Progress toward determining the light baryon SPECTRUM in lattice QCD

- 2005-06 SPECTRUM Collaboration of LHPC

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- Goal: Theoretical determination of the spectrum of baryons and other hadrons.

- Means: Lattice QCD
  - Variational method
  - Lots of interpolating field operators

Use quenched QCD to prove methods & look at light baryons

- $16^3 \times 64$ (167 configs) and $24^3 \times 64$ (239 configs) anisotropic lattices
- m_π = 490 MeV.
- Local operators
- One-link-displaced nonlocal operators
- Calculations performed using Jlab clusters

Outline

- Spin on the lattice
- A lattice view of the physical spectrum
- Variational method
- Stability of eigenvectors
- Pattern of low-lying states at a_s = 0.1 fermi
- Evidence for spin $\frac{5}{2}$
- Summary

Energies from correlation functions

$$C(t) = \sum_{\mathbf{x}} \langle 0 \big| B(\mathbf{x},t)\overline{B}(\mathbf{0},0) \big| 0 \rangle$$
$$= Z_1^2 e^{-E_1 t} + Z_2^2 e^{-E_2 t} + \cdots \quad (1)$$

Quantum numbers of states

- Spin quantum numbers $\mathbf{J}^2$ and J_z are not "good".
- Irreducible representations (irreps) of octahedral group are "good".

$$\sum_x \langle 0| \, T B_k^{(\Lambda\lambda)}(\mathbf{x},t) \overline{B}_{k'}^{(\Lambda',\lambda')}(0) \, |0\rangle =$$

$$C_{kk'}^{(\Lambda,\lambda)}(t) \delta_{\mathbf{\Lambda\Lambda'}} \delta_{\lambda\lambda'}$$

Double octahedral group, $\mathcal{O}^D$ and subduction of J.

- $G_{1g}(2)$, $H_g(4)$, $G_{2g}(2)$ irreps for positive parity.

- $G_{1u}(2)$, $H_u(4)$, $G_{2u}(2)$ irreps for negative parity.

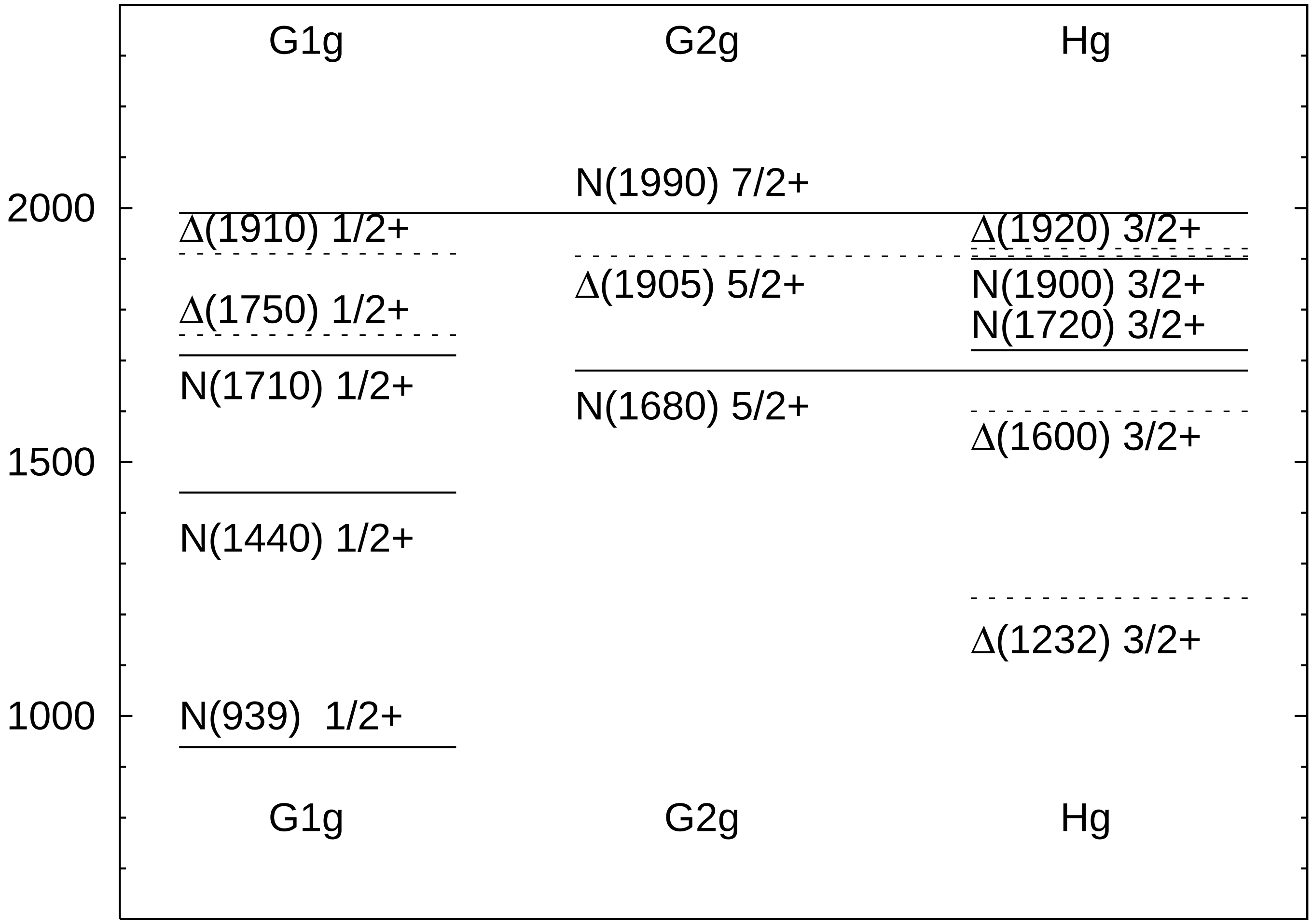
irrep of O^D	Spin J			
G_1	$1/2$			$7/2$
H		$3/2$	$5/2$	$7/2$
G_2			$5/2$	$7/2$

- Spin $\frac{1}{2}$ is in G_1 channel, $\frac{3}{2}$ in H channel
- Spin $\frac{5}{2}$ is in both G_2 and H

Example: As $a_s \to 0$, degenerate states

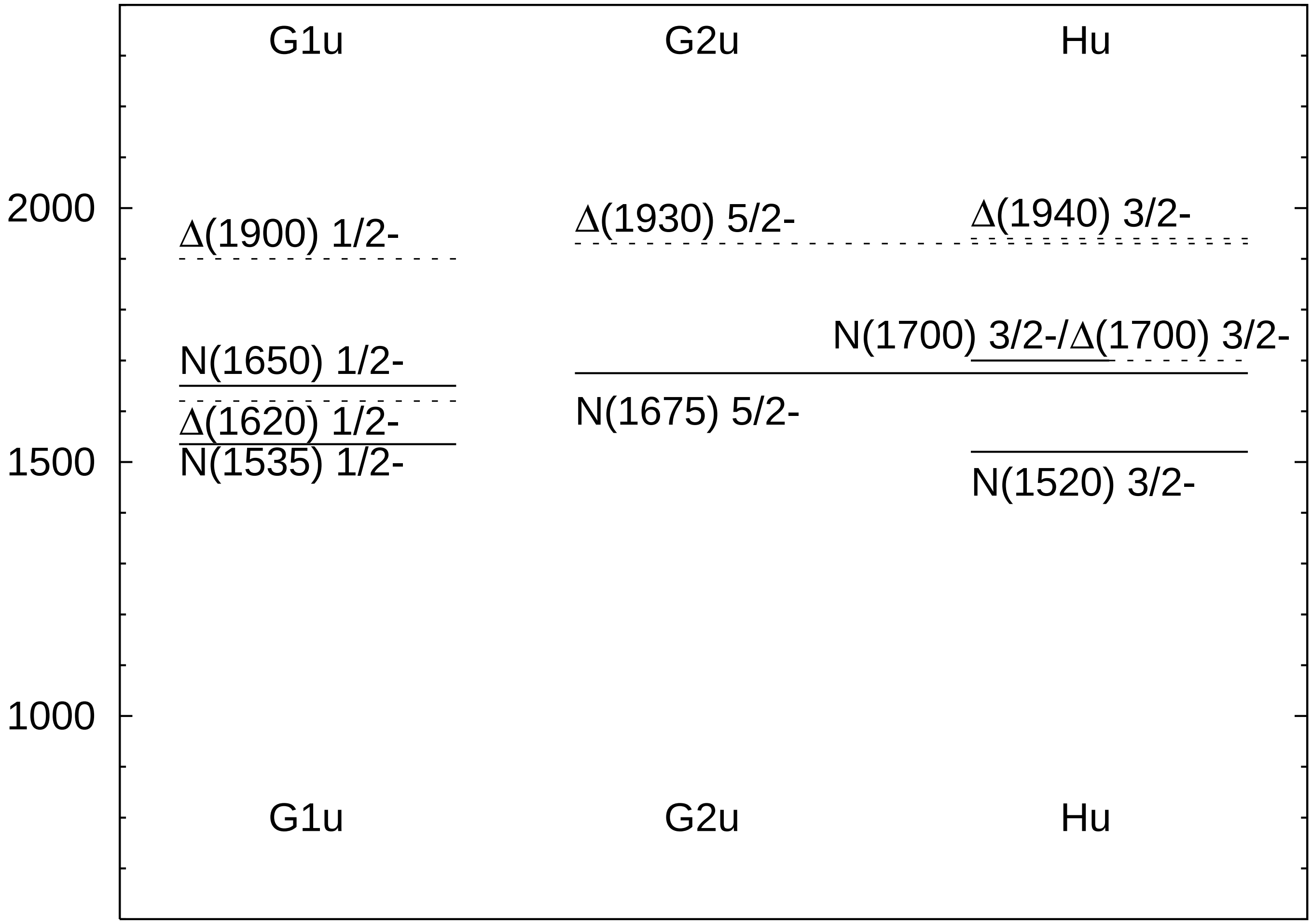
$$\begin{pmatrix} E_n(H) \\ E_m(G_2) \end{pmatrix} \longrightarrow E(J = 5/2),$$

Physical positive-parity spectrum subduced



- **Scattering states starting at $M_N + m_\pi$ become discrete states on lattice.**

Physical negative-parity spectrum subduced



- **With m_π= 490 MeV, $M_N + m_\pi$ s-wave state is close to $N(\frac{1}{2}^-, 1535)$.**

Need for nonlocal operators.

- Three spin $\frac{1}{2}$ local operators can produce $S = \frac{1}{2}$ or $S = \frac{3}{2}$.
- Example: $N_{121} = (u_1 d_2 - d_1 u_2) u_1$ is $S = \frac{1}{2}$ Nucleon operator
- No higher spins can be produced.
- No G_2 irreps can be produced.
- Nonlocal operators: discretizations of $L = 1$, $L = 2$, ...
- $L = 1 \oplus S = \frac{3}{2} \rightarrow J = \frac{5}{2} \rightarrow G_2$

Simplest nonlocal operators

- **Displace one quark relative to the other two.**
- **Form irreps of displacements:** $\mathcal{D}^{\Lambda_1}_{\lambda_1}$
- **Use Clebsch-Gordan coefficients for octahedral group to make overall irreps**

$$\overline{\mathcal{O}}_{\bar{k}}^{(\Lambda,\lambda)} = \sum_{\lambda_1,\lambda_2} C\begin{pmatrix} \Lambda & \Lambda_1 & \Lambda_2 \\ \lambda & \lambda_1 & \lambda_2 \end{pmatrix} \mathcal{D}^{\Lambda_1}_{\lambda_1} \overline{\Psi}_k^{(\Lambda_2,\lambda_2)}.$$

One-link displacement irrep operators $\mathcal{D}_\lambda^\Lambda$

$$\left(\hat{A}_1\overline{B} \quad \hat{D}_+\overline{B} \quad \hat{D}_-\overline{B} \quad \hat{D}_0\overline{B} \quad \hat{E}_0\overline{B} \quad \hat{E}_2\overline{B}\right)^T \equiv$$

$$\begin{pmatrix} \frac{1}{\sqrt{6}}(\hat{d}_x\overline{B} + \hat{d}_y\overline{B} + \hat{d}_z\overline{B} + \hat{d}_{-x}\overline{B} + \hat{d}_{-y}\overline{B} + \hat{d}_{-z}\overline{B}) \\ \frac{i}{2}[(\hat{d}_x\overline{B} - \hat{d}_{-x}\overline{B}) + i(\hat{d}_y\overline{B} - d_{-y}\overline{B})] \\ -\frac{i}{2}[(\hat{d}_x\overline{B} - \hat{d}_{-x}\overline{B}) - i(\hat{d}_y\overline{B} - \hat{d}_{-y}\overline{B})] \\ -\frac{i}{\sqrt{2}}(\hat{d}_z\overline{B} - \hat{d}_{-z}\overline{B}) \\ \frac{1}{\sqrt{12}}[2(\hat{d}_z\overline{B} + \hat{d}_{-z}\overline{B}) - (\hat{d}_x\overline{B} + \hat{d}_{-x}\overline{B}) - (\hat{d}_y\overline{B} + \hat{d}_{-y}\overline{B})] \\ \frac{1}{2}[(\hat{d}_x\overline{B} + \hat{d}_{-x}\overline{B}) - (\hat{d}_y\overline{B} + \hat{d}_{-y}\overline{B})] \end{pmatrix} \tag{2}$$

Combinations of displacements that transform according to the A_1, T_1, and E single-valued irreps of the octahedral group provide lattice discretizations of the spherical harmonics $Y_{\ell m}$,

$\hat{A}_1 \sim Y_{00}$,

$\hat{D}_{+,0,-} \sim Y_{11}, \quad Y_{10}, \quad Y_{1-1}$,

$\hat{E}_{0,2} \sim Y_{20}, \quad (Y_{22} + Y_{2-2})$.

Charge Conjugation

$$C^{(\Lambda\lambda)}_{kk'}(t) = \mathcal{P}_{k'}\delta_{\mathbf{P},0}\Big[\sum_n \theta(t)\langle 0|B^{(\Lambda\lambda)}_k|n\rangle\langle n|\overline{B}^{(\Lambda\lambda)}_{k'}|0\rangle e^{-E_n t}$$
$$-\sum_{\bar{n}} \theta(-t)\langle 0|\overline{B}^{(\Lambda\lambda)}_{k'}|\bar{n}\rangle\langle\bar{n}|B^{(\Lambda\lambda)}_k|0\rangle e^{E_{\bar{n}} t}\Big], \quad (3)$$

Charge conjugation relations $|\bar{n}\rangle = \mathcal{C}\,|n\rangle\, e^{i\phi}$ produce a relation between correlation functions

$$C^{(\Lambda\lambda)}_{kk'}(t) = \delta_{\mathbf{P},0}\sum_n \Big[\theta(t)\mathcal{P}^{(\Lambda)}_{k'}\langle 0|B^{(\Lambda\lambda)}_k|n\rangle\langle n|\overline{B}^{(\Lambda\lambda)}_{k'}|0\rangle e^{-E_n t}$$
$$-\eta_t\theta(T-t)\mathcal{P}^{(\Lambda_c)}_{k'}\langle 0|B^{(\Lambda_c\lambda_c)}_k|n\rangle^*\langle n|\overline{B}^{(\Lambda_c\lambda_c)}_{k'}|0\rangle^* e^{-E_{\bar{n}}(T-t)}\Big]. \quad (4)$$

The forward propagating signal of a correlation function is equal to the backward propagating signal of the parity-reversed, complex-conjugated correlation function within the factor $-\eta_t$, i.e.,

$$C^{(\Lambda\lambda)}_{kk'}(t) = -\eta_t C^{(\Lambda_c\lambda_c)*}_{kk'}(T-t). \quad (5)$$

Doubling of statistics

- For + parity operators use $\overline{B}^{(\Lambda)}$.
- For - parity operators use $\overline{B}^{(\Lambda_c)}$.
- Each provides a correlation function in $0 < t < T/2$ for BOTH parities.
- They are independent samples: $\approx$ 10 time slices apart.
- One is from $t < T/2$ and the other is from $t > T/2$.
- Number of samples is double the number of gauge configurations.
- $167 \rightarrow 334$ on 16^3, $239 \rightarrow 478$ on 24^3.

Variational method

- **Diagonalize matrices of correlation functions $C_{kk'}(t)$**

- **Extract spectrum of energies.**

1.) Renormalize: $\widetilde{C}^{(\Lambda)}_{kk}(t_0) = 1$.

$$\widetilde{C}^{(\Lambda)}_{kk'}(t) = N_k C^{(\Lambda)}_{kk'}(t) N_{k'},$$

2.) Solve generalized eigenvalue eq.

$$\sum_{k'} \widetilde{C}^{(\Lambda)}_{kk'}(t) v^{(n)}_{k'} = \alpha^{(n)}(t,t_0) \sum_{k'} \widetilde{C}^{(\Lambda)}_{kk'}(t_0) v^{(n)}_{k'},$$

3.) Obtain principal eigenvalues

$$\alpha^{(n)}(t,t_0) \simeq e^{-E_n(t-t_0)}\left(1 + \mathcal{O}(e^{-|\delta E|t})\right),$$

Eigenvectors

Eigenvectors diagonalize the renormalized correlation matrix:

$$v_k^{(n)T}(t,t_0)\widetilde{C}_{kk'}^{(\Lambda)}(t)v_{k'}^{(n')}(t,t_0) = \alpha^{(n)}(t,t_0)\delta_{nn'}.$$

Diagonal operators:

$$\overline{\mathcal{O}}_n^{(\Lambda\lambda)} = \sum_k \widetilde{v}_k^{(n)}(t,t_0)\overline{B}_k^{(\Lambda\lambda)}.$$

Normalization:

$$\widetilde{v}_k^{(n)}(t,t_0) \equiv Z_n(t)N_k v_k^{(n)}(t,t_0),$$

such that $\sum_k |\widetilde{v}_k^{(n)}(t,t_0)|^2 = 1.$

Diagonal correlation function:

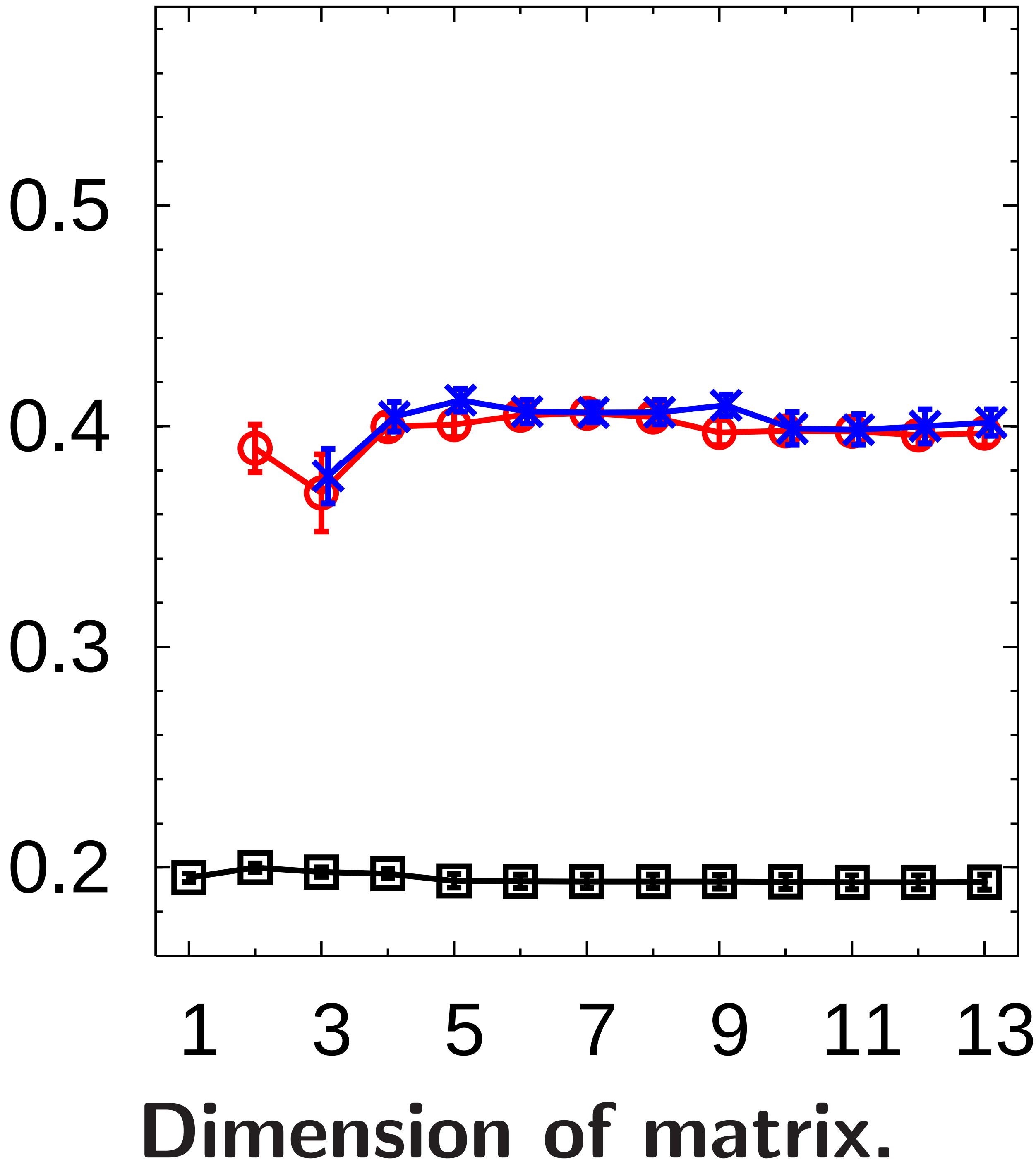
$$\widetilde{C}_{nn'}^{(\Lambda)}(t) = |Z_n(t)|^2\alpha^{(n)}(t,t_0)\delta_{n,n'}.$$

Weights of basis operators: $|\widetilde{v}_k^{(n)}(t,t_0)|^2$.

Volume dependence

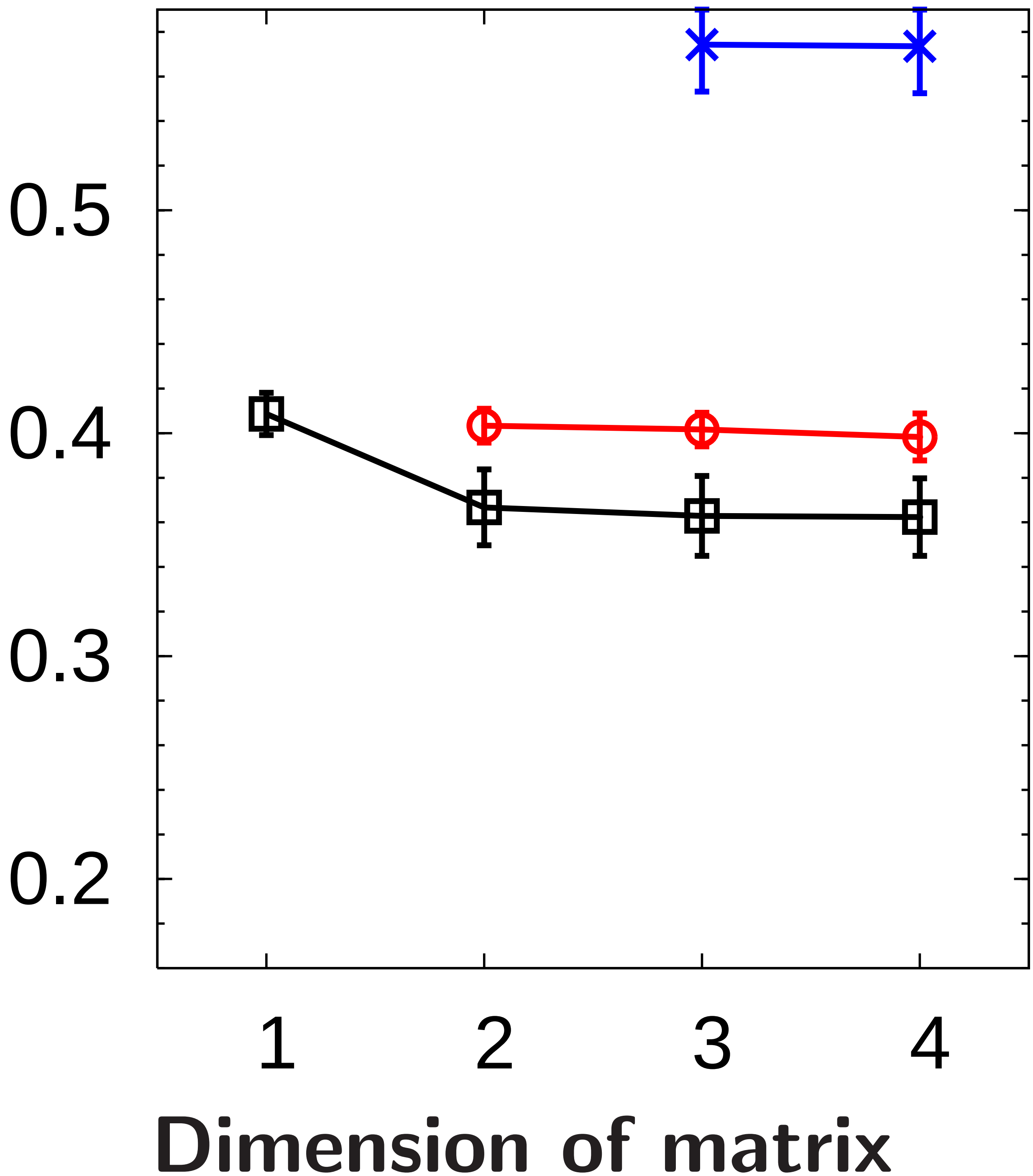
- **Single particle states at zero total momentum:**
- $C(L_1)/C(L_2) \approx 1.$
- **Two-particle states at zero total momentum:**
- $C(L_1)/C(L_2) \approx L_2^3/L_1^3 \to 3.37$
- **Does Z_n^2 help to identify scattering states?**

Stability check: N G_{1g}

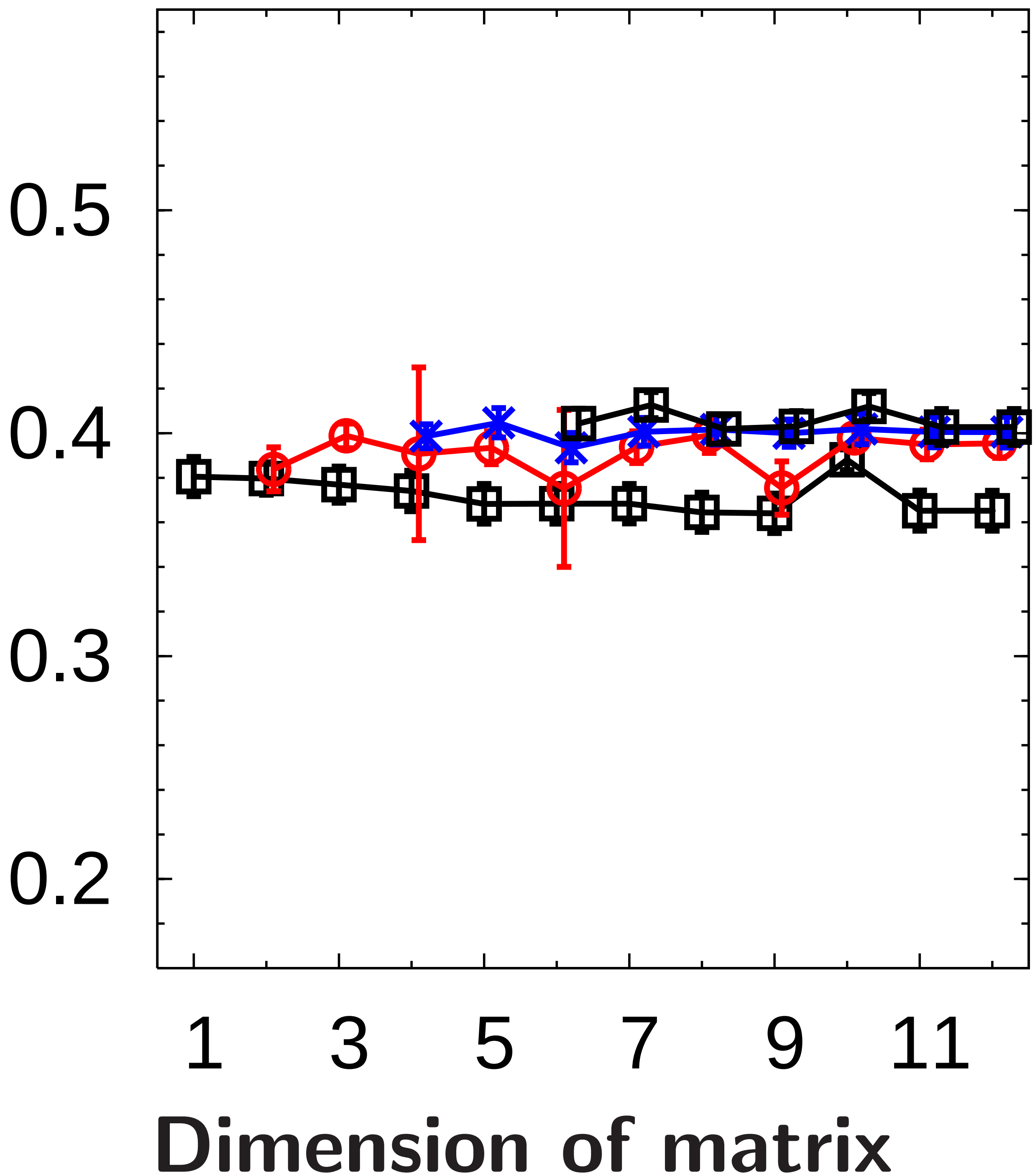


- $a_t = 6GeV^{-1}$

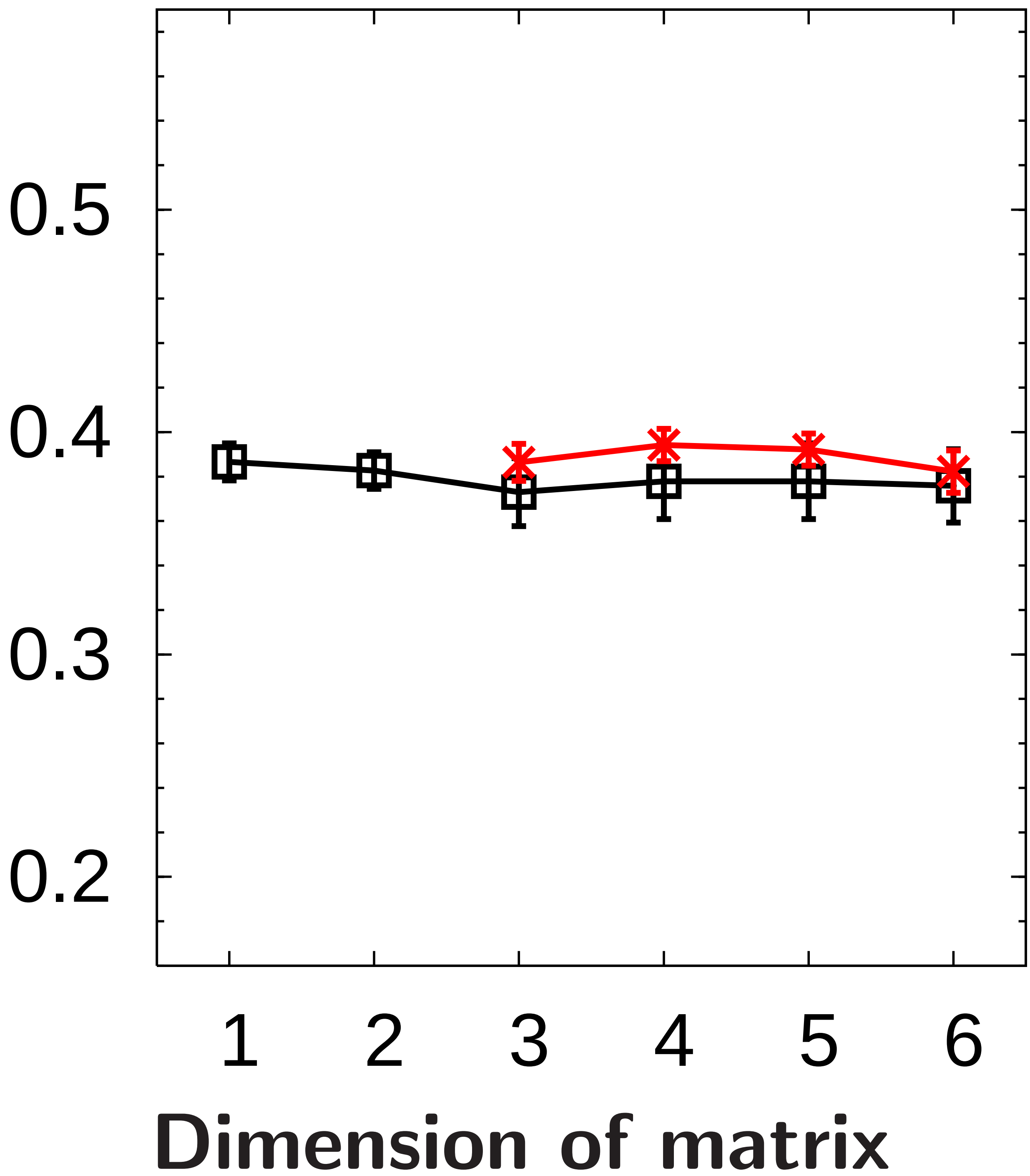
Stability check: N G_{2g}



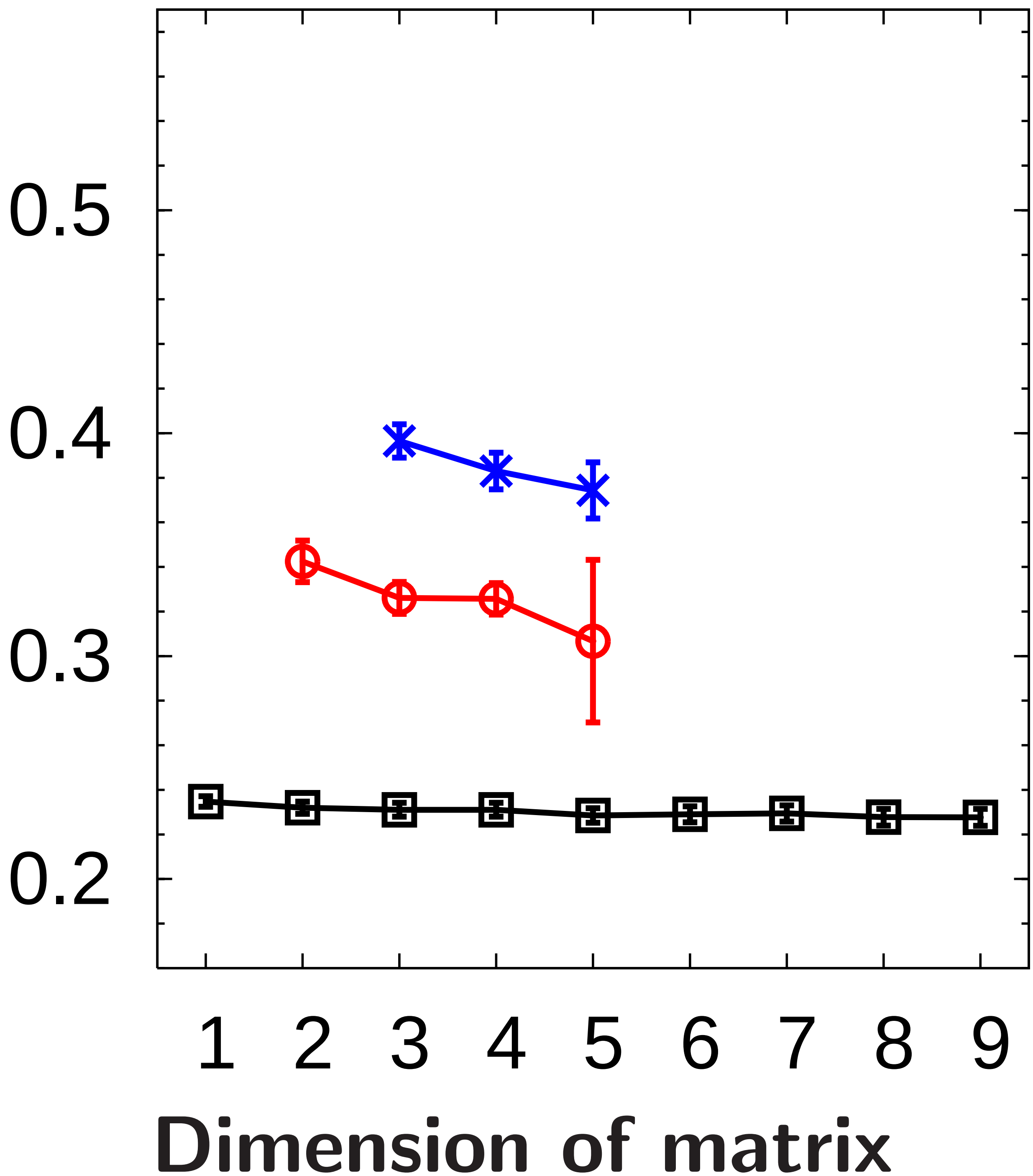
Stability check: N H_g



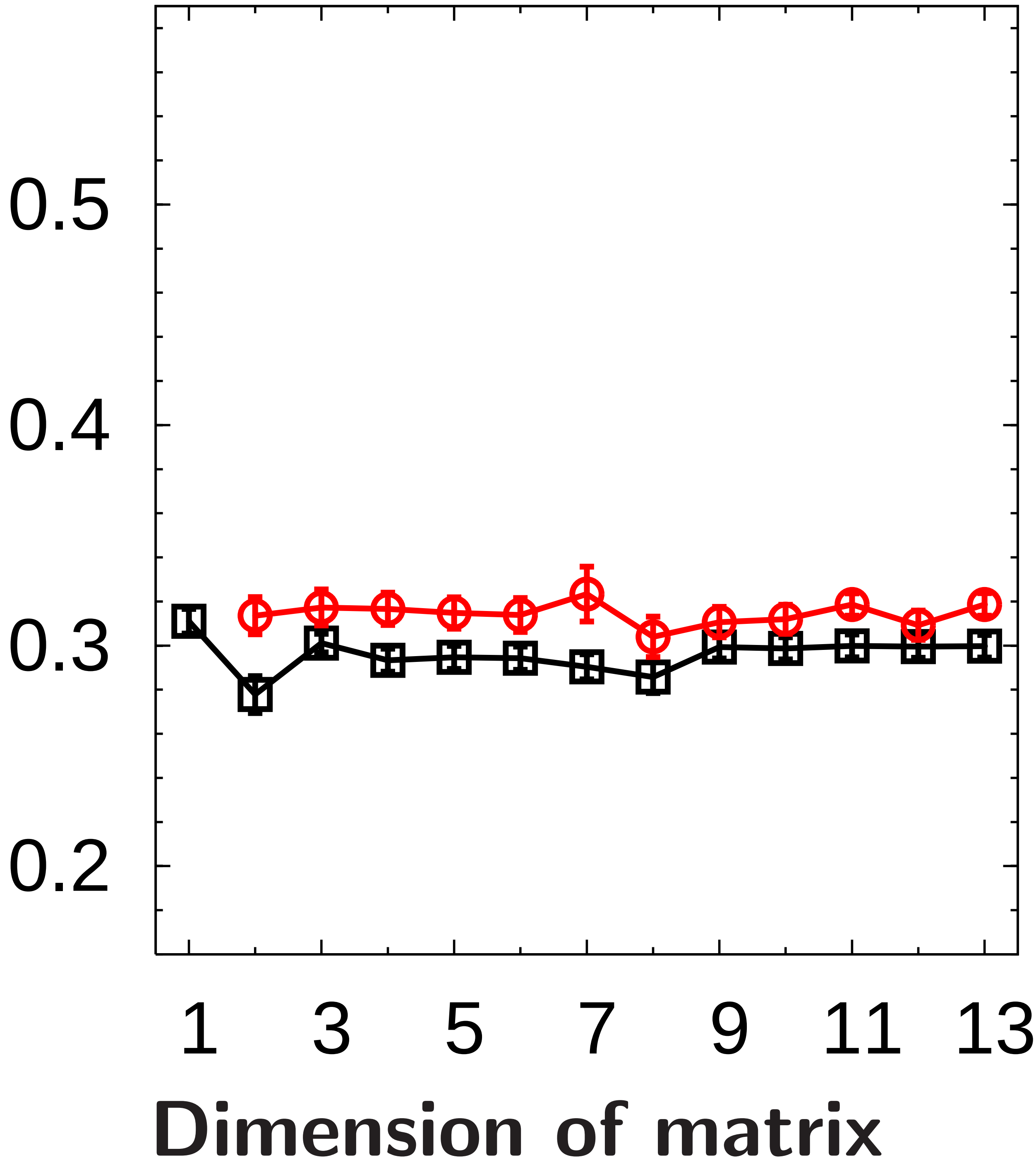
Stability check: $\Delta\ G_{1g}$



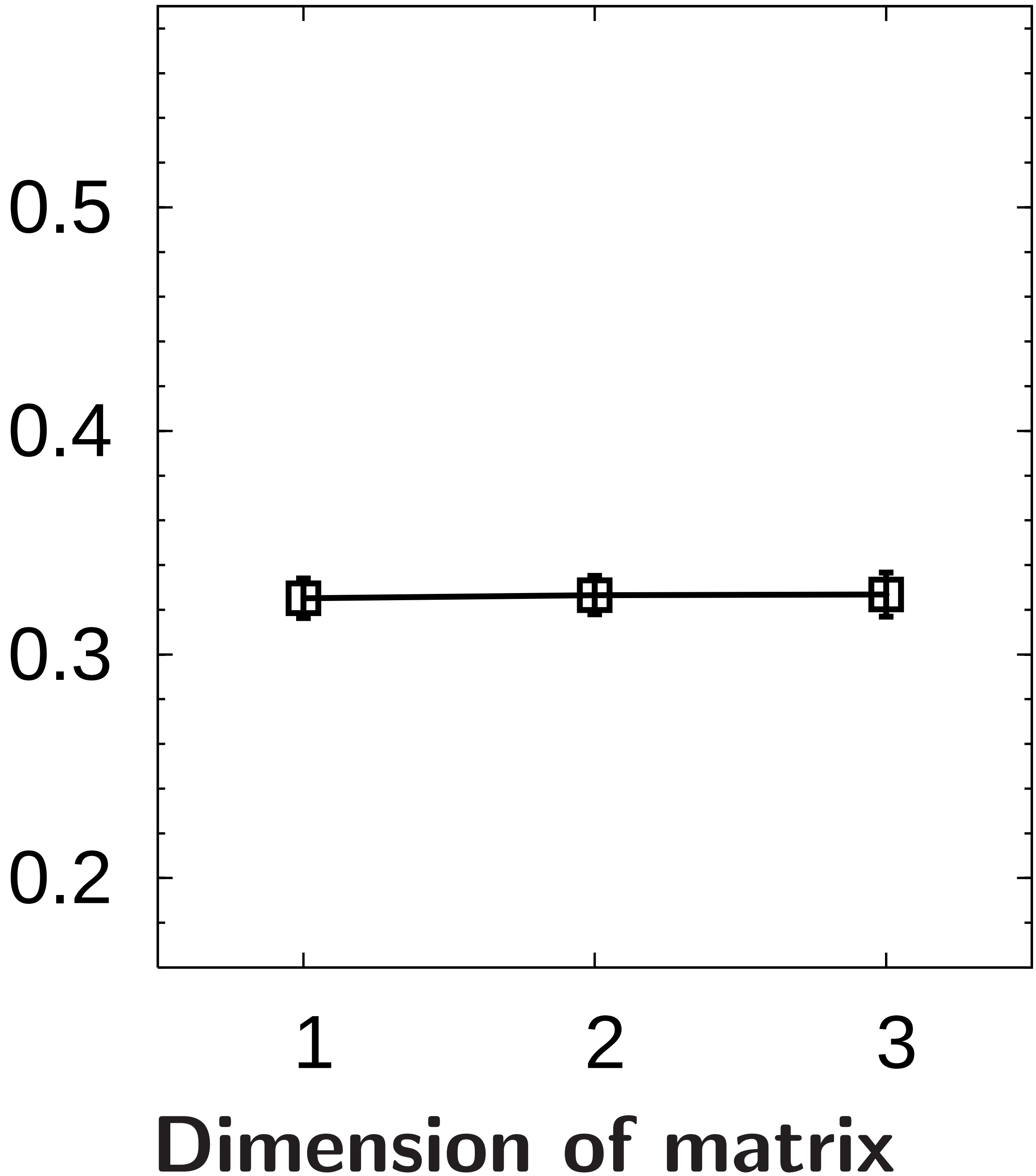
Stability check: ΔH_g



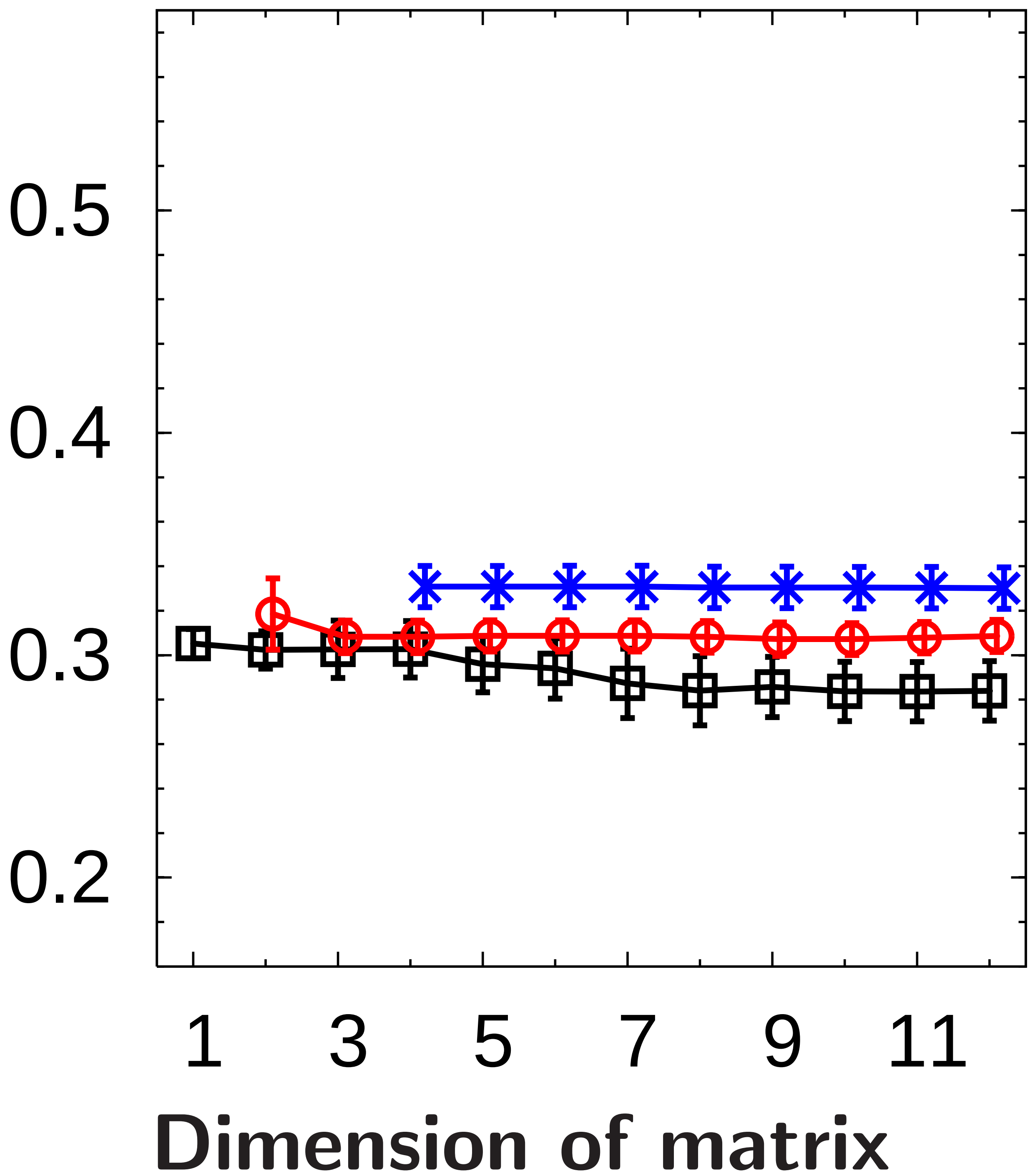
Stability check: N G_{1u}



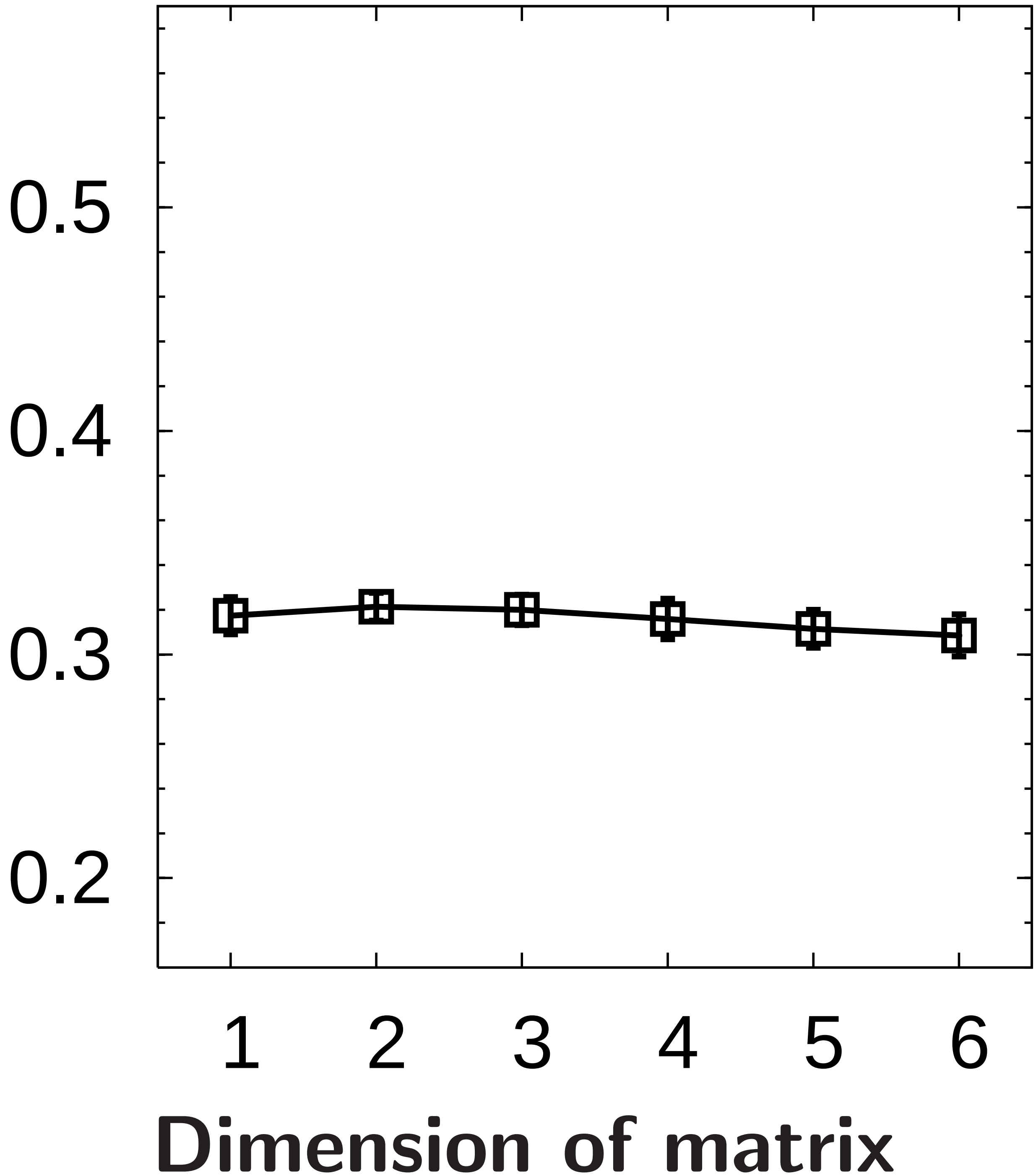
Stability check: N G_{2u}



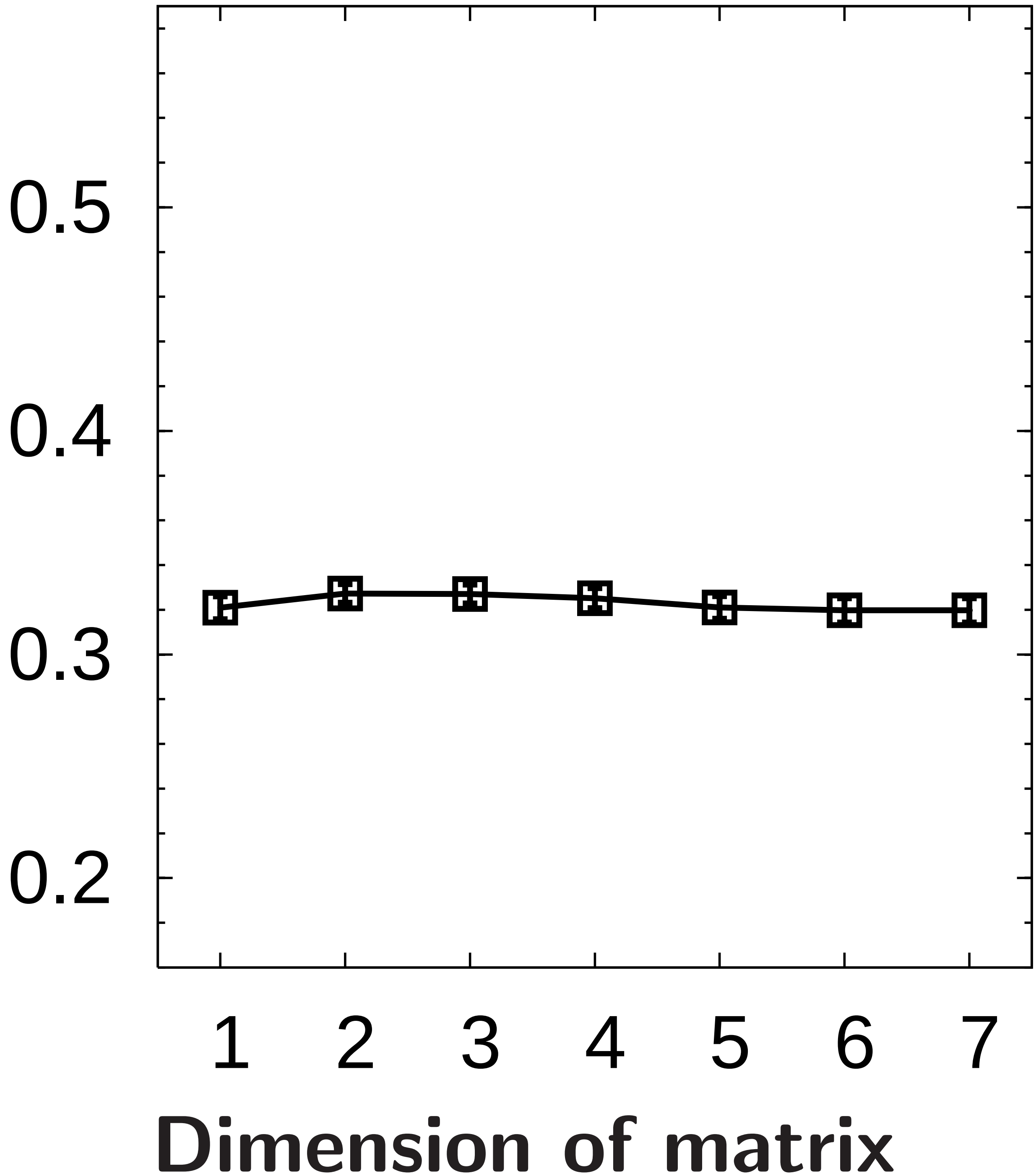
Stability check: N H_u



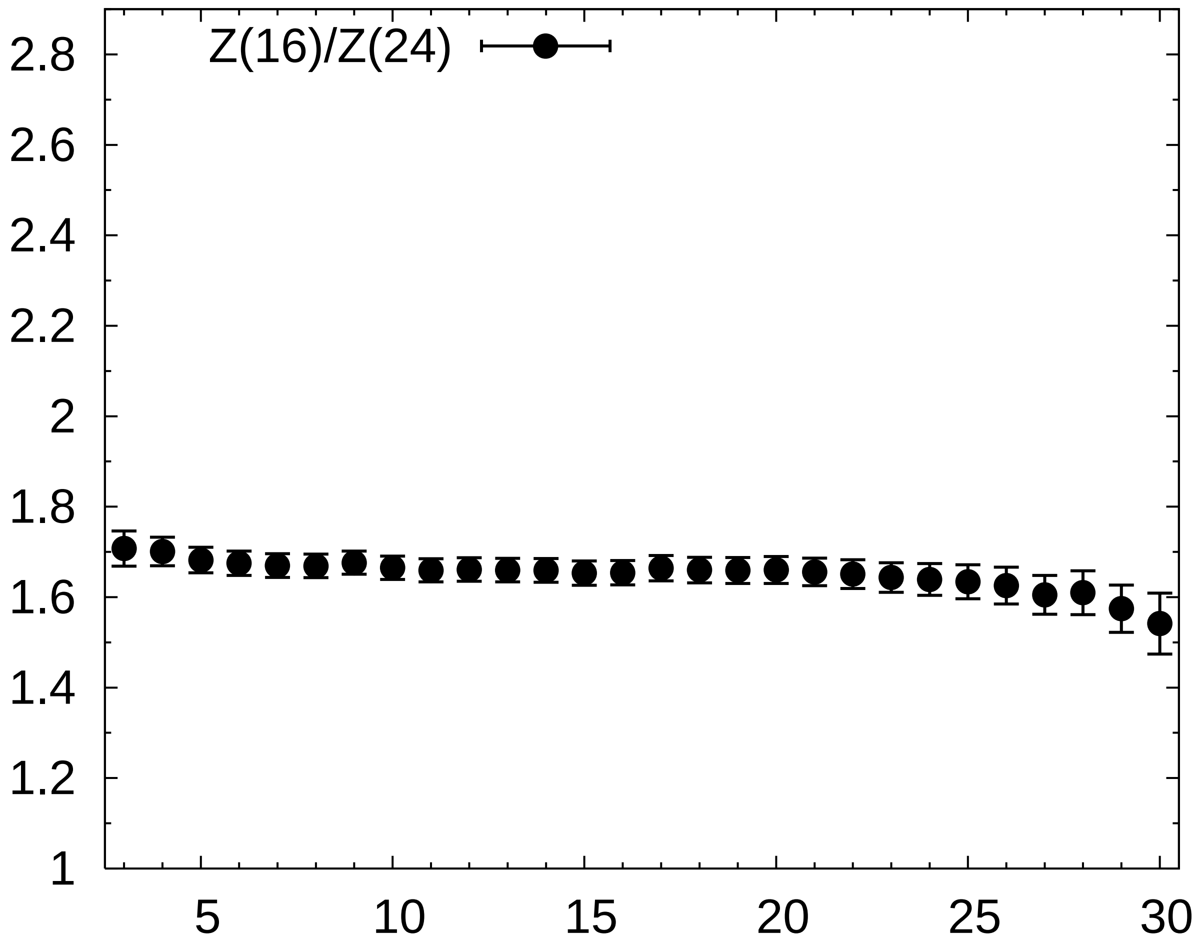
Stability check: $\Delta\ G_{1u}$



Stability check: ΔH_u



Volume dependence N G_{1g}

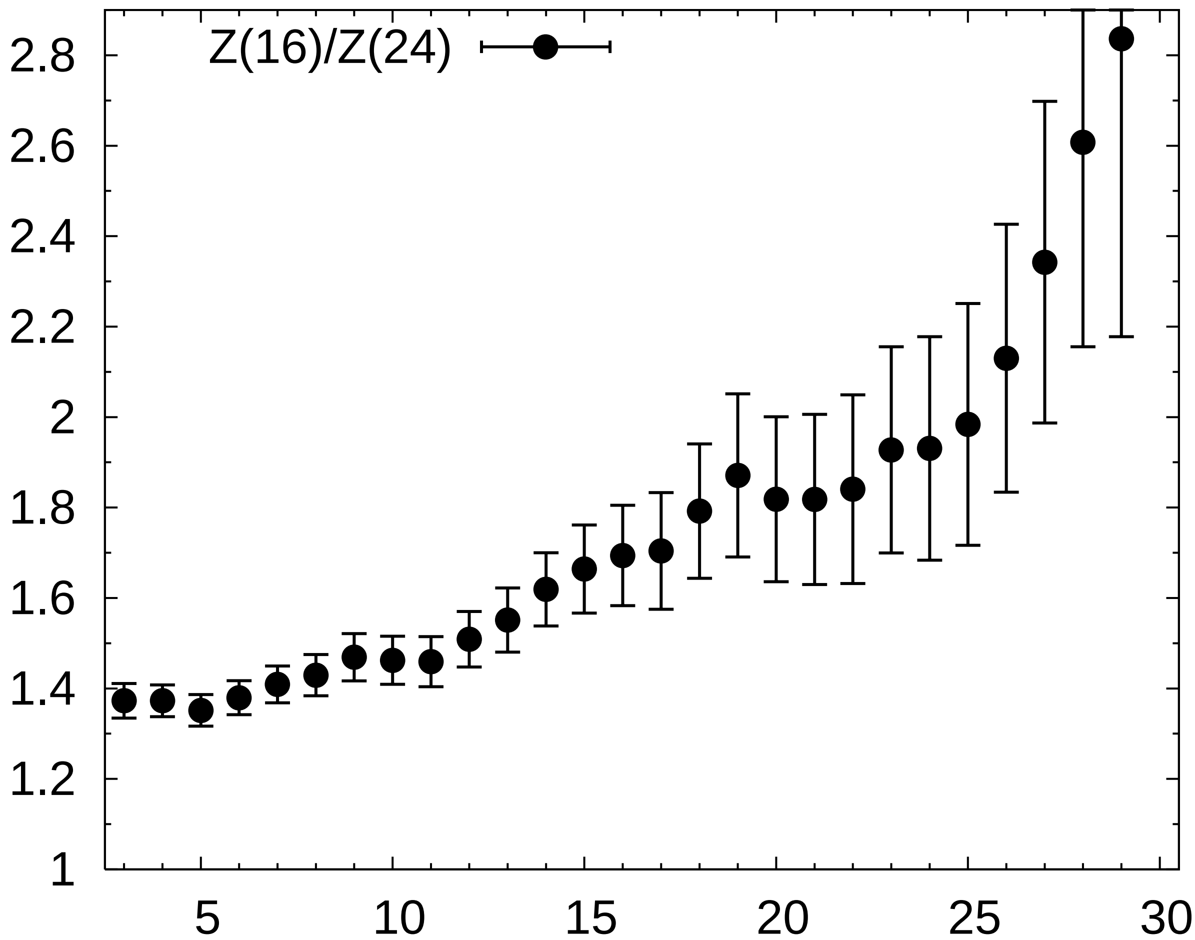


- **Gaussian smearing of quark field:** $\sigma = 3.0$, **N=20.**

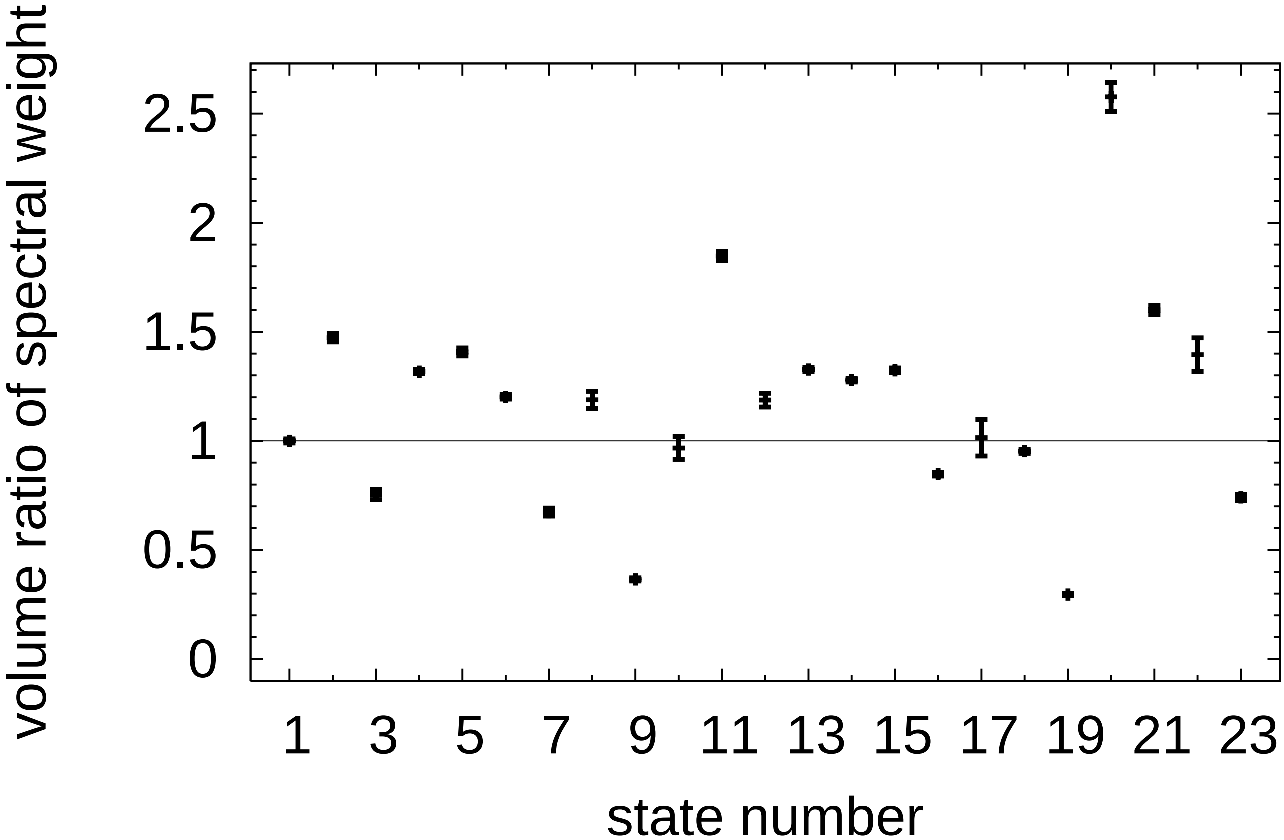
$$\hat{G}^{(N)}(x, x') = \sum_y (\delta_{x,y} + \sigma^2 \frac{\nabla^2_{x,y}}{4N}) \hat{G}^{(N-1)}(y, x'),$$

$$\hat{G}^{(0)}(x, x') = \delta_{x,x'},$$

Volume dependence N H_u

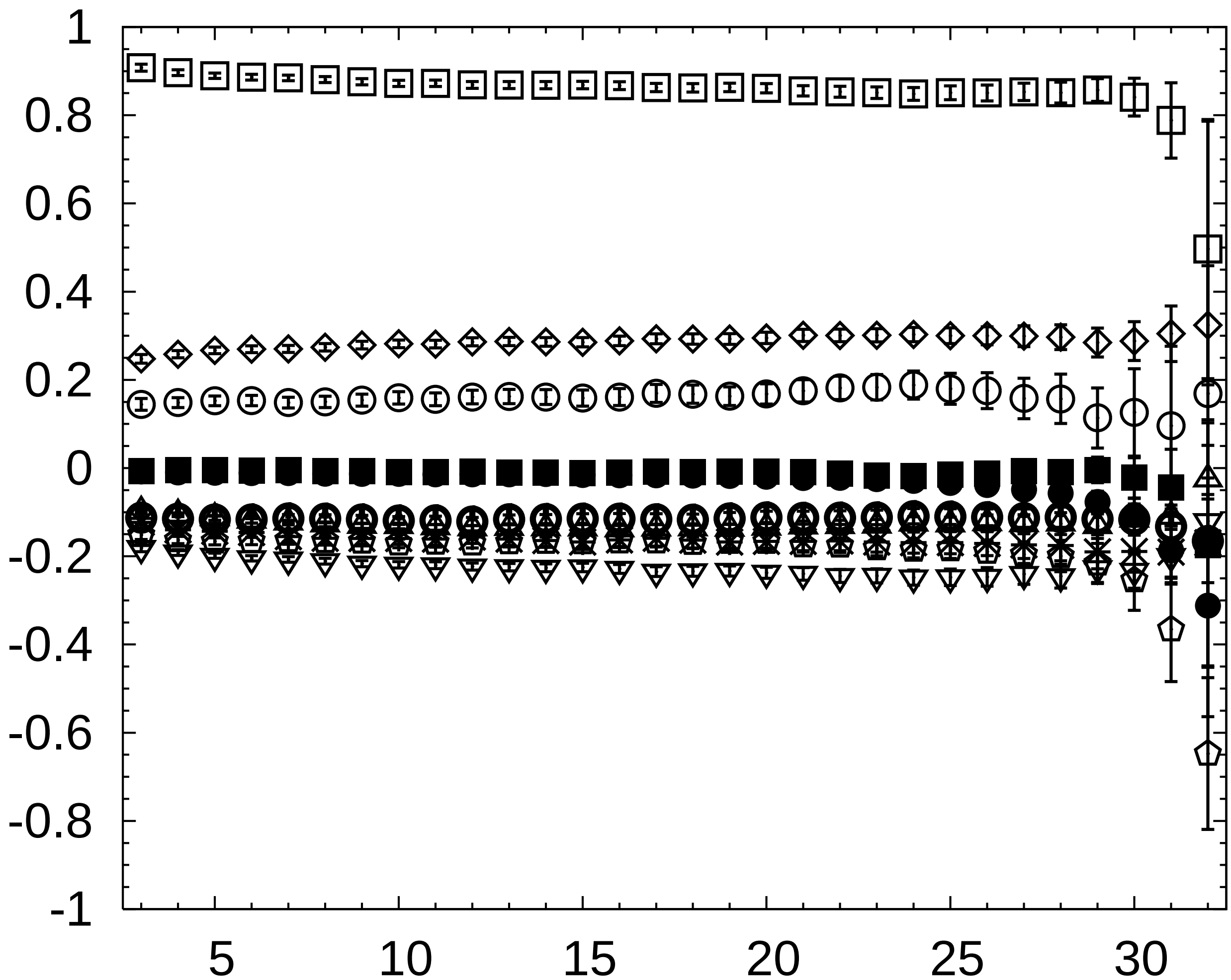


$$\left[Z^2(16)/Z^2(24)\right]/\left[Z_N^2(16)/Z_N^2(24)\right]$$



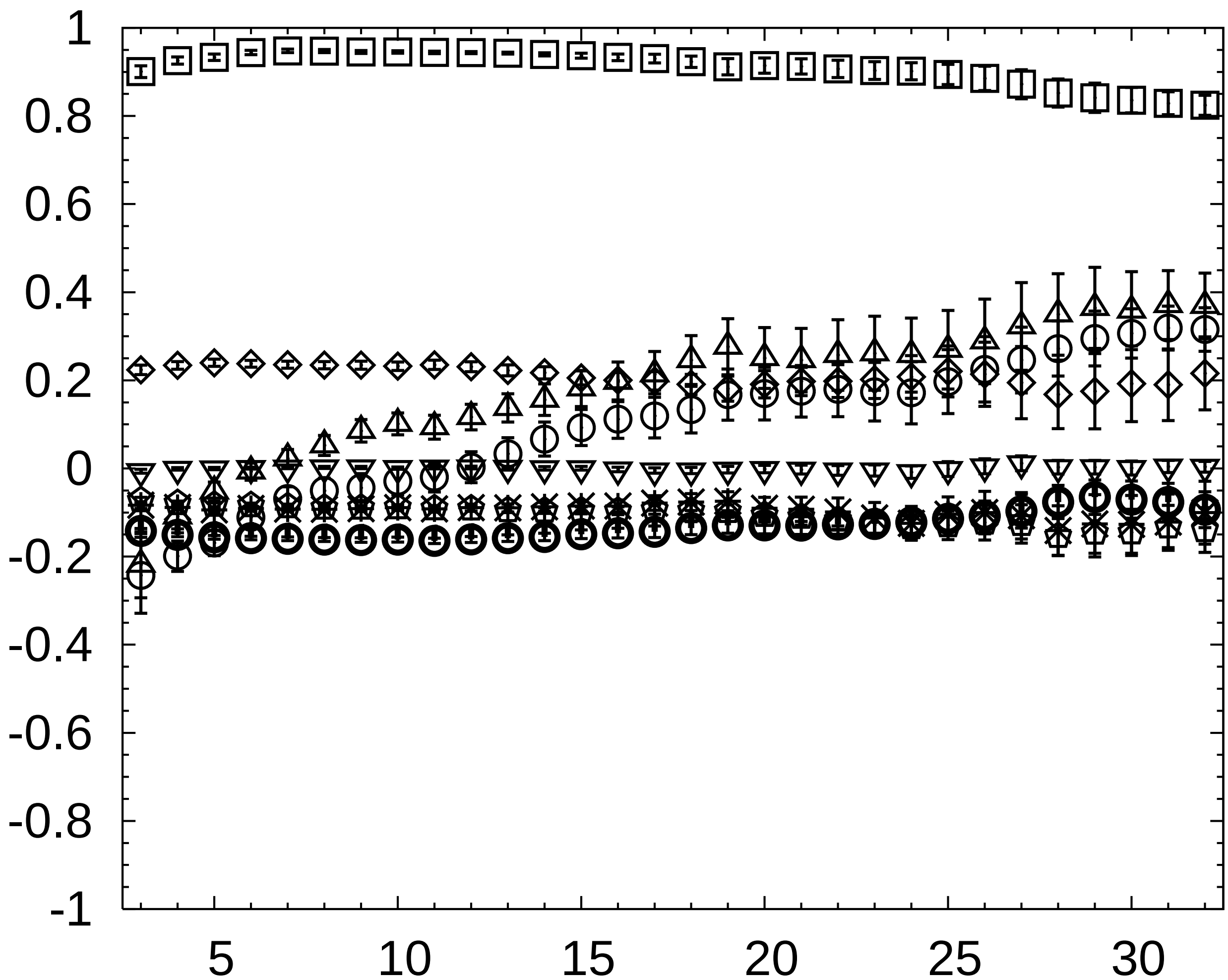
1:N(G_{1g}) gnd; 2:N(G_{1g}) 1st; 3:N(G_{1g}) 2nd; 4:N(G_{2g}) gnd; 5:N(G_{2g}) 1st; 6:N(G_{2g}) 2nd; 7:N(H_g) gnd; 8:N(H_g) 1st; 9:N(G_{1u}) gnd; 10:N(G_{1u}) 1st; 11:N(G_{2u}) gnd; 12:N(H_u) gnd; 13:N(H_u) 1st; 14:N(H_u) 2nd; 15:Δ(G_{1g}) gnd; 16:Δ(G_{1g}) 1st; 17:Δ(G_{2g}) gnd; 18:Δ(H_g) gnd; 19:Δ(H_g) 1st; 20:Δ(H_g) 2nd; 21:Δ(H_g) 3rd; 22:Δ(G_{1u}) gnd; 23:Δ(G_{2u}) gnd; 24:Δ(H_u) gnd.

Eigenvectors: N G_{1g}



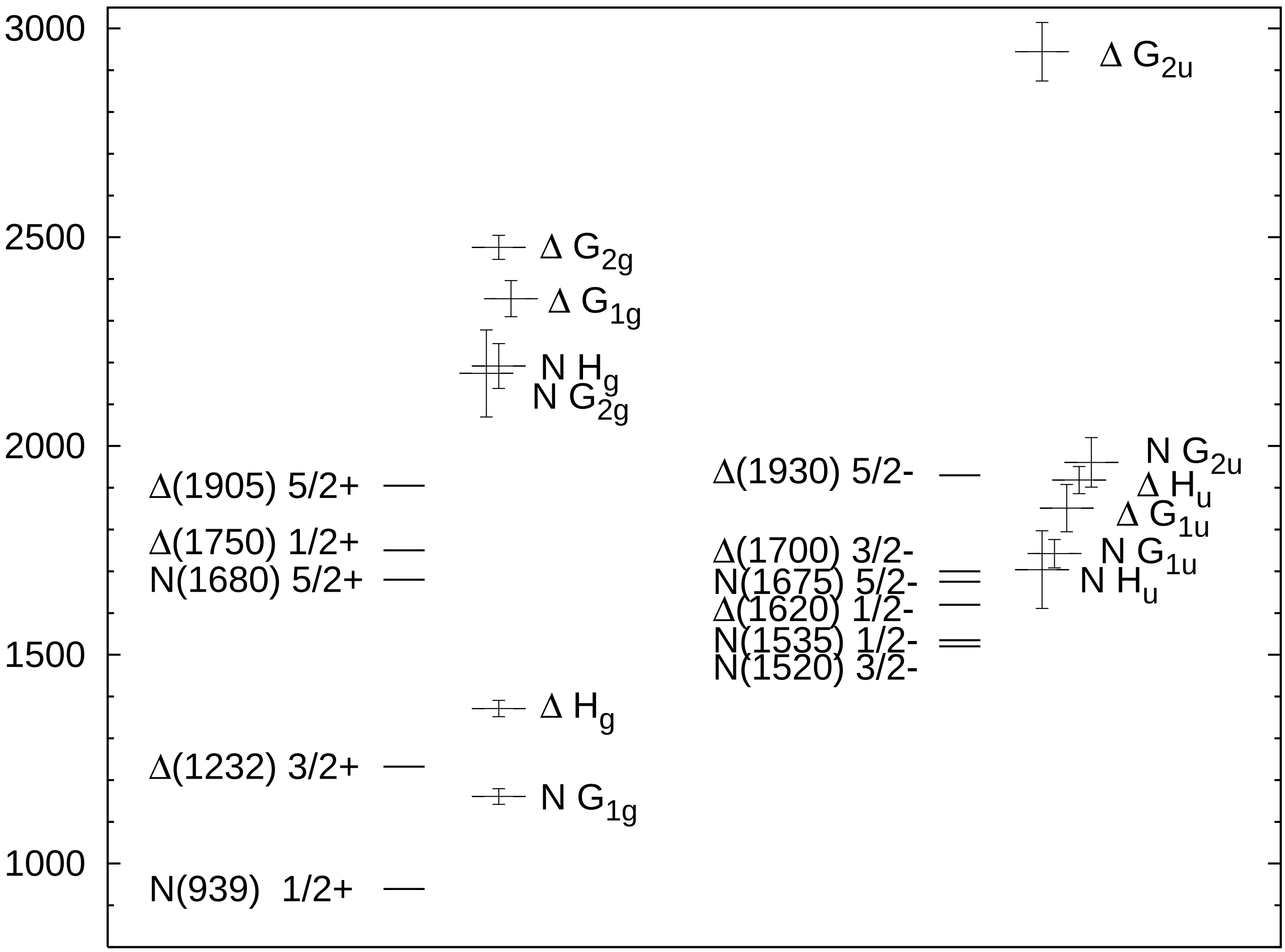
□	$\overline{N}^{G_{1g},1}_{\frac{1}{2},\frac{1}{2}}$	**MA**
◇	$\frac{1}{\sqrt{2}}\hat{D}^{-}\overline{N}^{H_u}_{\frac{3}{2},\frac{3}{2}} - \frac{1}{\sqrt{3}}\hat{D}^{0}\overline{N}^{H_u}_{\frac{3}{2},\frac{1}{2}} + \frac{1}{\sqrt{6}}\hat{D}^{+}\overline{N}^{H_u}_{\frac{3}{2},-\frac{1}{2}}$	**S**
▽	$\sqrt{\frac{2}{3}}\hat{D}^{+}\overline{N}^{G_{1u},1}_{\frac{1}{2},-\frac{1}{2}} - \frac{1}{\sqrt{3}}\hat{D}^{0}\overline{N}^{G_{1u},1}_{\frac{1}{2},\frac{1}{2}}$	**MA**
○	$\overline{N}^{G_{1g},2}_{\frac{1}{2},\frac{1}{2}}$	**MA**

Eigenvectors: N H_u



□	$\overline{N}^{H_u}_{\frac{3}{2},\frac{3}{2}}$	**MA**
△	$\sqrt{\frac{3}{5}}\hat{D}^0\overline{N}^{H_g}_{\frac{3}{2},\frac{3}{2}} - \sqrt{\frac{2}{5}}\hat{D}^+\overline{N}^{H_g}_{\frac{3}{2},\frac{1}{2}}$	**S**
◇	$\sqrt{\frac{3}{5}}\hat{D}^0\overline{N}^{H_g}_{\frac{3}{2},\frac{3}{2}} - \sqrt{\frac{2}{5}}\hat{D}^+\overline{N}^{H_g}_{\frac{3}{2},\frac{1}{2}}$	**MA**
○	$\hat{D}^+\overline{N}^{G_{1g},1}_{\frac{1}{2},\frac{1}{2}}$	**MA**

Pattern of lowest energies in each channel



- **Omits, e.g.,** $N(\frac{1}{2}, 1440)$.

Summary

- Quasi-local + One-link operators
- Lowest $I = \frac{1}{2}$ and $I = \frac{3}{2}$ energies
- 23 energies found on both lattice volumes
- Similar eigenvectors
- Volume ratios? Scattering states?
- Spin $\frac{5}{2}$ states seen in G_2 spectra.
- Partner H states are seen at $a_s = 0.1F$.
- See subduction pattern but want to confirm because scaling may be poor.
- Pattern of lowest energies is similar to the pattern of lowest physical resonance states.