Transverse spin densities from lattice QCD

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Introduction

looking for deeper insight into the (transverse) nucleon spin structure
and the relevant (tensor / quark helicity flip) generalized parton distributions

very difficult to directly access in experiment

yet at least qualitative relation to intrinsic transverse momentum dependent PDFs and to (single spin) asymmetries [M. Burkardt]

Outline

construction of transverse spin densities

lattice simulation and extraction of generalized form factors

numerical results

possible implications for single-spin and azimuthal asymmetries

(Transverse) spin structure of the nucleon

study general spin structure of the nucleon → spin-densities in impact parameter space

What are the ingredients?

1. nucleon states $|P, S\rangle = |P, \Lambda, S_\perp\rangle$

longitudinal $|P, \Lambda = \pm, S_\perp = 0\rangle = |P, \Lambda = \pm\rangle$

transvers(e)(ity)

$|P, \Lambda = 0, S_\perp = (1,0)\rangle = \sqrt{2}^{-1}\{|P,+\rangle + |P,-\rangle\}$

2. quark spin projection operators

$$\hat{\rho}_L(x) = \int_{-\infty}^{\infty} \frac{d\eta}{2\pi} e^{i\eta x} \bar{q}\Big(-\frac{\eta}{2}n\Big)\; n_\mu \gamma^\mu \big[1 + \lambda\gamma_5\big] \mathcal{U} q\Big(\frac{\eta}{2}n\Big)$$

$$\begin{aligned}\hat{\rho}_T(x) &= \int_{-\infty}^{\infty} \frac{d\eta}{2\pi} e^{i\eta x} \bar{q}\Big(-\frac{\eta}{2}n\Big)\; n_\mu \gamma^\mu \big[1 + (s_\perp \cdot \gamma_\perp)\gamma_5\big] \mathcal{U} q\Big(\frac{\eta}{2}n\Big) \\ &= \int_{-\infty}^{\infty} \frac{d\eta}{2\pi} e^{i\eta x} \bar{q}\Big(-\frac{\eta}{2}n\Big)\; n_\mu \big[\gamma^\mu + s_{\perp,j}\sigma^{\mu,j}\gamma_5\big] \mathcal{U} q\Big(\frac{\eta}{2}n\Big)\end{aligned}$$

1.+2.→3. now consider off-forward spin-diagonal matrix elements

$\langle P', \Lambda, S_\perp | \hat{\rho}_{L,T}(x) | P, \Lambda, S_\perp \rangle$

which can be completely parametrized by…

Generalized parton distributions

eight twist-two quark GPDs

$\{H(x,\xi,t),\, E\},\ \{\widetilde{H},\, \widetilde{E}\},\ \{H_T,\, \overline{E}_T,\, \widetilde{H}_T,\, \widetilde{E}_T\}$

vector/unpolarized/helicity-independent

$$\left\langle P',\Lambda'\right| \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}\Big(-\frac{\lambda}{2}n\Big)\, n_\mu \gamma^\mu \mathcal{U} q\Big(\frac{\lambda}{2}n\Big) \left|P,\Lambda\right\rangle = \overline{u}(P',\Lambda') n_\mu \Big\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} E(x,\xi,t)\Big\} u(P,\Lambda)$$

axial vector/polarized/helicity-dependent

$$\left\langle P',\Lambda'\right| \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}\Big(-\frac{\lambda}{2}n\Big)\, n_\mu \gamma^\mu \gamma_5 \mathcal{U} q\Big(\frac{\lambda}{2}n\Big) \left|P,\Lambda\right\rangle = \overline{u}(P',\Lambda') n_\mu \Big\{\gamma^\mu \gamma_5 \widetilde{H}(x,\xi,t) + \frac{\gamma_5 \Delta^\mu}{2m} \widetilde{E}(x,\xi,t)\Big\} u(P,\Lambda)$$

tensor/transversity/helicity-flip

$$\begin{aligned}\left\langle P',\Lambda'\right| \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}\Big(-\frac{\lambda}{2}n\Big)\, n_\mu \sigma^{\mu\nu} \gamma_5 \mathcal{U} q\Big(\frac{\lambda}{2}n\Big) \left|P,\Lambda\right\rangle &= \overline{u}(P',\Lambda') n_\mu \Big\{\sigma^{\mu\nu}\gamma_5 \big(H_T(x,\xi,t) - \frac{t}{2m^2}\widetilde{H}_T(x,\xi,t)\big) \\ &+ \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha \gamma_\beta}{2m} \overline{E}_T(x,\xi,t) + \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5 \Delta_\alpha}{2m^2} \widetilde{H}_T(x,\xi,t) \\ &+ \frac{\epsilon^{\mu\nu\alpha\beta}\overline{P}_\alpha \gamma_\beta}{m} \widetilde{E}_T(x,\xi,t)\Big\} u(P,\Lambda)\end{aligned}$$

slight change in notation compared to Diehl EPJC 19 (2001)

old $H_T,\, E_T,\, \widetilde{H}_T,\, \widetilde{E}_T$
new $H_T,\, \overline{E}_T = E_T + 2\widetilde{H}_T,\, \widetilde{H}_T,\, \widetilde{E}_T$

Diagonalization of matrix elements

Fourier-transform MEs to impact parameter space for $\xi = 0$

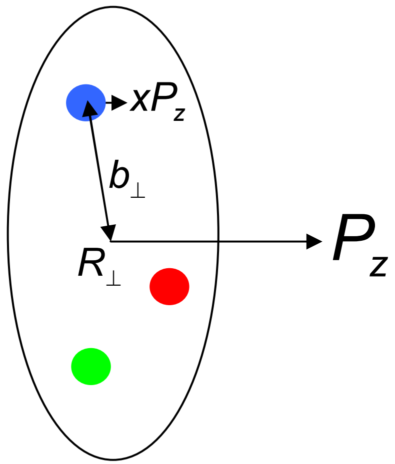
$\langle P', \Lambda, S_\perp | \hat{\rho}_{L,T}(x) | P, \Lambda, S_\perp \rangle$ → $\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp}$ ↔ $\Delta_\perp = P'_\perp - P_\perp$

$$\langle P^+, R_\perp = 0, \Lambda, S_\perp | \hat{\rho}_{L,T}(x, b_\perp) | P^+, R_\perp = 0, \Lambda, S_\perp \rangle$$

we find

- completely diagonal MEs — states are localized in the transverse plane
- impact parameter dependent density operator — $\hat{\rho}_{L,T}(x, b_\perp) = \int_{-\infty}^{\infty} \frac{d\eta}{2\pi} e^{i\eta x} \bar{q}\left(-\frac{\eta}{2} n, b_\perp\right) \mathcal{P}_{L,T} \mathcal{U} q\left(\frac{\eta}{2} n, b_\perp\right)$

$b_\perp$ is distance of the (active) quark to the center of momentum $R_\perp$ in a nucleon with large P_z



probability interpretation in transverse coordinate/ impact parameter space

Burkardt PRD 62 (2000)

Longitudinal quarks

$$\langle P^+, R_\perp = 0, \Lambda, S_\perp | \hat{\rho}_{L,T}(x, b_\perp) | P^+, R_\perp = 0, \Lambda, S_\perp \rangle$$

densities of quarks with x,$b_\perp$,λ, $s_\perp$ in a nucleon with P^+,$R_\perp$=0,Λ, $S_\perp$

case 1: longitudinally polarized quarks $s_\perp$=0

$$\langle P^+, 0_\perp, \Lambda, S_\perp | \hat{\rho}_L(x, b_\perp) | P^+, 0_\perp, \Lambda, S_\perp \rangle = \frac{1}{2}\left\{ H(x, b_\perp^2) + \lambda\Lambda \widetilde{H}(x, b_\perp^2) - \epsilon_{ij} S^i b_\perp^i \frac{\partial}{\partial b_\perp^2} E(x, b_\perp^2) \right\}$$

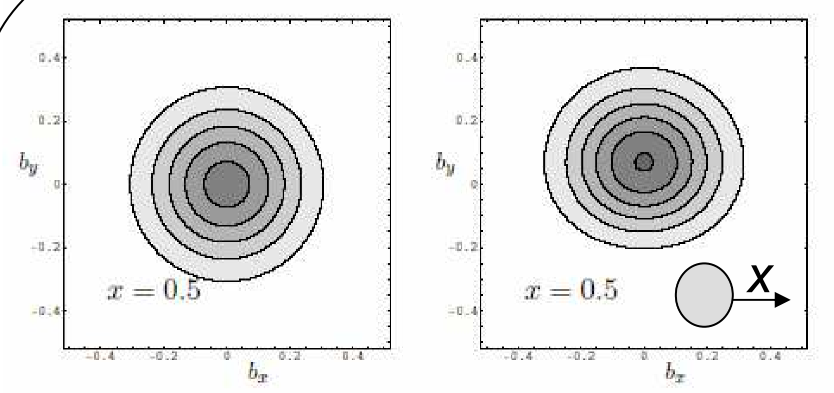
forward case — $\int d^2 b_\perp$ —

$$\langle P^+, \Lambda, S_\perp | \hat{\rho}_L(x) | P^+, \Lambda, S_\perp \rangle = \frac{1}{2}\{ q(x) + \lambda\Lambda \Delta q(x) \}$$

unpolarized quarks — $\lambda = 0, s_\perp = 0$

$$\langle P^+, 0_\perp, \Lambda, S_\perp | \hat{\rho}(x, b_\perp) | P^+, 0_\perp, \Lambda, S_\perp \rangle = \frac{1}{2}\left\{ H(x, b_\perp^2) - \epsilon_{ij} S^i b_\perp^i \frac{\partial}{\partial b_\perp^2} E(x, b_\perp^2) \right\}$$

deformed quark densities



Burkardt IJMP 14 (2003)

Transverse quarks

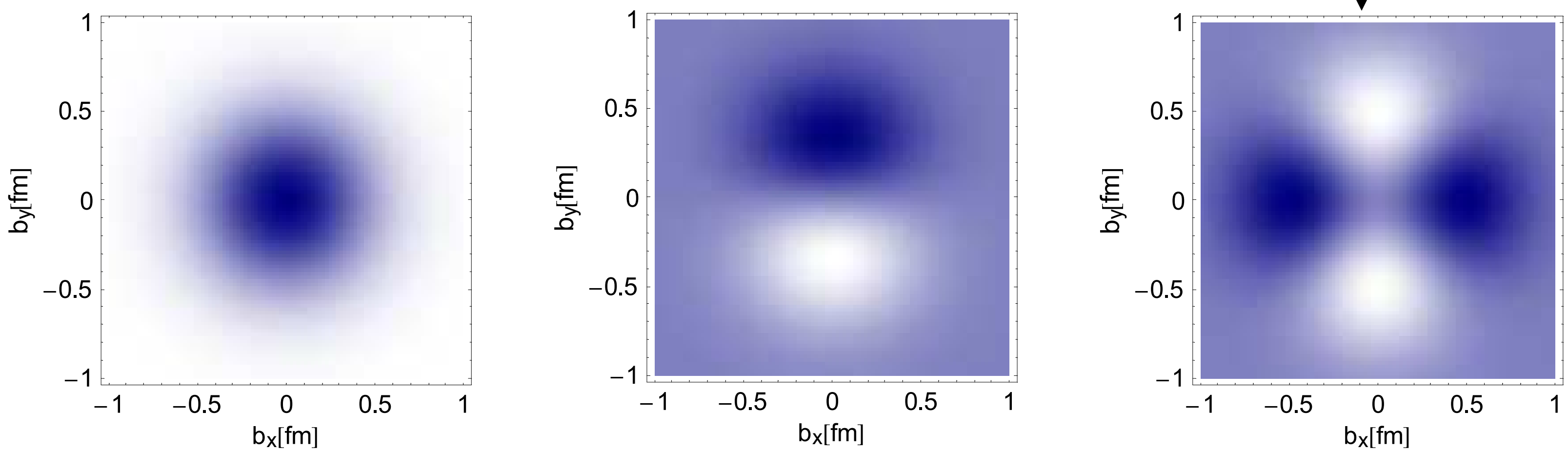
case 2: transversely polarized quarks λ=0

$$E' = \frac{\partial}{\partial b_\perp^2} E(x, b_\perp^2),\ \widetilde{H}''_T = \left(\frac{\partial}{\partial b_\perp^2}\right)^2 \widetilde{H}_T(x, b_\perp^2),$$

$$\Delta_b \widetilde{H}_T = \frac{\partial}{\partial b_\perp^i}\frac{\partial}{\partial b_\perp^i} \widetilde{H}_T(x, b_\perp^2)$$

Diehl / PhH EPJC 2005

$$\langle P^+, 0_\perp, \Lambda, S_\perp | \hat{\rho}_T(x, b_\perp) | P^+, 0_\perp, \Lambda, S_\perp \rangle$$
$$= \frac{1}{2}\Big\{ H + s^i S^i \Big(H_T - \frac{1}{4m^2} \Delta_b \widetilde{H}_T \Big) - \epsilon_{ij} S^i b_\perp^i \frac{1}{m} E' - \epsilon_{ij} s^i b_\perp^i \frac{1}{m} \overline{E}'_T + s^i (2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S^j \frac{1}{m^2} \widetilde{H}''_T \Big\}$$



monopole

dipole

quadrupole

all GPDs except $\widetilde{E}$, $\widetilde{E}_T$ are relevant for the analysis of probability densities!

Overview – computation of GPDs in Lattice QCD

continuum

- ~~bilocal LC-operators on the lattice~~
- Mellin-Moments
- matrix elements of local operators
- parametrization in terms of generalized form factors $A_{Tn0}(t)$, $\tilde{A}_{Tn0}(t)$, $\bar{B}_{Tn0}(t)\ldots$

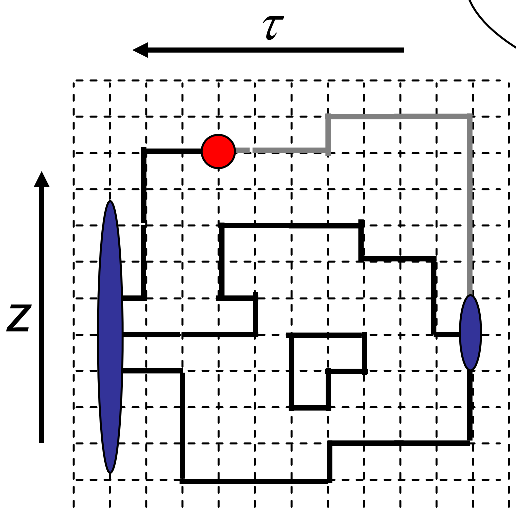
lattice

- calculation of two- and three-pt. functions
- construction of traceless & symmetric H(4) lattice operators
- plateau for 3pt/2pt
- lattice estimate of matrix elements of local operators
- renormalization lattice $\rightarrow$ $\overline{\mathrm{MS}}$

equate both sides

→ extract generalized form factors → moments of (G)PDs

→ moments of transverse spin densities



Ph. Hägler, LHP@Jlab 2006

Parametrization in terms of e.g. tensor GFFS

parametrization in terms of invariants only depending on $t=\Delta^2$ as dictated by Lorentz and P, T transformations

$2\lfloor n/2 \rfloor + n + 3$ GFFs ✓

$$A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \langle P'|\bar{q}(0)\sigma^{\mu\nu}\gamma_5 iD^{\mu_1}\ldots iD^{\mu_n} q(0)|P\rangle = \langle P'|O_T^{[\mu\{\nu]\mu_1\ldots\mu_n\}}|P\rangle =$$

$$A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \bar{U}(P')\Bigg\{ \sum_{i=0,\text{even}}^{n} \Bigg(\sigma^{\mu\nu}\gamma_5 \Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\left[A_{Tn+1i}(\Delta^2) - \frac{t}{2m^2}\tilde{A}_{Tn+1i}(\Delta^2)\right]$$

$$+\frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\bar{B}_{Tn+1i}(\Delta^2)$$

$$+\frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\tilde{A}_{Tn+1i}(\Delta^2)\Bigg)$$

$$+\sum_{i=0,\text{odd}}^{n}\frac{\epsilon^{\mu\nu\alpha\beta}\bar{P}_\alpha\gamma_\beta}{m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\ldots\bar{P}^{\mu_n}\tilde{B}_{Tn+1i}(\Delta^2)\Bigg\}U(P)$$

Ph.H., PLB 594(2004), see also Chen/Ji, PRD 71(2005)

how to get back the moments of the GPDs from the GFFs?

$$H_T{}^n(\xi,\Delta^2) = A_{Tn+10}(\Delta^2) + \sum_{i=2,\text{even}}^{n}(-2\xi)^i A_{Tn+1i}(\Delta^2),$$

$$\bar{E}_T{}^n = \bar{B}_{Tn+10}(\Delta^2) + \sum_{i=2,\text{even}}^{n}(-2\xi)^i \bar{B}_{Tn+1i}(\Delta^2) \quad \text{etc.}$$

H(4) tensor operator index combinations on the lattice for n=1,2

$n=1$ → $\tau_1^{(6)} : O_{T\mu\nu}^{n=1},\ \mu<\nu$ → $\#\,\text{of Ops} = 6$, $\#\,\text{of FFs} = 3$

$n=2$ →

$$\tau_1^{(8)} : O_{T1}^{n=2} = \frac{1}{2^{1/2}}\left(O_{T122} - O_{T133}\right),\ O_{T2}^{n=2} = \frac{1}{6^{1/2}}\left(O_{T122} + O_{T133} - 2O_{T144}\right)$$

$$O_{T3}^{n=2} = \frac{1}{2^{1/2}}\left(O_{T211} - O_{T233}\right),\ O_{T4}^{n=2} = \frac{1}{6^{1/2}}\left(O_{T211} + O_{T233} - 2O_{T244}\right)$$

$$O_{T5}^{n=2} = \frac{1}{2^{1/2}}\left(O_{T311} - O_{T322}\right),\ O_{T6}^{n=2} = \frac{1}{6^{1/2}}\left(O_{T311} + O_{T322} - 2O_{T344}\right)$$

$$O_{T7}^{n=2} = \frac{1}{2^{1/2}}\left(O_{T411} - O_{T422}\right),\ O_{T8}^{n=2} = \frac{1}{6^{1/2}}\left(O_{T411} + O_{T422} - 2O_{T433}\right)$$

$$\tau_2^{(8)} : O_{T1}^{n=2} = \left(\frac{2}{3}\right)^{1/2}\left(2O_{T\mu\{\nu\rho\}} + O_{T\nu\{\mu\rho\}}\right),\ O_{T2}^{n=2} = 2^{1/2} O_{T\nu\{\mu\rho\}},\quad \mu<\nu<\rho$$

→ $\#\,\text{of Ops} = 16$, $\#\,\text{of GFFs} = 4$

Göckeler, Horsley, Ilgenfritz,
Perlt, Rakow, Schierholz, Schiller,
Phys.Rev.D54:5705-5714,1996
Göckeler, 2005

NP renormalization constants available for n=1,2

be aware of operator mixing for n>2

The p-pole ansatz

p-pole ansatz $A(t) = \frac{A(0)}{\left(1 - t / p m_p^2\right)^p}$

FT

$A(b^2) = C(A(0), p)\, b^{p-1} K_{p-1}(\sqrt{p} m_p b)$

pro:

- describes almost all GFFs reasonably well over the whole range of momentum transfer
- simple relation to charge radius
- only 2(3) parameters
- simple Fourier-transform

contra:

- no sound theoretical foundation, purely phenomenological
- it is hard to determine m_p and p simultaneously from fit

(Mellin moments of) quark densities should be well behaved for $b \to 0$ and $b \to \infty$

for finite densities, we need

$H, \tilde{H}, H_T$: $p > 1$

$E, \overline{E}_T$: $p > \frac{3}{2}$

$\tilde{H}_T$: $p > 2$

Some simulation details

QCDSF/UKQCD - lattice - parameters:

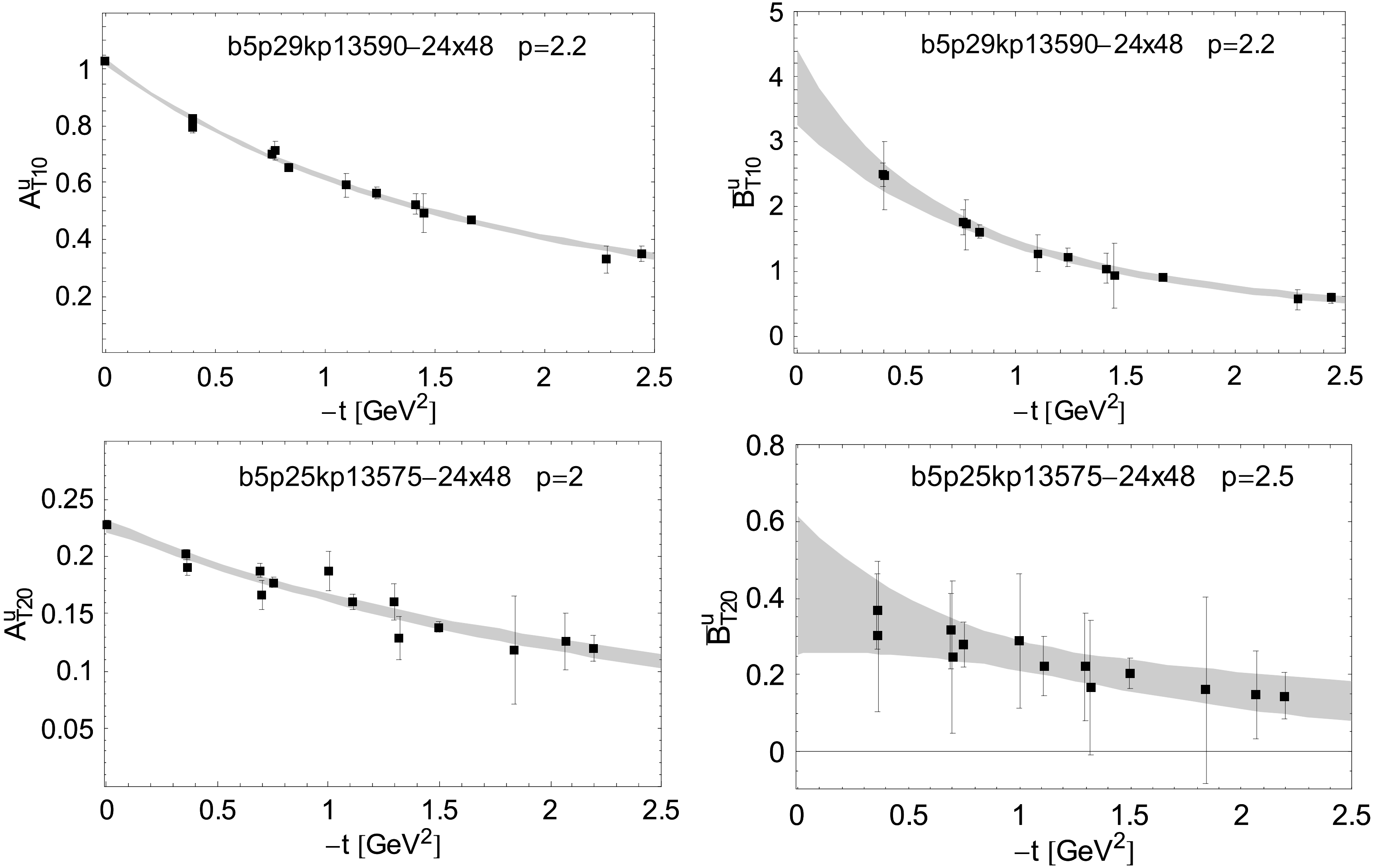
- $n_f = 2$ clover - improved Wilson - Fermions
- only connected contributions
- lattice spacing fixed using $r_0 = 0.467 fm$
- three projectors $\tilde{\Gamma}_{unpol} = \frac{1}{2}(1+\gamma_0), \tilde{\Gamma}_{1,2} = \frac{1}{2}(1+\gamma_0)\gamma_5\gamma_{1,2}$
- three sink - momenta $p' = (0,0,0),(1,0,0),(0,1,0)$

- renormalization is done non - perturbatively using the Rome - Southampton method
  $\text{lattice} \xrightarrow{NPT} \text{MOM} \xrightarrow{PT} \overline{\text{MS}}$
- results are presented for a scale of $4 GeV^2$
- lowest pion mass in this analysis is $\approx 400 MeV$
- finite volume data has not been used for this analysis

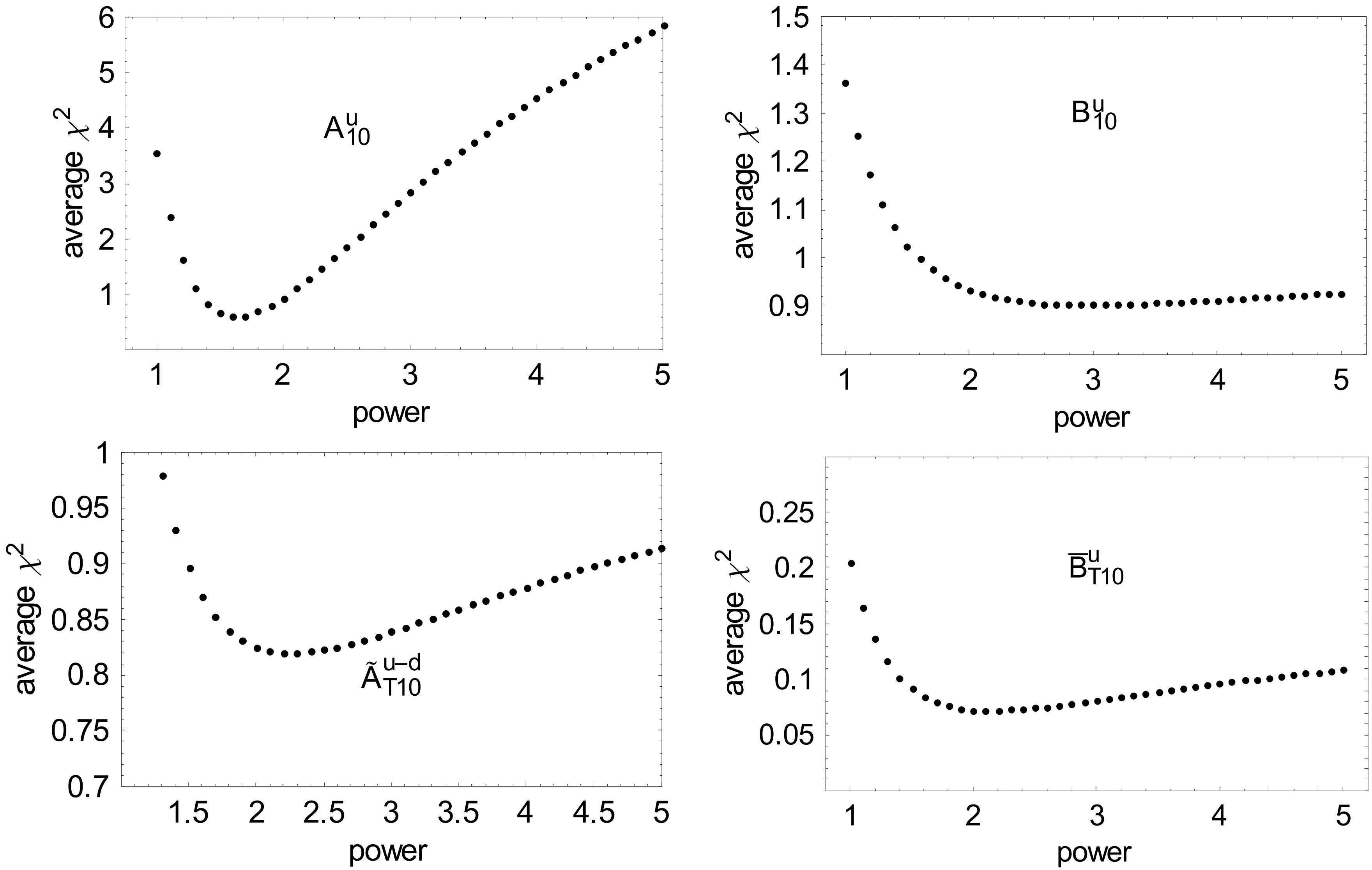
ParaNum	BetaVal	KappaVal	KappaValSea	Ls	Lt	mN	aLatt	aLattInv	mPion
1	5.20	.13420	.13420	16	32	1.1053	0.114545	1.7227	0.584732
2	5.20	.13500	.13500	16	32	0.864697	0.0982331	2.00876	0.414774
3	5.20	.13550	.13550	16	32	0.692691	0.0926403	2.13003	0.290712
4	5.25	.13460	.13460	16	32	0.945138	0.0985856	2.00158	0.493157
5	5.25	.13520	.13520	16	32	0.797921	0.0908914	2.17102	0.382066
6	5.25	.13575	.13575	24	48	0.606117	0.0844179	2.3375	0.255564
7	5.29	.13400	.13400	16	32	1.057290	0.0970289	2.03369	0.576715
8	5.29	.13500	.13500	16	32	0.835081	0.0893438	2.20863	0.420572
9	5.29	.13550	.13550	12	32	0.833103	0.0811327	2.43215	0.360479
10	5.29	.13550	.13550	16	32	0.728428	0.0839928	2.34933	0.333514
11	5.29	.13550	.13550	24	48	0.687432	0.0839023	2.35187	0.326959
12	5.29	.13590	.13590	12	32	0.81038	0.0761827	2.59017	0.3369150
13	5.29	.13590	.13590	16	32	0.6263690	0.080048	2.4651	0.2565
14	5.29	.13590	.13590	24	48	0.5656600	0.0799658	2.46763	0.23956
15	5.29	.13620	.13620	24	48	0.4682210	0.0774204	2.54877	0.1593870
16	5.40	.13500	.13500	24	48	0.755049	0.0766579	2.57413	0.403009
17	5.40	.13560	.13560	24	48	0.62409	0.073186	2.69624	0.312317
18	5.40	.13610	.13610	24	48	0.50987	0.0695562	2.83695	0.2208120

Preliminary results for moments of the quark helicity flip GPDs

QCDSF/UKQCD PLB 2005

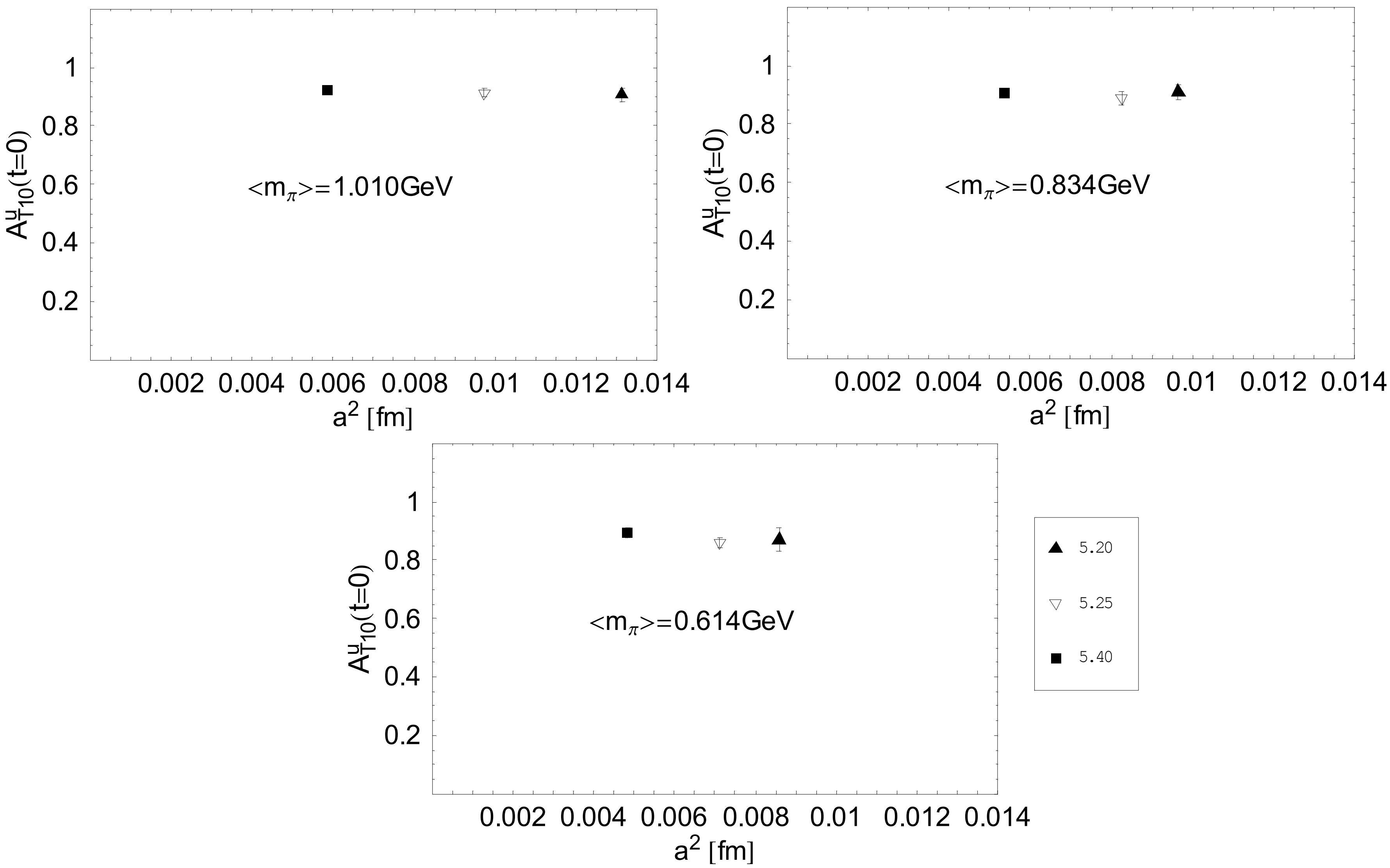


Determination of the „optimal“ power p



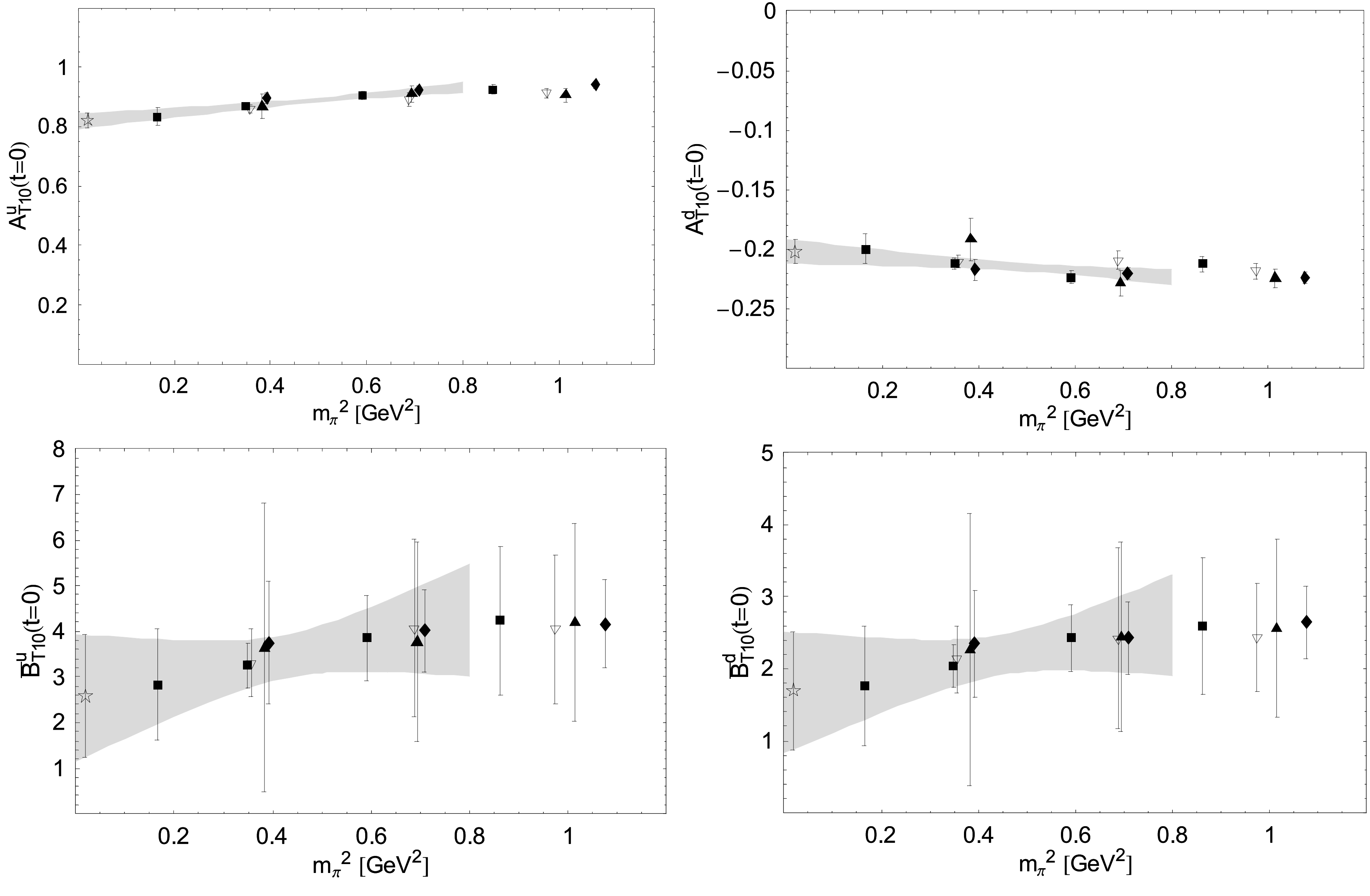
check p-dependence of final results later

Dependence on the lattice spacing



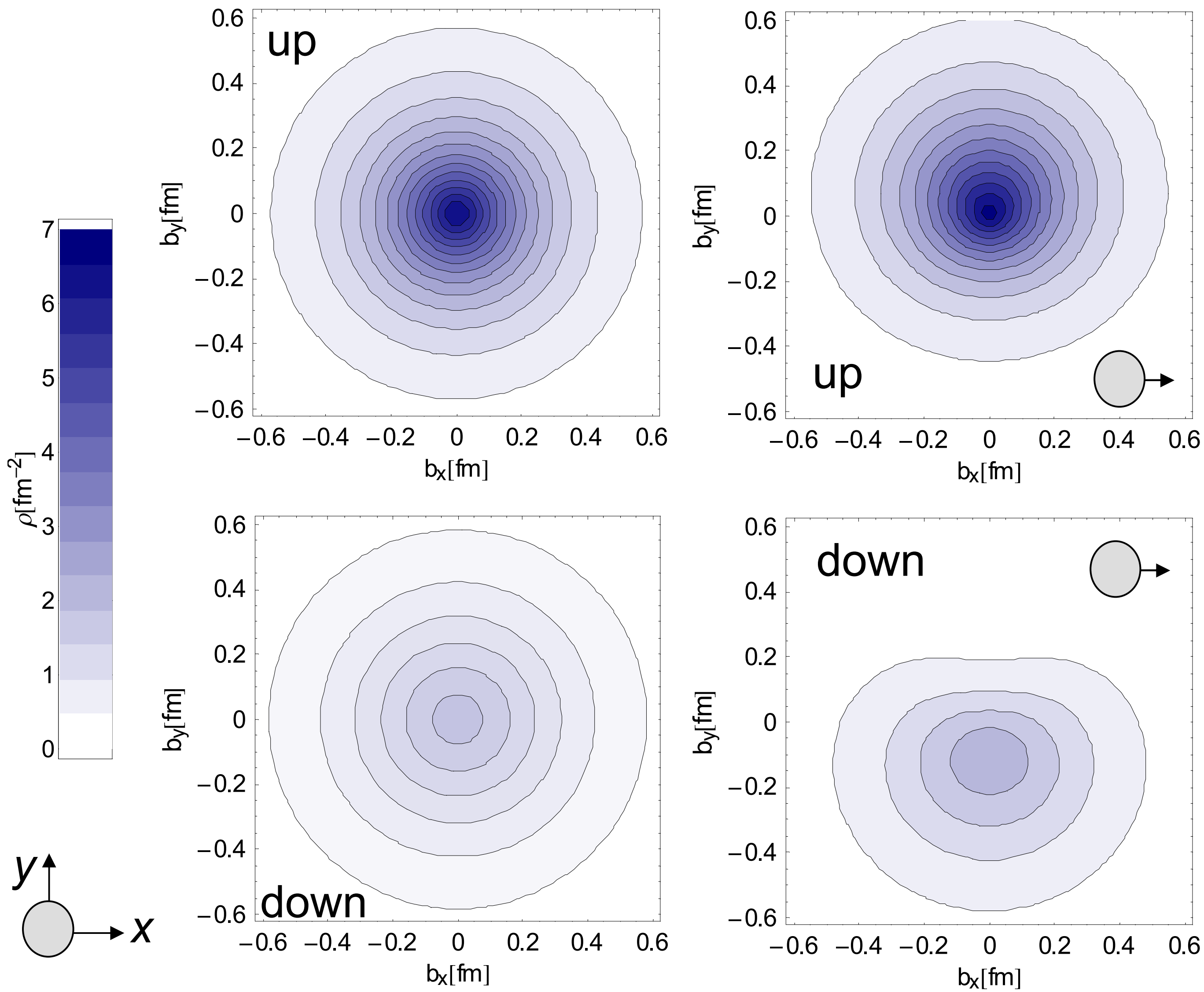
no dependence visible within statistical errors

Extrapolation in m_π



Results for the **lowest moment** of the up- and down quark densities

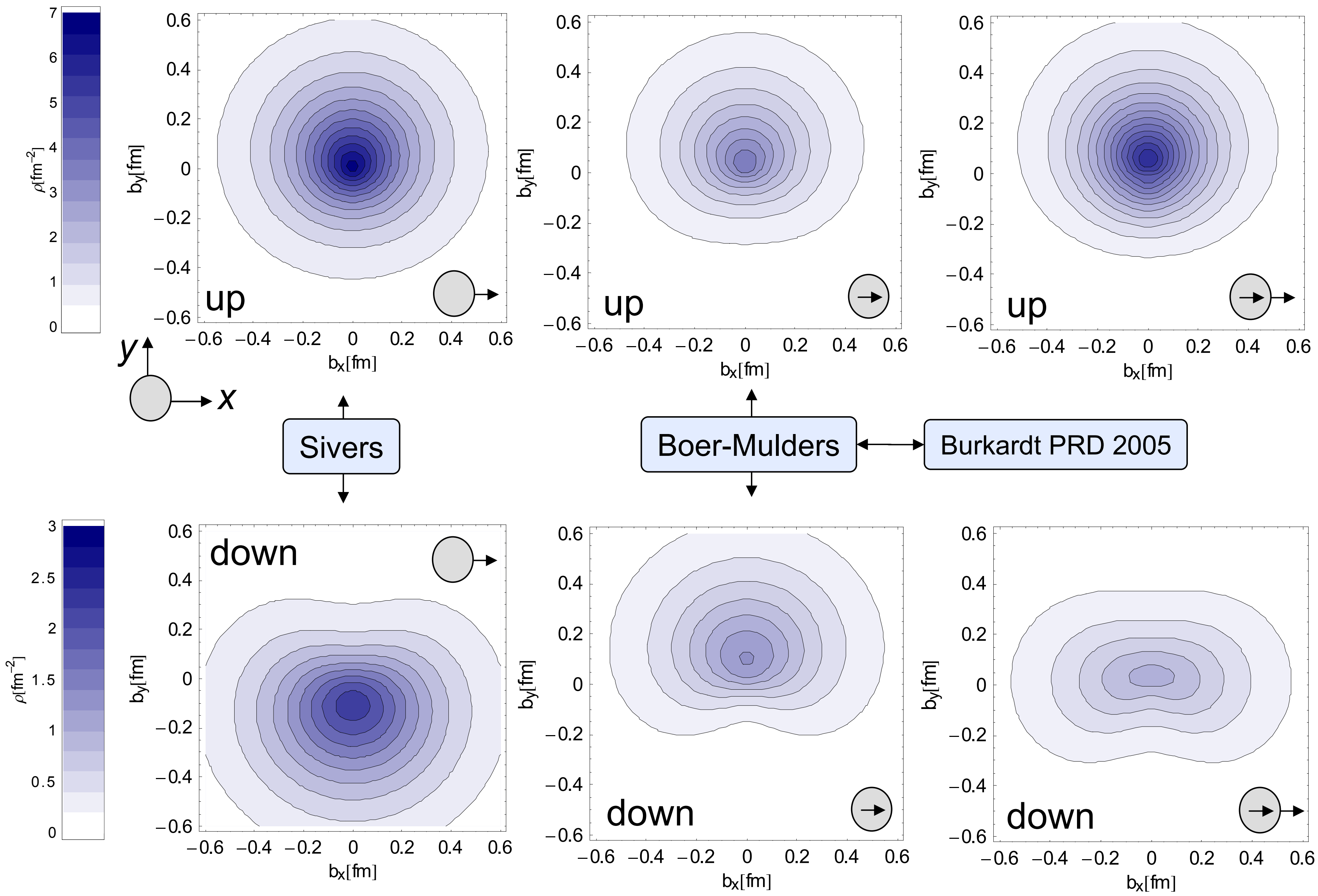
$$\rho^n(b_\perp) = \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp) = \rho_q^n(b_\perp) + (-1)^n \rho_{\bar{q}}^n(b_\perp)$$



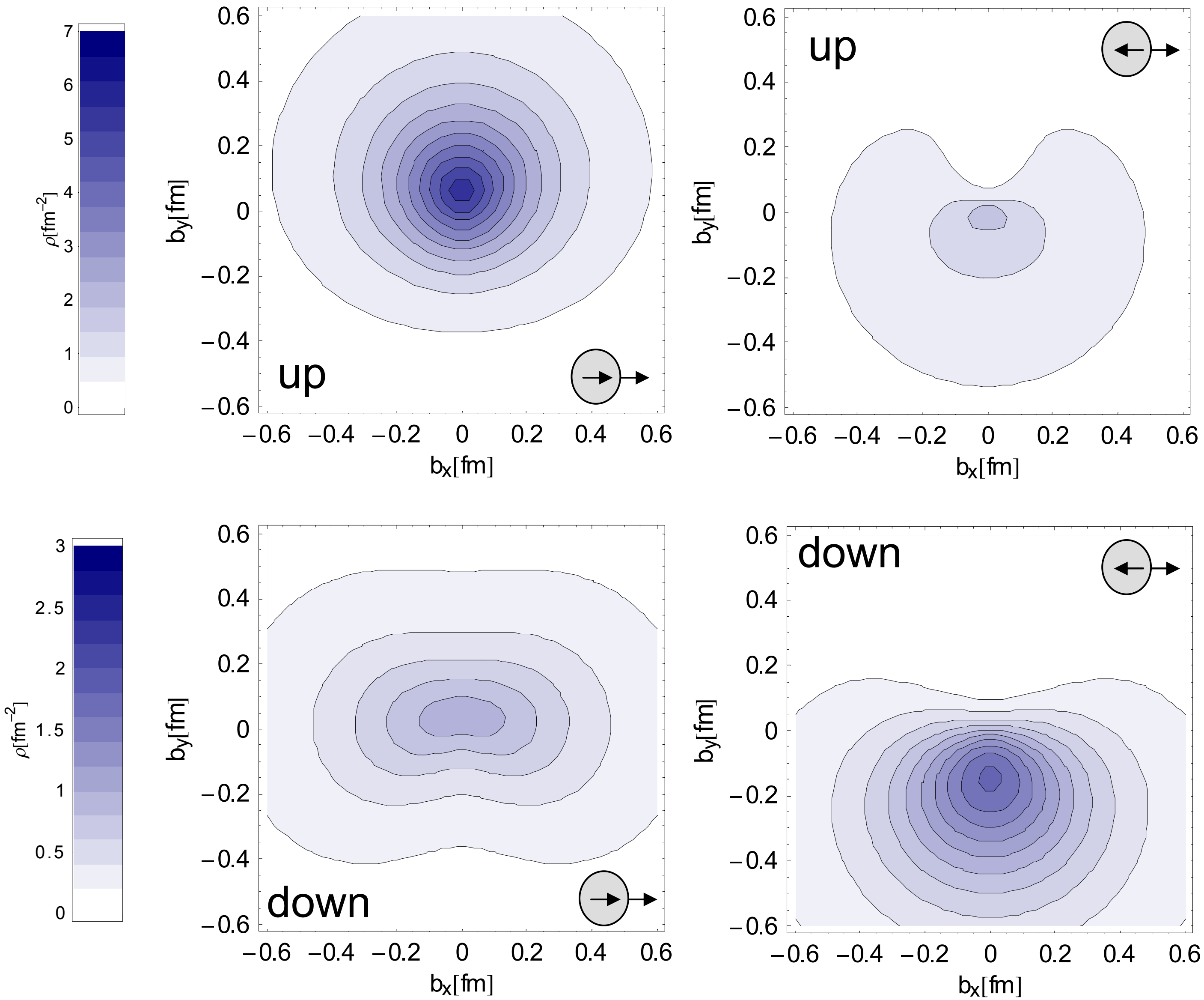
$$\int d^2 b_\perp \rho_u^{n=1}(b_\perp) = 2$$

$$\int d^2 b_\perp \rho_d^{n=1}(b_\perp) = 1$$

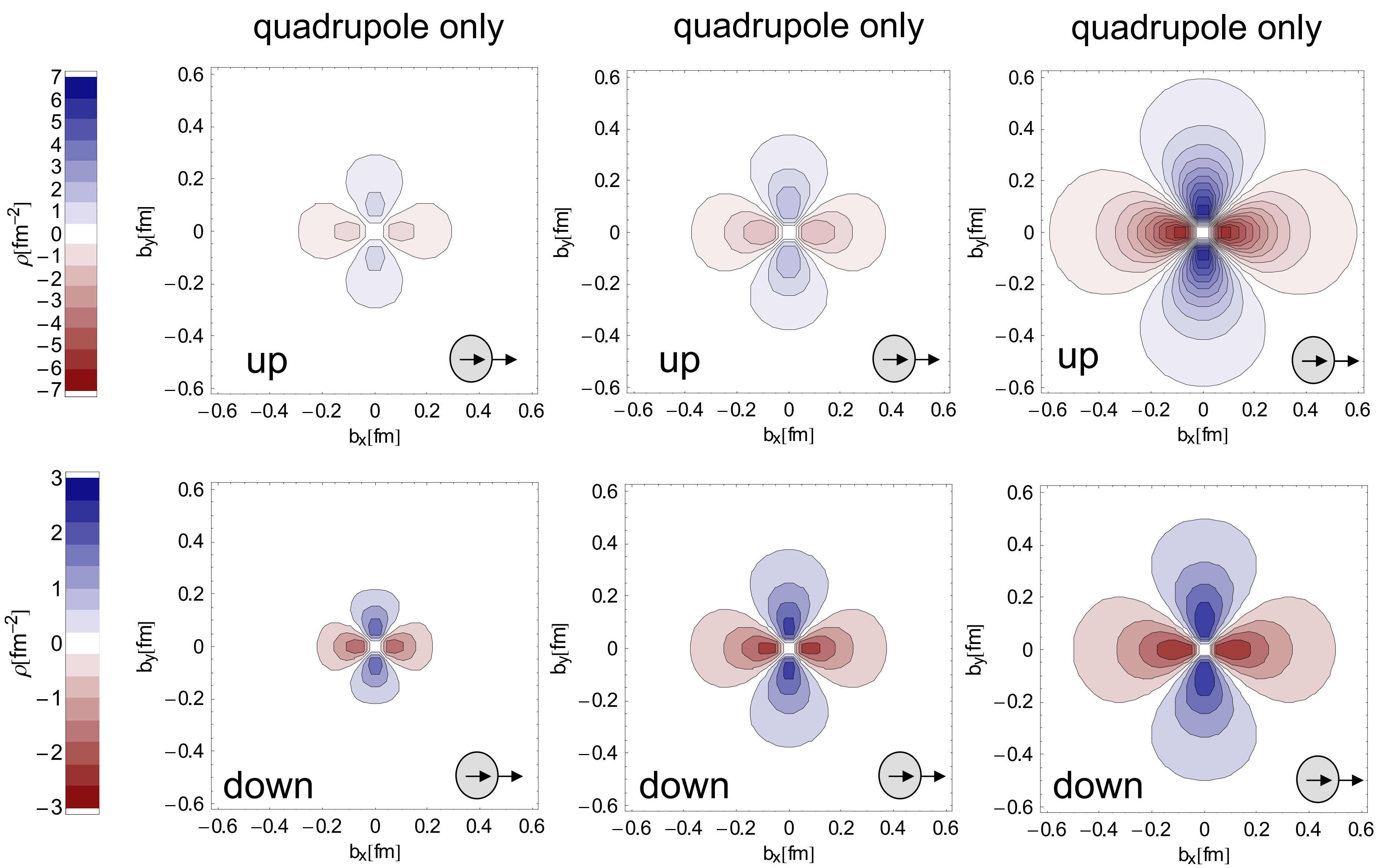
Preliminary results for the **lowest moment** of the transverse spin density



Aligned and anti-aligned transverse quark spin

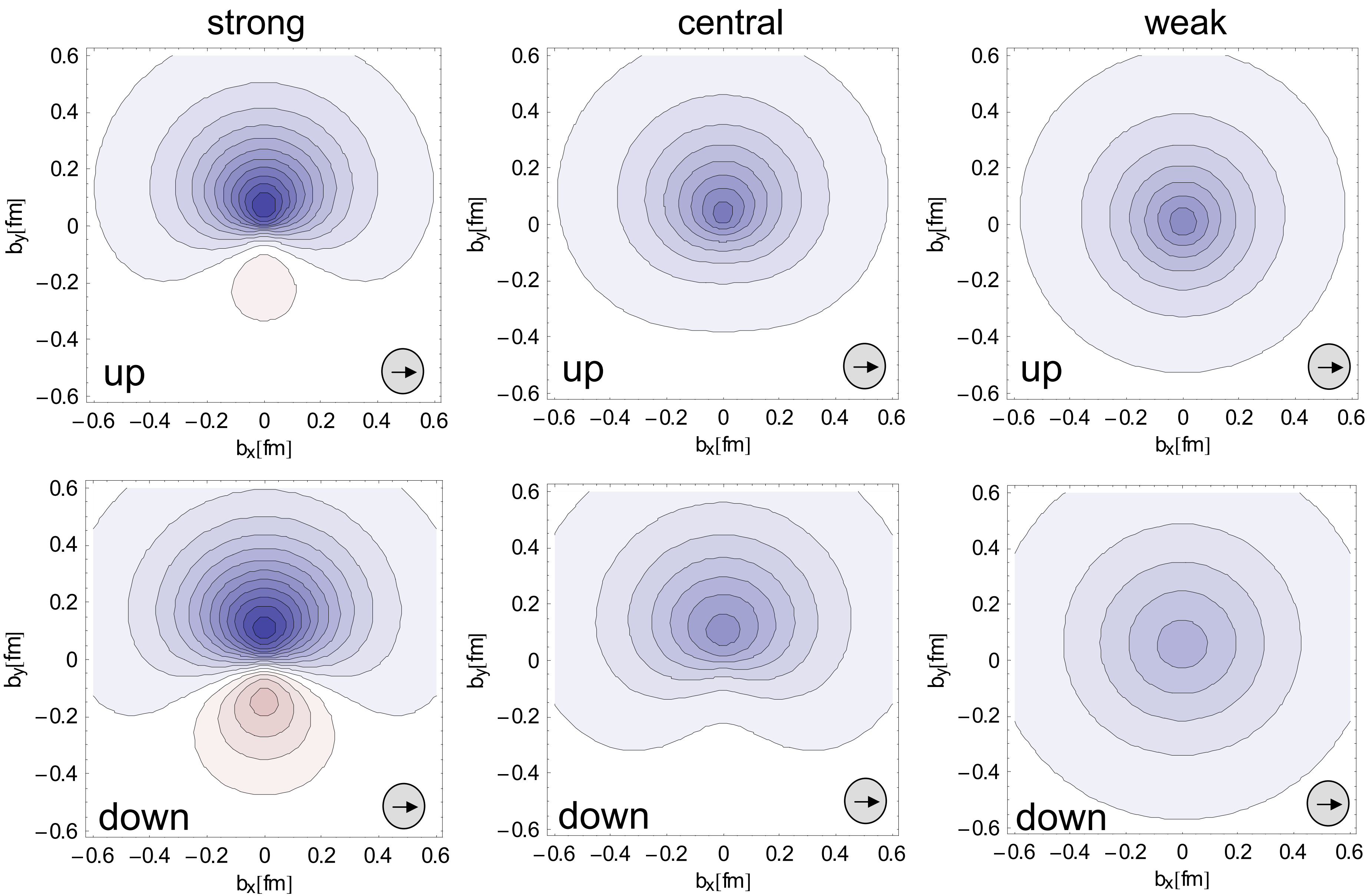


Quadrupole contributions (x10)

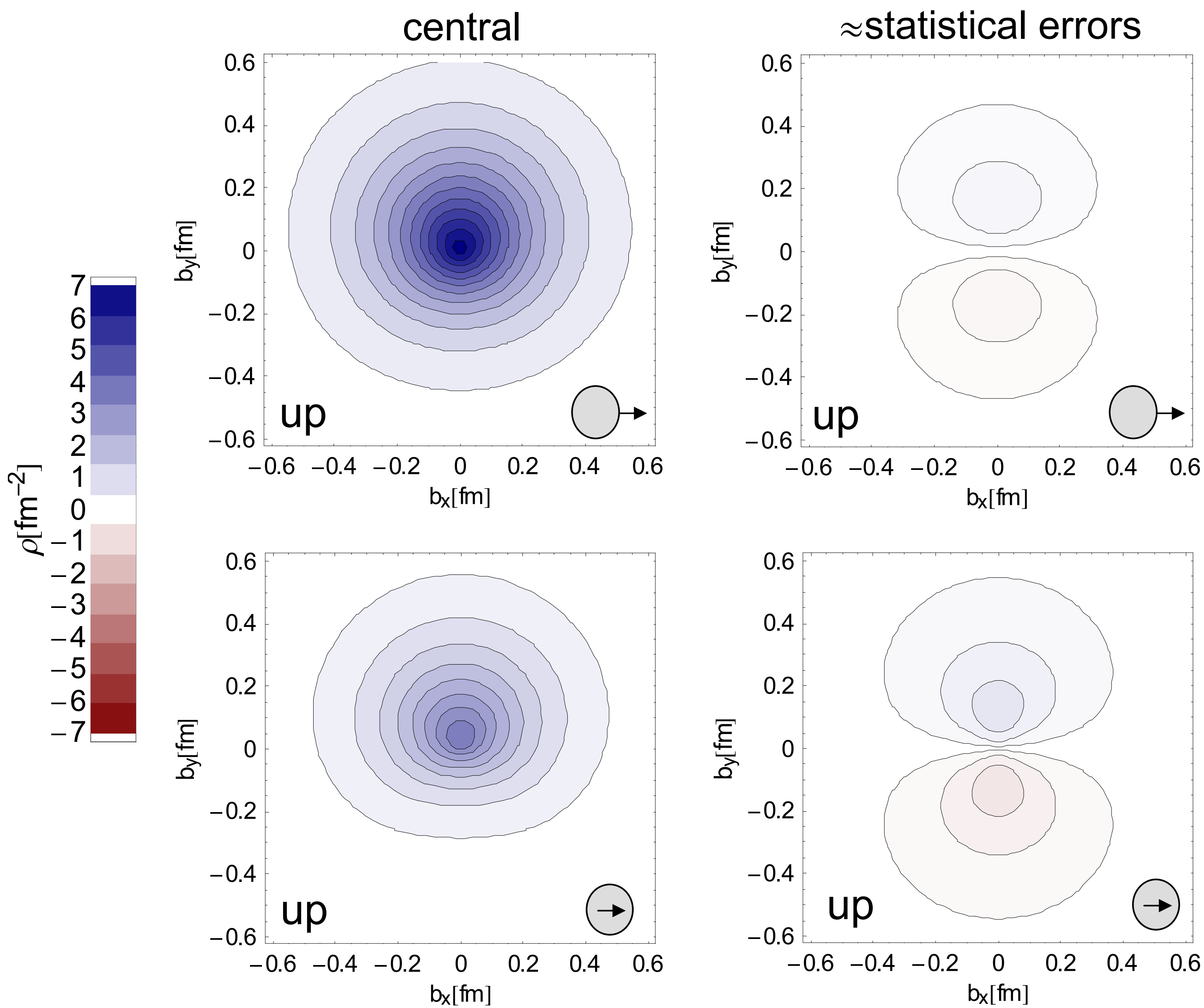


variations due to errors in $\tilde{A}_{T10}(t)$

Error analysis – statistical errors 1

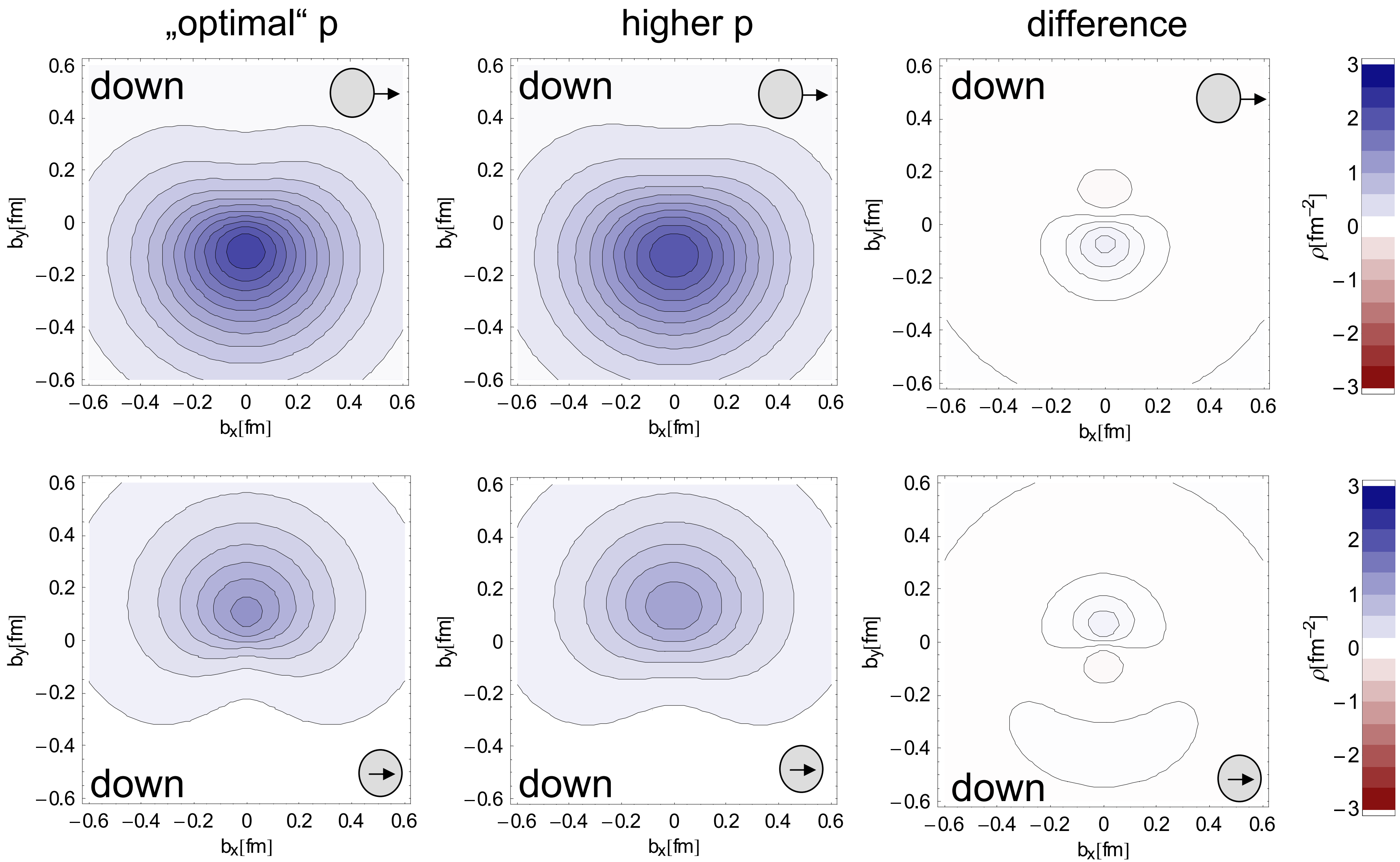


Error analysis – statistical errors 2

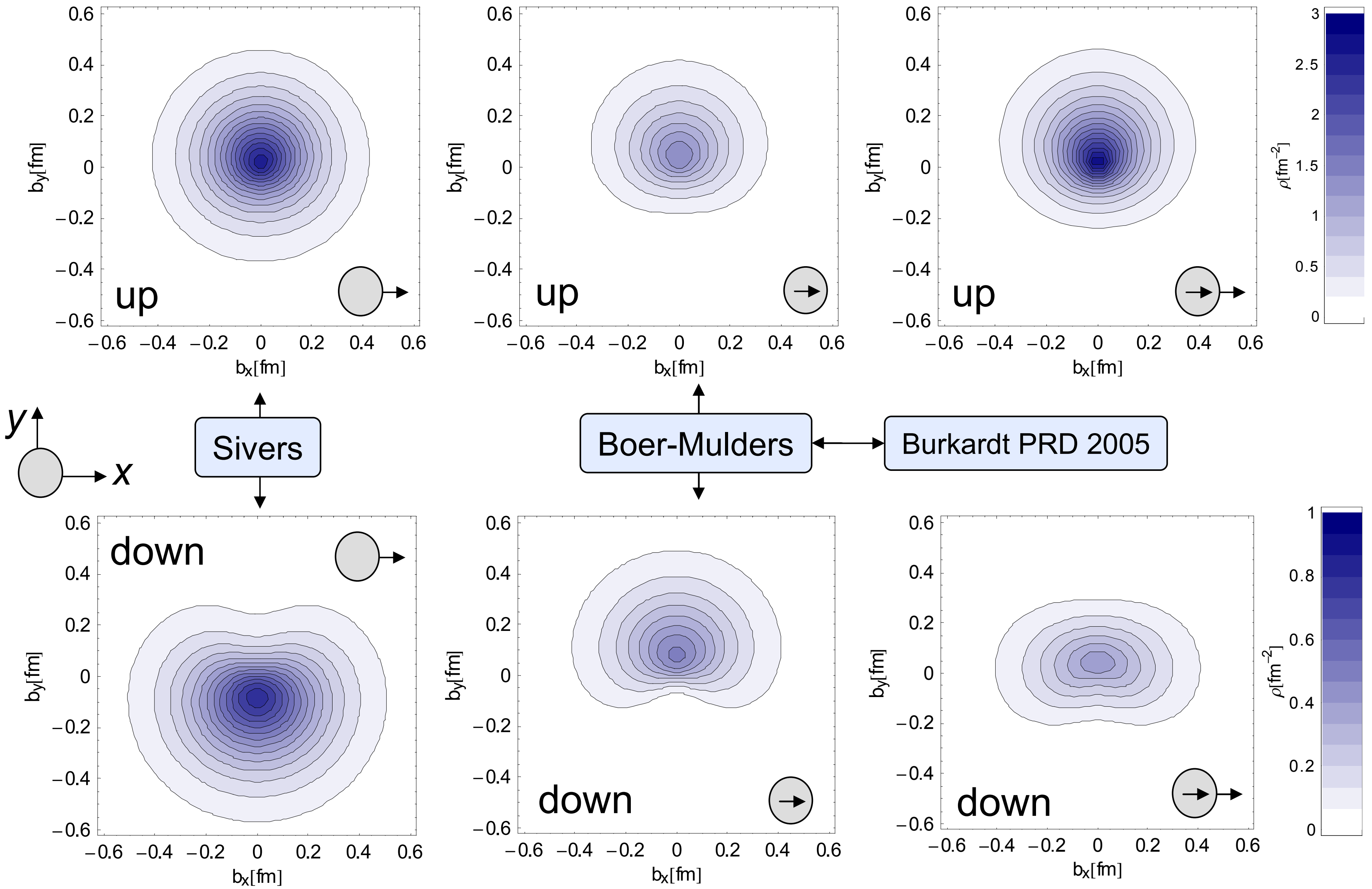


Error analysis – choice of p

$p[A_{10}] \to p[A_{10}] + 0.5$ $p[B_{10}] \to p[B_{10}] + 1$ $p[\bar{B}_{T10}] \to p[\bar{B}_{T10}] + 1$



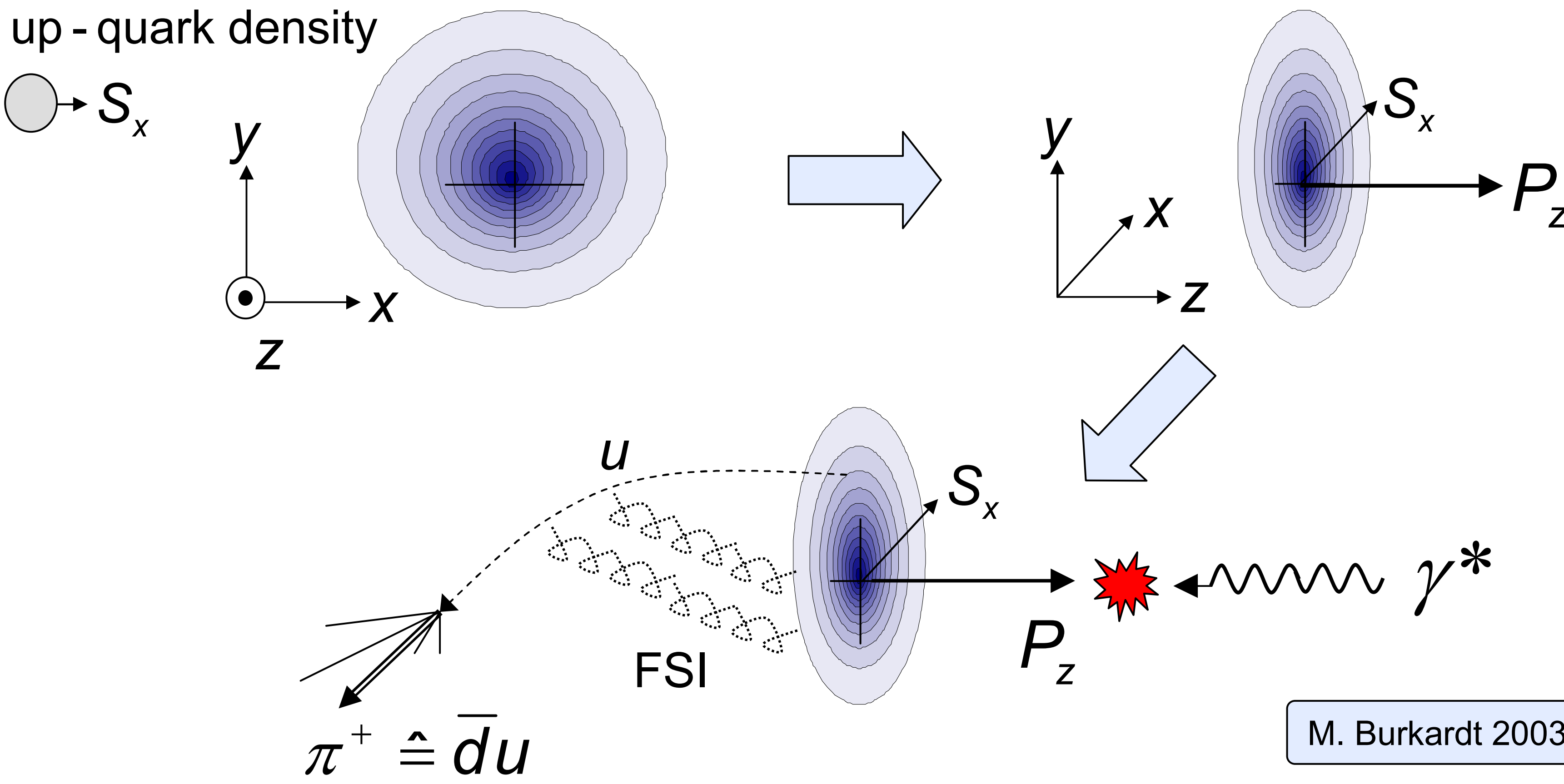
First results for the **n=2 moment** of the transverse spin density



very similar to lowest moment – more strongly peaked around the origin

statistical errors are large

Relation to SSAs and TMDPDFs

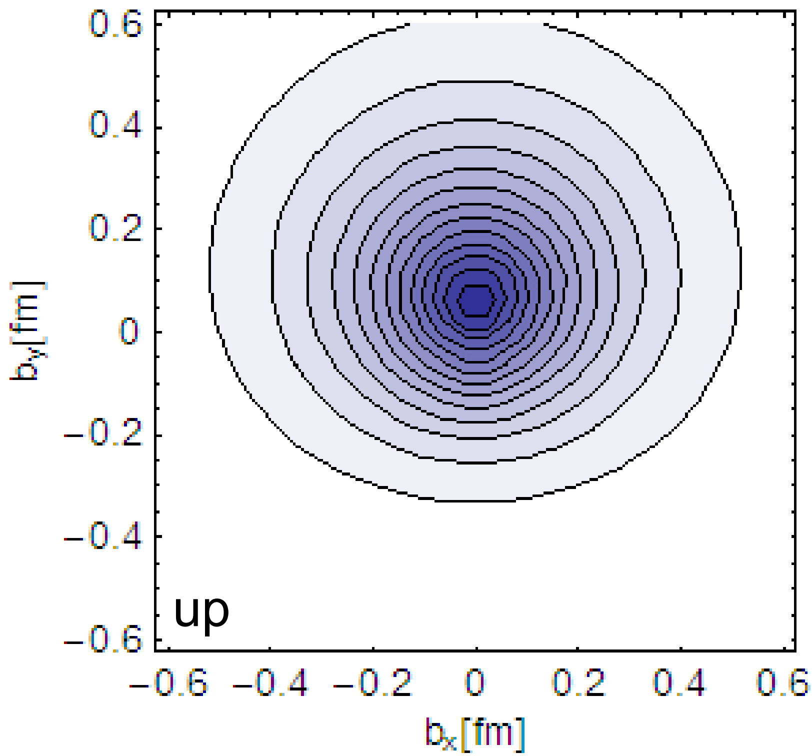


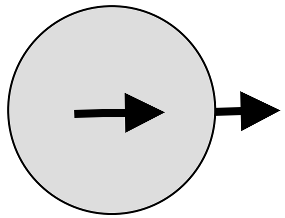
M. Burkardt 2003/2005

SSA in SIDIS — Sivers asymmetry/effect — $f_{1T}^{\perp} \sim -\int dx E(x,0,0) = -\kappa$ — $\kappa_u \approx -\kappa_d$

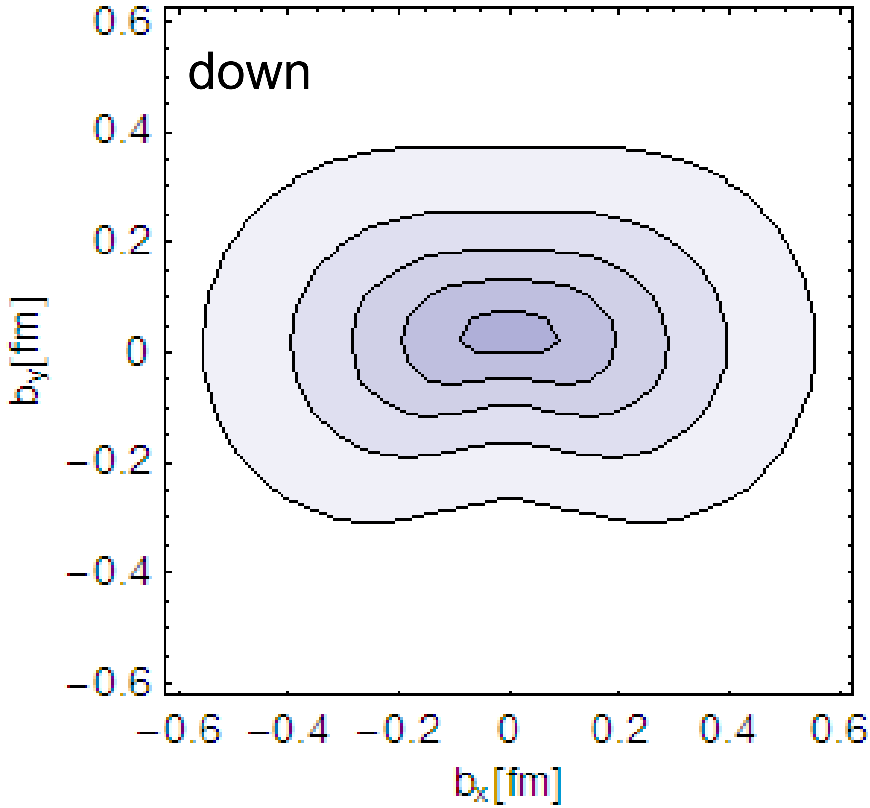
cos(2Φ) in SIDIS / unpol. DY — Boer-Mulders effect — $h_1^{\perp} \sim -\int dx \overline{E}_T(x,0,0) = -\kappa_T$ — $\kappa_{T,u} \approx \kappa_{T,d}$ — lattice

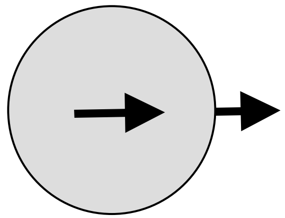
Up-quark density with rotating transverse quark spin





Down-quark density with rotating transverse quark spin





Summary

we find strongly distorted up- and down-quark densities
for transversely polarized quarks in transversely polarized nucleons

we presented first results for the n=2 moment of the transverse spin density

distortion patterns seem to be stable and weakly dependent on powers p

following M. Burkardt

deformed densities for unpolarized quarks in a transversely polarized nucleon lead to significant SSAs (seen by e.g. HERMES collaboration, PRL 2005)

deformed densities for transversely polarized quarks in an unpolarized nucleon can lead to significant azimuthal As (to be confirmed by e.g. Jlab, GSI/PANDA experiments)