Mesonic Systems with Ginsparg-Wilson valence quarks

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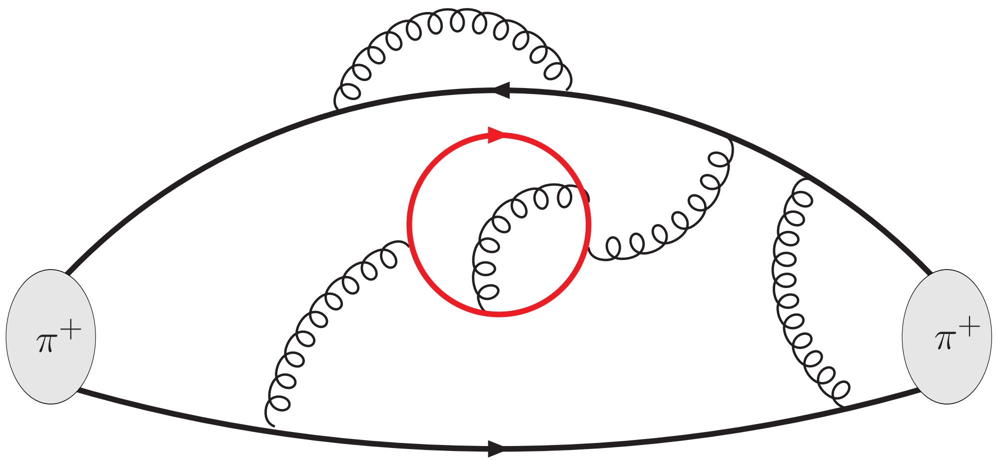
Preview

- Why use Mixed Action lattice QCD?
- I=2 Pi Pi scattering
- Mixed Action Effective Field Theory
- Two-Meson systems from Lattice QCD

Mixed Actions (MA) and Partial Quenching (PQ)

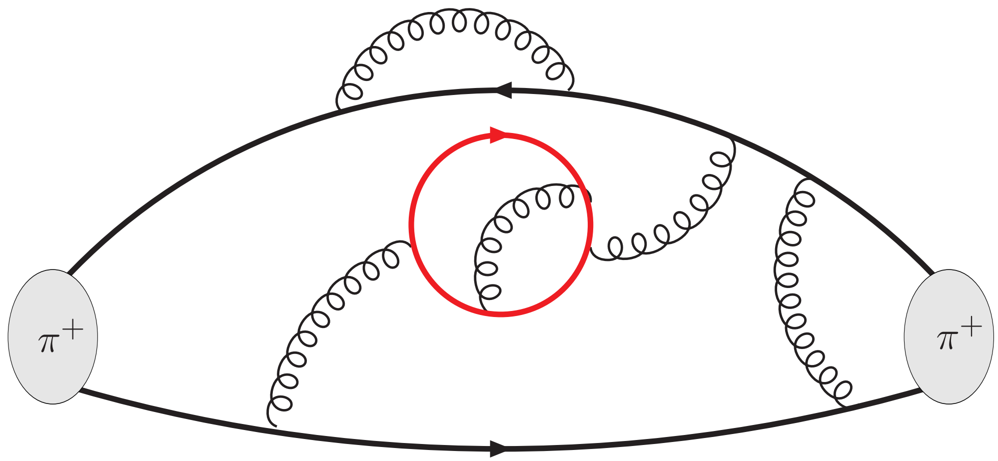


$$\langle \pi^\dagger(y)\pi(x)\rangle = \frac{1}{\mathcal{Z}[0]}\int \mathcal{D}\mathcal{A}\mathrm{Det}\left(\not{D}_{sea} + m_{sea}\right) e^{-S[\mathcal{A}]}$$

$$\times \mathrm{Tr}\left(\gamma_5 \left(\not{D}_{val} + m_{val}\right)^{-1}_{xy} \gamma_5 \left(\not{D}_{val} + m_{val}\right)^{-1}_{yx}\right)$$

$$\not{D}_{sea} - \not{D}_{val} = \mathcal{O}(a)$$

Mixed Actions (MA) and Partial Quenching (PQ)



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$\not{D}_{sea} - \not{D}_{val} = \mathcal{O}(a)$

$m_{sea} = m_{val}$: QCD

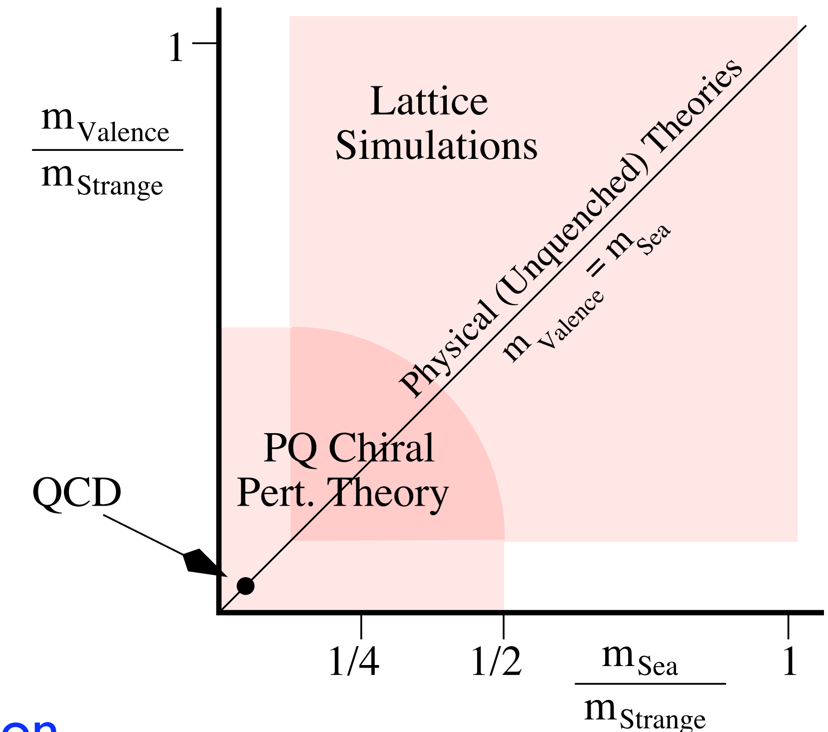
$\not{D}_{sea} = \not{D}_{val}$: Partially Quenched QCD

$m_{sea} = \infty$: Quenched QCD

Mixed Actions (MA) and Partial Quenching (PQ)

Why consider PQ or MA theories?

- simulating light sea quarks numerically costly: valence quarks are cheaper
- larger parameter space to match effective theory to: QCD limit of theory
- chiral symmetry of Ginsparg-Wilson quarks ideal: currently prohibitavely costly
- provide means to test effective field theories (EFT):
  do PQ and MA EFTs completely encode all the unitarity violation which is manifest in the low energy dynamics?



$I = 2$ $\pi\pi$ scattering

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$I = 2\ \pi\pi$ scattering

Adding mixed action and partial quenching effects

$$m_{uu}a_2 = -\frac{m_{uu}^2}{8\pi f^2}\left\{1 + \frac{m_{uu}^2}{(4\pi f)^2}\left[4\ln\left(\frac{m_{uu}^2}{\mu^2}\right) + 4\frac{\tilde{m}_{ju}^2}{m_{uu}^2}\ln\left(\frac{\tilde{m}_{ju}^2}{\mu^2}\right) + l'_{\pi\pi}(\mu) - \frac{\tilde{\Delta}_{PQ}^2}{m_{uu}^2}\left[\ln\left(\frac{m_{uu}^2}{\mu^2}\right)\right] - \frac{\tilde{\Delta}_{PQ}^4}{6m_{uu}^4}\right] + \frac{\tilde{\Delta}_{PQ}^2}{(4\pi f)^2}l'_{PQ}(\mu) + \frac{a^2}{(4\pi f)^2}l'_{a^2}(\mu)\right\}$$

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + f(a) - m_\pi^2$$

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + a^2\Delta_I - m_\pi^2$$ staggered sea

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + aW_0 - m_\pi^2$$ Wilson sea

Every sickness expected is apparent:

partial quenching

lattice discretization effects

$I = 2\ \pi\pi$ scattering

In physical parameters (mass and decay constant measured directly from correlators) the scattering length is given by

$$m_\pi a_2^{QCD} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 + l_{\pi\pi}(\mu)\right]\right\}$$

$I = 2$ $\pi\pi$ scattering

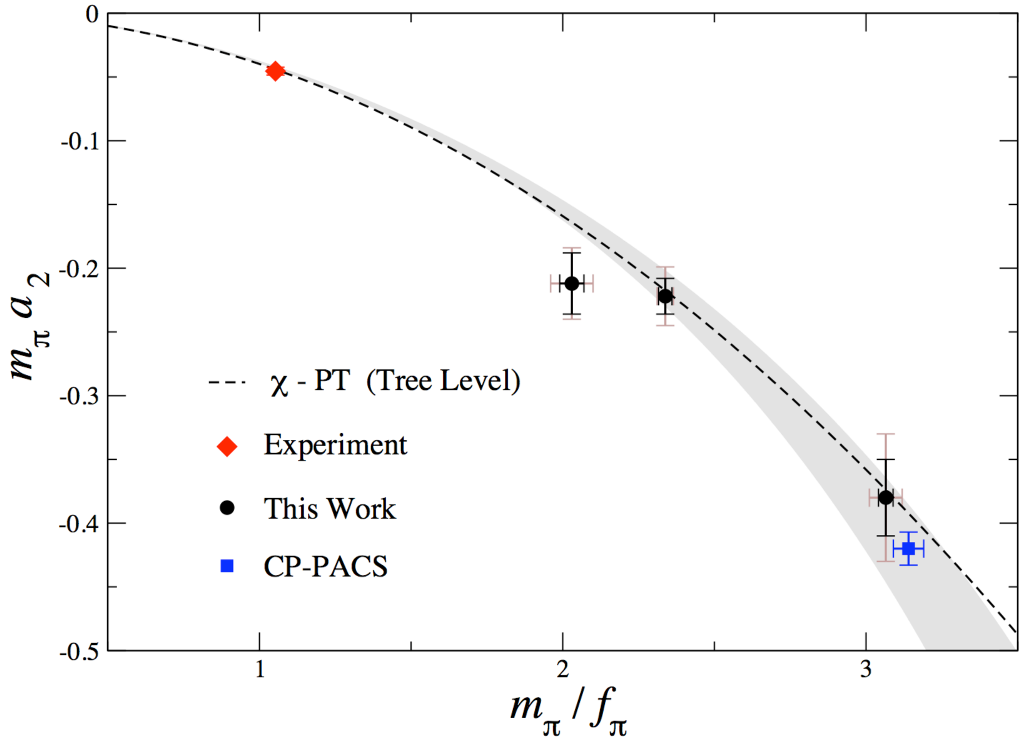
Adding mixed action and partial quenching effects,

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 + l_{\pi\pi}(\mu)\right] - \frac{m_\pi^2}{(4\pi f_\pi)^2}\frac{\tilde{\Delta}_{PQ}^4}{6m_\pi^4}\right\}$$

The explicit dependence on the lattice spacing has exactly cancelled - up to a calculable effect from the hairpin interactions!!!

This is independent of the type of sea-quarks

$I = 2\ \pi\pi$ scattering



NPLQCD:
Isospin 2 pion scattering length: Domain-wall valence quarks on staggered sea quarks.

S. Beane, P. Bedaque, K. Orginos, M. Savage PRD73 (2006)

Experimental point NOT used to constrain fit

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 + l_{\pi\pi}(\mu)\right] - \frac{m_\pi^2}{(4\pi f_\pi)^2}\frac{\tilde{\Delta}_{PQ}^4}{6m_\pi^4}\right\}$$

Postdiction and prediction here: form independent of sea quarks

→ $l_{\pi\pi}(\mu)$ largely insensitive to sea quarks and lattice spacing

Mixed Action Effective Field Theory

Discuss the Partially Quenched (PQ) and Mixed Action (MA) Lagrangians

Mesons

PQ

C. Bernard, M. Golterman, PRD 49 (1994)

S. Sharpe, PRD 56 (1997)

MA

O. Bar, G. Rupak, N. Shoresh PRD 67 (2003), PRD 70 (2004)

O. Bar, C. Bernard, G. Rupak, N. Shoresh PRD 72 (2005)

Mixed Actions (MA) and Partial Quenching (PQ)

$$\mathcal{L} = \frac{f^2}{8}\mathrm{str}\left(\partial_\mu \Sigma\, \partial^\mu \Sigma^\dagger\right) + \frac{f^2 B}{4}\mathrm{str}\left(\Sigma m_Q^\dagger + m_Q \Sigma^\dagger\right)$$

$$\Sigma = \exp\left(\frac{2\mathrm{i}\Phi}{\mathrm{f}}\right) \qquad \Phi = \begin{pmatrix} M & \chi^\dagger \\ \chi & \tilde{M} \end{pmatrix}$$

$$\tilde{M} = \begin{pmatrix} \tilde{\eta}_u & \tilde{\pi}^+ & \dots \\ \tilde{\pi}^- & \tilde{\eta}_d & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$M = \begin{pmatrix} \eta_u & \pi^+ & \dots & \phi_{uj} & \phi_{ul} & \dots \\ \pi^- & \eta_d & \dots & \phi_{dj} & \phi_{dl} & \dots \\ \vdots & \vdots & \ddots & \dots & \dots & \dots \\ \phi_{ju} & \phi_{jd} & \vdots & \eta_j & \phi_{jl} & \dots \\ \phi_{lu} & \phi_{ld} & \vdots & \phi_{lj} & \eta_l & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\chi = \begin{pmatrix} \phi_{\tilde{u}u} & \phi_{\tilde{u}d} & \dots & \phi_{\tilde{u}j} & \phi_{\tilde{u}l} & \dots \\ \phi_{\tilde{d}u} & \phi_{\tilde{d}d} & \dots & \phi_{\tilde{d}j} & \phi_{\tilde{d}l} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Mixed Actions (MA) and Partial Quenching (PQ)

more relevant operators

sea quarks - must add potential arising from the lattice spacing effects

mixing of quarks - must add potential which effects mesons of mixed valence-sea type

$$\mathcal{L} = a^2 C_{Mix} \left(T_3 \Sigma T_3 \Sigma^\dagger \right) \qquad T_3 = \mathcal{P}_S - \mathcal{P}_V$$

$\mathcal{P}_S$ sea projector

$\mathcal{P}_V$ valence projector

$$m_{vv}^2 = 2B_0 m_v$$

$$\tilde{m}_{ss}^2 = 2B_0 m_s + f(a) C_{sea}$$

$$\tilde{m}_{sv}^2 = B_0(m_v + m_s) + a^2 \Delta_{Mix} \qquad \Delta_{Mix} \equiv \frac{16 C_{Mix}}{f^2}$$

Partial Quenching (PQ)

Gasser-Leutwler Lagrangian

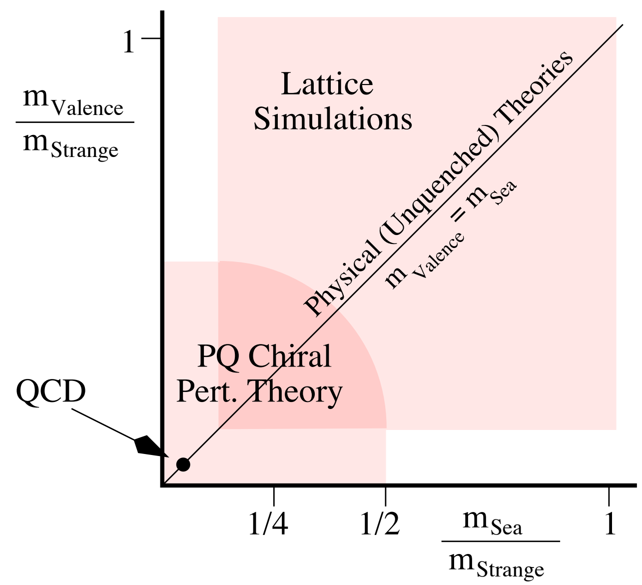
$$\mathcal{L} = L_1 \left[\mathrm{sTr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right)\right]^2 + L_2 \mathrm{sTr}\left(\partial_\mu \Sigma^\dagger \partial_\nu \Sigma\right) \mathrm{sTr}\left(\partial^\mu \Sigma^\dagger \partial^\nu \Sigma\right)$$

$$+ L_3 \mathrm{sTr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger \partial^\nu \Sigma \partial^\nu \Sigma^\dagger\right) + L_4\, 2B_0 \mathrm{sTr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) \mathrm{sTr}\left(m_q \Sigma^\dagger + \Sigma m_q^\dagger\right)$$

$$+ L_5\, 2B_0 \mathrm{sTr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \left(m_q \Sigma^\dagger + \Sigma m_q^\dagger\right)\right) + 4L_6\, B_0^2 \left[\mathrm{sTr}\left(m_q \Sigma^\dagger + \Sigma m_q\right)\right]^2$$

$$+ 4L_7\, B_0^2 \left[\mathrm{sTr}\left(m_q \Sigma^\dagger - \Sigma m_q\right)\right]^2 + 4L_8\, B_0^2 \left[\mathrm{sTr}\left(m_q \Sigma^\dagger m_q \Sigma^\dagger + \Sigma m_q \Sigma m_q\right)\right]^2$$

These coefficients, L_i have the same numerical value they do in chiral perturbation theory.



Mixed Actions (MA) and Partial Quenching (PQ)

Mixed Action breaks up the Gasser-Leutwyler operators; for example

$$4L_6\, B_0^2\ \left[\mathrm{sTr}\left(m_q\Sigma^\dagger + \Sigma m_q\right)\right]^2$$

$$\downarrow$$

$$4L_6\, C_6^{SS}\ \left[\mathrm{sTr}\left(\mathcal{P}_S B_0^S\left(m_Q\Sigma^\dagger + \Sigma m_Q^\dagger\right)\right)\right]^2 \quad + \quad 4L_6\, C_6^{VV}\ \left[\mathrm{sTr}\left(\mathcal{P}_V B_0^V\left(m_Q\Sigma^\dagger + \Sigma m_Q^\dagger\right)\right)\right]^2$$

$$+ \quad 8L_6\, C_6^{VS}\,\mathrm{sTr}\left(\mathcal{P}_V B_0^V\left(m_Q\Sigma^\dagger + \Sigma m_Q^\dagger\right)\right)\mathrm{sTr}\left(\mathcal{P}_S B_0^S\left(m_Q\Sigma^\dagger + \Sigma m_Q^\dagger\right)\right)$$

$$\begin{matrix} C_i^{VV} \\ C_i^{VS} \\ C_i^{SV} \\ C_i^{SS} \end{matrix} \quad = 1 + \mathcal{O}(a, a^2)$$

To the order we are interested, we can treat all the extra coefficients as 1

Mixed Actions (MA) and Partial Quenching (PQ)

In addition to these operators, there are also operators involving the lattice spacing, eg.

$$a^2\, L_{ma^2}\, \mathrm{sTr}\left(m_q \Sigma^\dagger\right)\ \mathrm{sTr}\left(\mathcal{P}_S\, \xi_5\, \Sigma\, \xi_5\, \Sigma^\dagger + \mathrm{p.c.}\right)$$

We find that all extra operators from mixed action Lagrangian can be absorbed into a field redefinition of f, and Bm_q. eg.

$$\frac{f^2 B}{4} \mathrm{str}\left(\Sigma m_Q^\dagger + m_Q \Sigma^\dagger\right)\Big(1 + a^2 L_{PQ} \mathrm{sTr}\left(\mathcal{P}_S\left(m_q \Sigma^\dagger + \Sigma m_q\right)\right) + a^2\, L_{ma^2}\, \mathrm{sTr}\left(\mathcal{P}_S\, \xi_5\, \Sigma\, \xi_5\, \Sigma^\dagger + \mathrm{p.c.}\right)\Big)$$

Mixed action is helping!!! “Factorization” of sea quark effects.

Mixed Actions (MA) and Partial Quenching (PQ)

Where does this break down?

Consider a correction to the L_6 operator. We found that the sea-quark effects from this operator cancelled - but the scattering length still depends upon this operator - from the valence quark contributions. Thus consider a correction to this operator, eg.

$$L_{6,a^2}\ \mathrm{sTr}\left(\mathcal{P}_V\left(m_q\Sigma^\dagger+\Sigma m_q\right)\right)\ \mathrm{sTr}\left(\mathcal{P}_V\left(m_q\Sigma^\dagger+\Sigma m_q\right)\right)\left(1+a^2\ \mathrm{sTr}\left(\mathcal{P}_S\,\xi_5\,\Sigma\,\xi_5\,\Sigma^\dagger\right)\right)$$

So we see there will be a non-vanishing a^2 contribution to the amplitude at the next order, $\mathcal{O}(a^2 m_\pi^4)$. However, this contribution can also be field redefined into L_6 - again a feature arising from the Ginsparg-Wilson symmetry in the valence sector.

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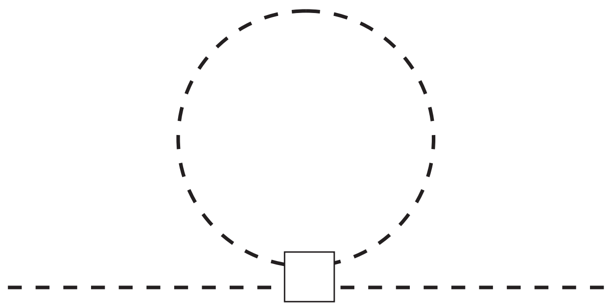
Taste Breaking in meson spectrum

$$8C_6^{VS} L_6 \,\mathrm{str}\left(\mathcal{P}_V\left(\Sigma m_Q + m_Q \Sigma^\dagger\right)\right) \mathrm{str}\left(\mathcal{P}_S\left(\Sigma m_Q + m_Q \Sigma^\dagger\right)\right)$$



$$\delta m_\pi^2 = -\frac{32 C_6^{VS} L_6\, m_\pi^2}{f^2} \sum_t \frac{n_t}{16} \frac{8B_0\, m_j}{(4\pi f)^2} \ln\left(\frac{m_t^2}{\mu^2}\right)$$

$$m_t^2 = 2B_0\, m_j + a^2\, \Delta_t$$

non-Physics of Partial Quenching

unitarity violation is same in mixed action and partially quenched theory at EFT level:

Explicitly display these sicknesses:
non-physics of partial quenching

$$\tilde{\Delta}^2_{ju} \equiv \tilde{m}^2_{jj} - m^2_\pi = 2B_0(m_j - m_u) + f(a)\Delta_{sea} + \dots$$

$$\tilde{\Delta}^2_{rs} \equiv \tilde{m}^2_{rr} - m^2_{ss} = 2B_0(m_r - m_s) + f(a)\Delta_{sea} + \dots$$

$$\tilde{\Delta}^2_{sj} \equiv \tilde{m}^2_{ss} - m^2_{jj} = 2B_0(m_s - m_j) - f(a)\Delta_{sea} + \dots$$

Two-Meson Systems

KK Scattering

KK scattering length has the same form as the $\pi\pi$
but more complicated algebra due to SU(3) breaking

Kπ Scattering

$K\pi$ scattering length, we observe a dependence upon the valence sea mesons, which introduces a new unknown into fit

KK scattering

$$
\begin{aligned}
\mathcal{T}_{K^+K^+} = &-\frac{4m_K^2}{f_K^2}+\frac{56m_K^4}{9(4\pi)^2f_K^4}-\frac{8m_K^4}{(4\pi)^2f_K^4}\ln\left(\frac{m_K^2}{\mu^2}\right)+\left(\frac{10m_K^2m_\pi^2}{9(4\pi)^2f_K^4}+\frac{m_\pi^4}{9(4\pi)^2f_K^4}\right)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\\
&-\left(\frac{m_\pi^4}{9(4\pi)^2f_K^4}+\frac{48m_\pi^2\tilde{m}_X^2}{45(4\pi)^2f_K^4}-\frac{46m_K^2\tilde{m}_X^2}{45(4\pi)^2f_K^4}+\frac{13\tilde{m}_X^4}{5(4\pi)^2f_K^4}\right)\ln\left(\frac{\tilde{m}_X^2}{\mu^2}\right)\\
&-\frac{8(2m_K^2+m_\pi^2)^2}{27(4\pi)^2f_K^4(\tilde{m}_X^2-m_\pi^2)}\left(\tilde{m}_X^2\ln\left(\frac{\tilde{m}_X^2}{\mu^2}\right)-m_\pi^2\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right)-m_K^4\,L_{KK}(\mu)-m_K^2m_\pi^2\,L_{K\pi}(\mu)\\
&+\frac{\tilde{\Delta}_{ju}^2\tilde{m}_X^2}{(4\pi)^2f_K^4}\mathcal{F}_1\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{\tilde{m}_X^2}{\mu^2}\right)+\frac{\tilde{\Delta}_{ju}^4}{(4\pi)^2f_K^4}\mathcal{F}_2\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{\tilde{m}_X^2}{\mu^2}\right)\\
&+\frac{\tilde{\Delta}_{ju}^6}{(4\pi)^2f_K^4\tilde{m}_X^2}\mathcal{F}_3\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2}\right)+\frac{\tilde{\Delta}_{ju}^8}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{F}_4\left(\frac{m_\pi^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{rs}^2\tilde{m}_X^2}{(4\pi)^2f_K^4}\mathcal{J}_1\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2},\frac{\tilde{m}_X^2}{\mu^2}\right)+\frac{\tilde{\Delta}_{rs}^4}{(4\pi)^2f_K^4}\mathcal{J}_2\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{ju}^2\tilde{\Delta}_{rs}^2}{(4\pi)^2f_K^4}\mathcal{J}_3\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2},\frac{\tilde{m}_X^2}{\mu^2}\right)+\frac{\tilde{\Delta}_{rs}^2\tilde{\Delta}_{sj}^2}{(4\pi)^2f_K^4}\mathcal{J}_4\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2},\frac{\tilde{m}_X^2}{\mu^2}\right)\\
&+\frac{\tilde{\Delta}_{ju}^4\tilde{\Delta}_{rs}^2}{(4\pi)^2f_K^4\tilde{m}_X^2}\mathcal{J}_5\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)+\frac{\tilde{\Delta}_{ju}^2\tilde{\Delta}_{rs}^4}{(4\pi)^2f_K^4\tilde{m}_X^2}\mathcal{J}_6\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{rs}^4\tilde{\Delta}_{sj}^2}{(4\pi)^2f_K^4\tilde{m}_X^2}\mathcal{J}_7\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)+\frac{\tilde{\Delta}_{ju}^2\tilde{\Delta}_{rs}^2\tilde{\Delta}_{sj}^2}{(4\pi)^2f_K^4\tilde{m}_X^2}\mathcal{J}_8\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{ju}^6\tilde{\Delta}_{rs}^2}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{J}_9\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)+\frac{\tilde{\Delta}_{ju}^4\tilde{\Delta}_{rs}^4}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{J}_{10}\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{ju}^4\tilde{\Delta}_{rs}^2\tilde{\Delta}_{sj}^2}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{J}_{11}\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)+\frac{\tilde{\Delta}_{ju}^2\tilde{\Delta}_{rs}^4\tilde{\Delta}_{sj}^2}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{J}_{12}\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)\\
&+\frac{\tilde{\Delta}_{rs}^4\tilde{\Delta}_{sj}^4}{(4\pi)^2f_K^4\tilde{m}_X^4}\mathcal{J}_{13}\left(\frac{m_\pi^2}{\tilde{m}_X^2},\frac{m_K^2}{\tilde{m}_X^2},\frac{m_{ss}^2}{\tilde{m}_X^2}\right)
\end{aligned}
$$

Kπ Scattering

Kaon-pion system has new effect not seen in KK or $\pi\pi$ system - at one-loop the presence of valence-sea mesons.

$$\mathcal{T}_{K^+\pi^+} \supset \frac{m_K m_\pi}{(4\pi)^2 f_K^2 f_\pi^2} \sum_F \left[C_{Fd} \ln\left(\frac{\tilde{m}_{Fd}^2}{\mu^2}\right) - C_{Fs} \ln\left(\frac{\tilde{m}_{Fs}^2}{\mu^2}\right) - 2 m_K m_\pi \mathrm{J}(m_{Fd}^2) + 4 m_K m_\pi \right]$$

$$C_{Fd} = \frac{4 m_K m_\pi^2 - \tilde{m}_{Fd}^2 (m_K + m_\pi)}{m_K - m_\pi} \qquad C_{Fs} = \frac{4 m_K^2 m_\pi - \tilde{m}_{Fs}^2 (m_K + m_\pi)}{m_K - m_\pi}$$

$$J(M) = 4 \frac{\sqrt{M^2 - m_\pi^2}}{m_K - m_\pi} \arctan\left[\frac{(m_K - m_\pi)\sqrt{M^2 - m_\pi^2}}{M^2 + m_K m_\pi - m_\pi^2} \right]$$

$a^2 \ln(\mu^2)$ still cancels - Ginsparg-Wilson chiral valence symmetry protects amplitude from these corrections

- counter term structure of scattering length is identical to that in QCD. Mixed mesons introduce an additional unknown Δ_{Mix}

Other Applications

Single Baryon Observables

- masses
- electromagnetic properties
- axial couplings

B. C. Tiburzi PRD 72(2005)

Nucleon-Nucleon Scattering

Hyperon-Nucleon Scattering

J-W. Chen, D. O'Connell, AW-L
hep-lat/06XXXXX

Conclusions

- Chiral properties of Ginsparg-Wilson fermions are very desirable
- Ginsparg-Wilson valence quarks suppress various sources of lattice spacing corrections - independent of type of sea quarks
  - Work with “lattice physical” parameters
  - Counter term structure of observables is identical to that in QCD, through NLO, up to perturbative corrections
  - Additional symmetry need to consider is SU(3)-valence symmetry, combined with projection onto initial and final states
- Arguments hold for other observables as well