Nucleon Structure from Dynamical DWF

Huey-Wen Lin

Columbia University

in collaboration with

Tom Blum, Konstantinos Orginos, Shigemi Ohta and Shoichi Sasaki

Outline

- Introduction
  - Isovector charge and structure functions
  - Lattice implementation
- Numerical results
  - Actions and parameters
  - Review of quenched RBC results
  - Ratios
  - Nonperturbative renormalization
  - Renormalized quantities
- Conclusion and Outlook

Nucleon Isovector Charge

- Nucleon axial charge:

$$\langle p|A_\mu^\dagger(0)|n\rangle = \overline{u}_p[\gamma_\mu\gamma_5 g_A(q^2) - iq_\mu\gamma_5 g_P(q^2)]u_n$$

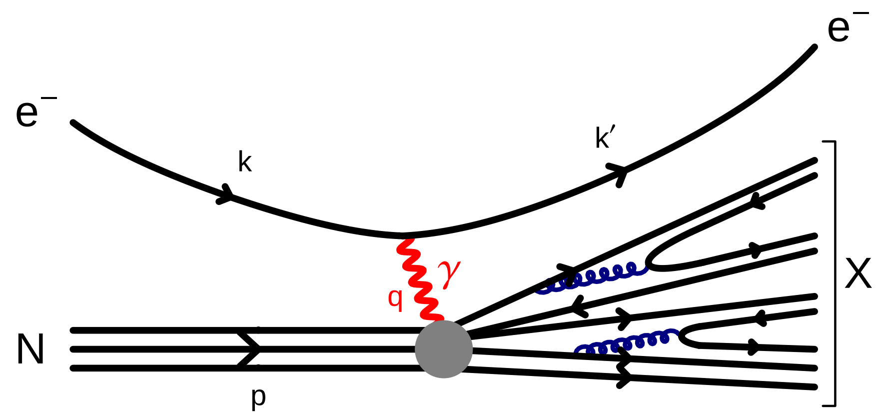
- Nucleon vector charge:

$$\langle p|V_\mu^\dagger(0)|n\rangle = \overline{u}_p[\gamma_\mu g_V(q^2) - q_\mu\sigma_{\mu\nu} g_T(q^2)]u_n$$

- Well measured experimentally g_A/g_V from neutron β decay
$\Rightarrow$ Good test of our understanding of nucleon structure

Moments of Structure Functions

- Deep Inelastic Scattering:



$$\sigma \sim L^{\mu\nu}W_{\mu\nu},$$
$$W_{\mu\nu} = i\int d^4x e^{iqx}\langle N|T\{J^\mu(x), J^\nu(0)\}|N\rangle,$$

- The symmetric, unpolarized, spin-average:

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(p^\mu - \frac{\nu}{q^2}q^\mu\right)\left(p^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu},$$

- The anti-symmetric, polarized:

$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma}q_\rho\left(\frac{s_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot sp_\sigma}{\nu^2}g_2(x,Q^2)\right).$$

Moments of Structure Functions (Cont.)

- Polarized

$$
\begin{aligned}
2\int dx\, x^n g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\triangle q} \\
2\int dx\, x^n g_2(x,Q^2) &= \frac{n}{(n+1)} \sum_{q=u,d} \Big[2e_{2,n}^{(q)}(\mu^2/Q^2, g(\mu)) d_n^q(\mu) \\
&+ e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\triangle q} \Big]
\end{aligned}
$$

- Unpolarized

$$
\begin{aligned}
2\int dx\, x^{n-1} F_1(x,Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q \\
\int dx\, x^{n-2} F_2(x,Q^2) = \sum_{q=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q
\end{aligned}
$$

- e_1, e_2, c_1, c_2 are the Wilson coefficients
- $\langle x^n\rangle_q$, $\langle x^n\rangle_{\triangle q}$, d_n are the forward nucleon matrix elements

Implementation on the Lattice

- Interpolating field

$$J_\alpha\left(\vec{p},t\right) = \sum_{\vec{x},a,b,c} e^{i\vec{p}\cdot\vec{x}}\epsilon^{abc}\left[u_a^T(y_1,t)C\gamma_5 d_b(y_2,t)\right]u_{c,\alpha}(y_3,t)\phi(y_1-x)\phi(y_2-x)\phi(y_3-x)$$

- Two-point correlation function

$$C_{2\text{pt}}\left(\vec{p},t\right) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_4}{2}\right)_{\alpha\beta}\langle J_\beta\left(\vec{p},t\right)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

- Three-point correlation function

$$C_{3\text{pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right) = \sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

- Operators

unpolarized	$\mathcal{O}^q_{\mu_1\mu_2\ldots\mu_n} = \left(\frac{i}{2}\right)^{n-1}\overline{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q - \text{trace}$	$\Gamma = \frac{1+\gamma_4}{2}$
polarized	$\mathcal{O}^{5q}_{\sigma\mu_2\ldots\mu_n} = \left(\frac{i}{2}\right)^{n}\overline{q}\gamma_\sigma\gamma_5\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q - \text{trace}$	$\Gamma = \frac{1+\gamma_4}{2}i\gamma_5\gamma_k$

Implementation on the Lattice

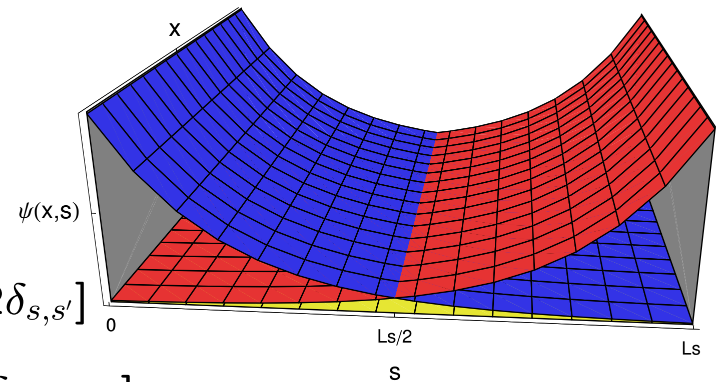
- List of operators (ignore disconnected diagram contribution)

$\langle x\rangle_q$	$\langle x\rangle_{\Delta q}$
momentum fraction	helicity distribution
$\mathcal{O}^q_{44} = \overline{q}\left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_k \gamma_k \overleftrightarrow{D}_k\right] q$	$\mathcal{O}^{5q}_{\{34\}} = i\overline{q}\gamma_5\left[\gamma_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_3\right] q$
$\mathbf{3}^+_1$	$\mathbf{6}^-_3$
$R_{\langle x\rangle_q} = \frac{C_{3pt}^{\Gamma,\mathcal{O}^q_{44}}}{C_{2pt}} = m_N\langle x\rangle_q$	$R_{\langle x\rangle_{\Delta q}} = \frac{C_{3pt}^{\Gamma,\mathcal{O}^{5q}_{\{34\}}}}{C_{2pt}} = m_N\langle x\rangle_{\Delta q}$
${\mathcal{P}^q_{44}}^{-1} = \gamma_4 p_4 - \frac{1}{3}\sum_{i=1,3}\gamma_i p_i$	${\mathcal{P}^{5q}_{34}}^{-1} = i\gamma_5\left(\gamma_3 p_4 + \gamma_4 p_3\right)$

$\langle 1\rangle_{\delta q}$	d_1
transversity	twist-3 matrix element
$\mathcal{O}^{\sigma q}_{34} = \overline{q}\gamma_5\sigma_{34}q$	$\mathcal{O}^5 q_{[34]} = i\overline{q}\gamma_5\left[\gamma_3 \overleftrightarrow{D}_4 - \gamma_4 \overleftrightarrow{D}_3\right] q$
$\mathbf{6}^+_1$	$\mathbf{6}^+_1$
$R_{\langle 1\rangle_{\delta q}} = \frac{C_{3pt}^{\Gamma,\mathcal{O}^{\sigma q}_{\{34\}}}}{C_{2pt}} = \langle 1\rangle_{\delta q}$	$R_{d_1} = \frac{C_{3pt}^{\Gamma,\mathcal{O}^{5q}_{[34]}}}{C_{2pt}} = d_1$
${\mathcal{P}^{\sigma q}_{34}}^{-1} = \gamma_5\sigma_{34}$	${\mathcal{P}^{5q}_{[34]}}^{-1} = i\gamma_5\left(\gamma_3 p_4 - \gamma_4 p_3\right)$

Domain Wall Fermions

- Formulation



$$D_{x,s;x',s'} = \delta_{x,x'} D^{\perp}_{s,s'} + \delta_{s,s'} D^{\parallel}_{x,x'}$$

$$\begin{aligned} D^{\perp}_{s,s'} &= \frac{1}{2}[(1-\gamma_5)\delta_{s+1,s'} + (1+\gamma_5)\delta_{s-1,s'} - 2\delta_{s,s'}] \\ &- \frac{m_f}{2}[(1-\gamma_5)\delta_{s,L_s-1}\delta_{0,s'} + (1+\gamma_5)\delta_{s,0}\delta_{L_s-1,s'}], \\ D^{\parallel}_{x,x'} &= \frac{1}{2}\sum_{\mu=1}^{4}\left[(1-\gamma_\mu)U_\mu(x)\delta_{x+\mu,x'} + (1+\gamma_\mu)U^{\dagger}_\mu(x')\delta_{x-\mu,x'}\right] + (M_5-4)\delta_{x,x'} \end{aligned}$$

- Controllable chiral symmetry breaking with L_s
  $\Rightarrow$ No complicated operator mixing
- Automatic $O(a)$ off-shell improvement
  $\Rightarrow$ Easy to implement RI/MOM NPR
  $\Rightarrow$ No extra $O(a)$ off-shell improved on either the action nor the operators

Gauge Action Choices

- $O(a^2)$-improved gauge actions:

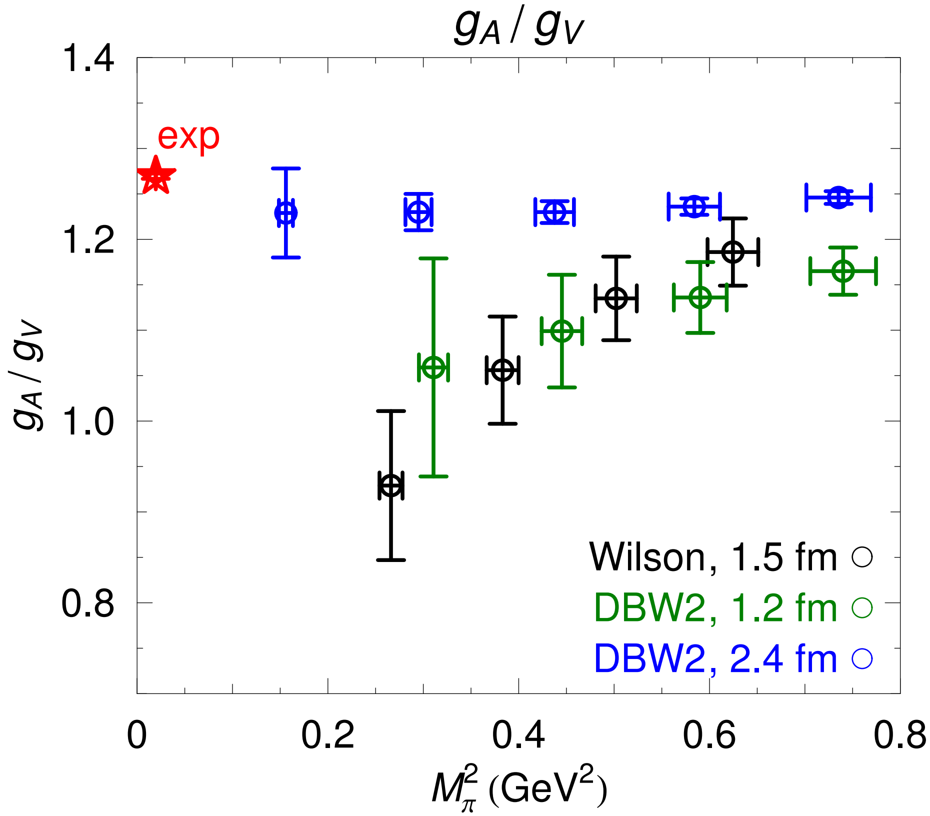
$$S_g = \frac{\beta}{3}\,\mathrm{ReTr}\left((1-8\,c_1)\left\langle \mathbb{1} - \square \right\rangle + c_1 \left\langle \mathbb{1} - \square\square \right\rangle \right)$$

- Constraint: $c_0 + 8c_1 = 1$
- Candidates:
  - Doubly blocked Wilson 2 (DBW2) with $c_1 = -1.40686$
  - Iwasaki action with $c_1 = -0.331$

- Why DBW2?
  - Smaller m_{res}
- Why not?
  - Suppress topology change
  - Scaling issues

RBCK Finite Volume Study

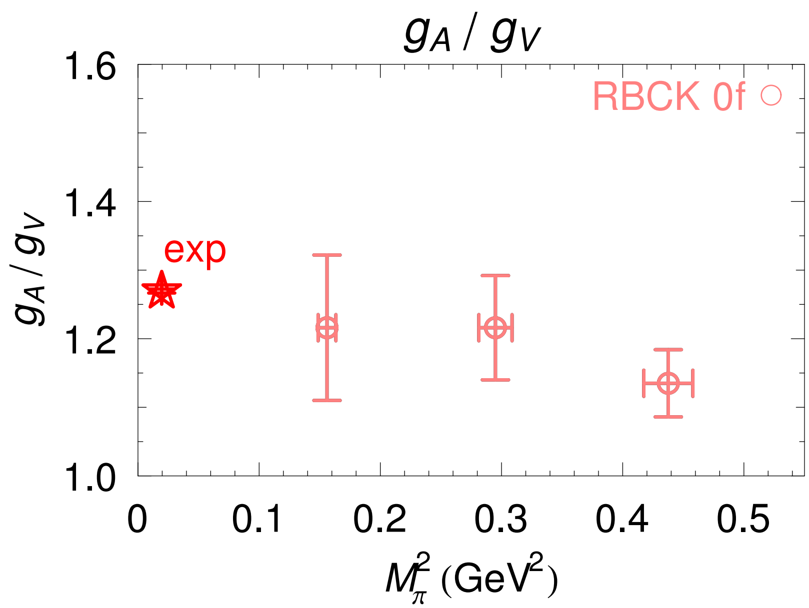
- 3 volumes: $(1.2\text{ fm})^3$, $(1.5\text{ fm})^3$, $(2.4\text{ fm})^3$ presented
- m_π ranges 395-765 MeV

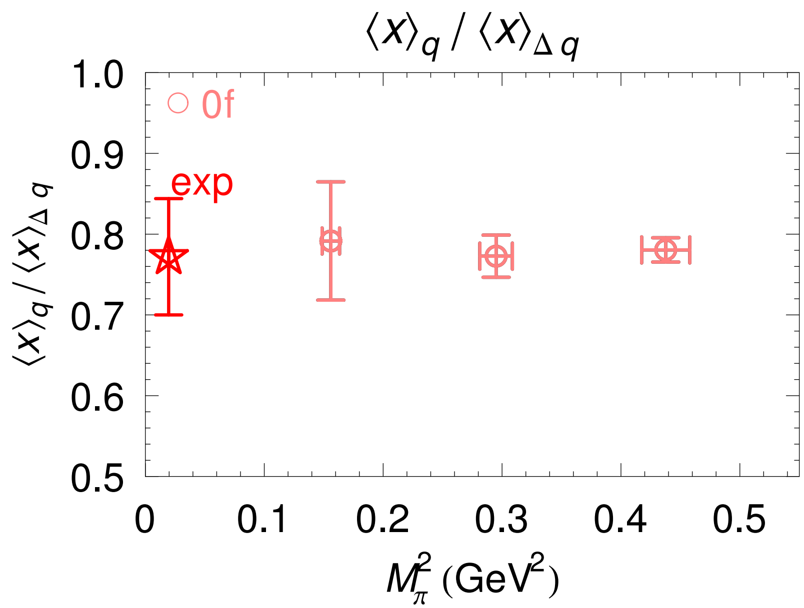


Blum, Ohta, Orginos, Sasaki '03

RBCK quenched results

- DBW2 $\beta = 0.87$, $a^{-1} = 1.31$ GeV, $(2.4\ \text{fm})^3$
- m_π ranges 395-765 MeV
- Result (concentrating on the lightest three)



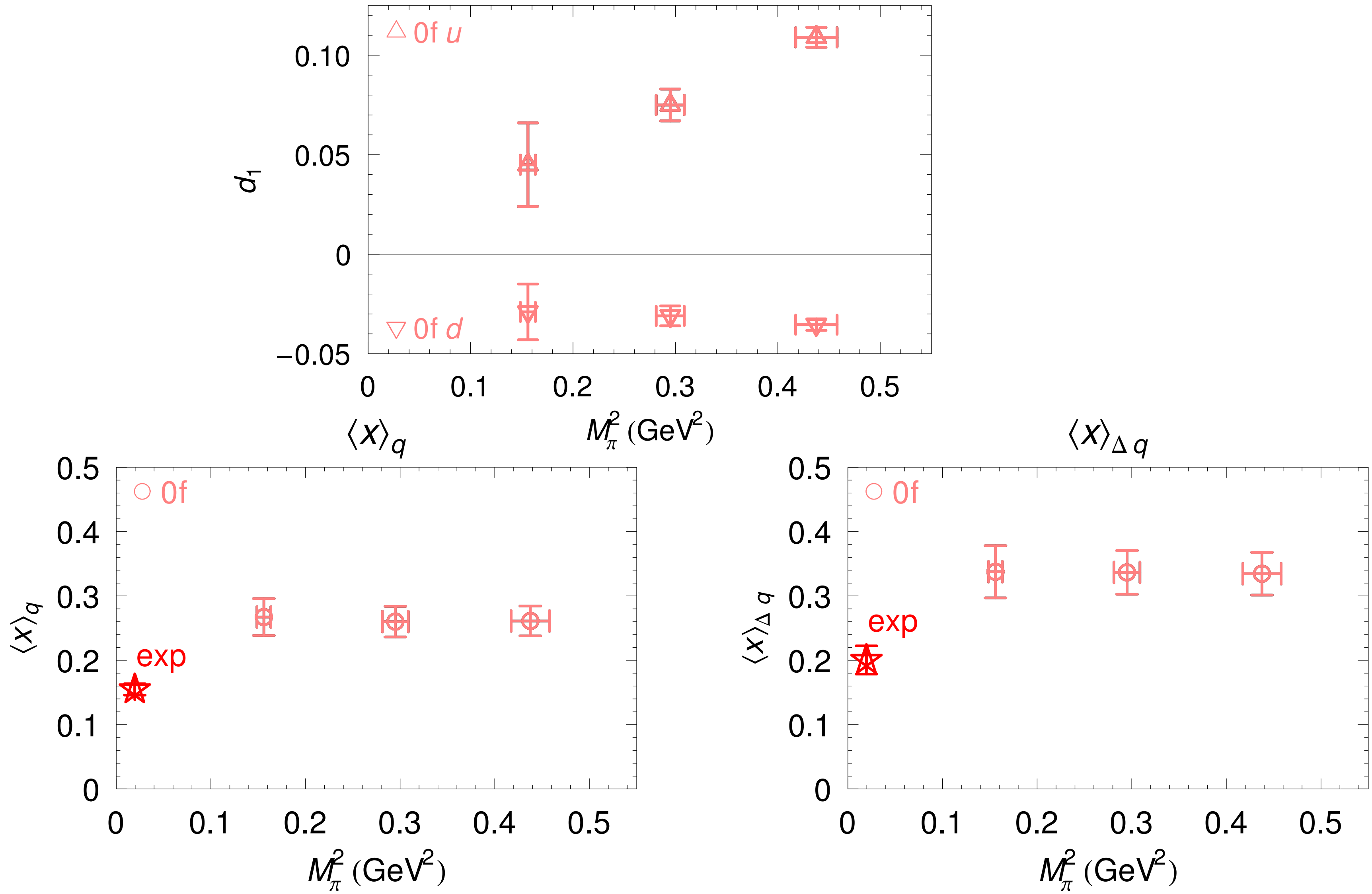


- $g_A = 1.212(27)_{\text{stat}}(24)_{\text{norm}}$
- Ratio is in very good agreement with experiment

Blum, Ohta, Orginos, Sasaki '03

Blum, Ohta, Orginos '05

RBCK quenched results (Cont.)



Blum, Ohta, Orginos '05

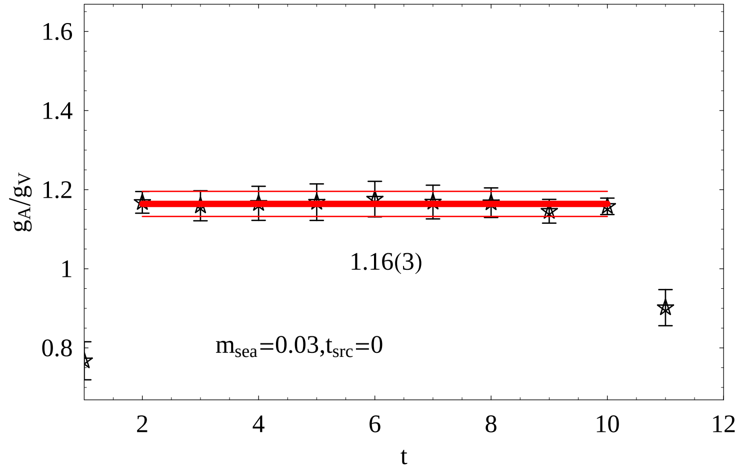
RBC/UKQCD Ensembles

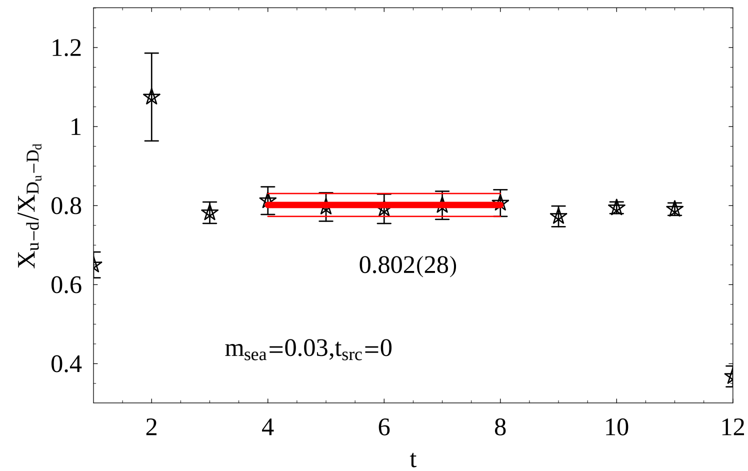
	2 flavor (RBC)	2+1 flavor (RBC/UKQCD)
Gauge action	DBW2	Iwasaki
β	0.80	2.13
Algorithm	HMC	RHMC
a^{-1}	$\approx$ 1.7 GeV	$\approx$ 1.6 GeV
Lattice Size	$16^3 \times 32 \times 12$	$16^3 \times 32 \times 16$, $24^3 \times 64 \times 16$
L	$\approx$ 2 fm	$\approx$ 2 fm, $\approx$ 3 fm
am_{res}	0.001372(49)	0.00307(3)
am_{sea}	{0.02,0.03,0.04}	{0.01,0.02,0.03}
	$[\frac{m_{\text{strange}}}{2}, m_{\text{strange}}]$	$[\frac{1}{4}m_{\text{strange}}, \frac{3}{4}m_{\text{strange}}]$
M_5	1.8	1.8
Measurements		
Smearing source	Gaussian	Gaussian
$\{t_{\text{src}}, t_{\text{snk}}\}$	{0,10}, {15,25}	{0,12}, {16,28}, {32,44},{48,60}
Conf.	175 , 220, 220	30, 25, 25
m_π(MeV)	{ 495.(4), 607.(4), 695.(4)}	{ 393.(4), 523.8(29), 611.7(26)}

- Unitary sea quark mass only
- Preliminary lightest 2f analysis
- Preliminary 2+1 analysis

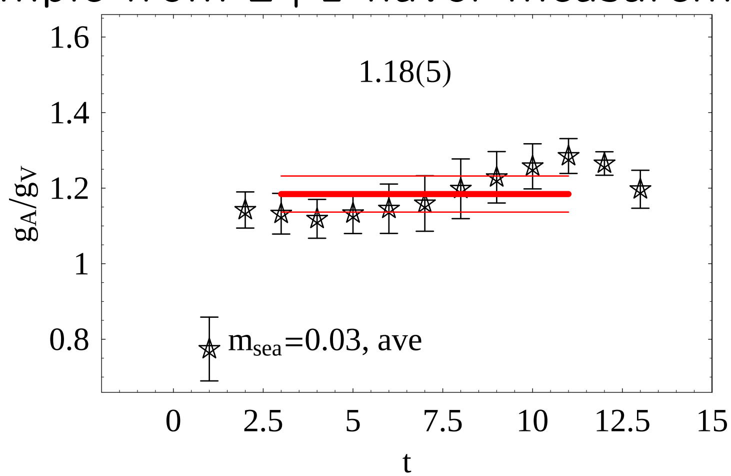
Ratios: Plateau

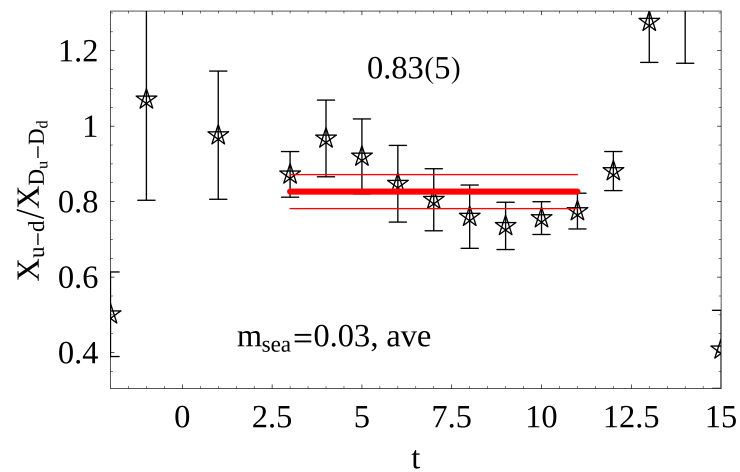
- Example from 2 flavor measurement





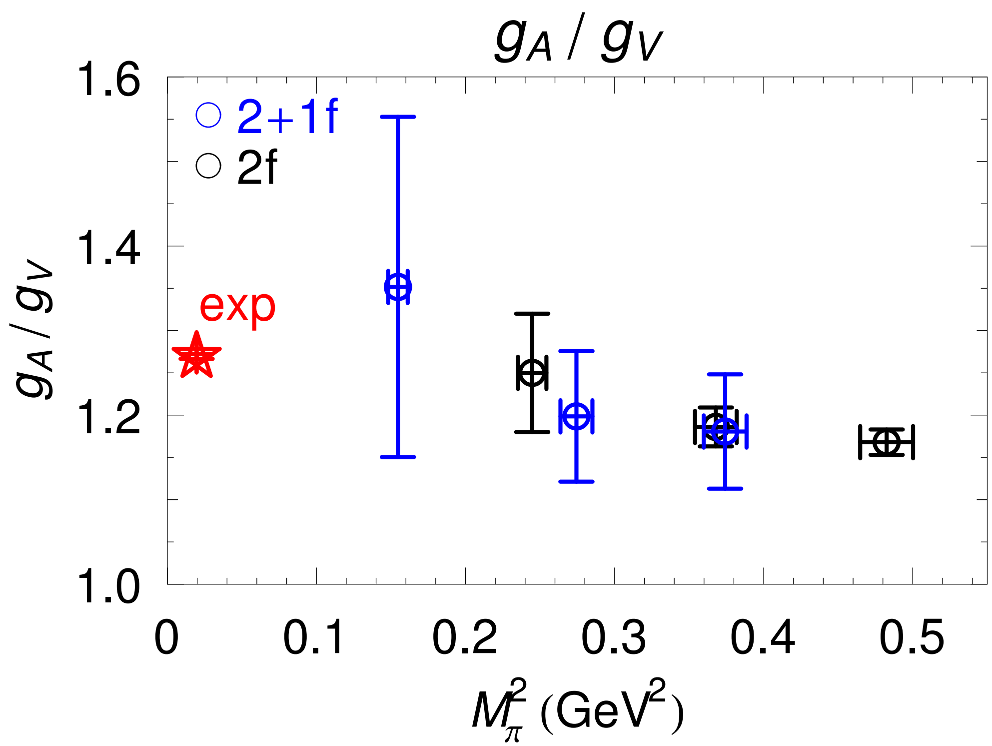
- Example from 2+1 flavor measurement





Ratios: g_A/g_V

- Preliminary 2+1 data set; ≤ 30 each ensemble



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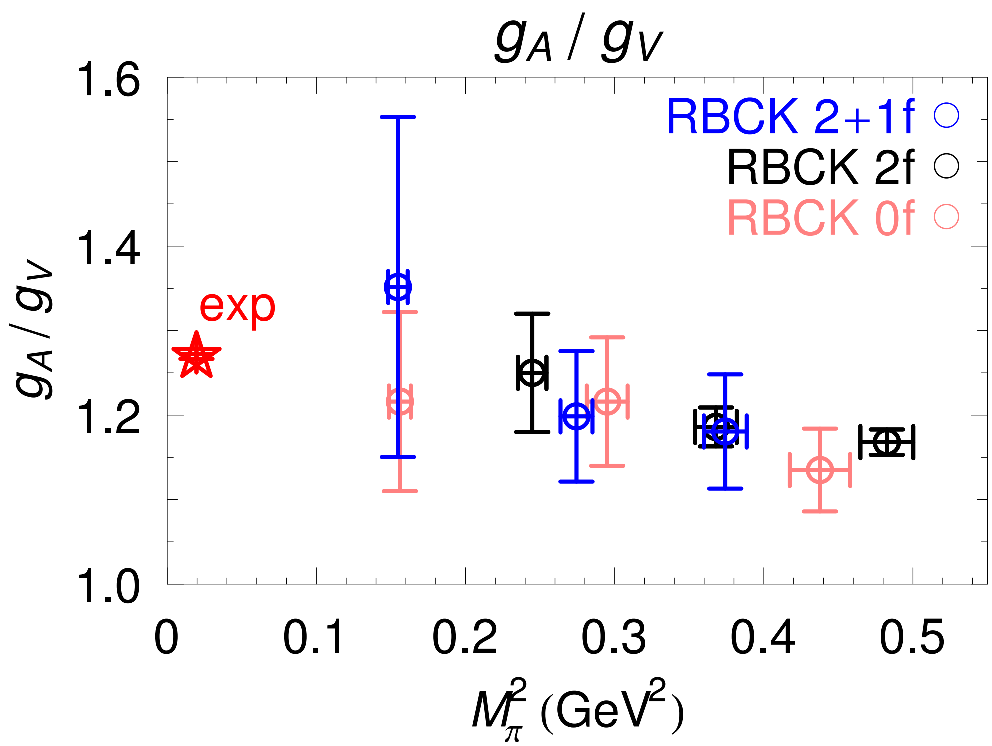

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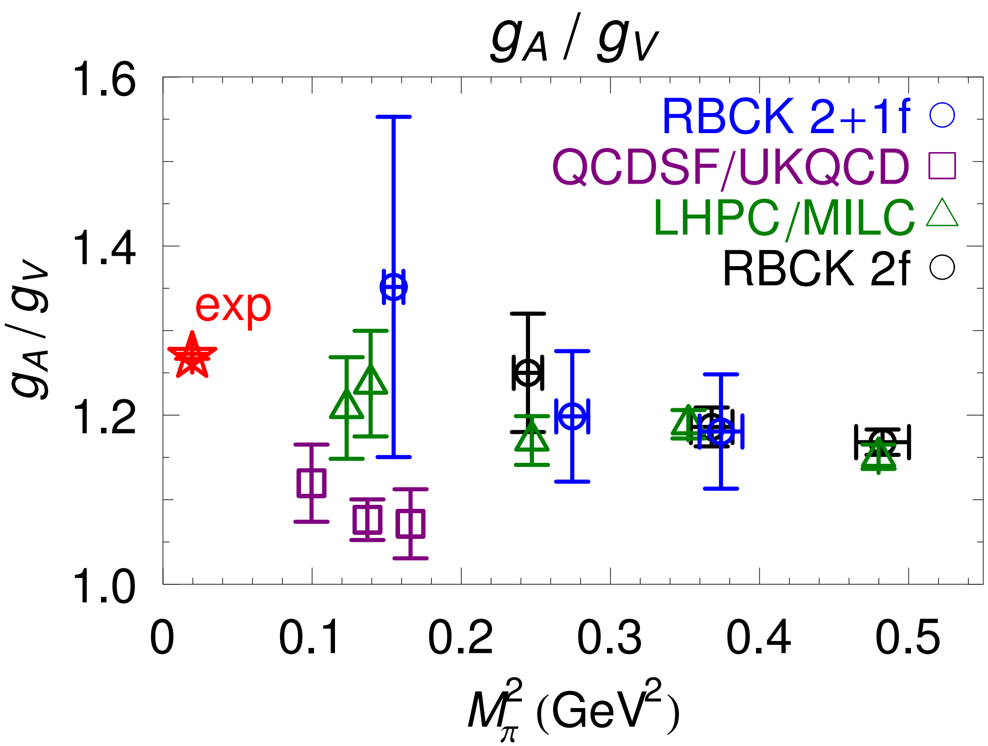

Ratios: $\langle x \rangle_q / \langle x \rangle_{\Delta q}$

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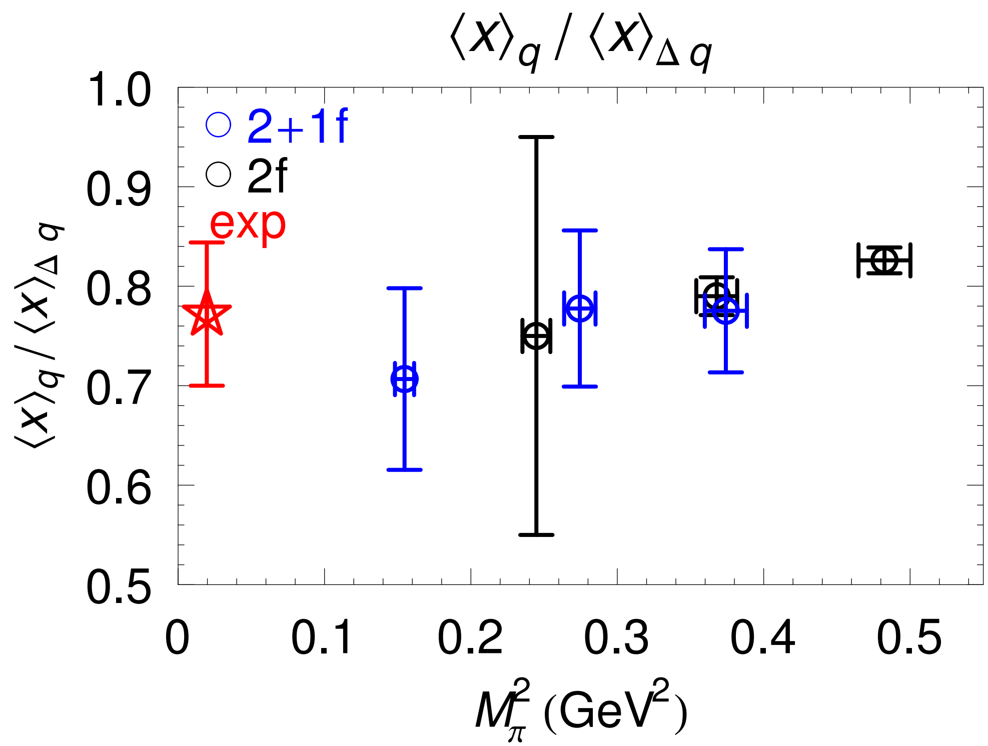

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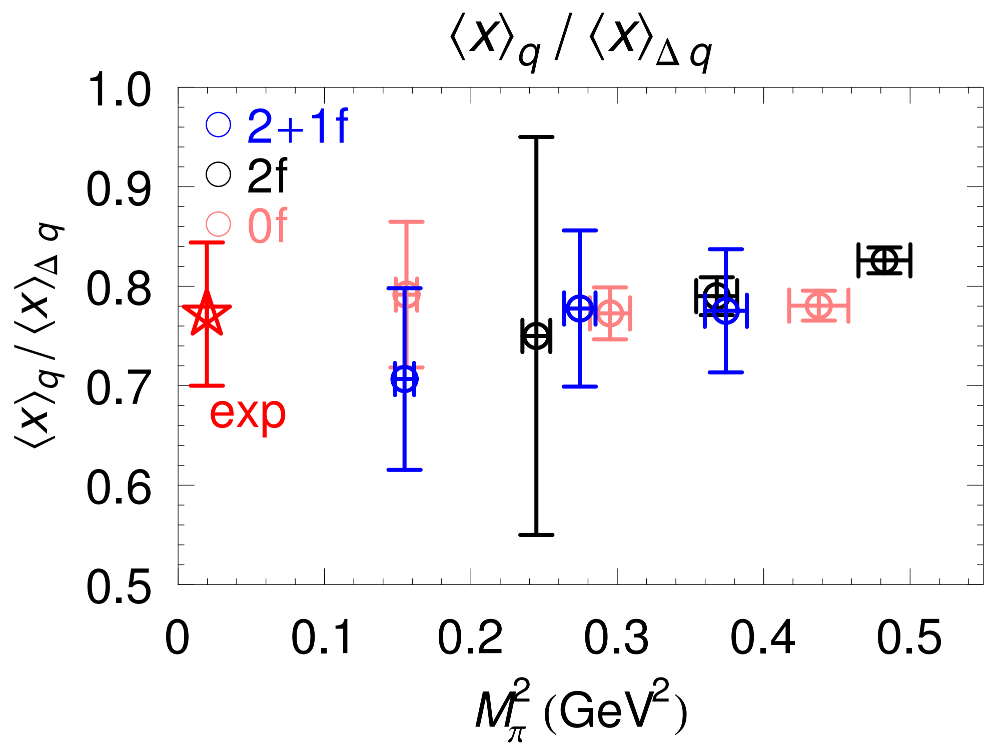
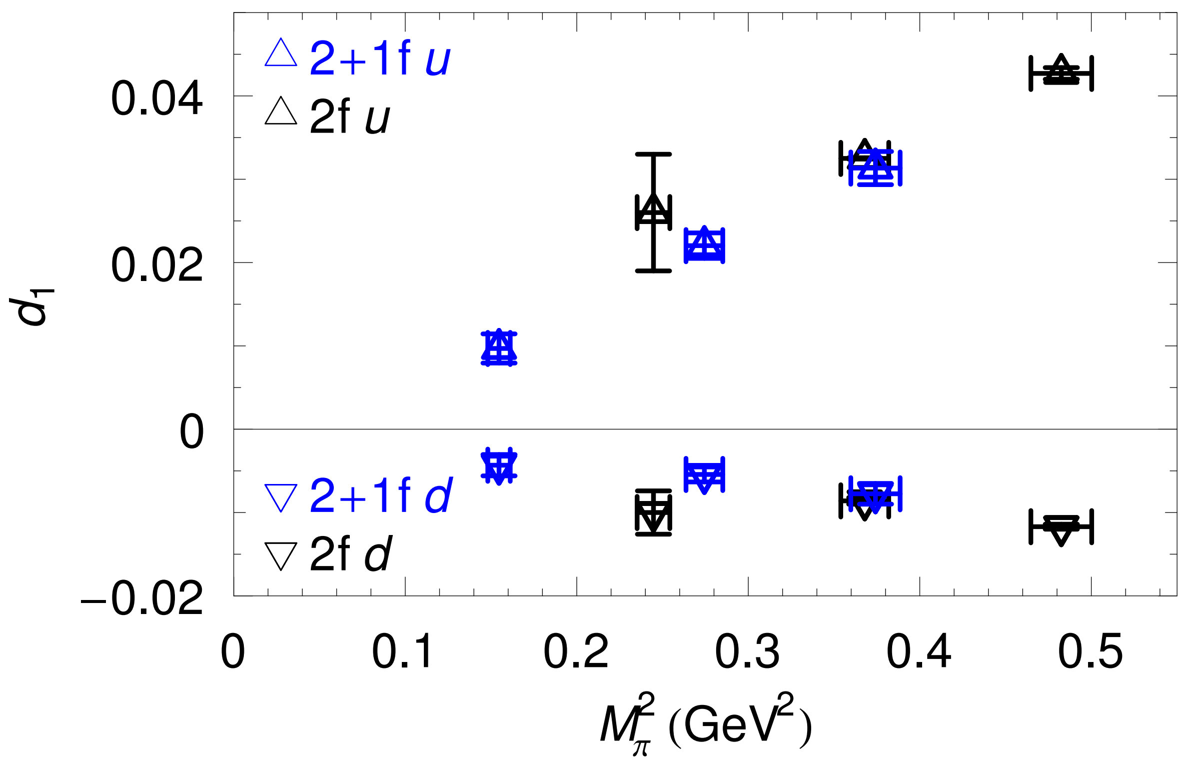


Bare d_1

- Preliminary 2+1 data set; ≤ 30 each ensemble



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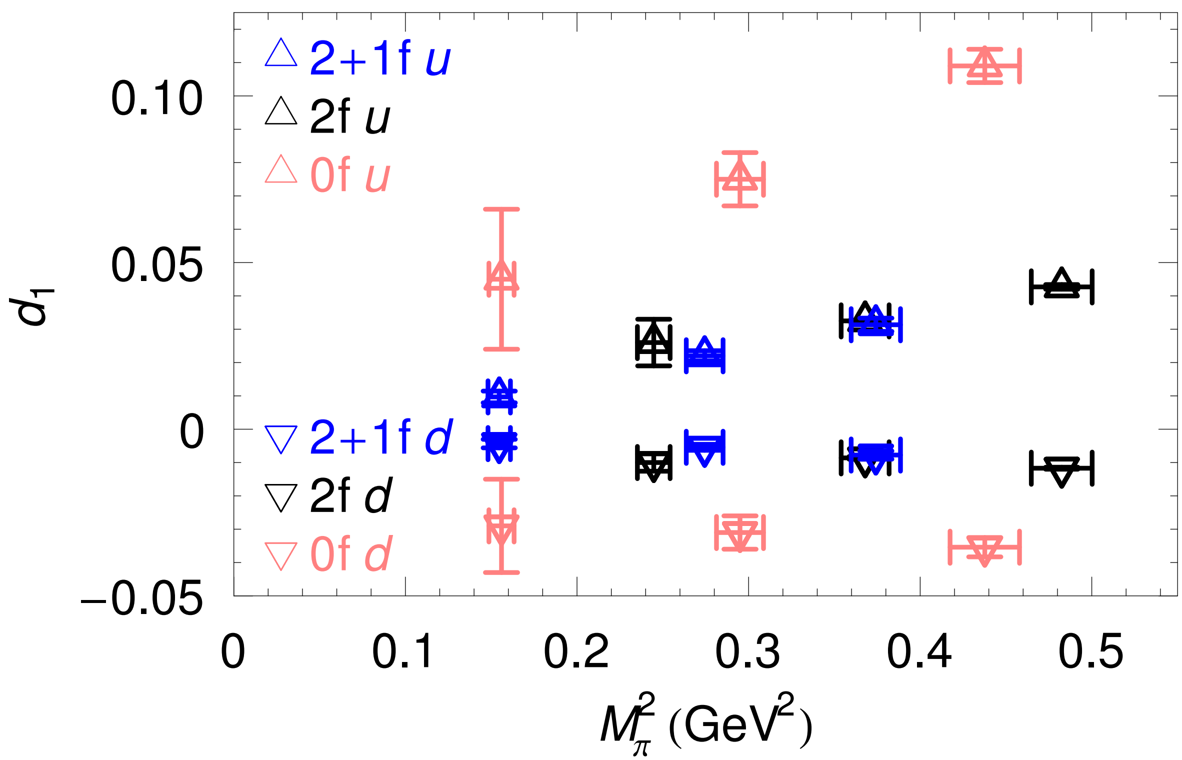

Bare d_1

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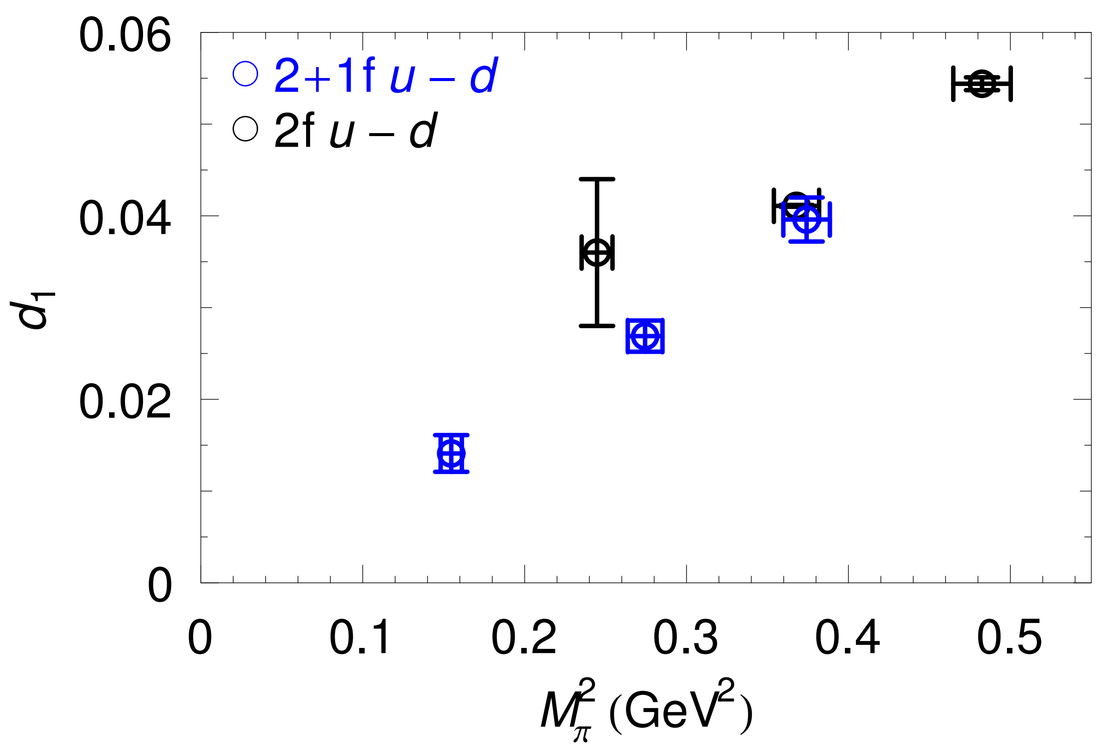

Bare d_1

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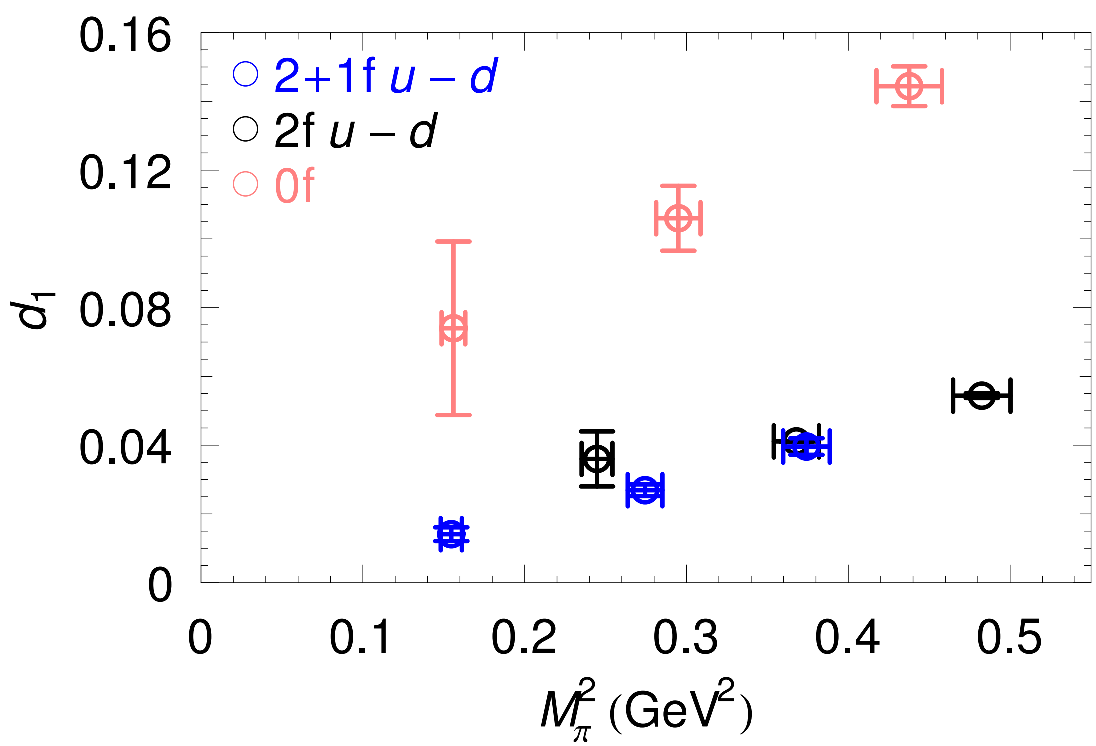


Nonperturbative Renormalization

- In general,

$$\mathcal{O}_i(\mu) = Z_i(\mu, a)\mathcal{O}_i(\mu) + \sum_{i \neq j} Z_{ij}(\mu; a)\mathcal{O}_j(\mu)$$

Chiral fermions are free of mixing problem

- Fourier transform Green function

$$G_{O_\Gamma}(p; a) = \sum_{x,y} e^{-ip\cdot(x-y)} \langle \psi(x) O_\Gamma(0) \overline{\psi}(y) \rangle$$

- Calculate point ($S(pa; 0)$) and point-split ($D_\mu S(pa; 0)$) source propagator

$$\begin{aligned} S(pa; 0) &= \sum_x e^{-ip\cdot x} S(x; 0) \\ D_\mu S(pa; 0) &= \sum_x \frac{1}{2} e^{-ip\cdot x} \left[S(x; -\hat{\mu}) U_\mu(-\hat{\mu}) - S(x; \hat{\mu}) U_\mu^\dagger(0) \right] \end{aligned}$$

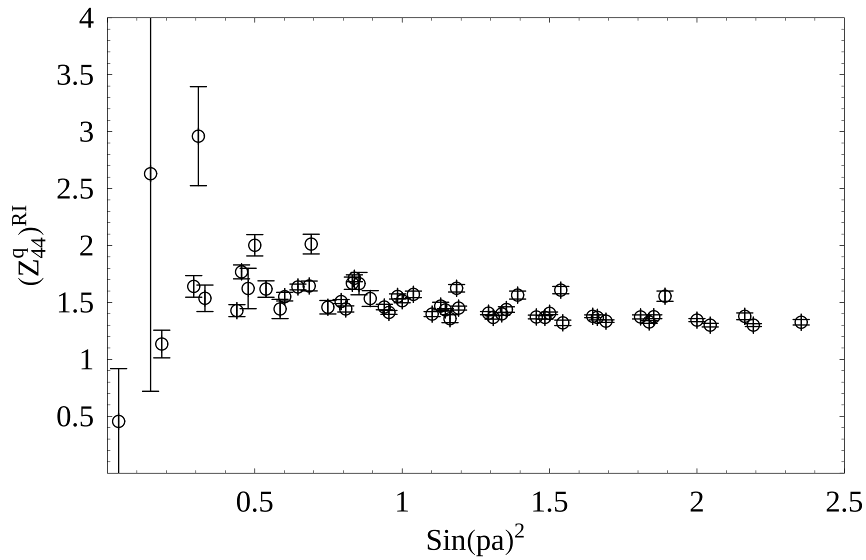
- Truncate the external quark propagator legs

$$\Lambda_{O_\Gamma}(p; a) = S(p; a)^{-1} G_{O_\Gamma}(p; p'; a) S(p'; a)^{-1}$$

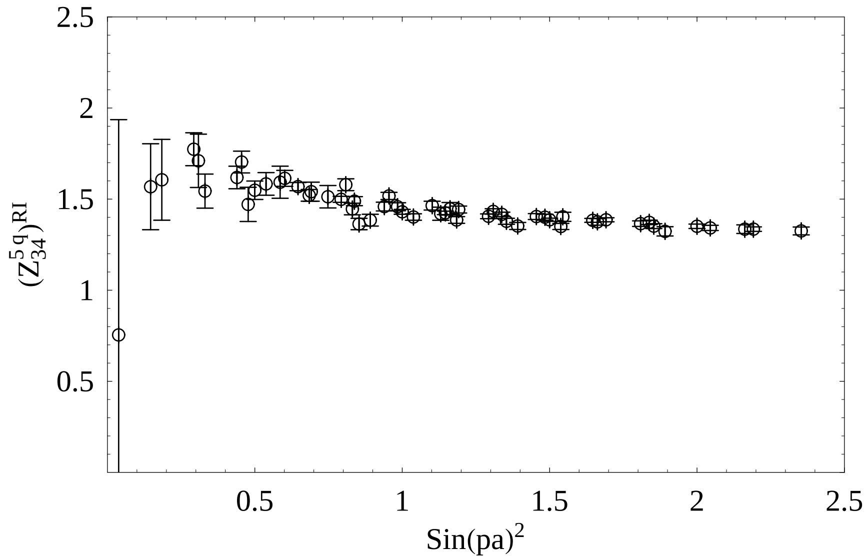
Nonperturbative Renormalization (Cont.)

- Example from 2 flavor case with $am_{\mathrm{sea}} = 0.02$

$\mathcal{O}^{q}_{44}$



$\mathcal{O}^{5q}_{34}$



- Z^{RI} obtained after projection

$$Z_{O_\Gamma}(\mu;a)^{-1} Z_q(\mu;a) = \frac{1}{12} \mathrm{Tr}\left(\Lambda_{O_\Gamma}(p;a) P_\Gamma\right)|_{p^2=\mu^2},$$

μ must fall inside the renormalization window

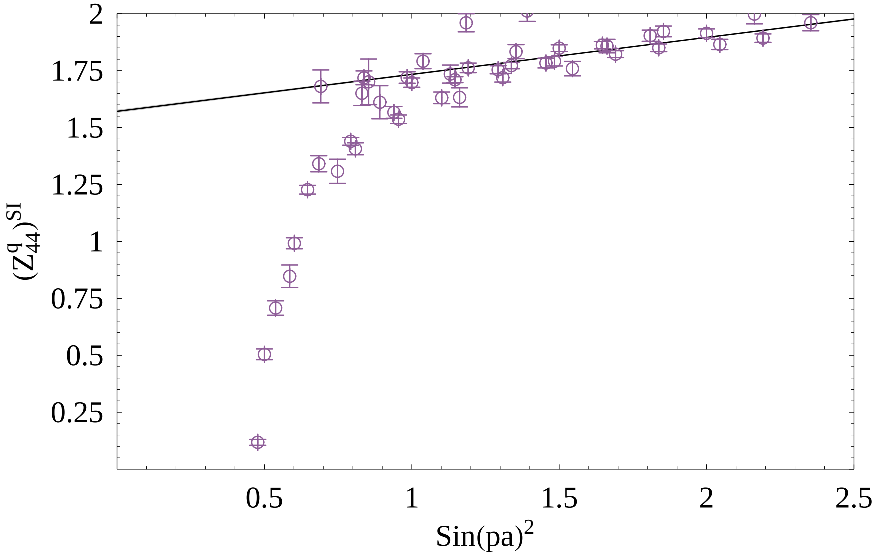
$$\Lambda_{\mathrm{QCD}} \ll \mu \ll 1/a$$

- Convert to $\overline{\mathrm{MS}}$ scheme

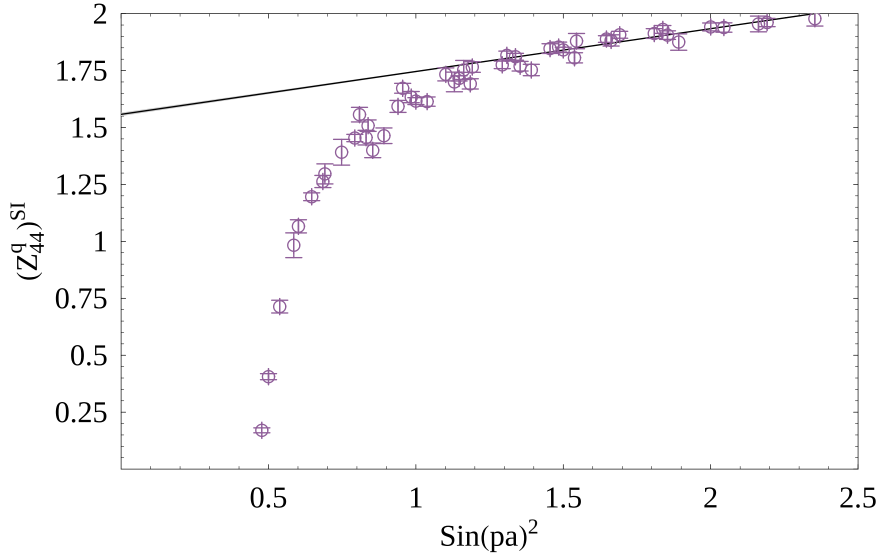
Nonperturbative Renormalization (Cont.)

- Do RGI running to remove $(ap)^2$ dependence
  (Example from 2 flavor case with $am_{\text{sea}} = 0.02$)

$\mathcal{O}^q_{44}$



$\mathcal{O}^{5q}_{34}$



- Renormalized at 2 GeV
  $\Rightarrow Z_{\mathcal{O}^q_{44}} = 0.983(4)$ and $Z_{\mathcal{O}^{5q}_{34}} = 0.975(5)$.

Summary of the Renormalization Factors

(Preliminary)

	2 flavor	2+1 flavor
Conf.	{94, 111, 90}	{76, 76, 76}
$Z_{\mathcal{O}^q_{44}}$	{0.983(4), 0.962(3), 0.970(5)}	{0.877(13), 0.891(10), 0.866(10)}
$Z_{\mathcal{O}^{5q}_{34}}$	{0.975(5), 0.999(3), 0.988(3)}	{0.997(7), 0.918(4), 0.936(4)}
$Z_{\mathcal{O}^{\sigma q}_{34}}$	{0.707(1), 0.711(1),0.720(1)}	{0.676(2), 0.675(2), 0.663(2)}

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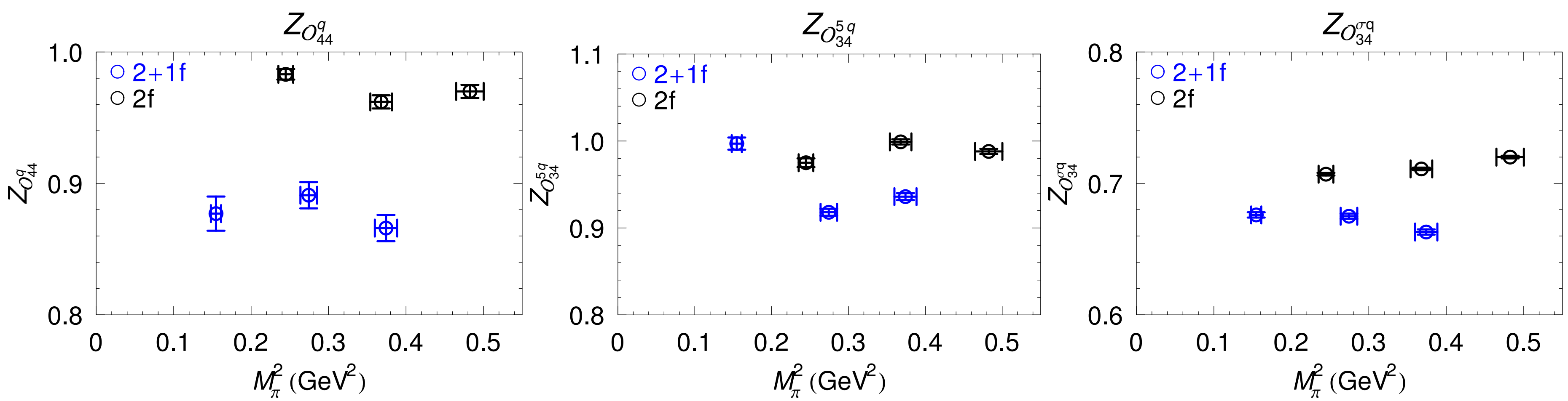
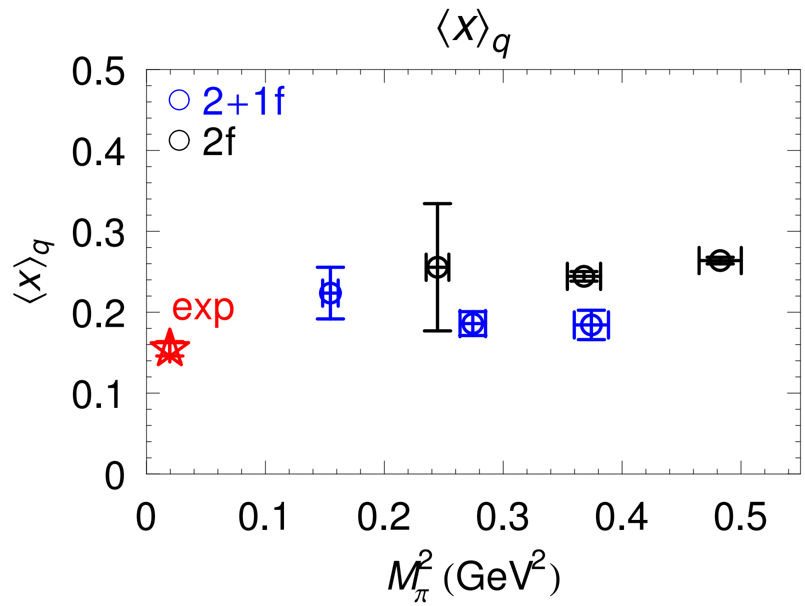
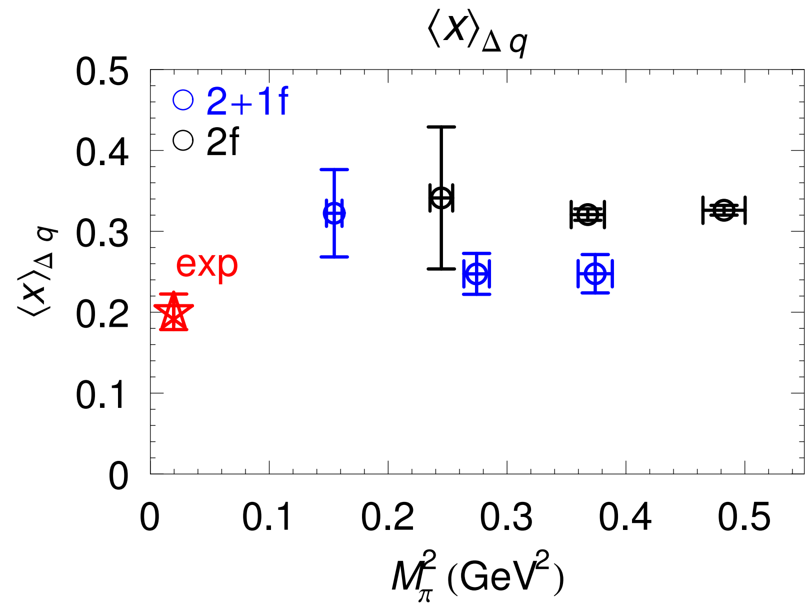


Renormalized $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$

(Preliminary)

- Results



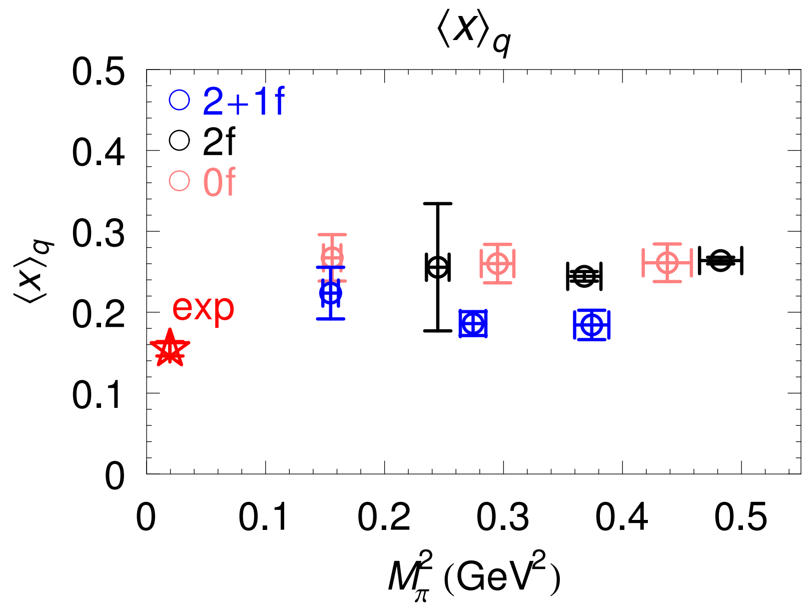


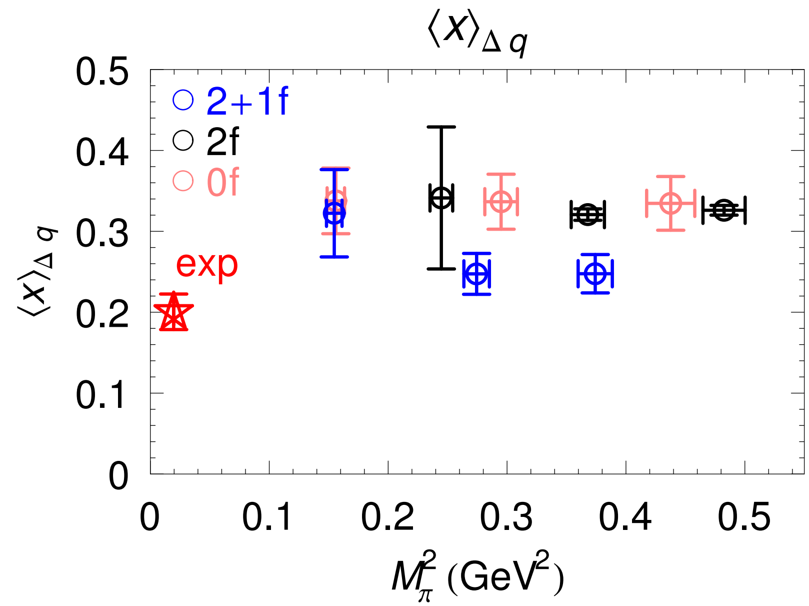
- Not yet extrapolated

Renormalized $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$

(Preliminary)

- Results



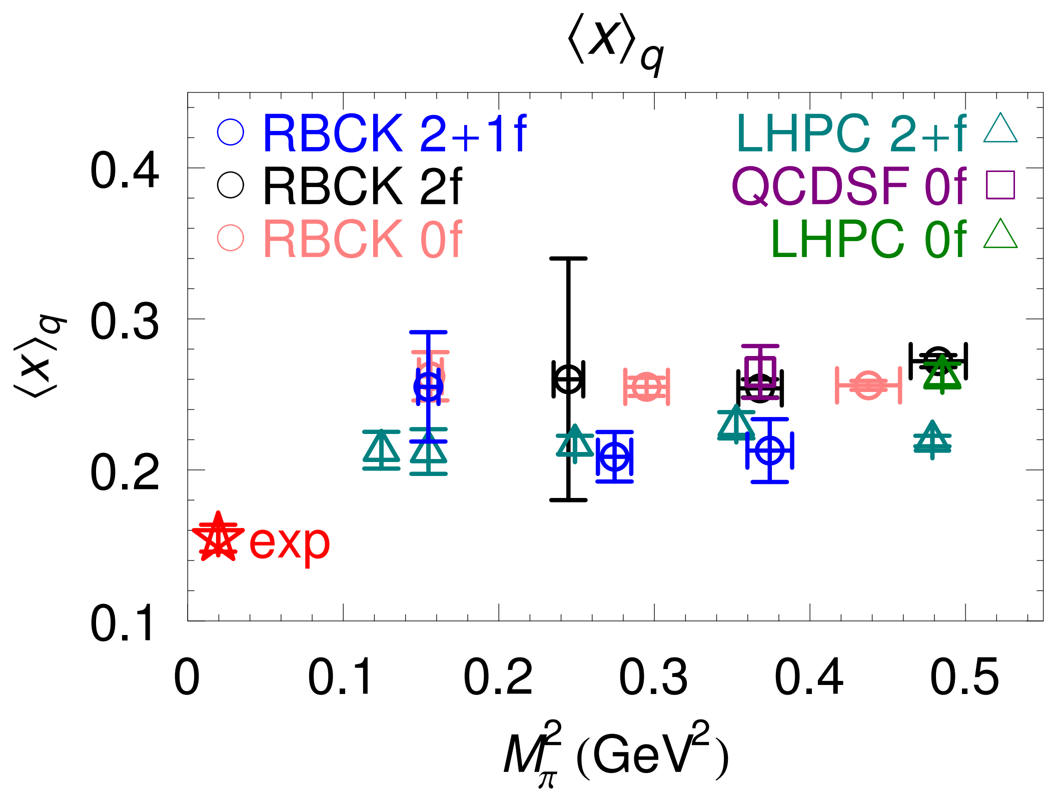


- Not yet extrapolated

Renormalized $\langle x \rangle_q$

(Preliminary)

- Comparisons with other calculations

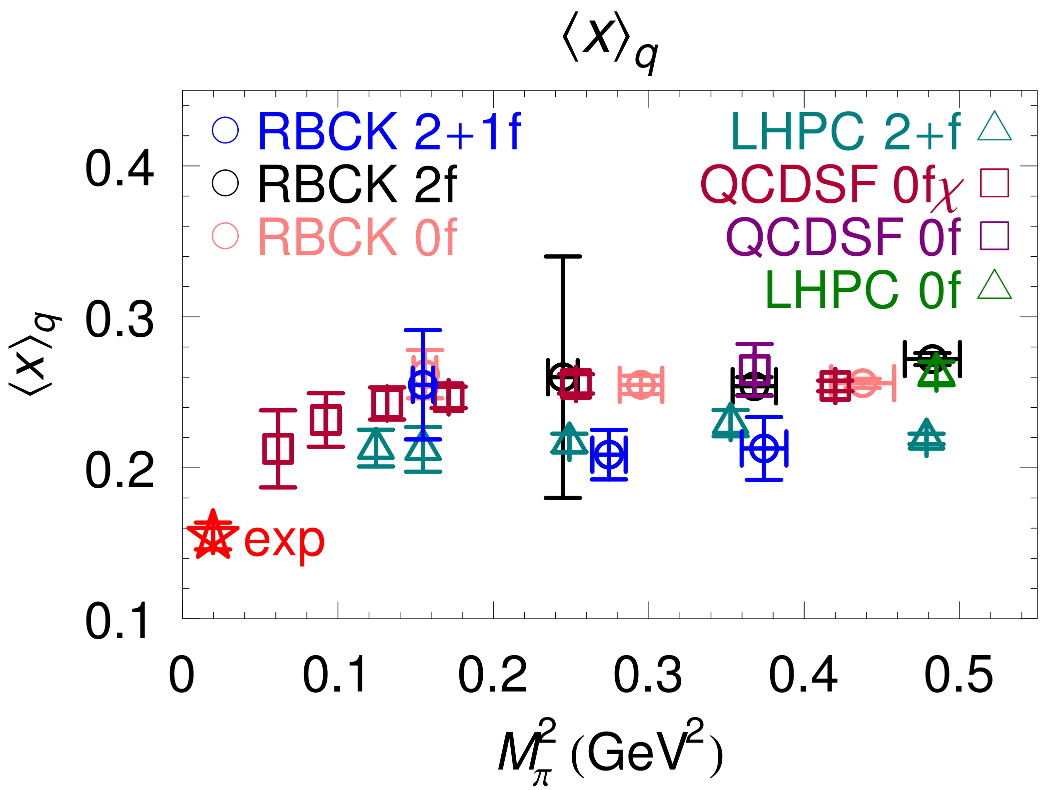


- Data taken from hep-ph/0509101 (PT renormalized)

Renormalized $\langle x \rangle_q$

(Preliminary)

- Comparisons with other calculations

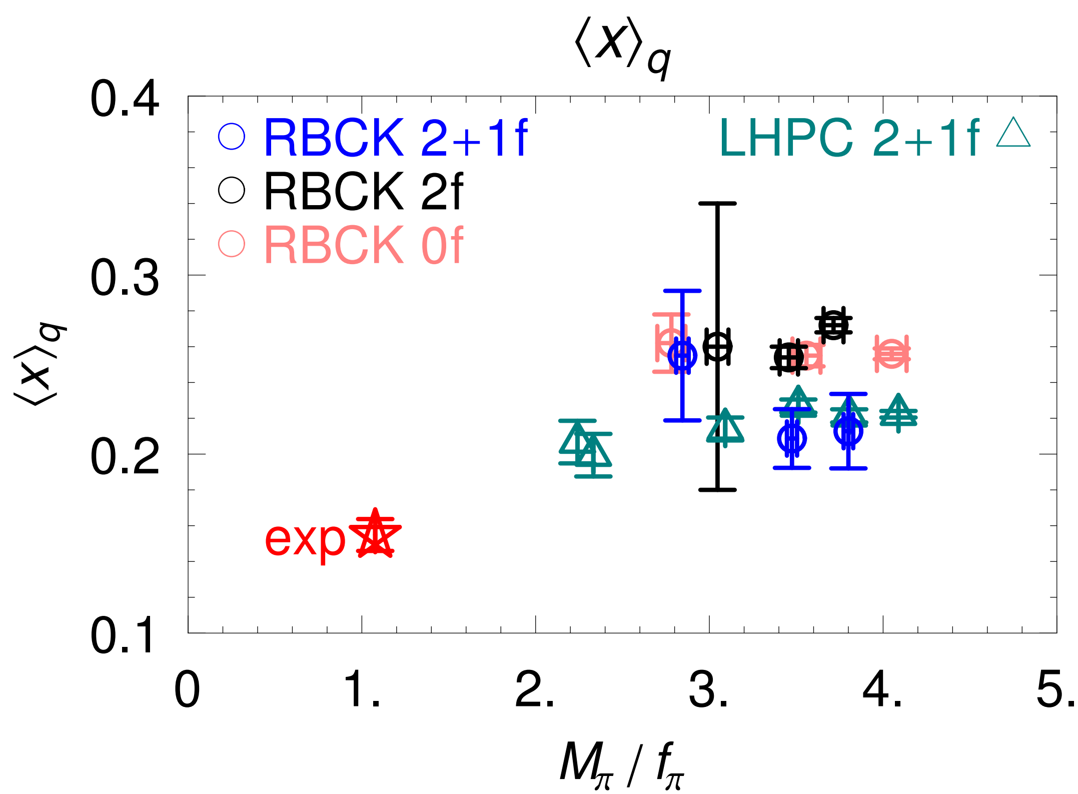


- Data taken from hep-ph/0509101 (PT renormalized)
- Quenched overlap data from G. Schierholz Trento'06

Renormalized $\langle x \rangle_q$

(Preliminary)

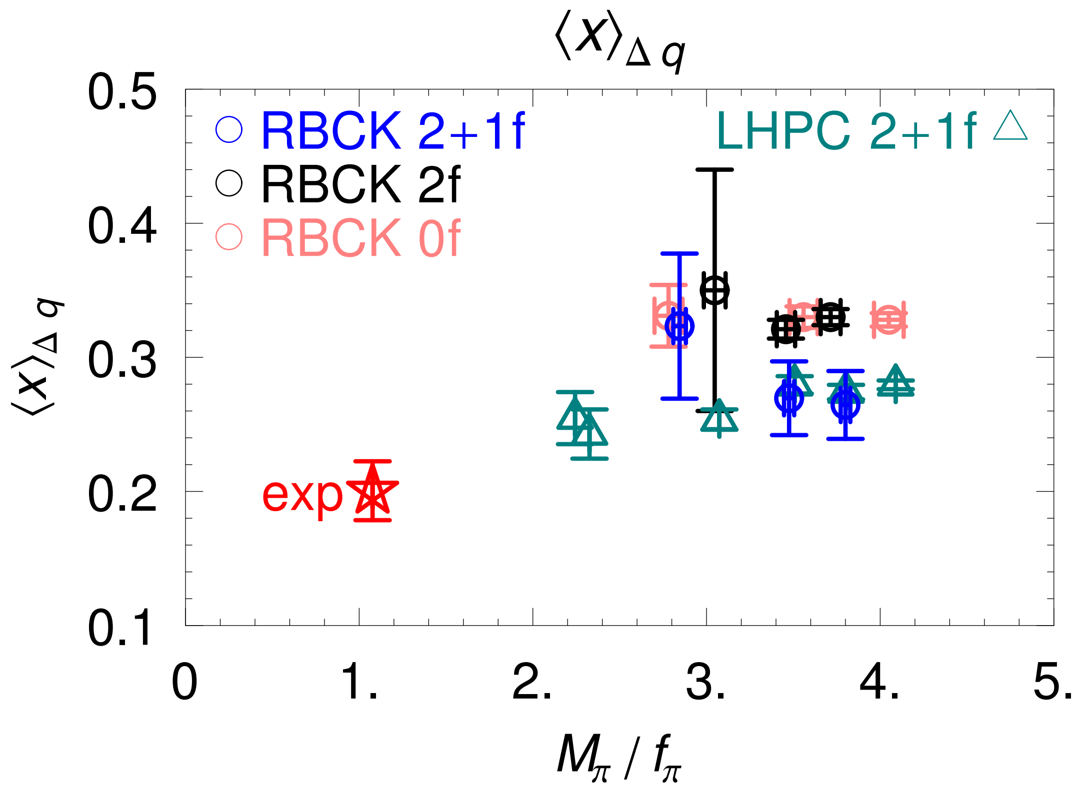
- Compare with DWF on 2+1 staggered sea; PT renormalization. (Renner Lat'06)



Renormalized $\langle x \rangle_{\Delta q}$

(Preliminary)

- Compare with DWF on 2+1 staggered sea; PT renormalization. (Renner Lat'06)



Conclusion and Outlook

- Preliminary on lightest 2 flavor and all 2+1 flavor data

From our current data...

- Experimentally consistent ratios of g_A/g_V and $\langle x \rangle_q / \langle x \rangle_{\Delta q}$
- (Preliminary) Renormalized $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$ are up to 50% higher than experimental values
- Our 2 flavor and 2+1 flavor data line up nicely
- No curvature observed so far

More to do:

- One more sea quark measurement to finish in 2 flavor case
- More statistics needed from 2+1 flavor case
- Finalize the NPR analysis
- Momentum dependence of the axial and vector charge
- Transversity analysis

In the near future...

- New ensembles with $am_{\text{sea}} = 0.005$ ($m_\pi \approx 290$ MeV) with $L \approx$ 3 fm will further explore the lighter m_{PS} region