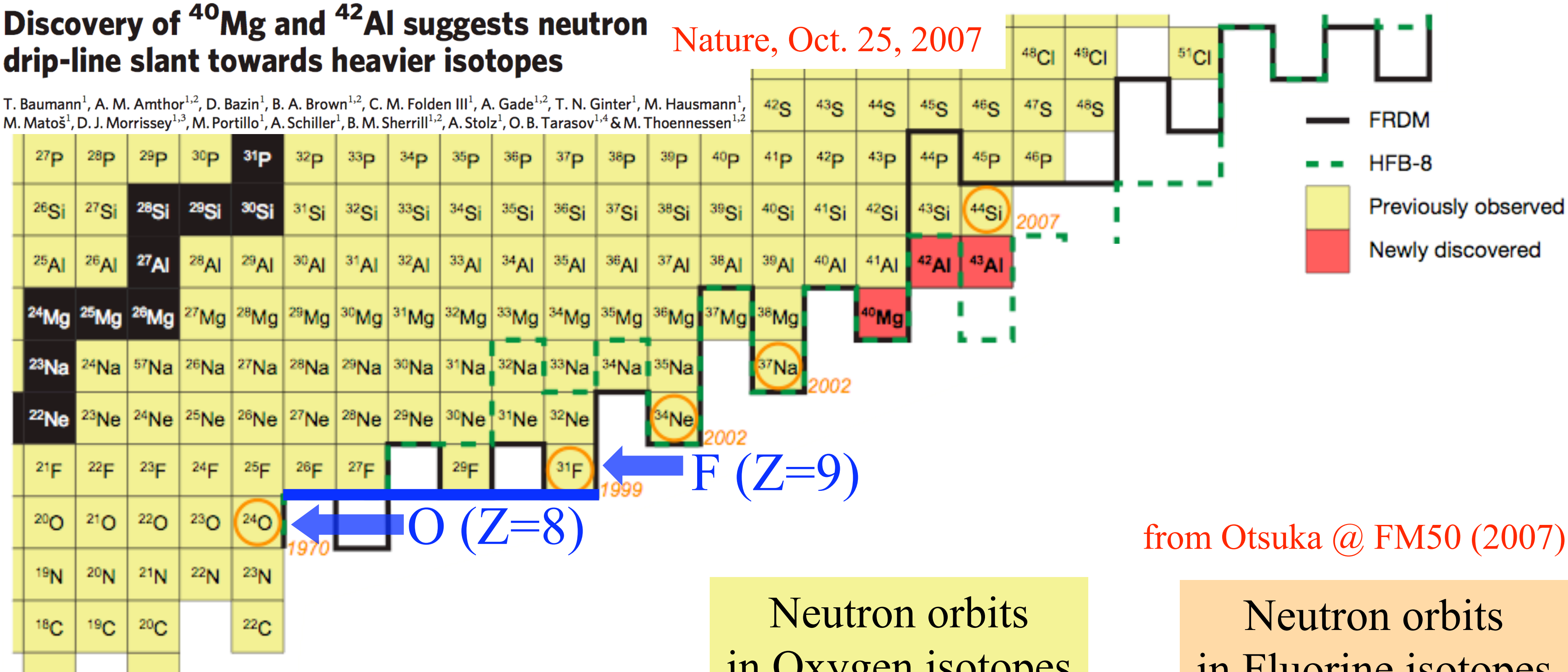
large S_E at low density due to clustering

large S_E confirmed in near Fermi-energy HI collisions

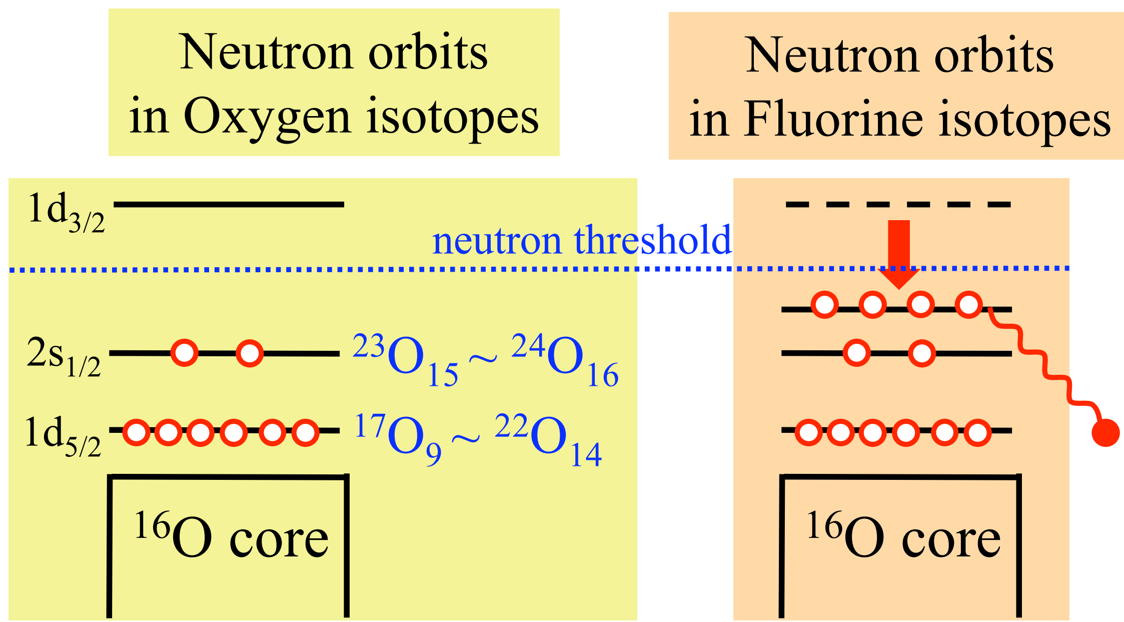
triangles: ^{64}Zn+^{92}Mo/^{197}Au (Y_p=0.44) Kowalski et al. TAMU (2006)



Neutron-rich nuclei and 3N interactions



from Otsuka @ FM50 (2007)



neutron $d_{3/2}$ - proton $d_{5/2}$
interaction binds
$d_{3/2}$ neutrons in Fluorine

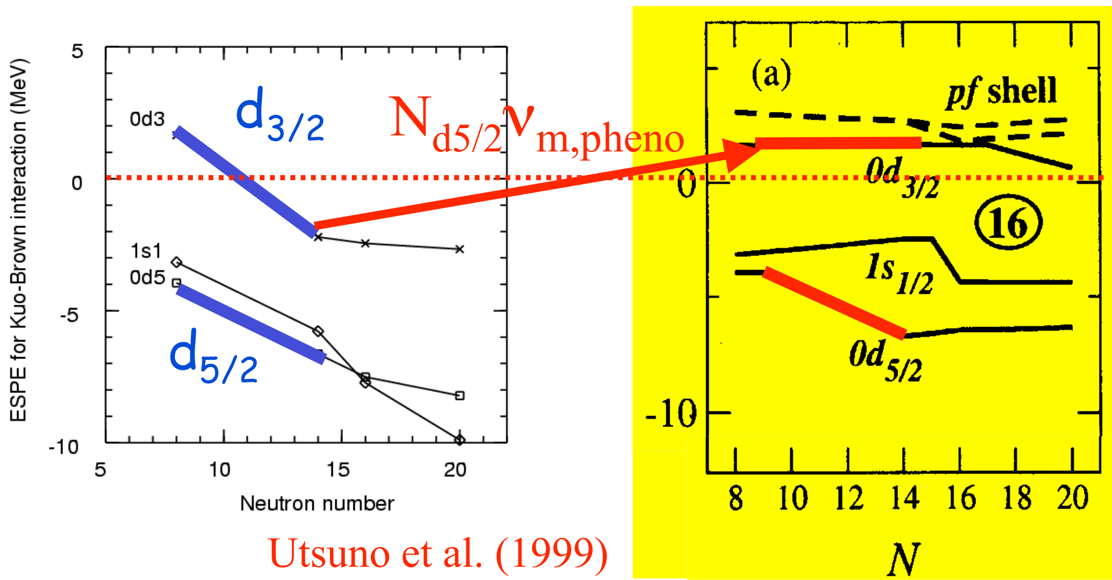
Why do $d_{5/2}$ neutrons not
bind $d_{3/2}$ in oxygen?

Oxygen isotopes without 3N interactions

Monopole part of nuclear forces $\mathcal{V}_{st}^{T} = \frac{\sum_J \mathcal{V}_{stst}^{JT}(2J+1)[1-(-)^{J+T}\delta_{st}]}{\sum_J (2J+1)[1-(-)^{J+T}\delta_{st}]}$

determines interaction of s with t orbit → change in $d_{3/2}$ by $N_{d5/2}\,\nu_m$

⇒ small changes in monopoles enhanced by number of neutrons



Utsuno et al. (1999)

microscopic NN interactions bind $d_{3/2}$ neutrons,
require phenomenological repulsive contribution to T=1 monopoles

→ neutron $d_{3/2}$ remains unbound, dripline at N=16 for Oxygen

first results indicate that $\nu_{m,pheno}$ due to 3N interactions

First results with 3N interactions

monopoles from $V_{\text{low k}}(\Lambda)$ + 2nd order (6hw)

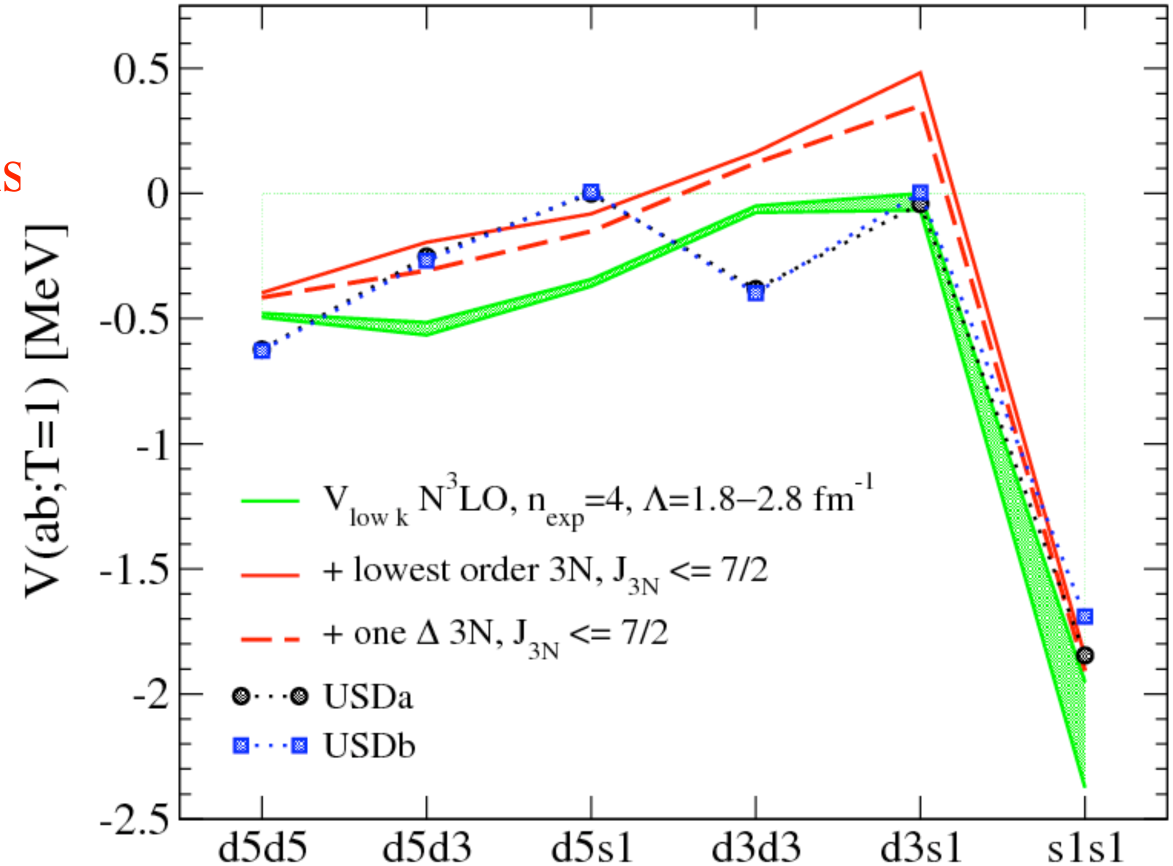
cutoff independent in T=1

find repulsive contributions from 3N interactions

dominated by c_i terms
→ Δ-hole contributions

Holt, Otsuka, AS, Suzuki, in prep.

reproduces hierarchy of orbitals and N=16 dripline



Holt, Otsuka, AS, Suzuki, in prep.

correlation of
low-momentum 3N interaction c_3 part with location of neutron drip line

similar results for monopoles in pf shell

Fermi gases with population imbalance

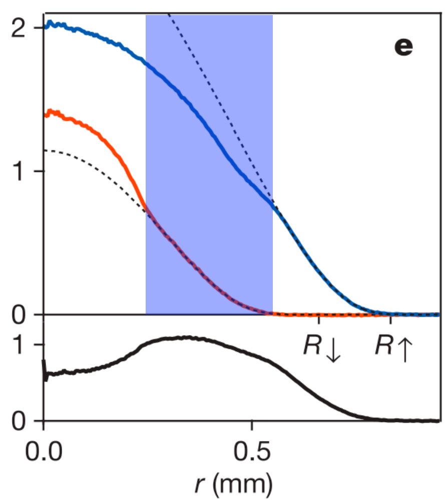
Superfluid core with equal densities and normal excess fermion wings

Zwierlein et al., Shin et al. (MIT, 2006), Partridge et al. (Rice, 2006)

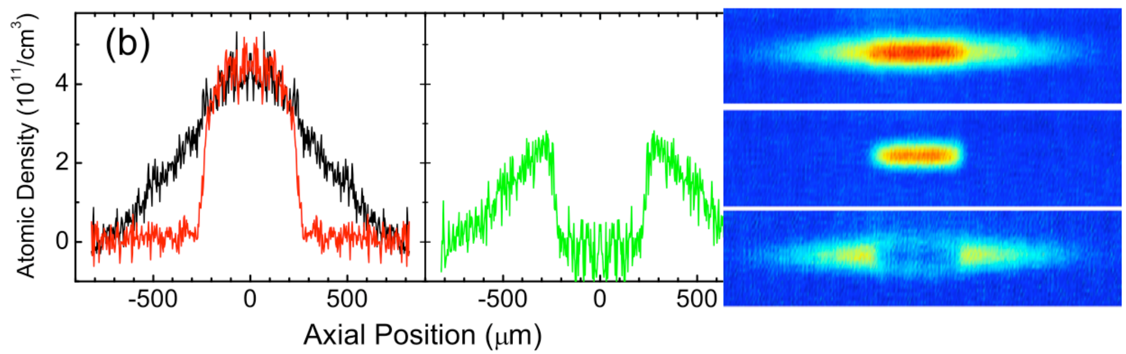
Difference in existence of intermediate phase

could be due to different trap aspect ratio, different N of particles Ku, Braun, AS, in prep.

MIT



Rice



Are spin skins similar to neutron skin?

cold atoms:

attractive short-range S-wave interaction favors equal spin densities

pairing is strong (very weak shell effects)

Thanks to collaborators

TRIUMF S. Bacca, S. Baroni, J. Braun, J.D. Holt, M. Ku, E. O'Connor

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Iowa State University P. Maris, J.P. Vary

University of Oslo M. Hjorth-Jensen

東京大学 The University of Tokyo T. Otsuka

日本大学 T. Suzuki

Summary

For microscopic NN and 3N calculations, a measurement of the neutron skin in lead constrains 3N (many-N?) interactions via the symmetry energy. This correlates with the location of the neutron dripline, as shown for oxygen.