Electromagnetic and spin polarisabilities from lattice QCD

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[WD, BC Tiburzi and A Walker-Loud, PRD73, 114505]

I: How to extract EM and spin polarisabilities from lattice QCD using external fields

II: How to relate lattice measurements to the polarisabilities of the real world

Hadron polarisabilities

- Hadron polarisabilities describe the deformation of a particle in an external (EM) field

- Quadratic energy shifts from effective Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 - \vec{\mu} \cdot \vec{B} - 2\pi\alpha|\vec{E}|^2 - 2\pi\beta|\vec{H}|^2 - 2\pi\gamma_1\vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \ldots$$

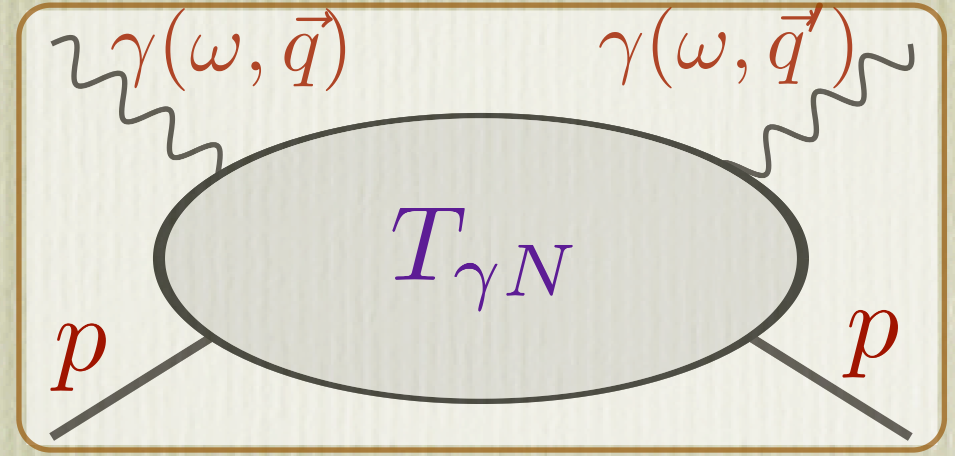
Magnetic moment ($\vec{\mu}$), Electric pol (α), Magnetic pol (β), First spin pol (γ_1)

- Electric and magnetic polarisabilities: ability to align with or against the applied field

- Spin and higher order polarisabilities are less intuitive: more detailed view of EM structure

Compton scattering

- Experimentally measured in the low frequency limit of real Compton scattering



- Thomson limit and Low–Gell-Mann–Goldberger LET determined by Born terms (charge and magnetic moment)

$$T_{\gamma N} = f(\underbrace{\omega, \vec{q}, \vec{q}', \vec{\epsilon}, \vec{\epsilon}', \vec{\sigma}}_{\text{Kinematics}}; Z, \mu, \underbrace{\alpha, \beta,} \underbrace{\gamma_{1,\ldots,4}}) + \mathcal{O}(\omega^4)$$

- Next order given in terms of EM and spin polarisabilities

Experiment

- MAMI, Saskatoon, *JLab*, OOPS, ELSA, HIγS
- EM and 2 combinations of spin polarisabilities are measurable for the proton but *difficult* experiments
- Neutron accessed via (quasi-)elastic Compton scattering on the deuteron - *even more difficult*

$$\alpha_p = 12.0(6), \quad \beta_p = 1.9(6), \quad \alpha_n = 13(2), \quad \beta_n = 3(2) \quad 10^{-4}\,\mathrm{fm}^3$$

$$\gamma_\pi^{(p)} = -39(2), \quad \gamma_0^{(p)} = -1.0(1), \quad \gamma_\pi^{(n)} = 59(4), \quad 10^{-4}\,\mathrm{fm}^4$$

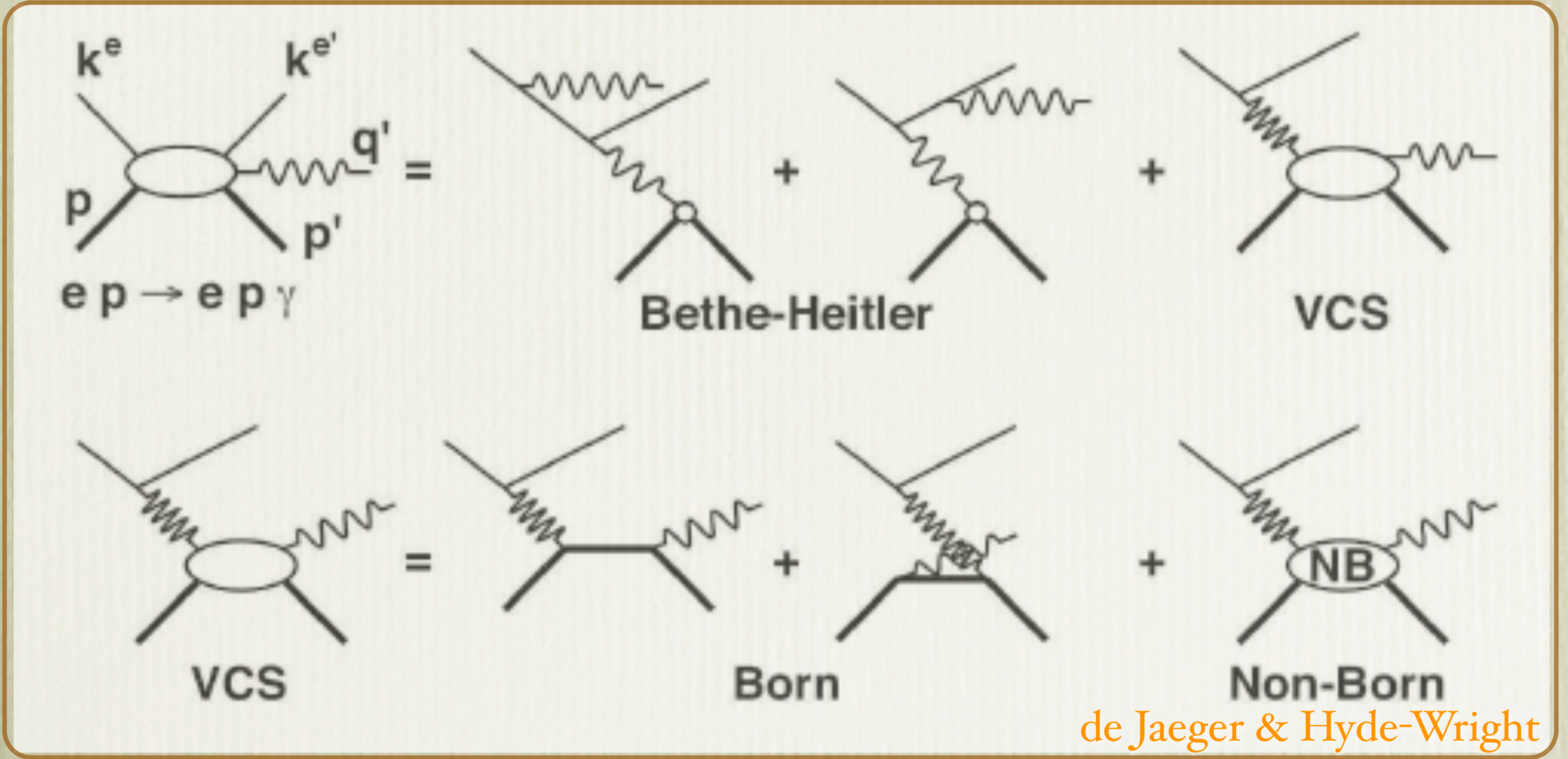
[de Jaeger & Hyde-Wright 05]

- Sign indicates diamagnetic nature of nucleon
- Small size of polarisabilities indicates tightly bound relativistic system - hard to deform

Further polarisabilities

- Higher orders in the frequency expansion gives higher order polarisabilities [Holstein *et al.* '99]

- Virtual and doubly virtual Compton scattering leads to generalised polarisabilities [Guichon, Liu & Thomas '95]



de Jaeger & Hyde-Wright

Lattice approaches

1. Four point correlators
    - Analogous to experimental measurement
    - Difficult - many disconnected contractions
2. Energy shifts in two point correlators in external U(1) field
    - *Quenched QCD*: external field can be added after gauge configurations are generated
    - *QCD*: external field must be known during gauge field generation - costly but multipurpose

External field method

- Quenched external fields simple to apply:

$$U_\mu^a(x) \to U_\mu^a(x) \cdot U_\mu^{\text{ext}}(x)$$

  - E.g.: magnetic field $\vec{B} = (0, 0, B)$

Quantised for periodic links

$$U_0^{\text{ext}} = U_2^{\text{ext}} = U_3^{\text{ext}} = 1,\ U_1(x) = e^{ieBx_2}$$

  - Look for shift in energy quadratic in |B|

$$C_{\uparrow\uparrow}(\tau, B) = \sum_{\vec{x}} \langle 0|\chi_\uparrow(\vec{x}, t)\overline{\chi}_\uparrow(0)|0\rangle$$
$$= \exp\left[-(M - \mu|B| + 2\pi\beta|B|^2)\tau\right] + \mathcal{O}(|B|^3)$$

Magnetic polarisability (β)

Magnetic moment (μ)

Field constraints

- Field values are restricted by a number of constraints
  - Perturbative in EFT: $|eB|, |eE| < m_\pi^2$
  - Periodicity of box: e.g. magnetic field

$$U_\mu(x + L\hat{\nu}) = U_\mu(x)$$
$$a^2|eB| = \frac{2\pi n}{L}, \quad n \in Z$$

  - Landau levels well represented
- *Existing calculations do not satisfy these constraints*

External field method

- Can study more than energy shifts - hadronic correlator analysis ≡ effective field theory
  - matching behaviour of QCD correlator to EFT correlator (not just in ε-regime)
- E.g.: charged particle in constant electric field

$$C_{ss'}(\tau;E) = \sum_{\vec{x}} \langle 0|\chi_s(\vec{x},t)\overline{\chi}_{s'}(0)|0\rangle$$

$$= \delta_{s,s'} \exp\left[-(M+2\pi\alpha|E|^2)\tau - \frac{q^2|E|^2}{6M}\tau^3\right] + \dots$$

Acceleration of proton at large times

Electric polarisability

- Valid for $L^{-1} < m_\pi,\ |eE| < m_\pi^2$

External field method

- All six polarisabilities can be calculated
  - utilise all information in hadron correlators including spin-flip matrix elements
  - Spin polarisabilities require space/time varying U(1) fields: E.g. γ_{E1E1}

$$U_\mu^{\mathrm{ext}} = e^{iaeA_\mu(x)},\ A_\mu(x) = \left(-\frac{a_6 t^2}{2a}, \frac{-ib_6 t}{2}, 0, 0\right)$$

$$\frac{C_{\uparrow\uparrow}(\vec{p},\tau;A)}{C_{\downarrow\downarrow}(\vec{p},\tau;A)} = \exp\left[\frac{2\pi}{a}\, a_6\, b_6\, \gamma_{E_1E_1}\, \tau\right] + \ldots$$

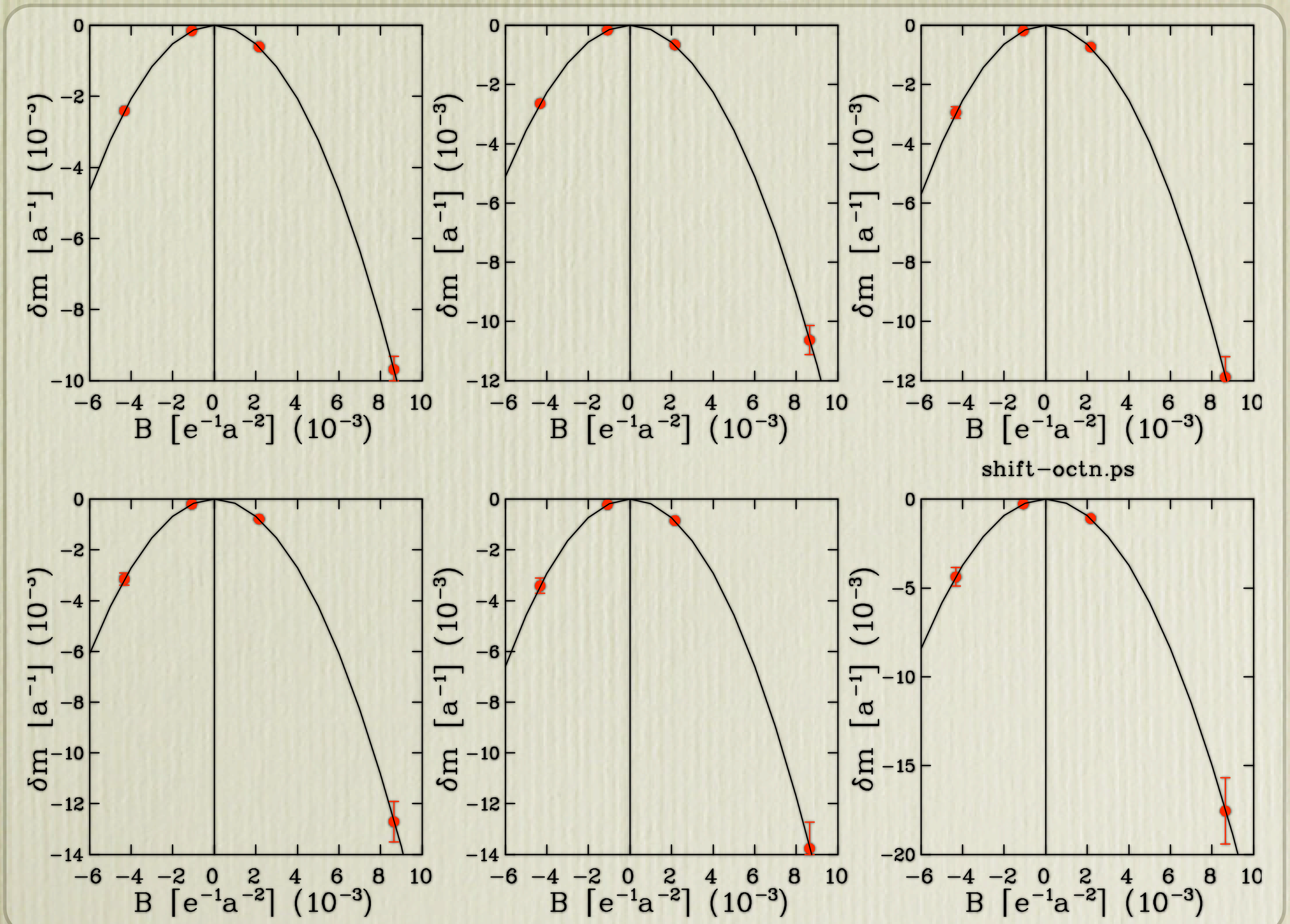
Quenched lattice polarisabilities

- External field calculations of magnetic moments and EM polarisabilities have a long history
  - Martinelli *et al.*, Bernard *et al.*: μ for n, p, Δ [83]
  - Fiebig *et al.*: α for neutron [89]
  - Christensen *et al.*: α for uncharged particles [05]
  - Lee *et al.*: μ for baryons [05]
  - Lee *et al.*: β for many baryons and mesons [05]
  - Preliminary work on spin polarisabilities

Quenched magnetic polarisabilities

[Lee *et al.*, hep-lat/0509065]

- Use four field values (average over +/-)

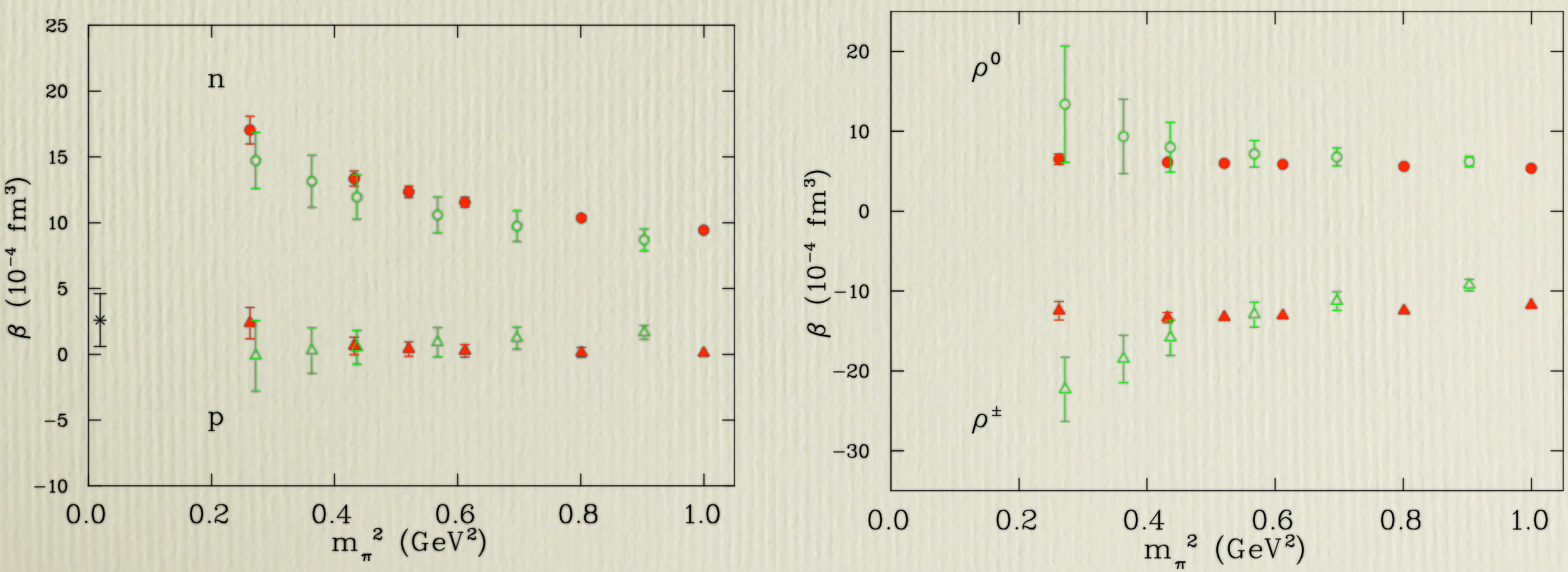


Quenched magnetic polarisabilities

[Lee *et al.*, hep-lat/0509065]

- Calculated for many hadrons

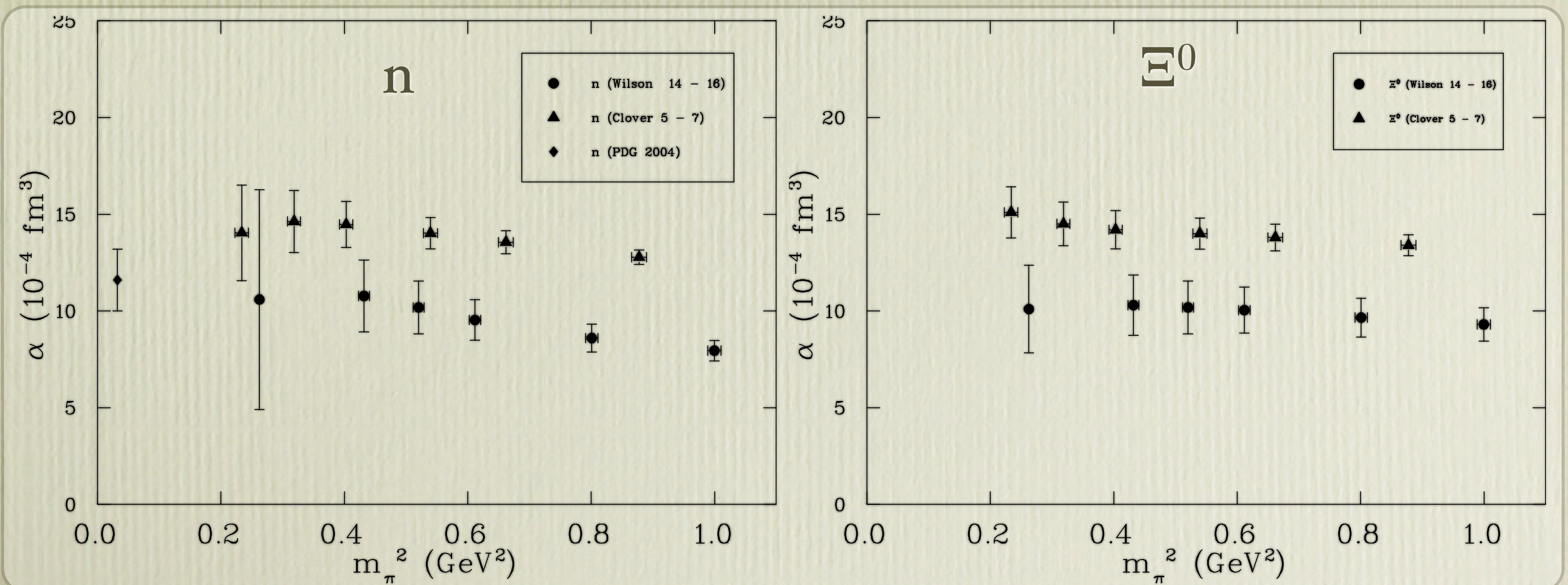
$n,\ p,\ \Sigma^{\pm,0}, \Xi^{0,-},\ \Delta^{++,\pm,0}\ \Sigma^{*\pm,0}, \Xi^{*0,-},\ \Omega,\ \pi^{\pm,0},\ K^{\pm,0},\ \rho^{\pm,0},\ K^{*\pm,0}$



Quenched electric properties

[Christensen *et al.*, hep-lat/0408024]

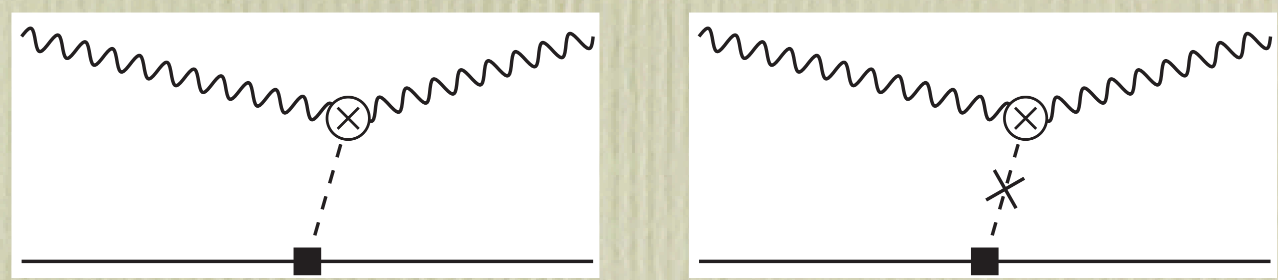
- Also do calculations with four field values (pos/neg)
- Neutral particles n, Σ^0, Ξ^0, Δ^0 Σ^{*0}, Ξ^{*0}, π^0, K^0, ρ^0 , K^{*0}



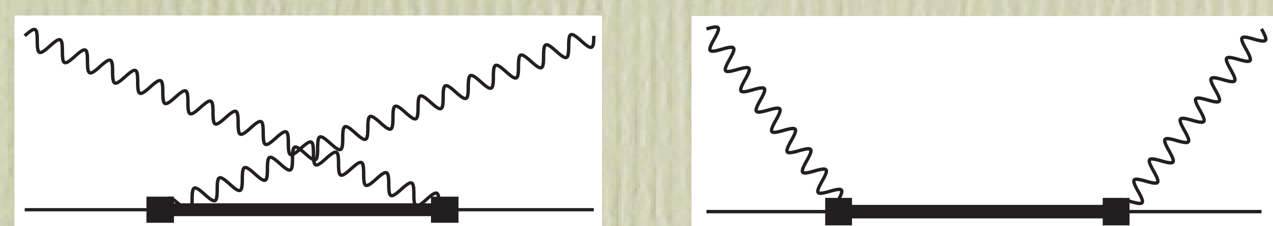
Chiral perturbation theory

- Many studies of nucleon polarisabilities in the context of chiral perturbation theory (χPT)
- Extended to partially-quenched χPT at finite volume using heavy baryon formalism
- No undetermined LECs at NLO - loops are more important than counter-terms
- Functional form similar to χPT, but couplings change

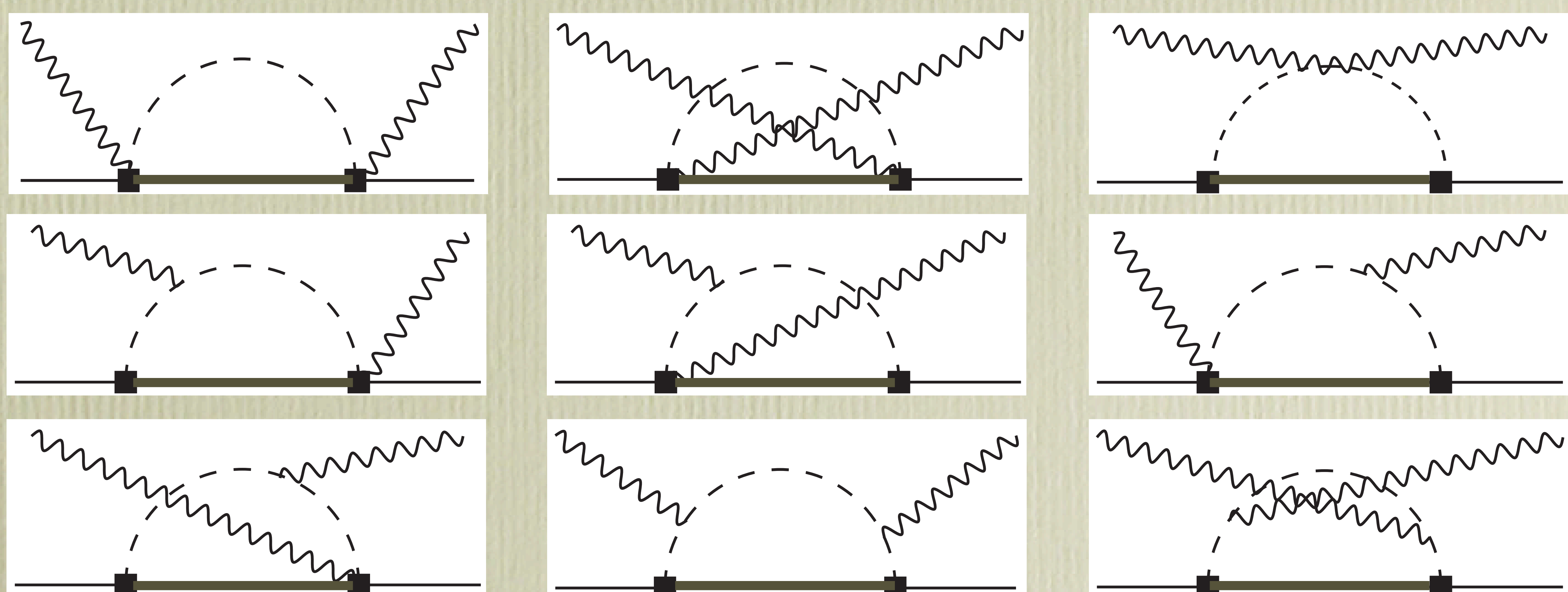
PQχPT contributions



Anomalous $\pi^0 \rightarrow \gamma\gamma$



Δ-pole graphs



LOOPS

Wess-Zumino-Witten

- Chiral anomaly contributes through π°→γγ

$$\mathcal{L}^{PQ}_{\pi^0\gamma\gamma} = -\frac{3e^2}{16\pi^2 f}\mathrm{tr}\left[\Phi\mathcal{Q}^2\right]\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Extension to partially quenched QCD non-trivial

$$\mathcal{L}^{PQ}_{\pi^0\gamma\gamma} \propto \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\Big[\mathrm{a}_1\,\mathrm{str}\left[\Phi\mathcal{Q}^2\right] + \mathrm{a}_2\,\mathrm{str}\left[\Phi\mathcal{Q}\right]\mathrm{str}\left[\mathcal{Q}\right] + \mathrm{a}_3\,\mathrm{str}\left[\Phi\right]\mathrm{str}\left[\mathcal{Q}\right]^2 + \mathrm{a}_4\,\mathrm{str}\left[\Phi\right]\mathrm{str}\left[\mathcal{Q}^2\right]\Big]$$

$$\mathcal{L}^{PQ}_{\pi^0\gamma\gamma} = -\frac{3e^2}{16\pi^2 f}\mathrm{str}\left[\Phi\mathcal{Q}^2\right]\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- No need to extend Witten's global quantisation construction to graded Lie groups

Infinite volume results

- Proton electric polarisability

Involve axial couplings and quark charges

$$\alpha_p = \frac{e^2}{4\pi f^2}\left[\frac{5G_B}{192\pi}\frac{1}{m_\pi} + \frac{5G'_B}{192\pi}\frac{1}{m_{uj}} + \frac{G_T}{72\pi^2}F_\alpha(m_\pi) + \frac{G'_T}{72\pi^2}F_\alpha(m_{uj})\right]$$

Singular in chiral limit

Non-analytic function involving Δ isobar

$$F_\alpha(m) = \frac{9\Delta}{\Delta^2 - m^2} - \frac{\Delta^2 - 10m^2}{2(\Delta^2 - m^2)^{3/2}}\ln\left[\frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}\right]$$

- Results for other polarisabilities similar but also have contributions from anomaly and poles

Finite volume effects

- Polarisabilities are very sensitive to infrared scales

  ➡ Expect large FV effects in lattice calculations

- Easily included in EFT for p-regime
  - Momentum integrals ⇒ mode sums

$$\int \frac{d^d k}{(2\pi)^d} \Rightarrow \frac{1}{L^3} \int \frac{d\,k_0}{2\pi} \sum_{\vec{k}}$$

where $\vec{k} = \frac{2\pi}{L}\vec{n}$ for $n_i \in \mathbb{Z}$

- 10% FV effects even at m_π= 500 MeV

Electric polarisability again

$$\alpha(L) = \frac{e^2}{1152\pi f^2}\int_0^\infty d\lambda\Big[3G_B\mathcal{F}_\alpha(\mathcal{M}_{uu}) + 3G'_B\mathcal{F}_\alpha(\mathcal{M}_{uj}) + 8G_T\mathcal{F}_\alpha(\mathcal{M}^\Delta_{uu}) + 8G'_T\mathcal{F}_\alpha(\mathcal{M}^\Delta_{uj})\Big]$$

$$\mathcal{M}^\Delta_{ab} = \sqrt{m^2_{ab} + 2\lambda\Delta + \lambda^2}$$

$$\mathcal{F}_\alpha(m) = 180\lambda^2\mathcal{I}_{\frac{7}{2}}(m) + 190\mathcal{J}_{\frac{7}{2}}(m) - 280\lambda^2\mathcal{J}_{\frac{9}{2}}(m) - 455\mathcal{K}_{\frac{9}{2}}(m) + 315\lambda^2\mathcal{K}_{\frac{11}{2}}(m) + 252\mathcal{L}_{\frac{11}{2}}(m)$$

$$\mathcal{I}_\beta(M) = \frac{1}{L^3}\sum_{\vec{k}}\frac{1}{\left[|\vec{k}|^2 + M^2\right]^\beta}$$

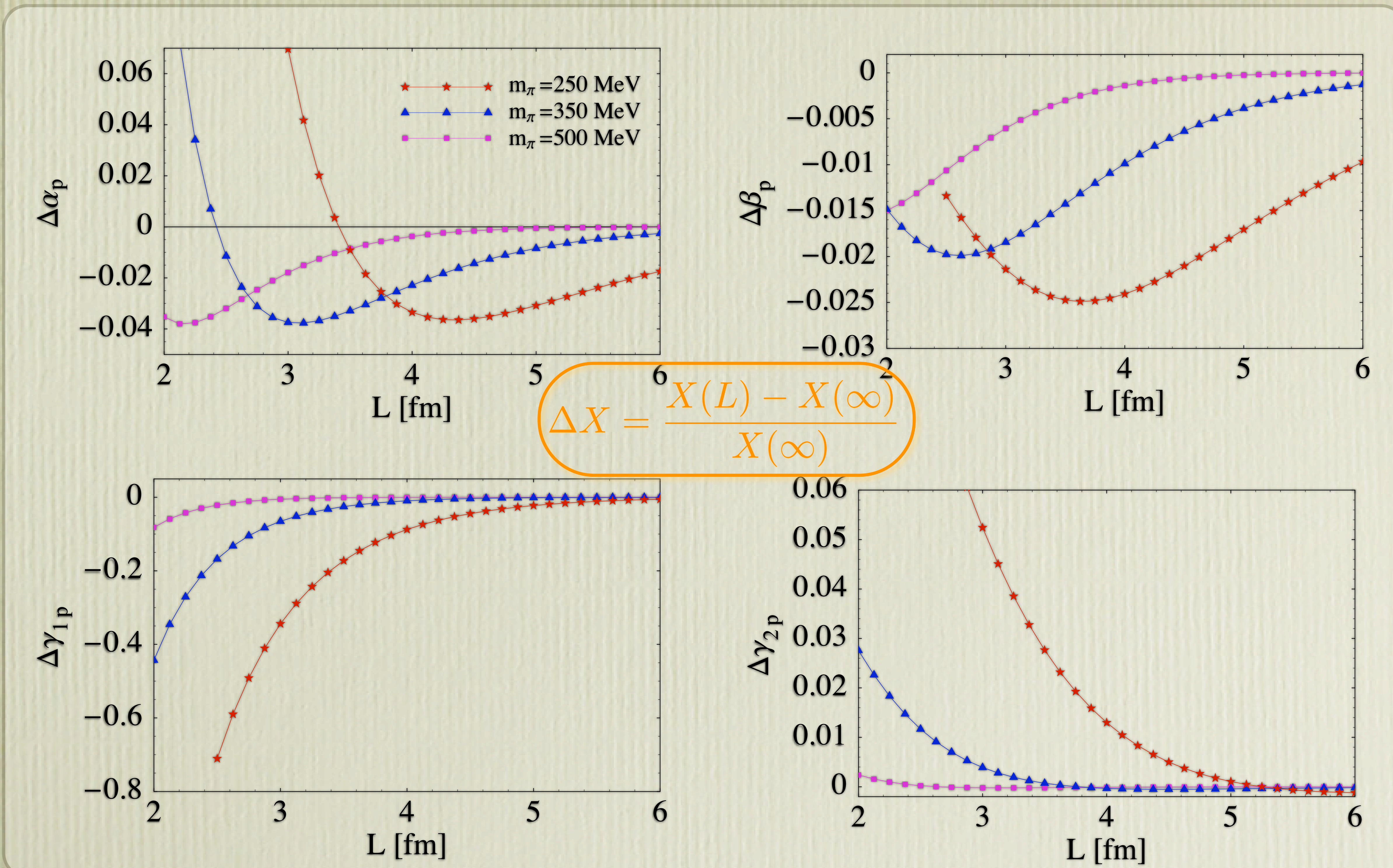
$$\mathcal{I}_\beta(M) = \sum_{\vec{n}}\frac{E_{1-\beta}(|\vec{n}|^2 + x^2)}{L^3\Gamma(\beta)} + \frac{\pi^{\frac{3}{2}}}{\Gamma(\beta)L^3}\int_0^1 dt\, t^{\beta-5/2}e^{-t\,x^2}\left[\sum_{\vec{n}\neq 0} e^{-\frac{\pi^2|\vec{n}|^2}{t}} + 1\right]$$

$$\mathcal{L}_\beta(M) = \mathcal{I}_{\beta-3}(M) - 3M^2\mathcal{I}_{\beta-2}(M) + 3M^4\mathcal{I}_{\beta-1}(M) - M^6\mathcal{I}_\beta(M)$$

$$\mathcal{K}_\beta(M) = \mathcal{I}_{\beta-2}(M) - 2M^2\mathcal{I}_{\beta-1}(M) + M^4\mathcal{I}_\beta(M)$$

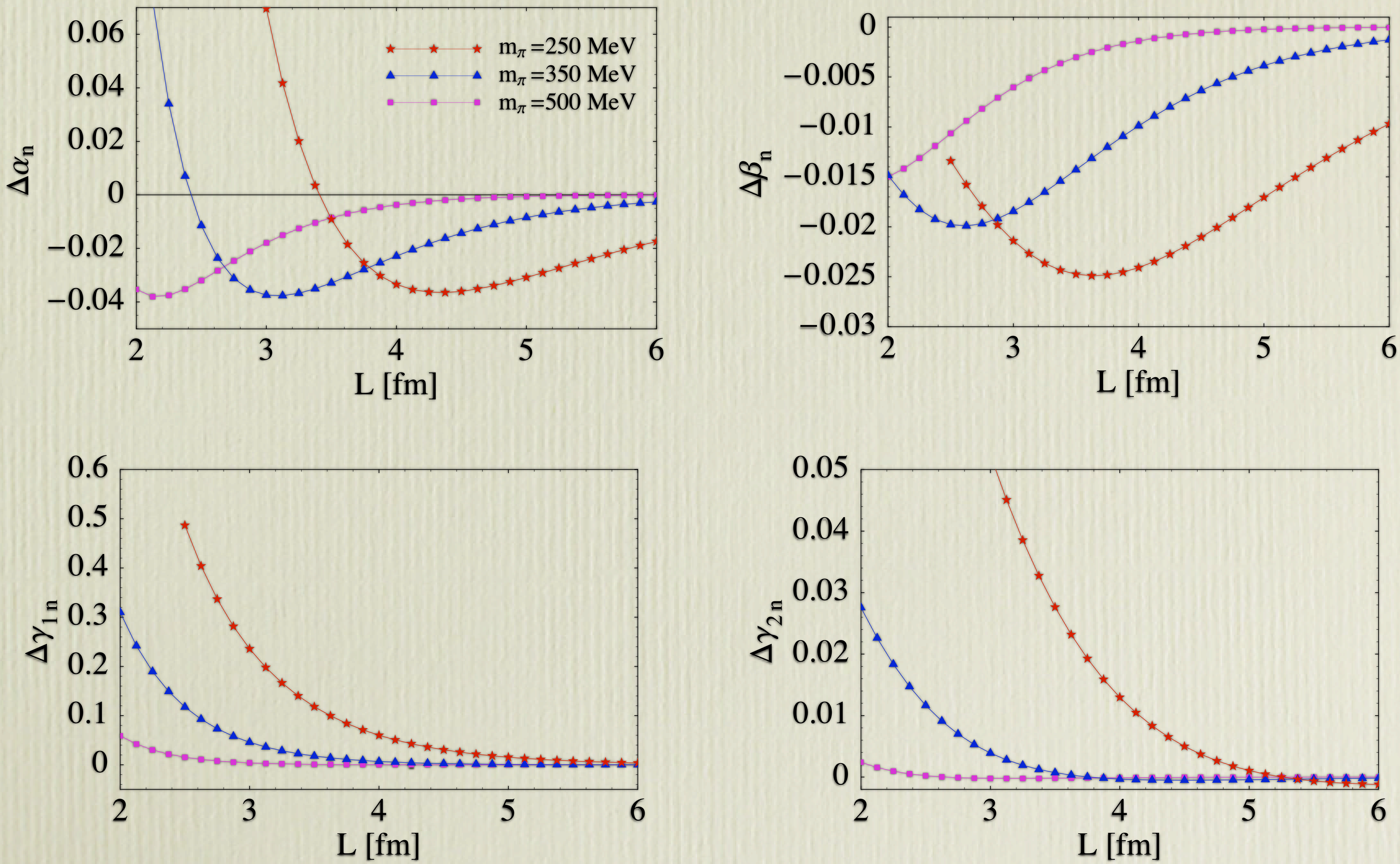
$$\mathcal{J}_\beta(M) = \mathcal{I}_{\beta-1}(M) - M^2\mathcal{I}_\beta(M)$$

Volume Dependence: Proton

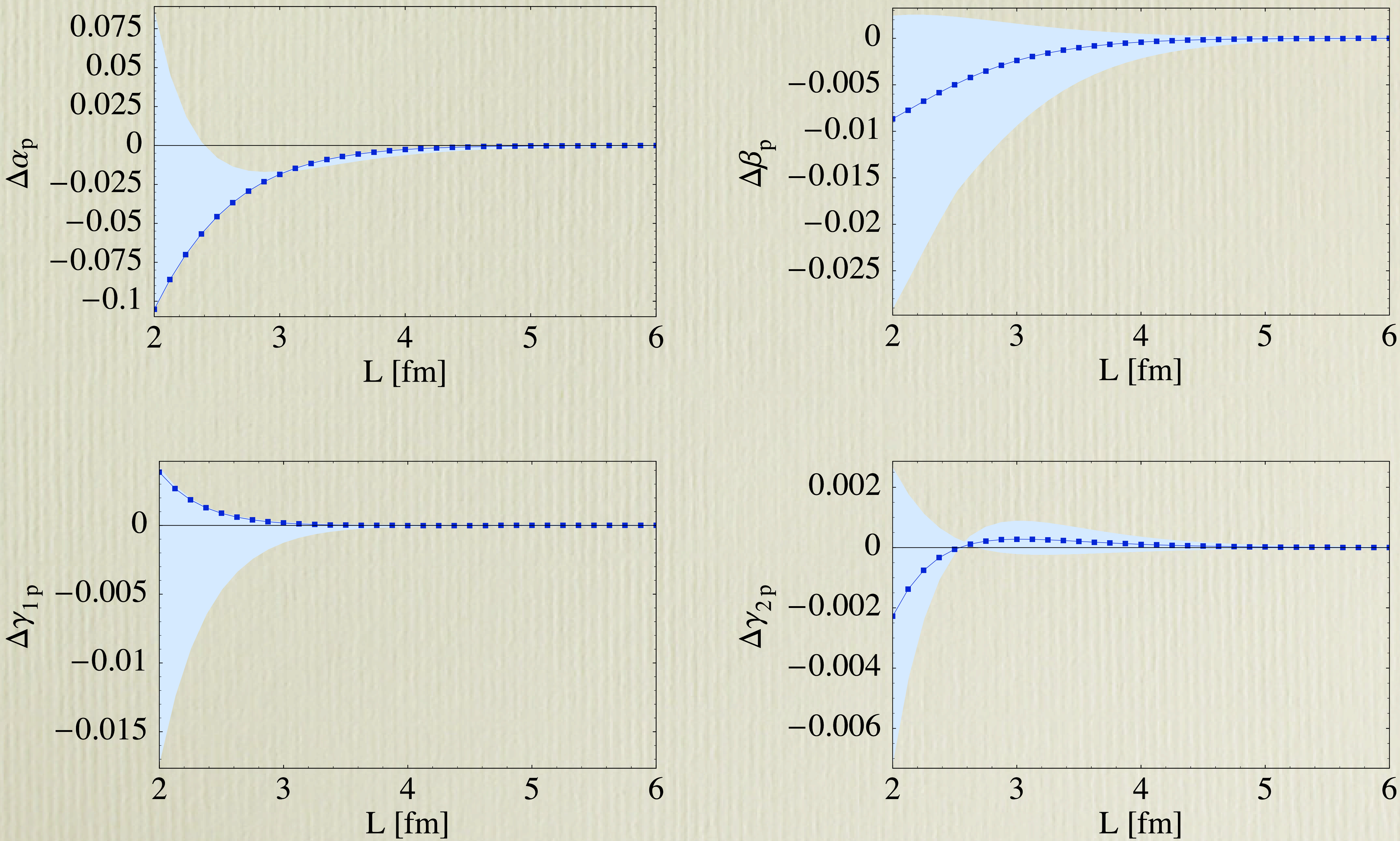


$$\Delta X = \frac{X(L) - X(\infty)}{X(\infty)}$$

Volume Dependence: Neutron



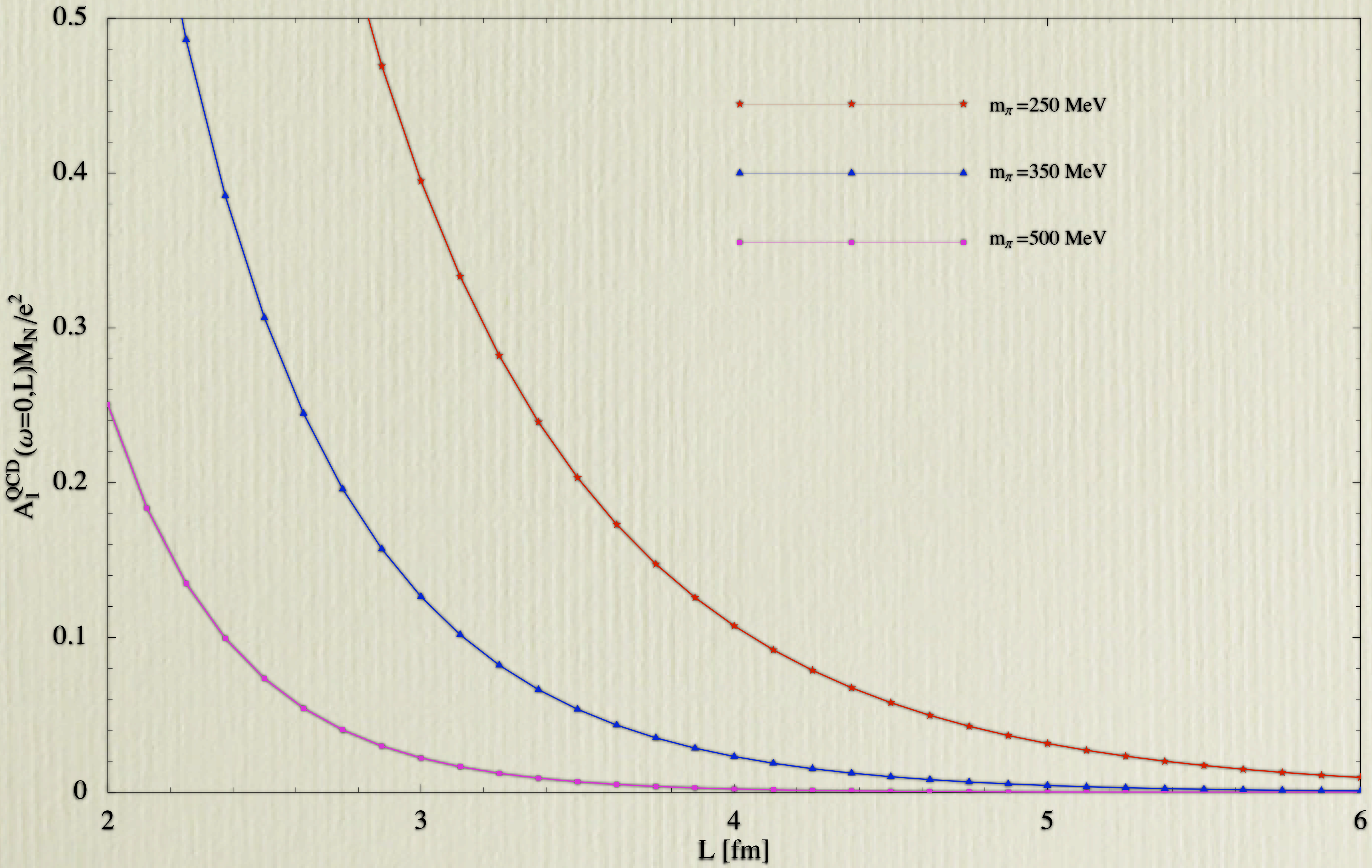
Volume Dependence: Quenched



m_π = 500 MeV

Thomson Limit ($\omega = 0$)

- Thomson limit for photon-neutron scattering
  - ➡ Vanishes at infinite volume!

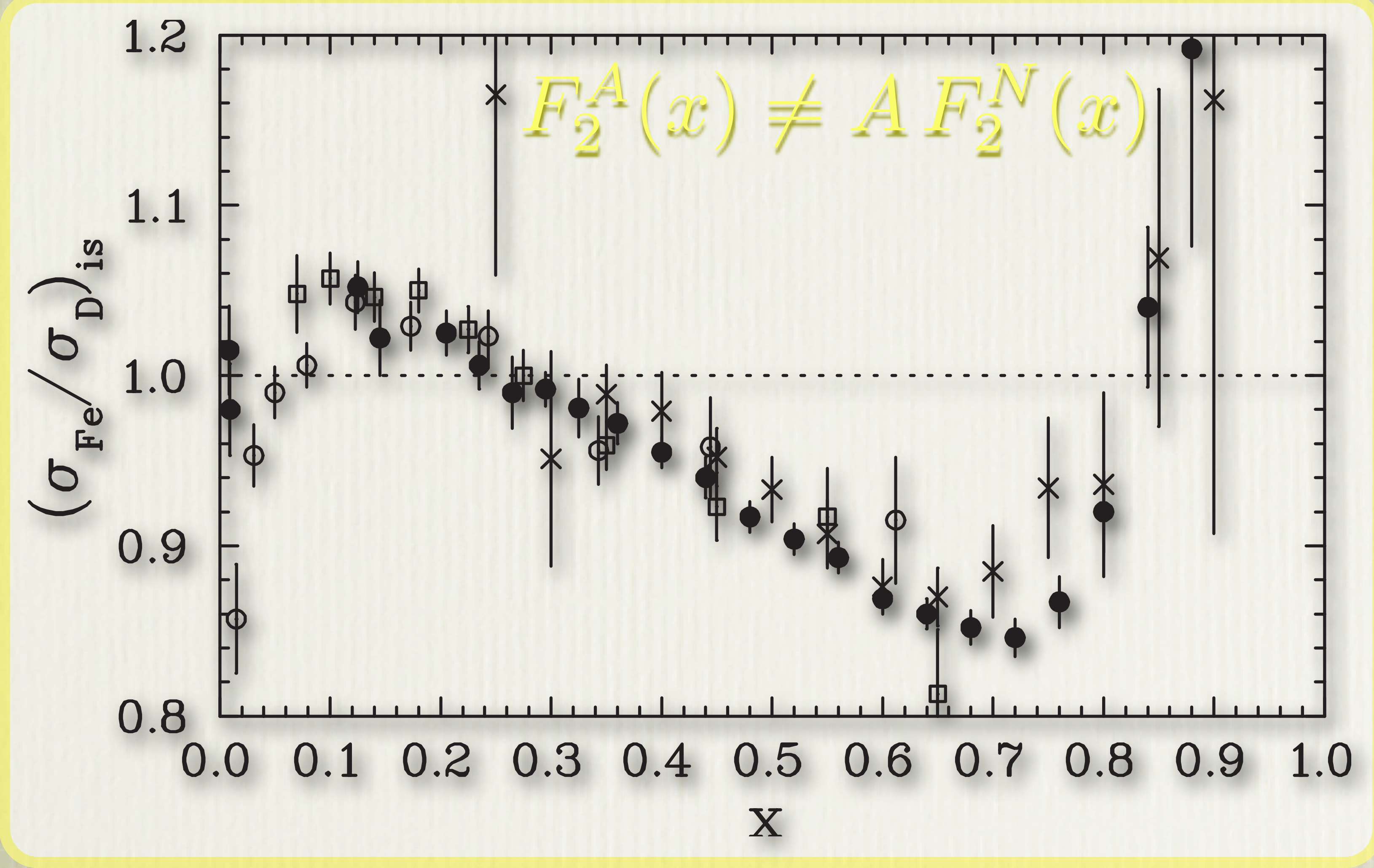


Other external fields

- Not limited to physical EM fields
- Can also use for any other quark bilinear
  - twist-two matrix elements (PDFs, GPDs), spin content (momentum injection)
  - EMC effect from lattice QCD (nuclear effects in parton distributions)
  - neutrino breakup of the deuteron
  - (flavour) twisted boundary conditions

EMC effect

- EMC 1983: Modification of PDFs in nuclei



- Was a surprise since $\epsilon/M \sim 1\%$

EMC on the lattice

- Simplest manifestation:

$$R^d(x,Q^2) = \frac{F_2^d(x,Q^2)}{F_2^p(x,Q^2)+F_2^n(x,Q^2)} \neq 1$$

- Lattice methods *can* be used to investigate the EMC effect

- Measure two-particle energy levels in external field coupled to twist-two operators

- Determine 2-body coefficients in $\langle d|\mathcal{O}^{\mu_1\cdots\mu_n}|d\rangle$

  ➡ Leading medium modification of moments

[WD Phys Rev D71 054506, WD & JW Chen Phys Lett B625,165]

Future Prospects

- All EM and spin polarisabilities can be measured with external fields
- Preliminary lattice calculations underway for spin polarisabilities
- Large volume effects and strong mass dependence require large volumes and small masses!
- Higher order and generalised polarisabilities [(doubly)-virtual Compton scattering] are also measurable
- Parity violating polarisabilities??

To be more specific...

$$T_{\gamma N} = A_1(\omega,\theta)\,\vec{\epsilon}\,' \cdot \vec{\epsilon} + A_2(\omega,\theta)\,\vec{\epsilon}\,' \cdot \hat{k}\,\vec{\epsilon}\cdot\hat{k}' + i\,A_3(\omega,\theta)\,\vec{\sigma}\cdot(\vec{\epsilon}\,'\times\vec{\epsilon}) + i\,A_4(\omega,\theta)\,\vec{\sigma}\cdot(\hat{k}'\times\hat{k})\,\vec{\epsilon}\,'\cdot\vec{\epsilon}$$
$$+ i\,A_5(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}\,'\times\hat{k})\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k}')\,\vec{\epsilon}\,'\cdot\hat{k}\right] + i\,A_6(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}\,'\times\hat{k}')\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k})\,\vec{\epsilon}\,'\cdot\hat{k}\right]$$

$$A_1(\omega,\theta) = -Z^2\frac{e^2}{M_N} + \frac{e^2}{4M_N^3}\left(\mu^2(1+\cos\theta) - Z^2\right)(1-\cos\theta)\,\omega^2 + 4\pi(\alpha + \beta\,\cos\theta)\omega^2 + \mathcal{O}(\omega^4)$$

$$A_2(\omega,\theta) = \frac{e^2}{4M_N^3}(\mu^2 - Z^2)\omega^2\cos\theta - 4\pi\beta\omega^2 + \mathcal{O}(\omega^4)$$

$$A_3(\omega,\theta) = \frac{e^2\omega}{2M_N^2}\left(Z(2\mu - Z) - \mu^2\cos\theta\right) + 4\pi\omega^3(\gamma_1 - (\gamma_2 + 2\gamma_4)\,\cos\theta) + \mathcal{O}(\omega^5)$$

$$A_4(\omega,\theta) = -\frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_2 + \mathcal{O}(\omega^5)$$

$$A_5(\omega,\theta) = \frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_4 + \mathcal{O}(\omega^5)$$

$$A_6(\omega,\theta) = -\frac{e^2\omega}{2M_N^2}Z\mu + 4\pi\omega^3\gamma_3 + \mathcal{O}(\omega^5)$$

EFT correlators

- Pionless effective field theory: cutoff $p < m_\pi$
- Lagrangian

$$\mathcal{L}_{\rm eff}(\vec{x},\tau;A) = \Psi^\dagger(\vec{x},\tau)\Bigg[\left(\frac{\partial}{\partial\tau} + i\,q\,A_4\right) + \frac{(-i\vec{\nabla} - \,q\,\vec{A})^2}{2M} - \mu\,\vec{\sigma}\cdot\vec{H}$$
$$+2\pi\left(\alpha\,\vec{E}^2 - \beta\,\vec{H}^2\right) - 2\pi i\Big(-\gamma_{E_1E_1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}}$$
$$+\gamma_{M_1M_1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} + \gamma_{M_1E_2}\sigma^i E^{ij}H^j + \gamma_{E_1M_2}\sigma^i H^{ij}E^j\Big)\Bigg]\Psi(\vec{x},\tau) + \ldots$$

- Resum interactions with external field