# Recent Results for Nucleon Form Factors

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QCDSF Collaboration

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# Outline

# Content

# Electromagnetic form factors

$$\langle p', s' | J^\mu | p, s \rangle = \overline{\psi}(p', s') \left[ \gamma_\mu F_1(q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2(q^2) \right] \psi(p, s)$$

- $q = p' - p$ ... momentum transfer
- We will consider, e.g.

Proton form factors: $\frac{2}{3}\overline{u}\gamma^\mu u - \frac{1}{3}\overline{d}\gamma^\mu d$

Isovector form factors: $\overline{u}\gamma^\mu u - \overline{d}\gamma^\mu d$

$\rightarrow$ Disconnected terms cancel

# Matrix elements on the lattice

$$R(t,\tau,\vec{p}\,',\vec{p}\,) = \frac{C_3(t,\tau,\vec{p}\,',\vec{p}\,)}{C_2(t,\vec{p}\,')} \times \left[\frac{C_2(\tau,\vec{p}\,')C_2(t,\vec{p}\,')C_2(t-\tau,\vec{p}\,)}{C_2(\tau,\vec{p}\,)C_2(t,\vec{p}\,)C_2(t-\tau,\vec{p}\,')}\right]^{1/2}$$

where

$$C_2(t,\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

and

$$C_3(t,\tau,\vec{p}\,',\vec{p}\,) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,'\,)\mathcal{O}(\tau)\bar{B}_\beta(0,\vec{p}\,)\rangle$$

We use the local vector current: $\overline{\psi}(x)\,\gamma_\mu\,\psi(x)$

# Renormalisation and improvement

$$V_\mu = Z_V(1 + b_V a m_q) \left[\bar{\psi}\gamma_\mu\psi + \mathrm{i} c_V a \partial_\lambda(\bar{\psi}\sigma_{\mu\lambda}\psi)\right]$$

- $Z_V$ and $b_V$ have been determined non-perturbatively [QCDSF 2002]
- $c_V$ known only perturbatively ➔ neglected here

# Simulation details

Configurations with $N_\mathrm{f} = 2$ O(a)-improved dynamical quarks generated by UKQCD+QCDSF.

$$m_{\mathrm{PS,sea}} = 340, ..., 1170 \text{ MeV} \qquad a = 0.07, ..., 0.11 \text{ fm}$$
$$m_{\mathrm{PS,val}} = 340, ..., 1240 \text{ MeV} \qquad V = 1.4, ..., 2.6 \text{ fm}$$

- Simulations much closer to the physical quark mass
- Reasonably small lattice spacing

# Momenta and polarisations

- 3 initial state momentum:

$$\frac{L}{2\pi}\vec{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

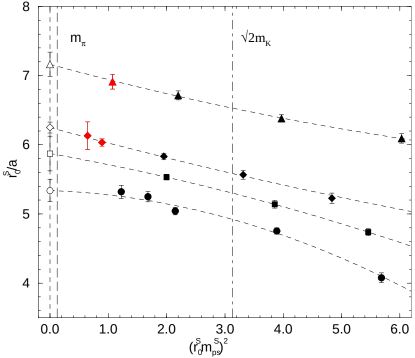
- 3 choices for polarisations:

$$\begin{aligned} \Gamma &= \frac{1}{2}(1+\gamma_4) \\ \Gamma &= \frac{1}{2}(1+\gamma_4)\mathrm{i}\gamma_5\gamma_1 \\ \Gamma &= \frac{1}{2}(1+\gamma_4)\mathrm{i}\gamma_5\gamma_2 \end{aligned}$$

- 17 different choices of $\vec{q} = \vec{p}\,' - \vec{p}$

# Scale definition

- $r_0$ can be determined with good precision on the lattice
  → Good for scaling lattice results
- Experimental value less well known
  → Use nucleon mass for conversion into physical units
  → $r_0 = 0.467$ fm



# Content

# Parametrisation and $q^2$ scaling

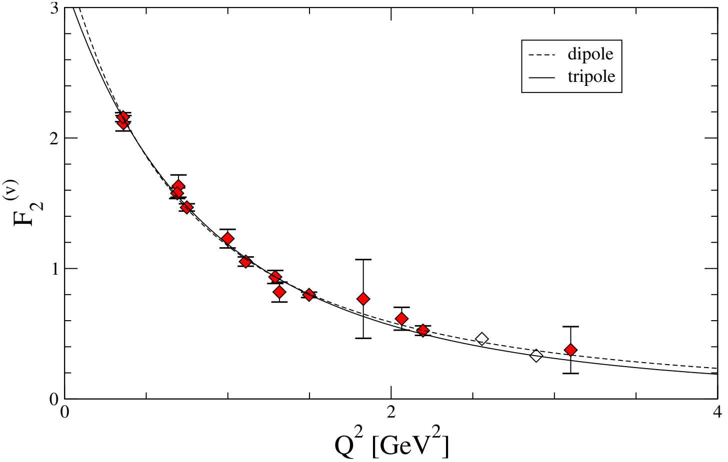
Lattice (and experimental) data can be (well?) described by

$$F_i(q^2) = \frac{A_i}{(1 - q^2/M_i^2)^p}$$

Naive expectation from dimensional counting:

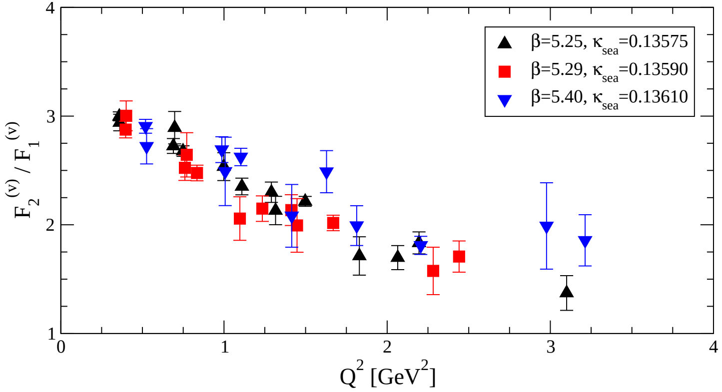
- $F_1 \propto q^{-4} \quad \rightarrow p = 2$
- $F_2 \propto q^{-6} \quad \rightarrow p = 3$

# Let the data tell?



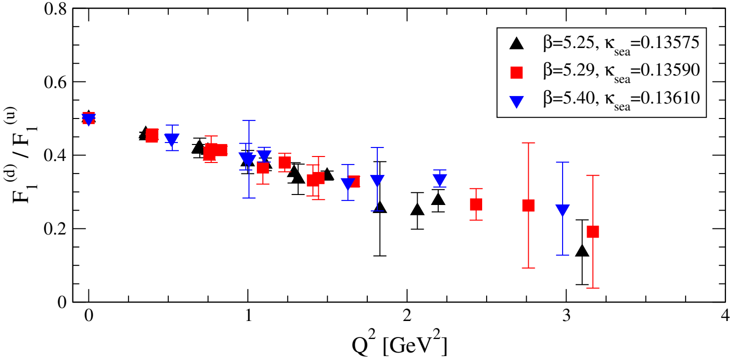

- Difference of fits small wrt to statistical errors

# Scaling of $F_2^{(v)}/F_1^{(v)}$




- $m_{\mathrm{PS}} \approx 600$ MeV, $a = 0.084, \ldots, 0.070$ fm

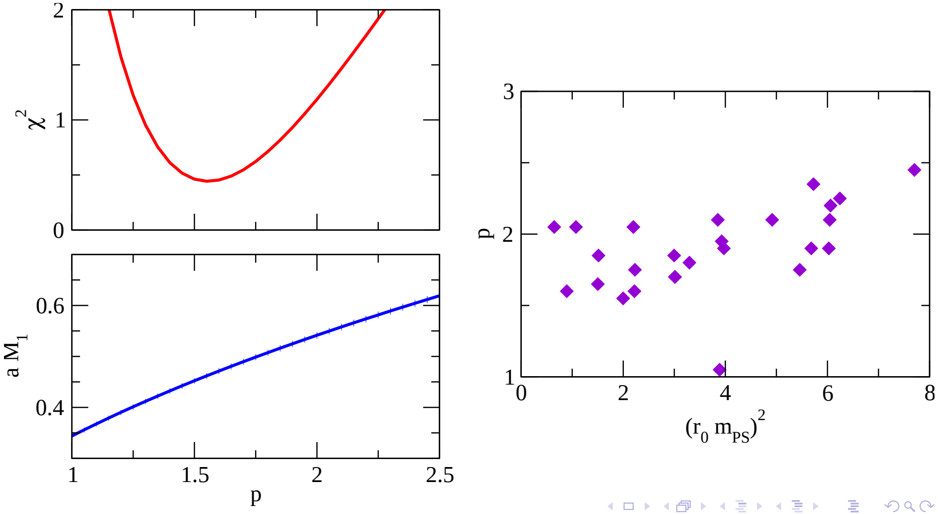
# Scaling of $F_1^{(d)}/F_1^{(u)}$




- Data suggests $p = 2$ and $p = 3$ to fit $F_1^{(u)}$ and $F_1^{(d)}$, respectively
- Flavour dependence also observed in fits to experimental data [Diehl et al., 2005]

# Exploring *p*

Consider $\chi^2$ as a function of *p*:



# Form factor radii and magnetic moment

Definitions:

- Form factor radii $r_i$:

$$F_i(q^2) = F_i(0)\left[1 + \frac{1}{6}\,\mathbf{r_i^2} q^2 + \mathcal{O}(q^4)\right]$$

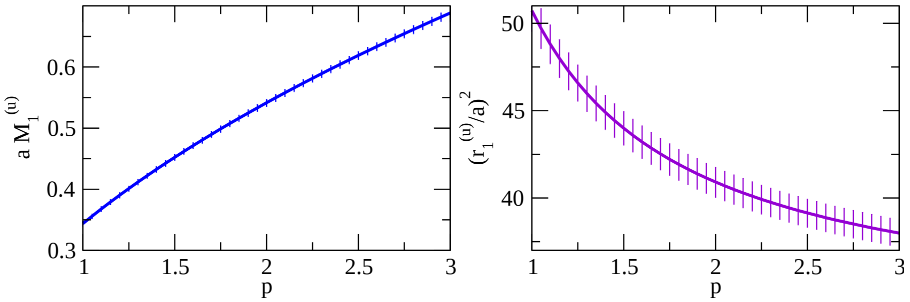
- Magnetic moment $\mu$ / anomalous magnetic moment $\kappa$:

$$\mu = 1 + \kappa = F_1(0) + F_2(0)$$

For comparison with effective theories (and experimental numbers) and we will use

$$\kappa^{(v)\mathrm{norm}} = \kappa^{(v)}\, m_\mathrm{N}(m_\pi)/m_\mathrm{N}(m_\mathrm{PS})$$

# $p$-Dependence of form factor radii




- Choice of $p$ has less impact on form factor radii

# Parametrisation of lattice results

Fit data to

$$F_i(q^2) = \frac{A_i}{(1 - q^2/M_i^2)^p}$$

where

- $p = 2$ for $F_1^{(u)}$
- $p = 3$ for $F_1^{(d)}$, $F_2^{(u)}$, $F_2^{(d)}$

This allows us to determine:

- $F_1^{(q)}(0)$, $F_2^{(q)}(0)$ ($q = u, d$)
- Di-/tripole masses or, equivalently, the form factor radii

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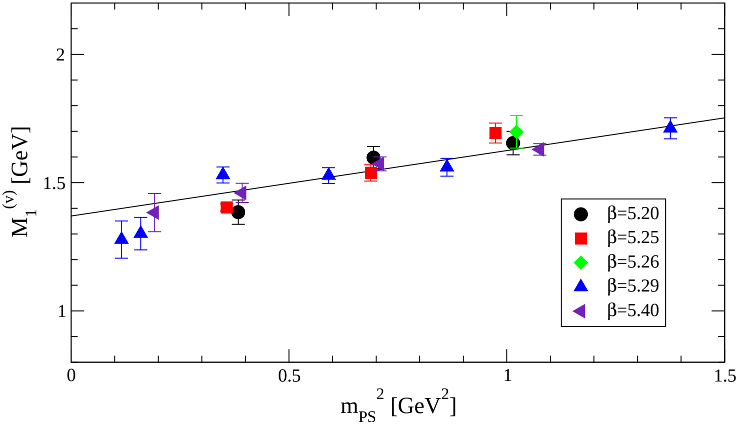
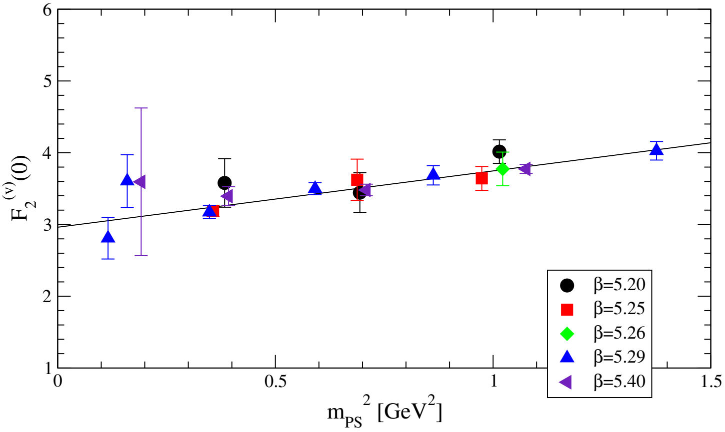
# “Naive” extrapolation of masses




- Di-/tripole masses appear to change linearly with the quark mass

# “Naive” extrapolation of anomalous magnetic moment




- Result in chiral limit smaller than experimental number

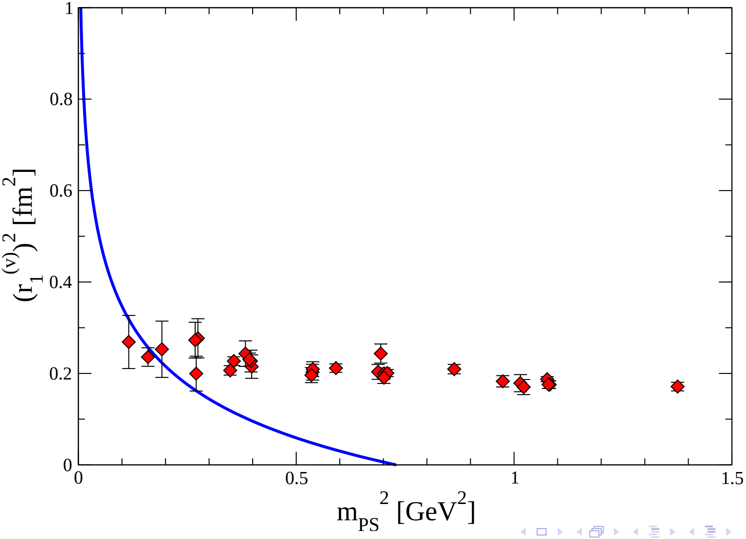
# ChEFT result for $[r_1^{(v)}]^2$

[Hemmert and Weise, 2002; QCDSF 2003]

$$\left(r_1^{(v)}\right)^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7{g_A}^2 + \left(10g_A^2 + 2\right)\log\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]\right\}$$

$$+\frac{{c_A}^2}{54\pi^2 F_\pi^2}\left\{26+30\log\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]+30\frac{\Delta}{\sqrt{\Delta^2 - m_{\mathrm{PS}}^2}}\log\left[\frac{\Delta}{m_{\mathrm{PS}}} + \sqrt{\frac{\Delta^2}{m_{\mathrm{PS}}^2} - 1}\right]\right\}$$

# $[r_1^{(v)}]^2$: comparison ChEFT vs. lattice



# ChEFT result for $[r_2^{(v)}]^2$

$$\left(r_2^{(v)}\right)^2 = \frac{g_A{}^2 M_N}{8F_\pi{}^2\kappa^{(v)}(m_{\mathrm{PS}})\pi m_{\mathrm{PS}}} + \frac{c_A{}^2 M_N}{9F_\pi^2\kappa^{(v)}(m_{\mathrm{PS}})\pi^2\sqrt{\Delta^2 - m_\pi^2}} \log\left[\frac{\Delta}{m_{\mathrm{PS}}} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right] + \frac{24M_N}{\kappa^{(v)}(m_{\mathrm{PS}})} B_{c2}$$

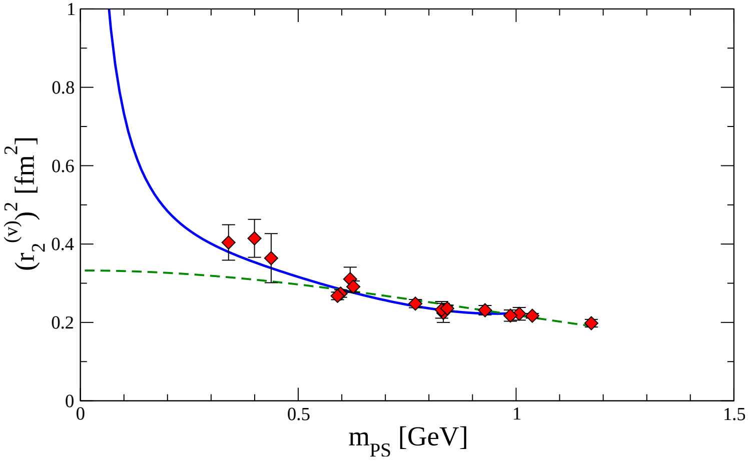
# ChEFT result for $\kappa^{(v)}$

$$\kappa^{(v)}(m_{\rm PS}) = \kappa^{(v)0} - \frac{{g_A}^2\, m_{\rm PS} M_N}{4\pi {F_\pi}^2} + \frac{2{c_A}^2 \Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1-\frac{m_{\rm PS}^2}{\Delta^2}}\log R(m_{\rm PS}) + \log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$
$$- 8E_1^{(r)}(\lambda) M_N m_{\rm PS}^2 + \frac{4c_A c_V g_A M_N m_{\rm PS}^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_{\rm PS}^3}{27\pi F_\pi^2 \Delta}$$
$$- \frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1-\frac{m_{\rm PS}^2}{\Delta^2}\right)^{3/2}\log R(m_{\rm PS}) + \left(1-\frac{3m_{\rm PS}^2}{2\Delta^2}\right)\log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$

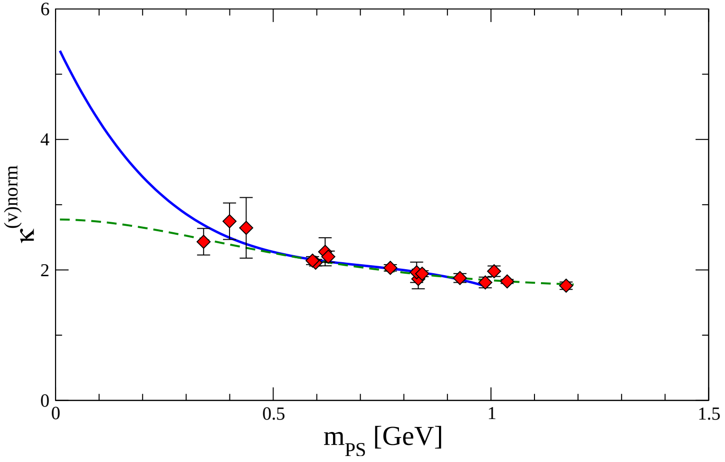
where $R(m) = \frac{\Delta}{m} + \sqrt{\frac{\Delta^2}{m^2} - 1}$

# $[r_2^{(v)}]^2$: comparison ChEFT vs. lattice

- Joined fit to $[r_2^{(v)}]^2$ and $\kappa^{(v)}$:

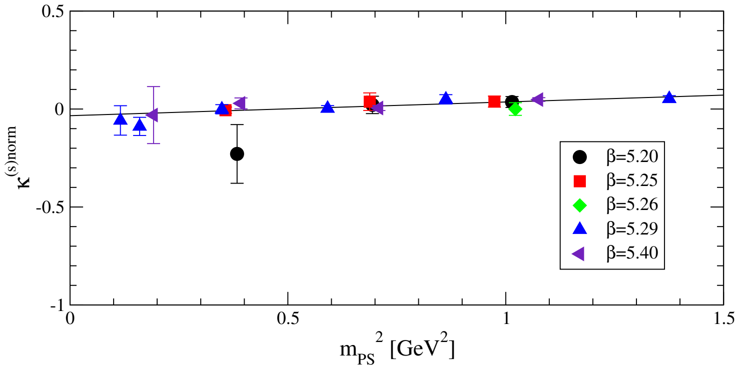


# $\kappa^{(v)}$: comparison ChEFT vs. lattice



# ChEFT result for $\kappa^{(s)}$

$$\kappa^{(s)}(m_{\mathrm{PS}}) = \kappa^{(s)0} - 8E_2 m_{\mathrm{N}} m_{\mathrm{PS}}^2$$

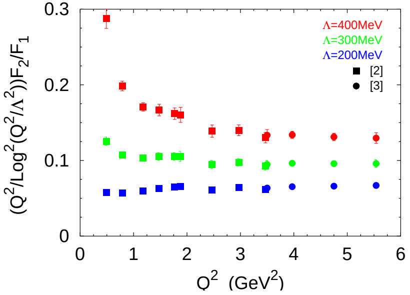


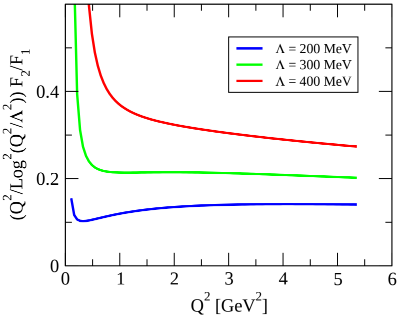
# Content

# $F_2^{(p)}$ vs. $F_1^{(p)}$

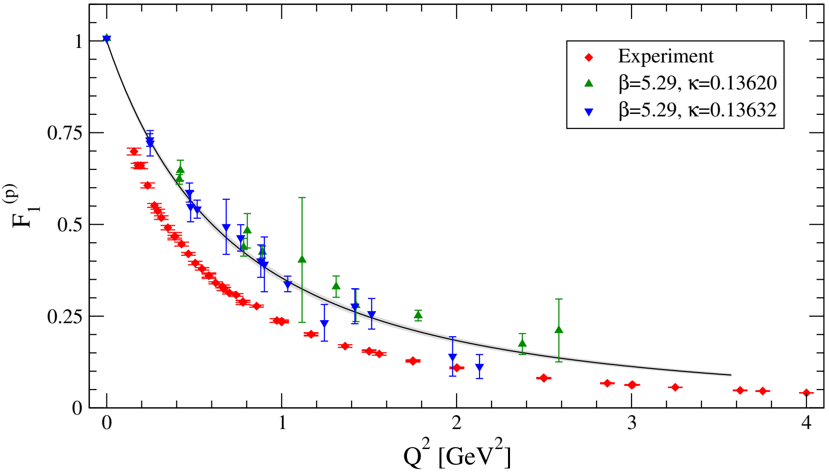
Perturbative QCD [Belitsky et al., 2003]:

$$\left(Q^2/\log(Q^2/\Lambda)^2\right) F_2(Q^2)/F_1(Q^2) \propto \text{const}$$



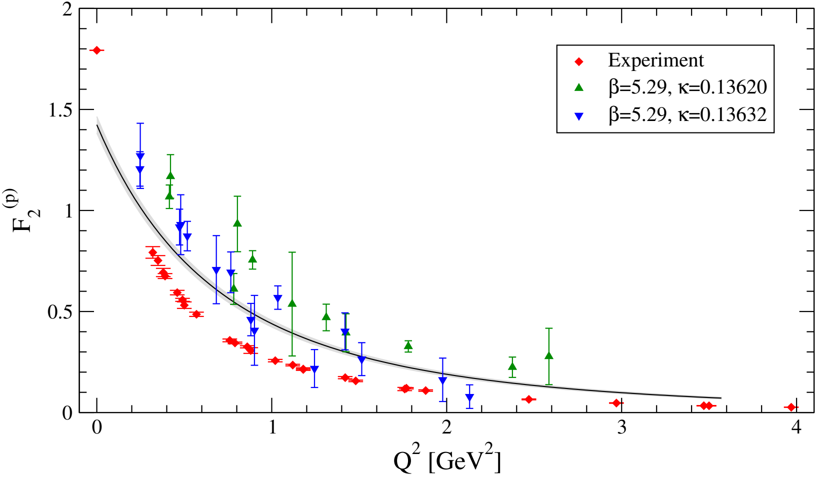



# $F_1^{(p)}$: Lattice vs. Experiment




- Lattice results at $m_{\text{PS}} \gtrsim 300$ MeV → too small Dirac radius

# $F_2^{(p)}$: Lattice vs. Experiment



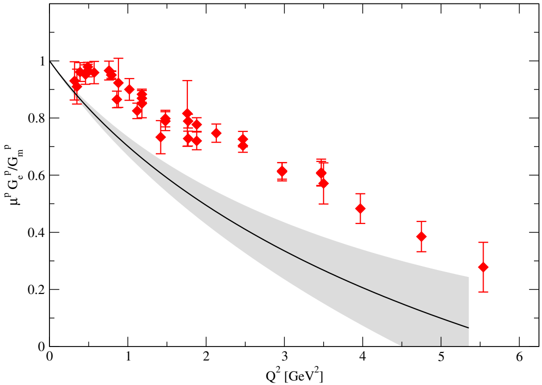

- Better agreement for $m_{\mathrm{PS}} \simeq 300$ MeV?

# Quantitative Comparison

| | Experiment | "Naive" | ChEFT |
|---|---|---|---|
| $(r_1^{(v)})^2$ [fm$^2$] | 0.585 | 0.31(3) | 0.71 |
| $(r_2^{(v)})^2$ [fm$^2$] | 0.797 | 0.34(3) | 0.60 |
| $\kappa^{(u)}$ | 1.67 | 1.4(1) | |
| $\kappa^{(d)}$ | $-2.03$ | $-1.6(1)$ | |
| $\kappa^{(v)}$ | 3.70 | 2.9(1) | 3.95 |
| $\kappa^{(s)}$ | $-0.12$ | $-0.03(1)$ | $-0.03$ |

# Calculation of $\mu^{(p)} G_e^{(p)}(q^2)/G_m^{(p)}(q^2)$ in chiral limit

$$
\begin{aligned}
G_e(q^2) &= F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2) \\
G_m(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$



# Summary and Conclusions

- Parametrisation of the form factors:
  - Large uncertainties remain, as lattice data is not precise enough to fix $q^2$ dependence
  - Qualitative agreement with experimental data found, e.g. flavour dependence of $F_1$
- Chiral extrapolations:
  - First indications for strong effects at light quark masses
  - Lattice data much closer to the chiral limit is crucial (and starting to become available)
- Other systematic errors:
  - Within the statistical errors and above systematic errors, discretisation effects seem negligible.
  - Contributions from disconnnected terms have not been taken into account