Partially-quenched chiral extrapolation of the vector meson

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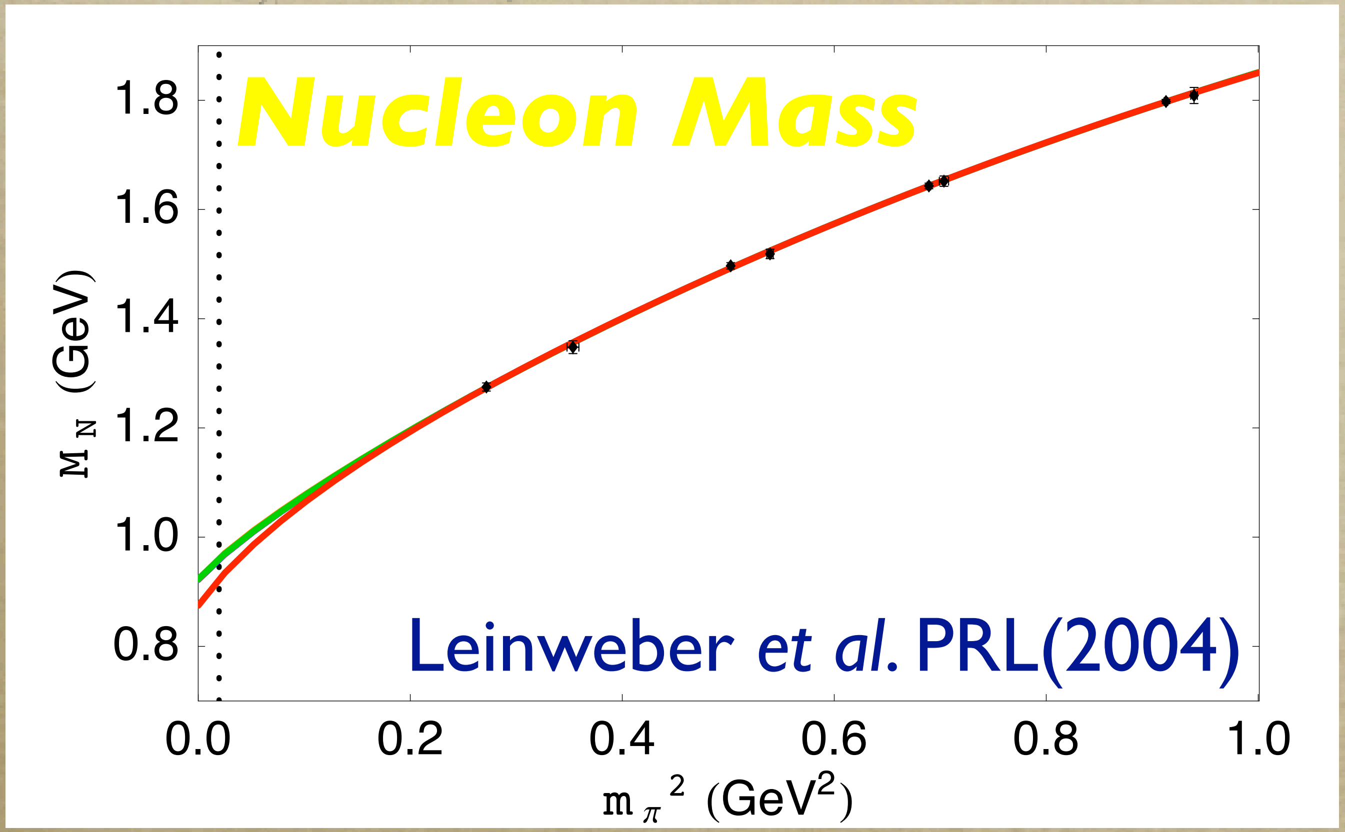
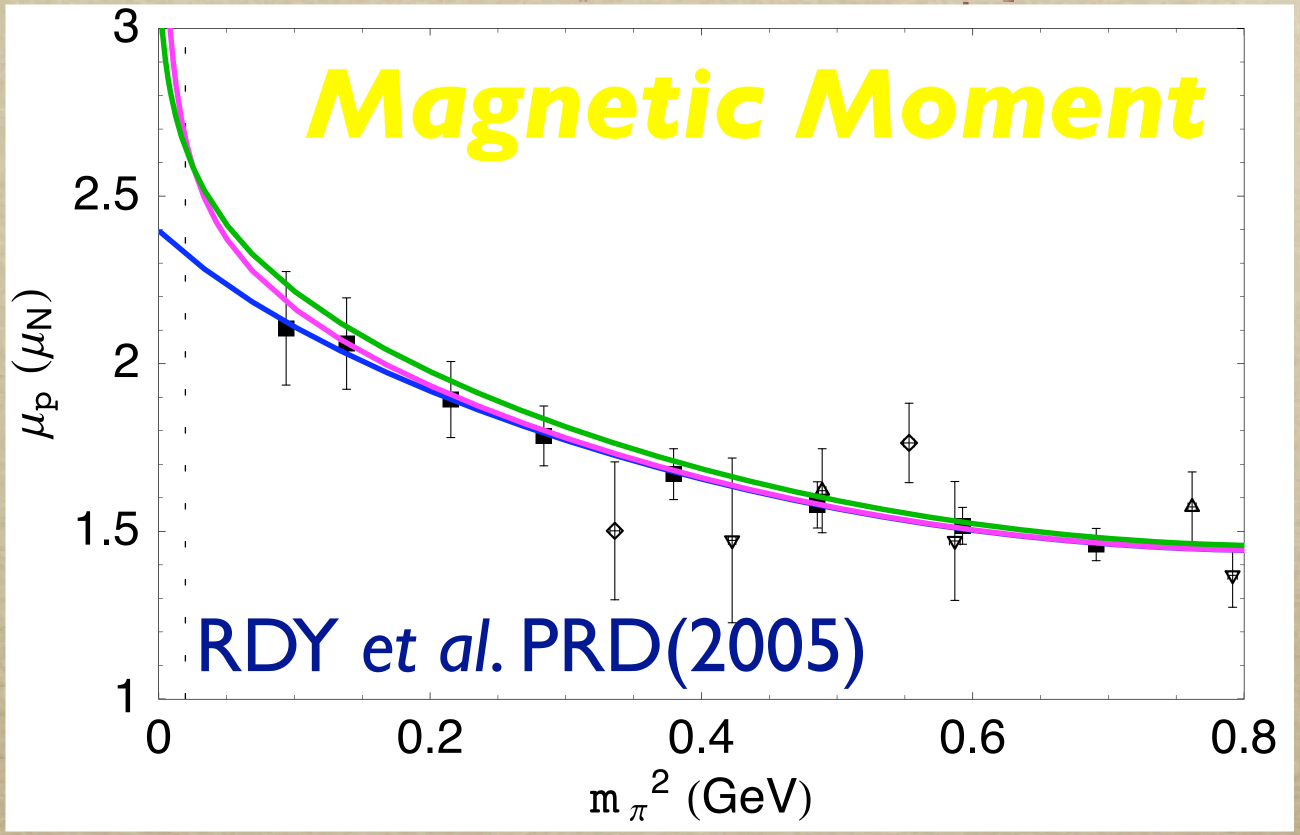
THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

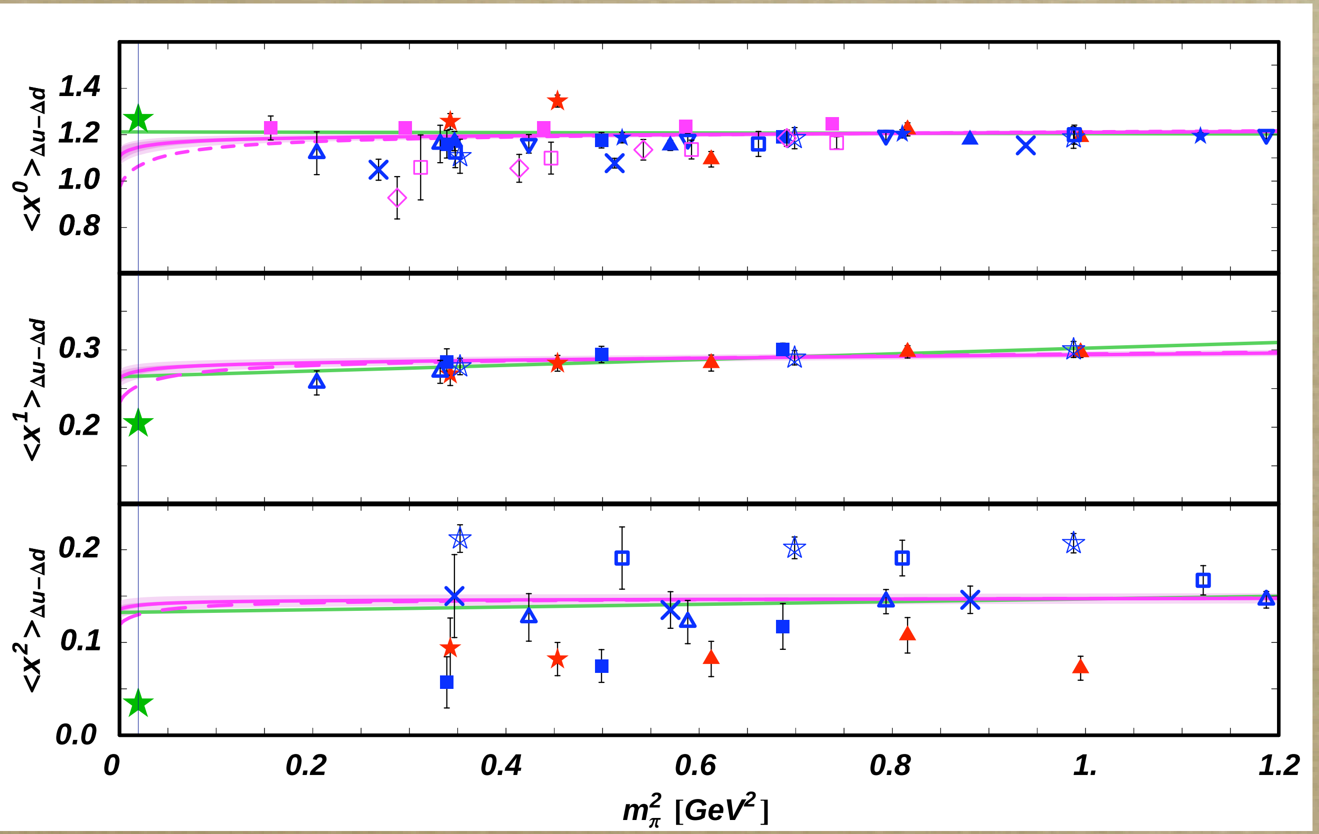
Outline

- *Chiral extrapolation for lattice QCD*
  - *Finite-Range Regularisation (FRR)*
- *Application to PQ sector*
  - *Constructing expansion for rho-meson*
- *Global analysis: discretisation, FV, PQ*

Chiral Extrapolation







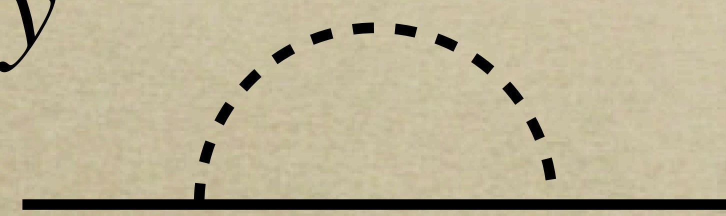
Isovector Moments

Detmold et al. PRD(2002)

Chiral Symmetry

Gell-Mann–Oakes–Renner $m_\pi^2 \propto m_q$

Nucleon self-energy



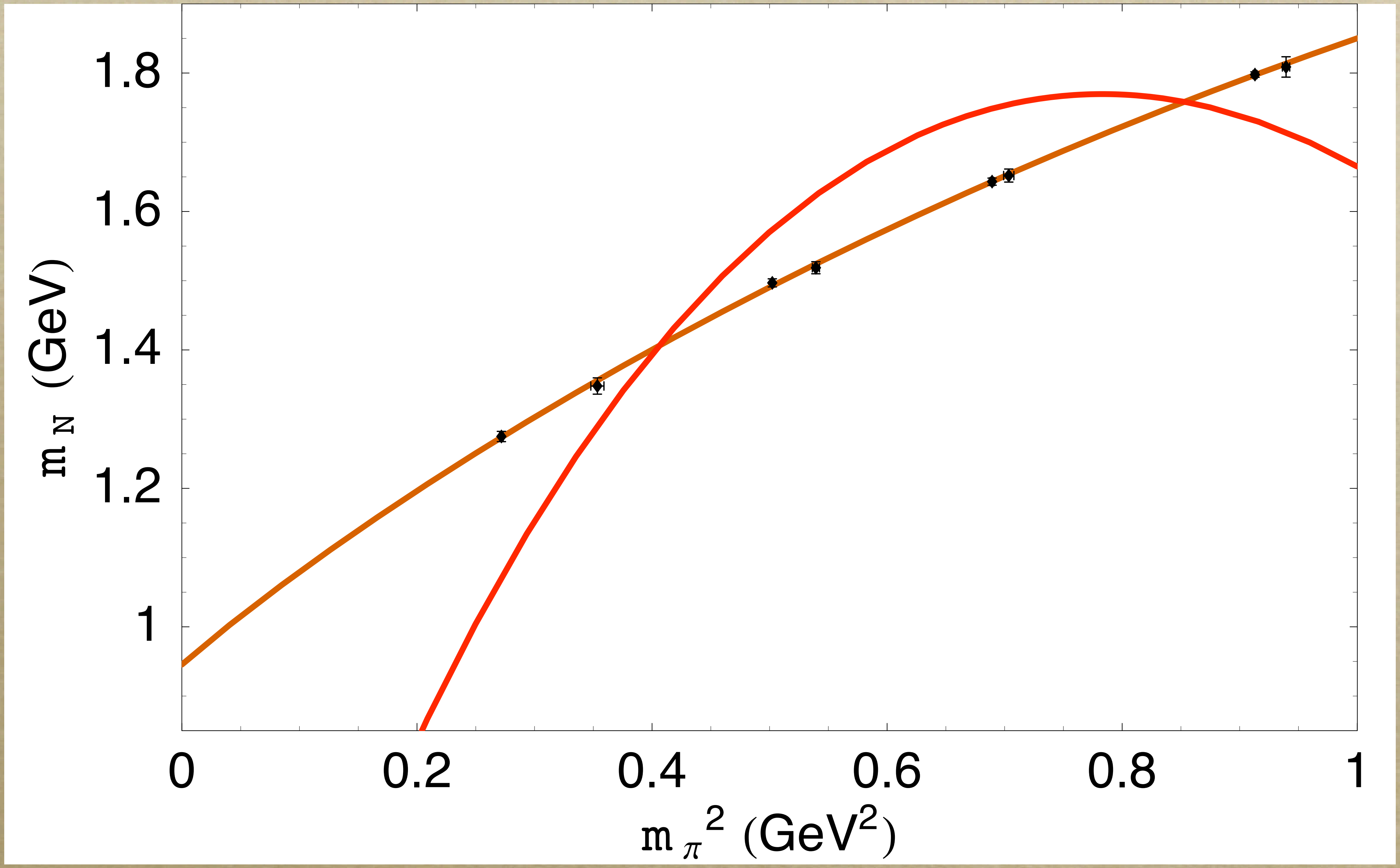
$\sim m_\pi^3$

Chiral expansion of nucleon mass:

$$M_N = c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + \ldots$$

$$\chi_\pi = -\frac{3g_A^2}{32\pi f_\pi^2} \simeq -5.5\mathrm{GeV}^{-2}$$

Fit Lattice Results

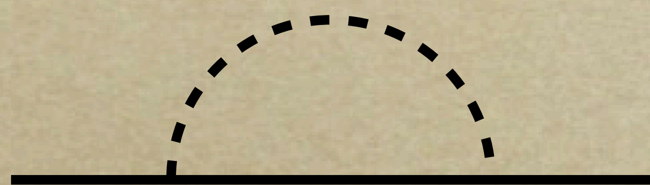


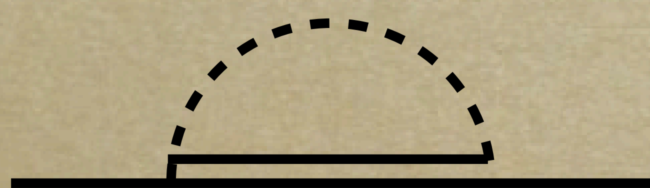
Fit 1: $\chi_\pi \simeq -0.63\mathrm{GeV}^{-2}$ **too small!**

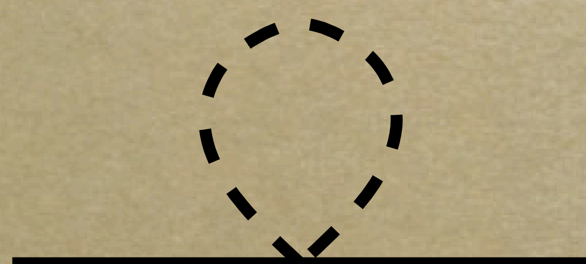
Fit 2: $\chi_\pi = -5.5\mathrm{GeV}^{-2}$ **!%#@*!**

Nucleon Mass Expansion

$$m_N = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + a_6 m_\pi^6$$
$$+ \chi_\pi I_\pi + \chi_{\pi\Delta} I_{\pi\Delta} + \chi_{\text{tad}} I_{\text{tad}}$$

$I_\pi =$ 

$I_{\pi\Delta} =$ 

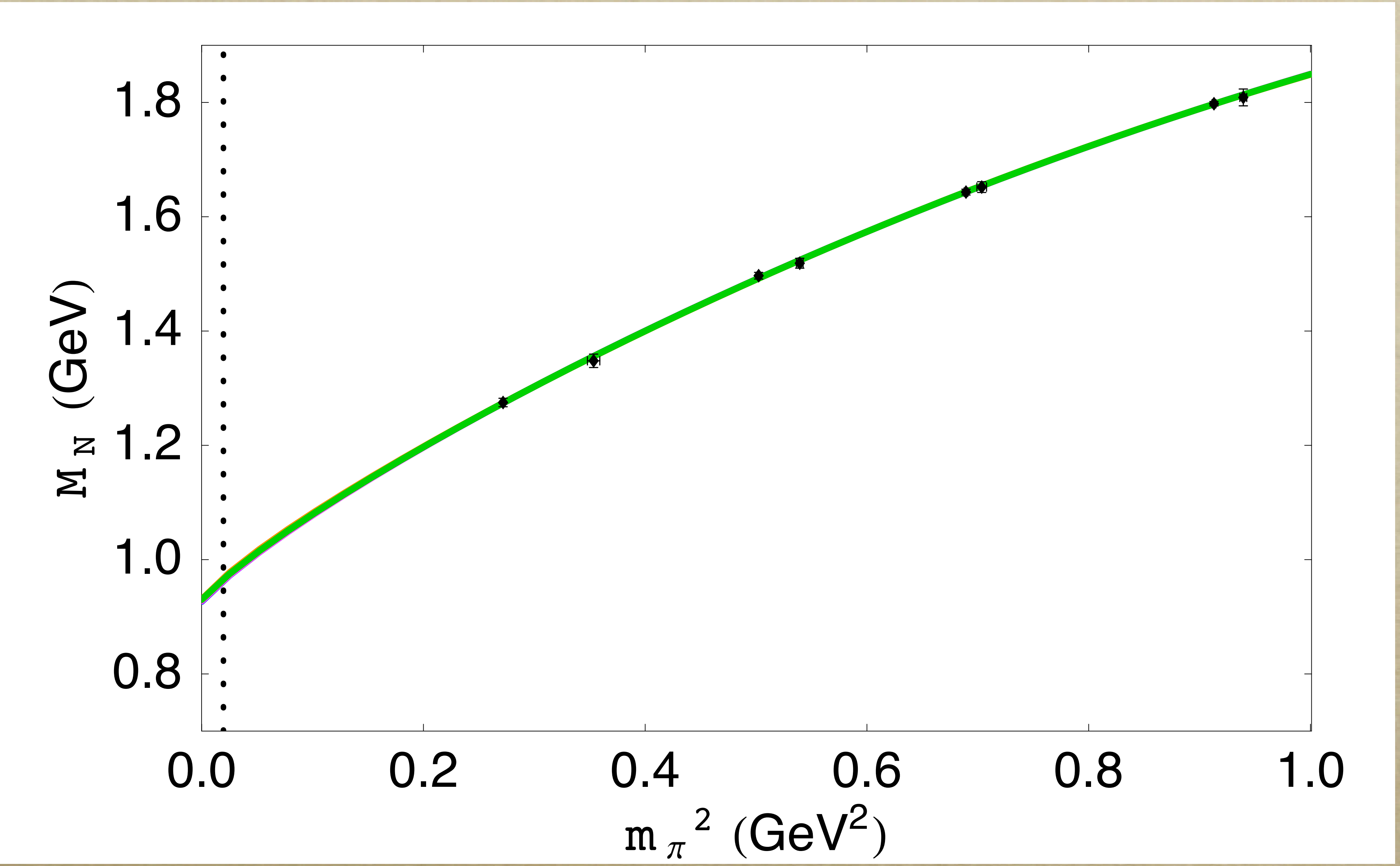
$I_{\text{tad}} =$ 

Loop integrals require regularisation

Introduce cutoff:
Finite-Range Regularisation
(FRR)

Mathematically equivalent to dim. reg.

Chiral Extrapolation



Dipole

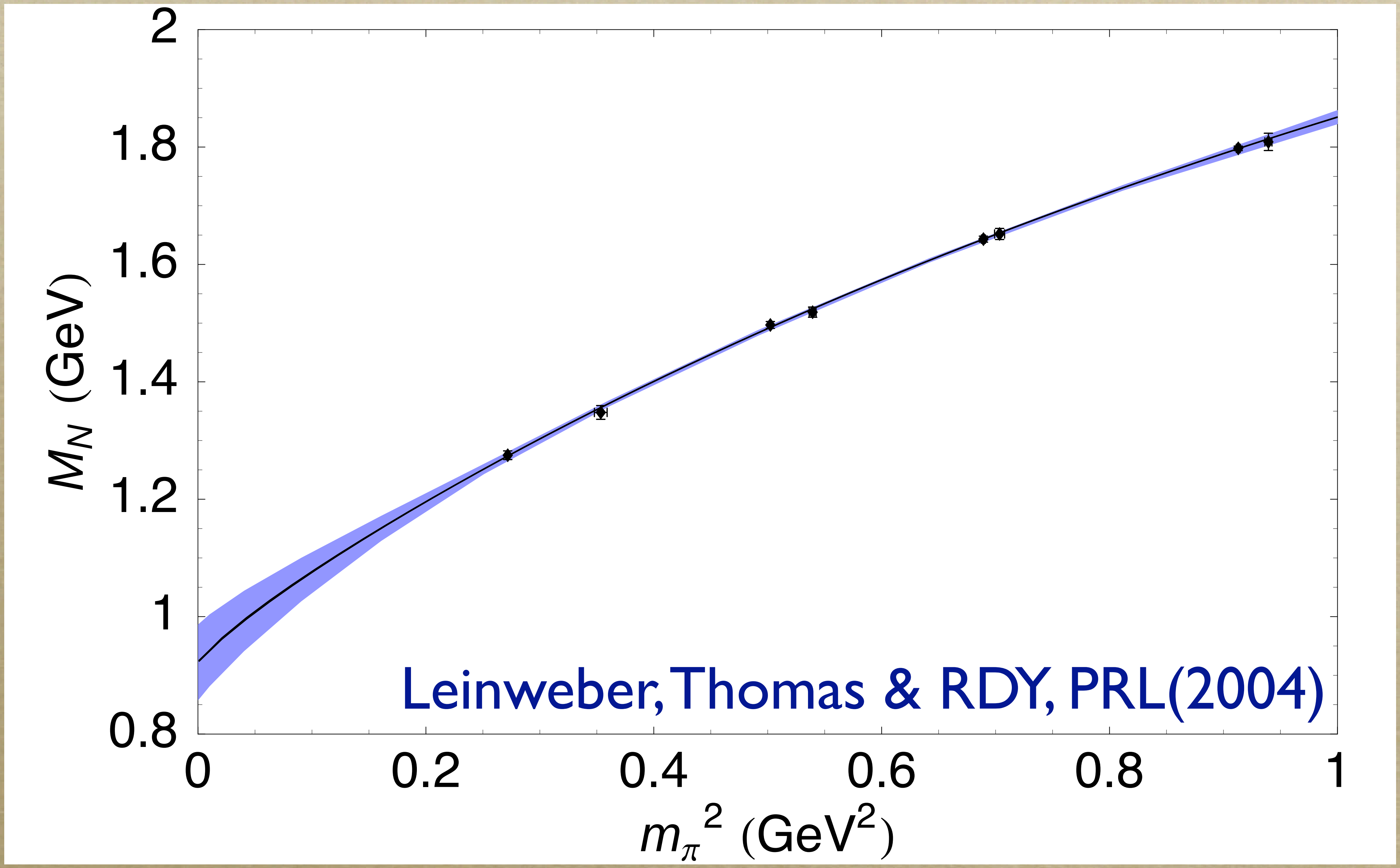
Theta

Monopole

Gaussian

Leinweber, Thomas & RDY, PRL(2004)

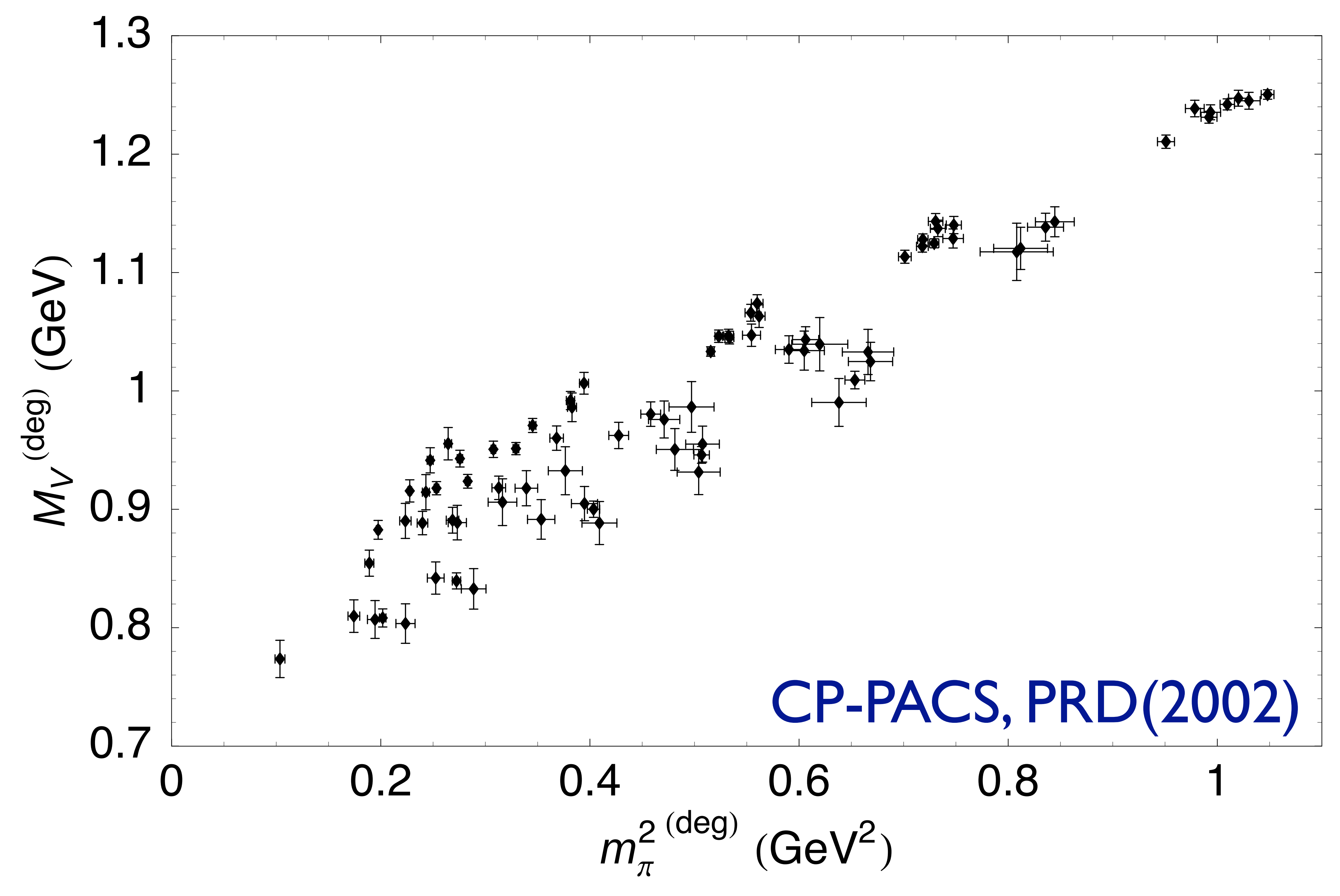
Statistical Uncertainty



Extrapolated result ~7% stat. uncertainty

Partially-Quenched Results

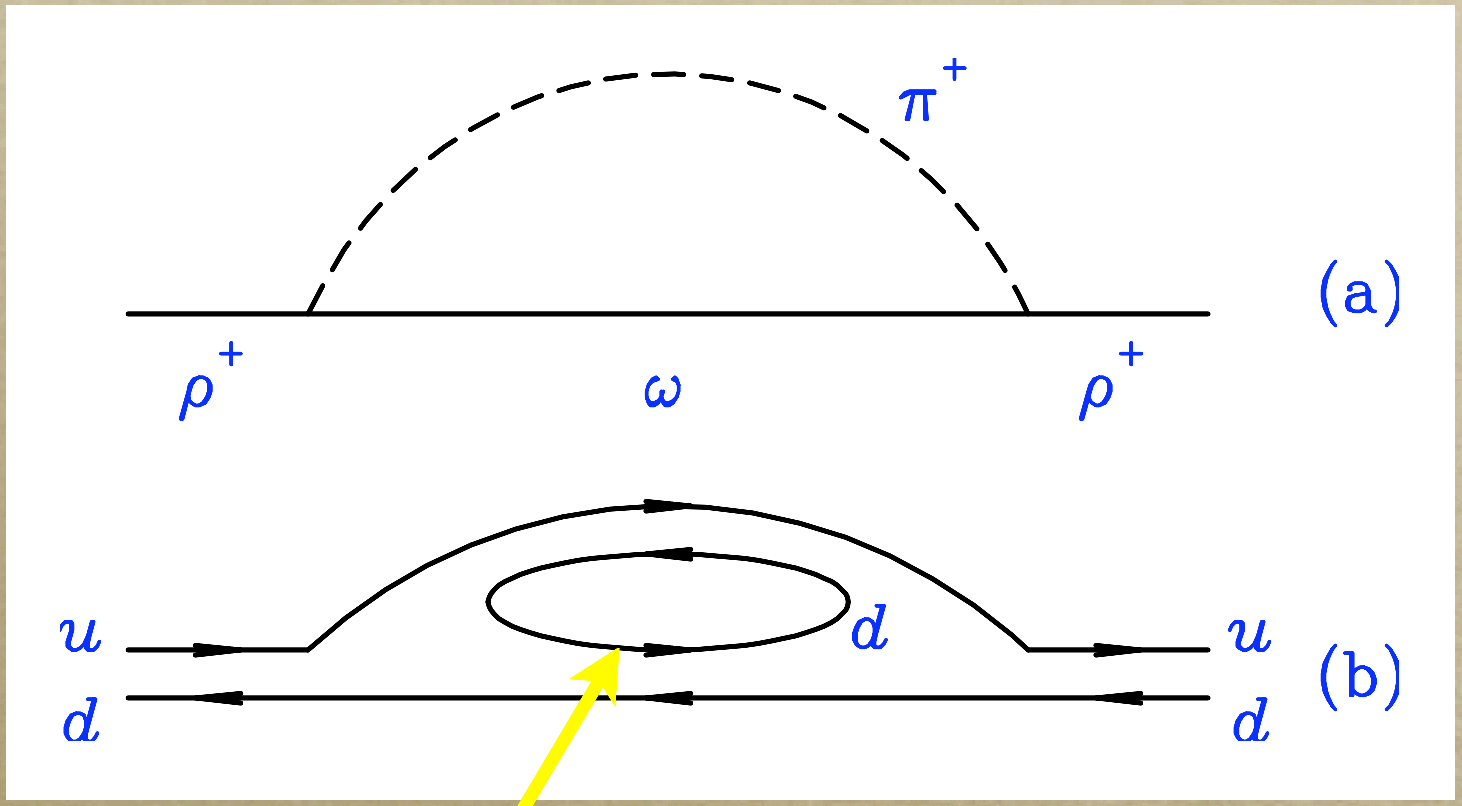
Vector meson mass



$4\ \beta;\ 4\ \kappa_{sea};\ 5\ \kappa_{val} \Longrightarrow$ 80 data points

Vector Meson in EFT

Leading-loop contribution to M_V



$\rho \to \omega\pi$

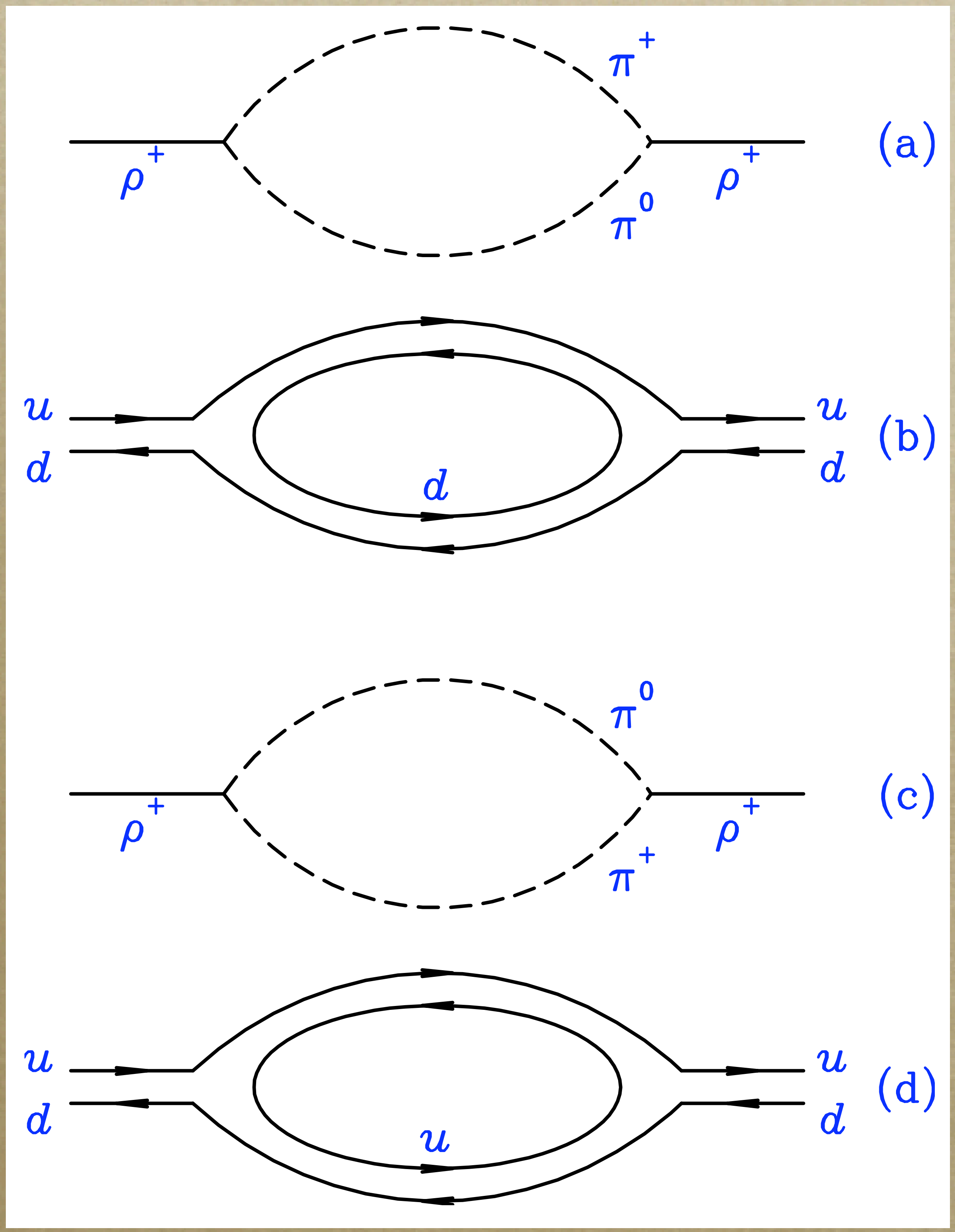
$LNA \sim m_\pi^3$

Partially-quenched: sea-quark mass differs

$$m_\pi[m_v, m_v] \Rightarrow m_\pi[m_v, m_s]$$
$$M_V[m_v, m_v] \Rightarrow M_V[m_v, m_s]$$

For full evaluation, essential to have K and K type masses calculated!*

Beyond Leading Order



ρ → ππ *decay channel*

$M_V > 2m_\pi$

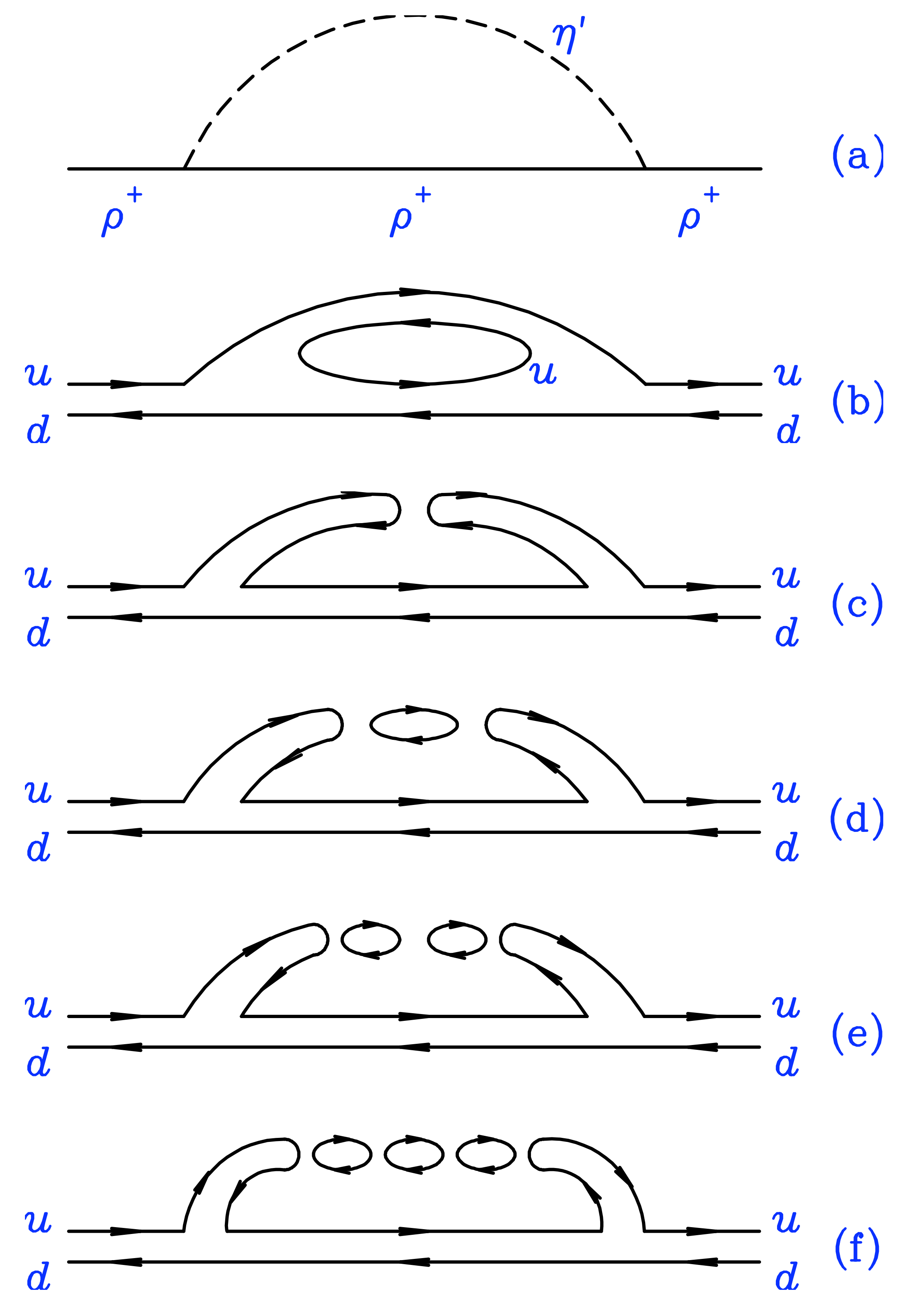
Imaginary part involves 2 energetic pions

Outside scope of EFT!

We fix coupling to reproduce width at physical point

Masses modified in PQ

Another Quenched Artifact



$\rho \to \rho\eta'$

η' *behaves like a GB*

Formally decouples from EFT as $m_{val} \to m_{sea}$

Double-pole structure
$\propto (m_{sea} - m_{val})$

μ_0 *integrates out*

Fit Formula

$$M_V^2 = (\alpha_0 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4 + \alpha_6 m_\pi^6)^2 + \Sigma^{TOT}$$

Discretisation artifacts:

$$\alpha_0 \rightarrow \alpha_0 + X_1 a + X_2 a^2$$

$$\alpha_2 \rightarrow \alpha_2 + X_1' a + X_2' a^2$$

Finite-volume:

Σ^{TOT}

Discrete momentum sums

Fit Formula

$$M_V^2 = (\alpha_0 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4 + \alpha_6 m_\pi^6)^2 + \Sigma^{TOT}$$

Consistent with zero

Discretisation artifacts:

$$\alpha_0 \to \alpha_0 + X_1 a + X_2 a^2$$

$$\alpha_2 \to \alpha_2 + X_1' a + X_2' a^2$$

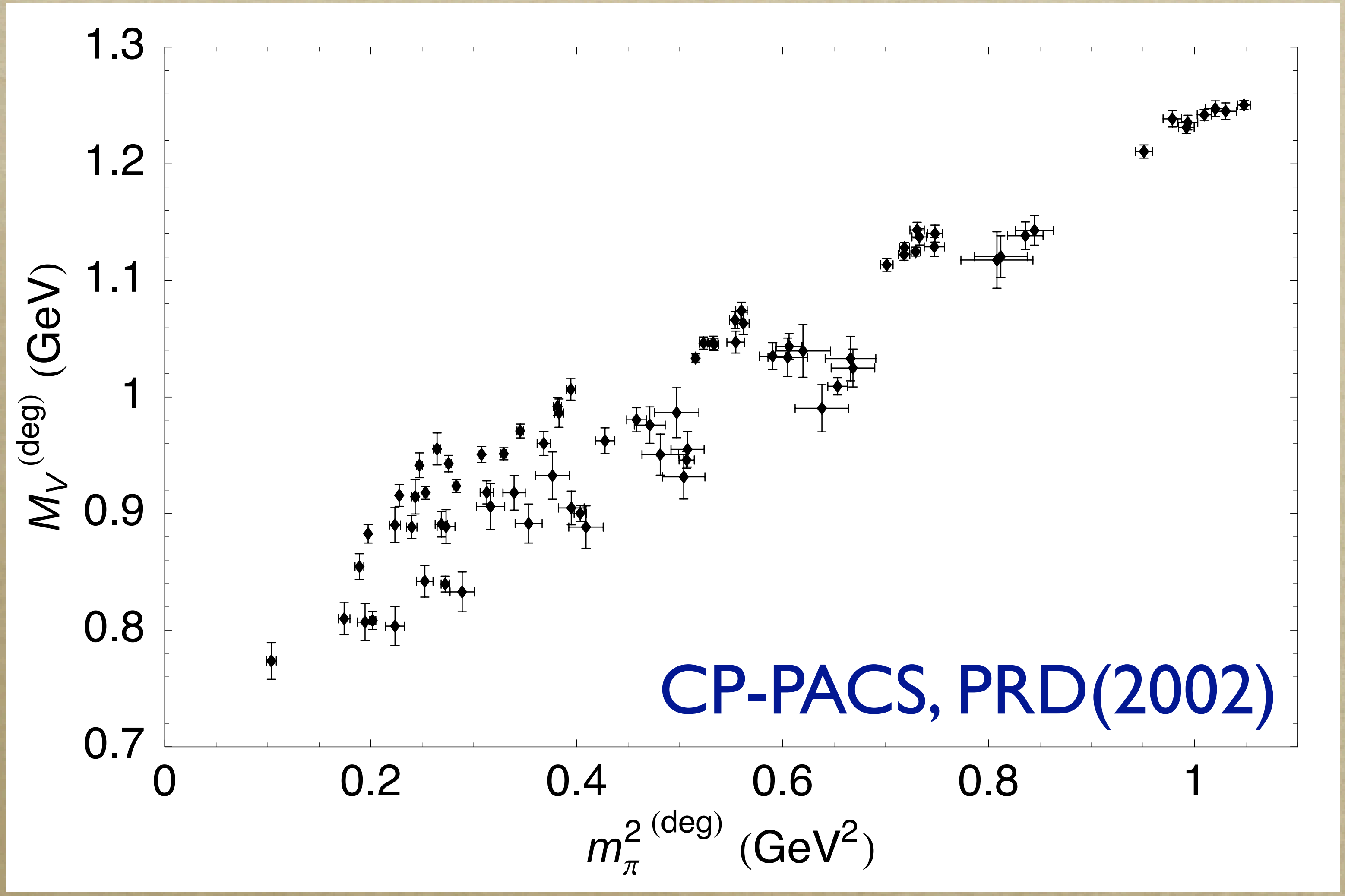
No effect observed

Finite-volume:

Σ^{TOT}

Discrete momentum sums

Global Fit

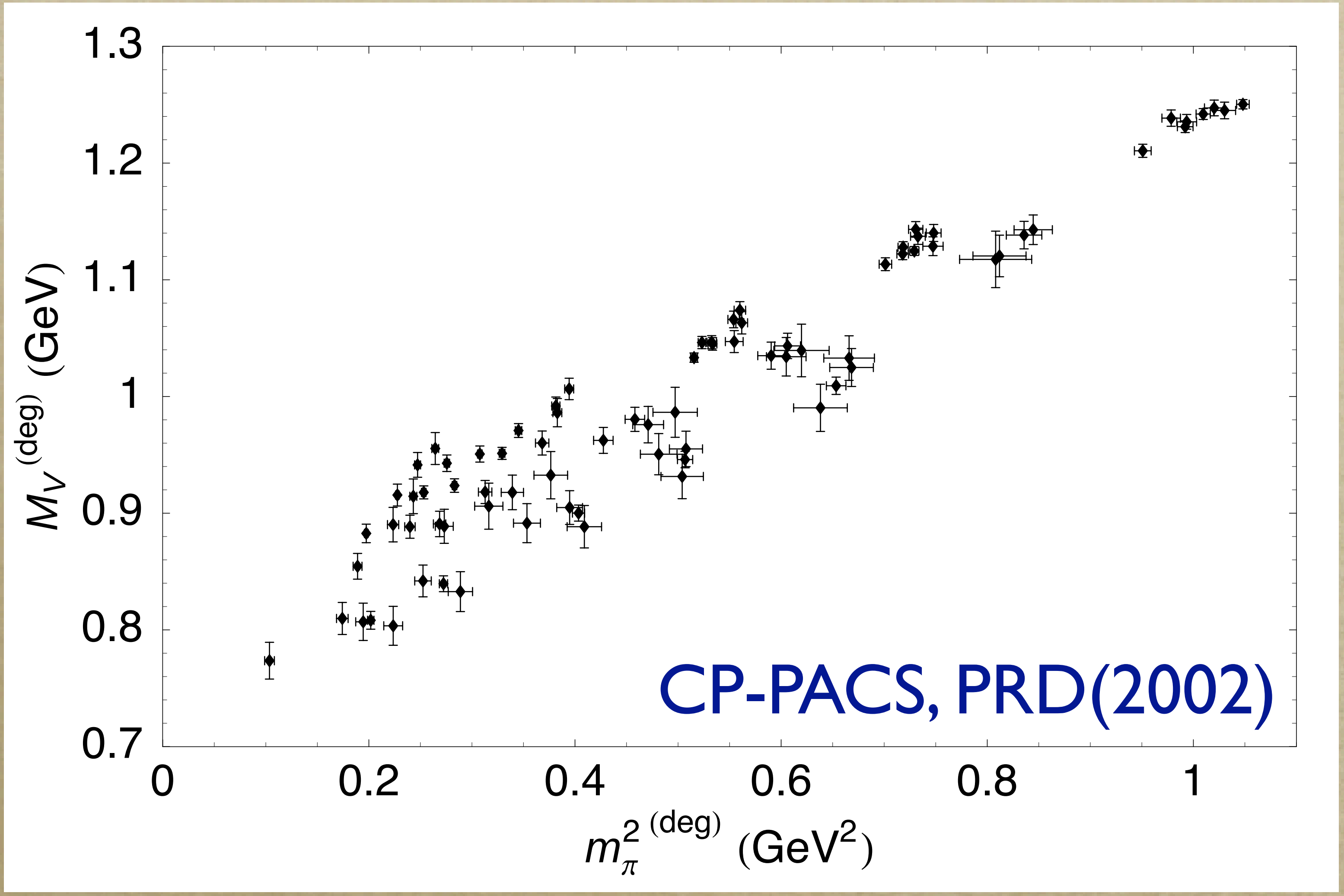


$$M_V^2 = (\alpha_0 + X_2 a^2 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4)^2 + \Sigma^{TOT}(L)$$

α_0	X_2	α_2	α_4	χ^2/dof
0.832(9)	*-1.40(3)*	*0.494(12)*	*-0.061(9)*	*39/75*

$620 < \Lambda < 690\,\mathrm{MeV}$

Global Fit



Compare linear fits

$\Sigma \to 0$

best fit

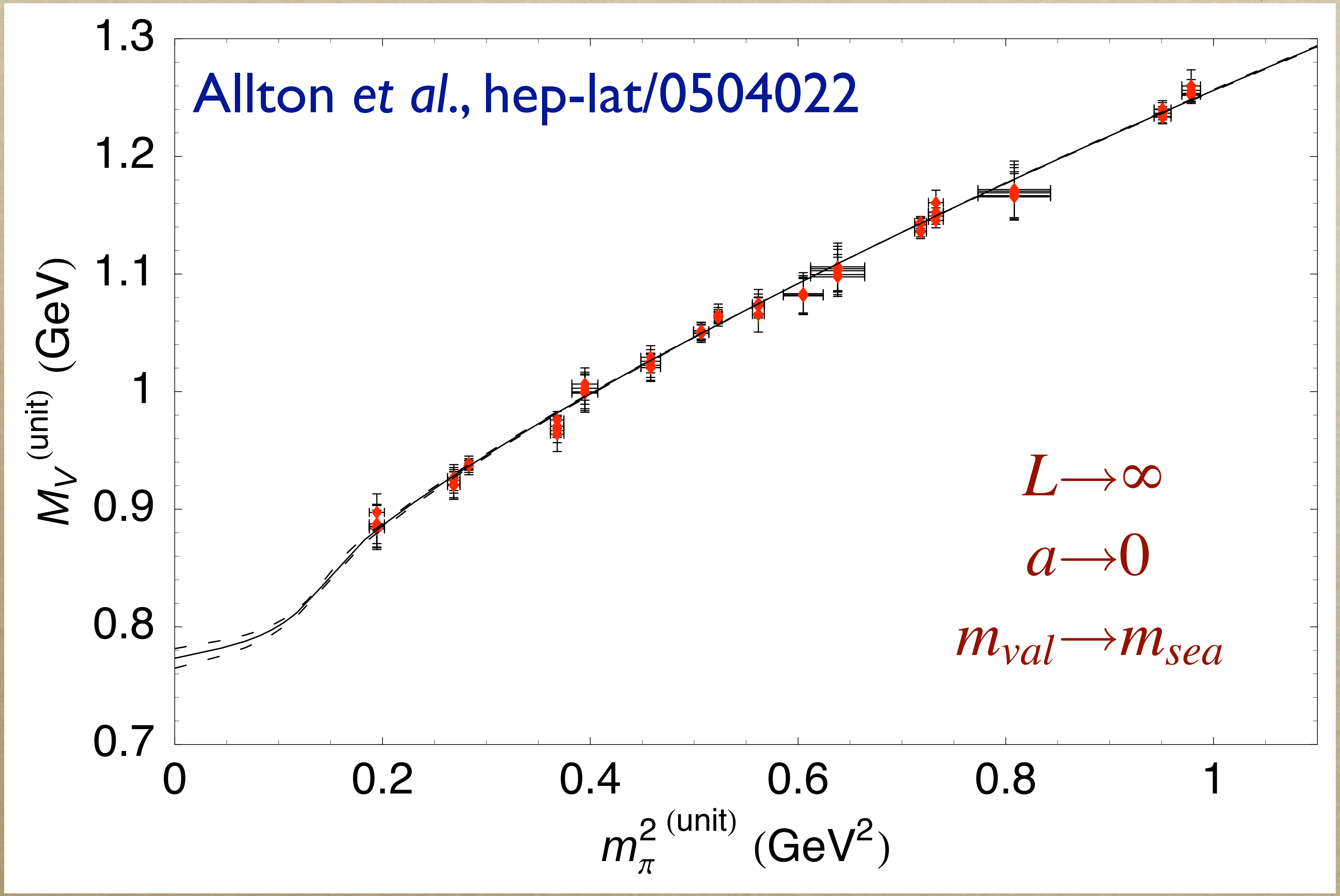
$\chi^2/dof = 77/74$

$$M_V^2 = (\alpha_0 + X_2 a^2 + \alpha_2 m_\pi^2 + \alpha_4 m_\pi^4)^2 + \Sigma^{TOT}(L)$$

α_0	X_2	α_2	α_4	χ^2/dof
0.832(9)	-1.40(3)	0.494(12)	-0.061(9)	39/75

$620 < \Lambda < 690\,\text{MeV}$

Physical Determination



Extrapolation $M_V = 777(4)^{+16}_{-6}(8)\,\mathrm{MeV}$

Conclusions

- *FRR offers solution to chiral extrapolation with moderate quark masses*
- *Rho-meson decay beyond EFT*
    - *constraint from physical width*
- *Use of partially-quenched data dramatically increases statistical precision*

Thanks

Chris Allton
Wes Armour
Derek Leinweber
Tony Thomas

References

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