Flavor Twisting for Isovector Form Factors

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Duke University

LHP2006, August 1st

Outline

Flavor Twisted Boundary Conditions
and
Isovector Form Factors

- Quantized momentum and form factors

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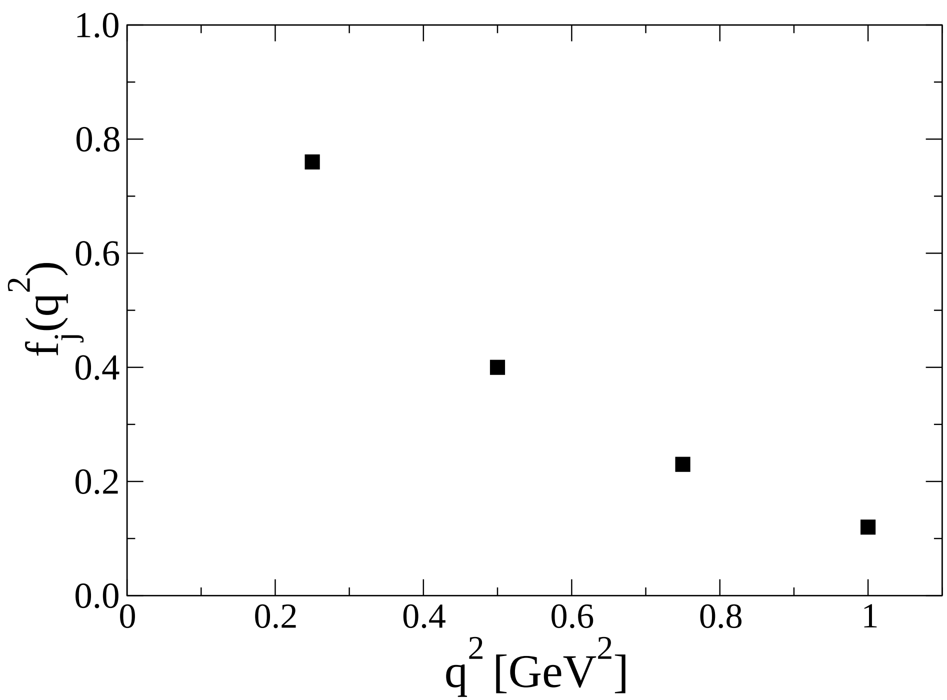

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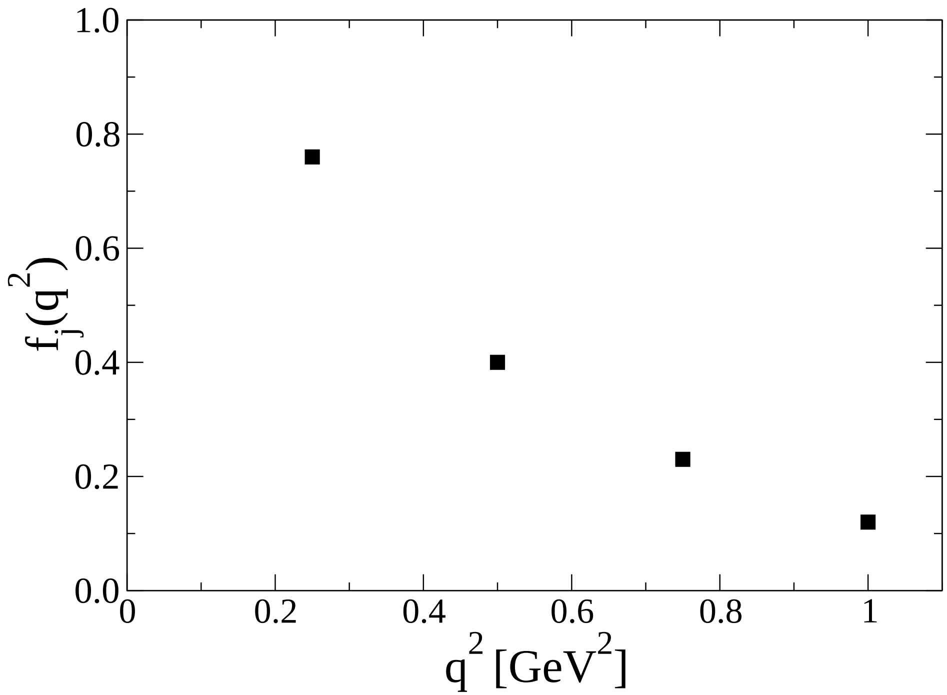

- $O_j \propto q$
- $L = 24a$, $a = 2\,\texttt{GeV}^{-1}$: $q_{\text{min}} = 2\pi/L \sim 500\,\texttt{MeV}$

Nucleon isovector form factor

- Definition & Chiral expansion
  $F_2(q^2) = G_M^p(q^2) - G_M^n(q^2) = \mu^{\text{iso}} + f(q^2/4m_\pi^2)$

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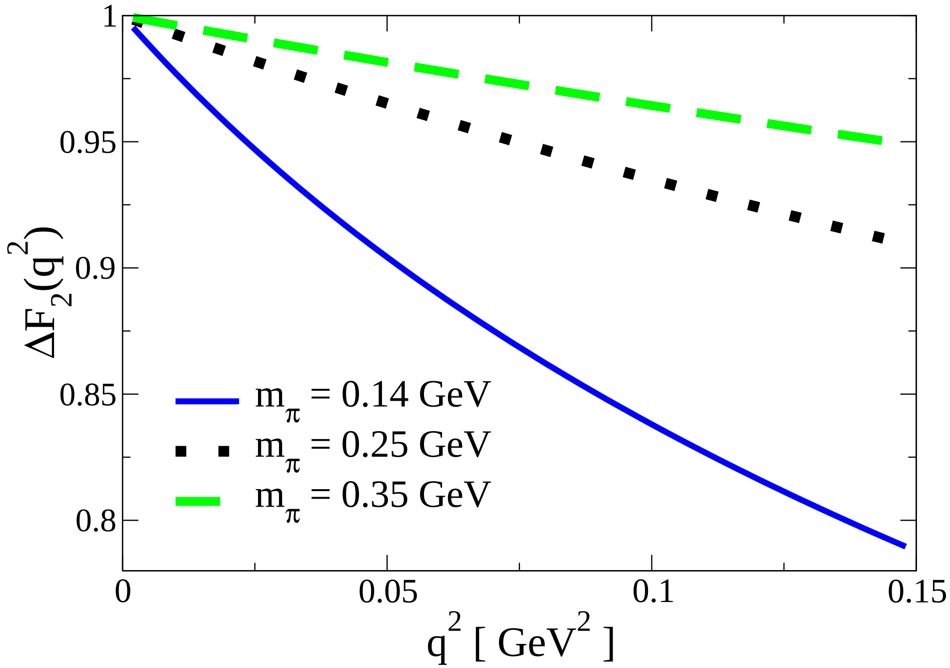

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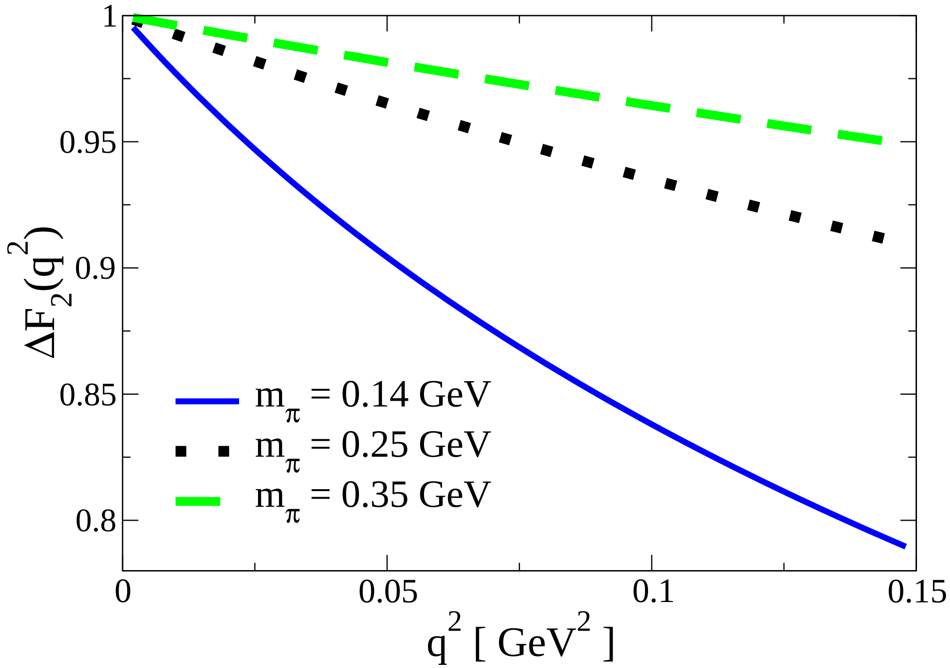

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- Chiral corrections, recoil corrections, lattice point

Twisted boundary conditions

$U^\dagger U = 1$ global symmetry of action, e.g. $U = \exp i\, \theta_i^a\, T_C^a$

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- Partial twisting $\quad T_C^a \in SU(N|N)_{\text{val}} \in SU(N+M|N)$

Field momenta of hadrons

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Sachrajda & Villadoro PLB 609

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- Baryons $\tilde{D}_\mu B^{ijk} = D_\mu B^{ijk} + i(B_\mu^i + B_\mu^j + B_\mu^k)B^{ijk}$

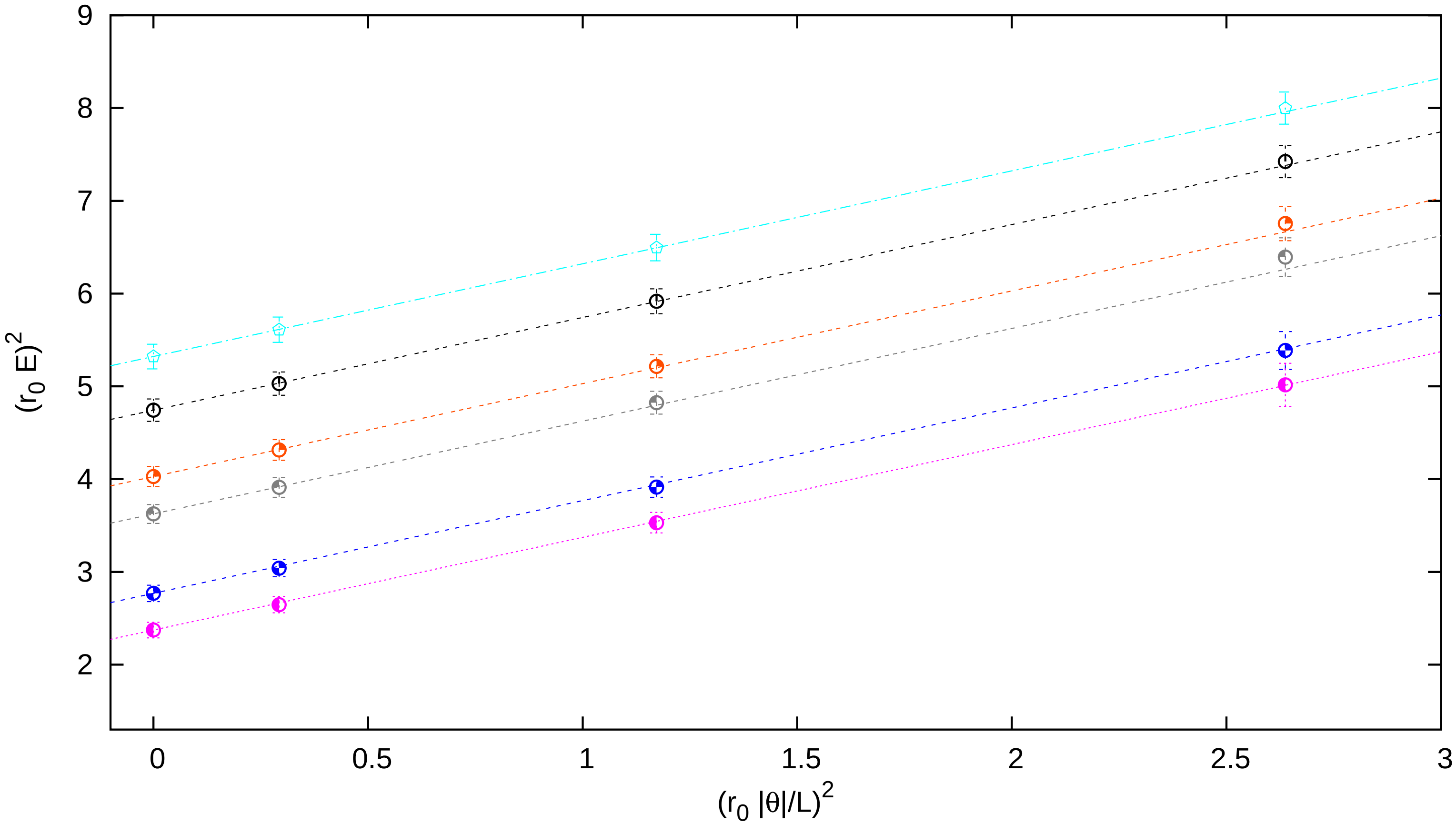
$$E_p = M_p + \frac{\boldsymbol{B}_p^2}{2M_p}, \qquad B_p = 2B_u + B_d$$

Tiburzi PLB 617

Numerical investigations

Meson dispersion relations

Quenched de Divitiis, Petronzio & Tantalo PLB595



Dynamical partially twisted Flynn, Jüttner & Sachrajda PLB632

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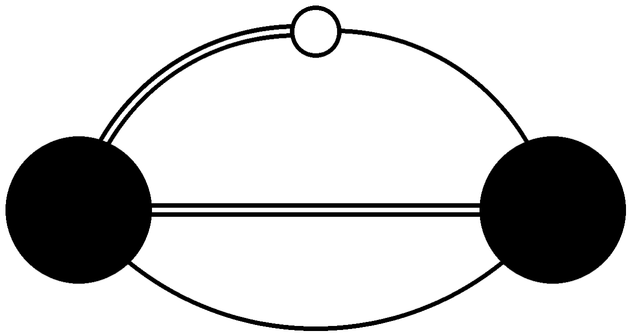
- 

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$$C(t,t') = \sum_{x,x'} \langle 0|\tilde{\mathcal{P}}(x,t)\tilde{J}^{+}_{5\mu}(x',t')\tilde{\mathcal{N}}(0,0)|0\rangle$$

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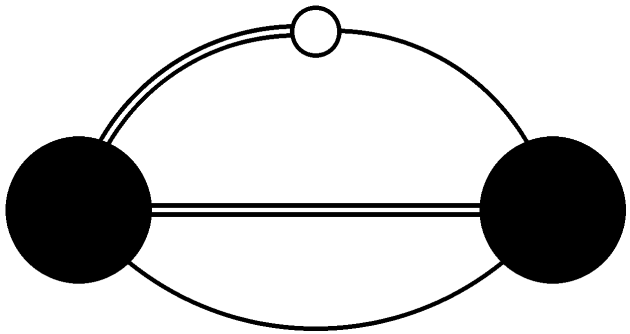
- 

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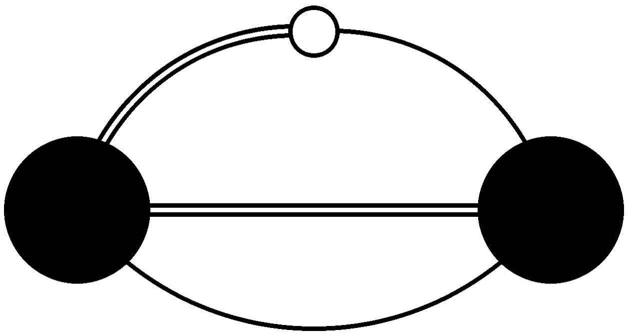
- 

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- Momentum transfer $\boldsymbol{q} = \boldsymbol{B}_p - \boldsymbol{B}_n = \boldsymbol{B}_{\pi^+}$ Tiburzi PLB 617

Further numerical investigations

Meson decay constants Flynn, Jüttner & Sachrajda PLB632

$$\langle \tilde{\pi}^+(\mathbf{0}) \,|\, \tilde{\boldsymbol{J}}_5^+ \,|\, 0 \rangle = i f_\pi \boldsymbol{B}_{\pi^+}$$

f_π reliably extracted

$|\Delta S| = 1$ currents Guadagnoli, Mescia and Simula PRD73

$$\langle \pi^-(p') \,|\, \bar{s}\gamma_\mu u \,|\, K^0(p) \rangle$$

$K \to \pi$ form factors

Systematics?

Isospin relations

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$$\langle p \,|\, \overline{u}\, \Gamma\, d \,|\, n \rangle = \langle p \,|\, \overline{u}\, \Gamma\, u \,|\, p \rangle - \langle n \,|\, \overline{u}\, \Gamma\, u \,|\, n \rangle$$

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- Vector current $\rightarrow$ electromagnetic current

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- Shifts $\sim 1\%$ for $m_\pi \sim 300$ MeV, 2.5 fm

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- Ideal testing ground • TwBCs • Chiral physics

Isovector magnetic moment

- Take $B_2^u = B \neq 0$ and $J^+_{\mu=3}$

$$\langle \tilde{p}(\mathbf{0}) \,|\, \tilde{J}_3^+ \,|\, \tilde{n}(\mathbf{0}) \rangle = -\frac{iB}{2M} F_2(B^2) + L_{\mathsf{FV}} + K_{\mathsf{FV}}$$

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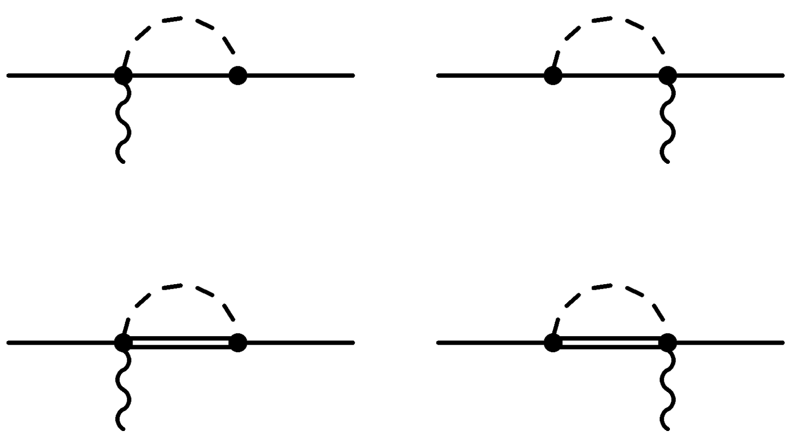

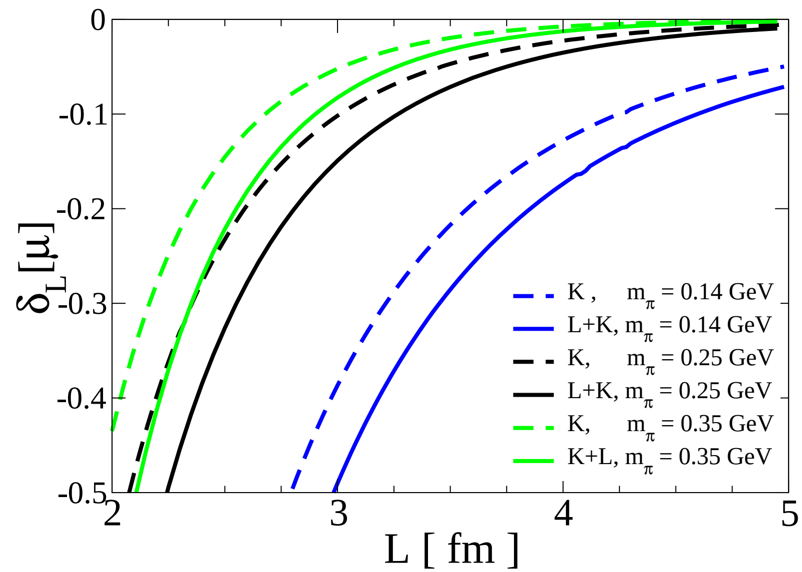

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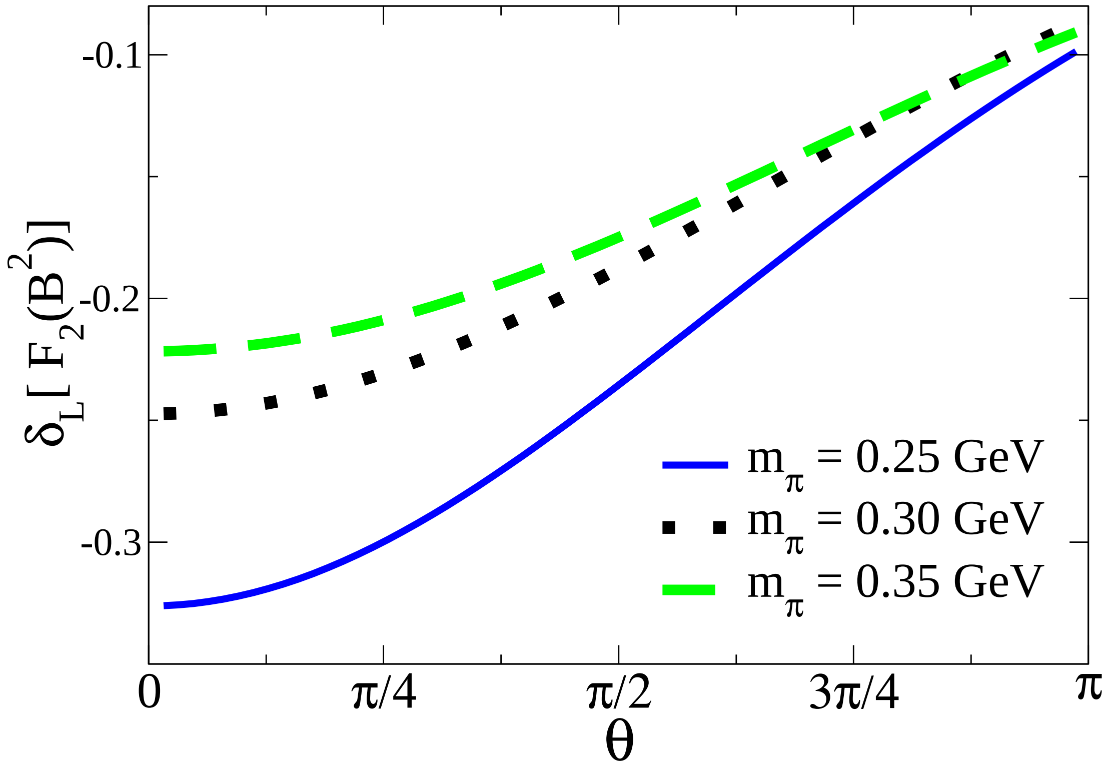
Extracting $F_2(B^2)$: Volume Effects, B^2 resolving power



$q \sim 500$ MeV $\to 25$ MeV in fixed volume

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- BFs isoscalar can be done time consumingly
  TwBCs ???

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