Perturbative Expansions of Wilson Loops in Full QCD from MC Simulations

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Contents

- need higher-order perturbation theory
  e.g.: precise determination of α_s (HPQCD, PRL **95**, 052002, 2005)
- lattice perturbation theory $\rightarrow$ difficult
- alternative: measure perturbative coefficients from Monte Carlo simulations at high-β (weak coupling)
- challenges
- results: Wilson loops in full QCD (MILC action)
- preliminary study for NRQCD
- outlook

Need Higher-Order PT

- e.g.: precise determination of α_s (HPQCD Collaboration)
- key ingredient: expansion of WL

$$-\ln W = c_1\alpha + c_2\alpha^2 + c_3\alpha^3 + \ldots$$

measure this … then solve for α

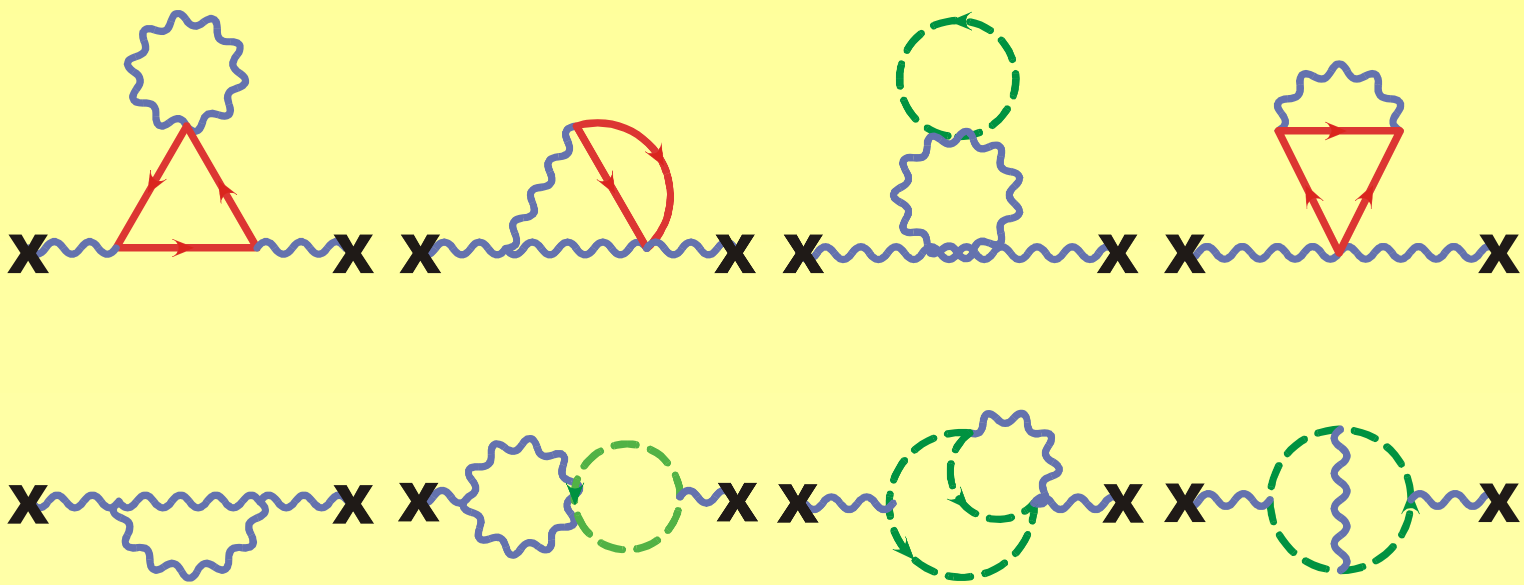
- to what order in PT → a few % error?

$$\alpha(a \approx 0.1\text{fm}) \sim 0.2 - 0.3$$

need 1-loop (α^2) or 2-loop (α^3) calculations

Difficult! (particularly for complicated actions)

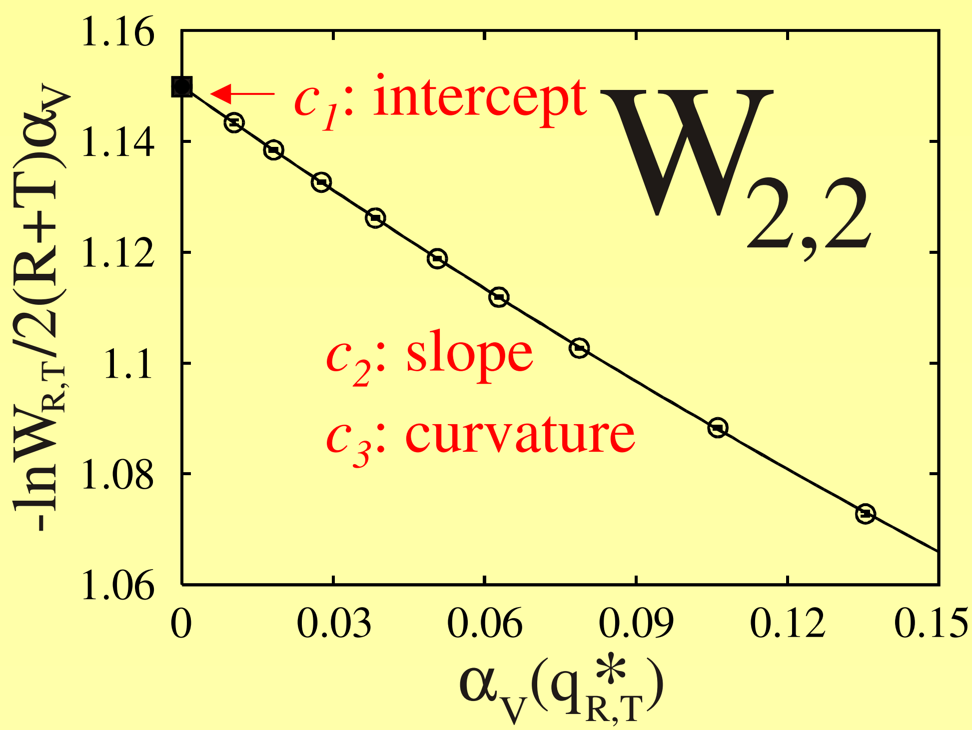
analytic calculation (MILC action): Q. Mason & H. Trottier



taken from Q. Mason's Ph.D Thesis (Cornell Electronic Library)

Alternative: High-β Simulations

- simulate the quantities of interest (Wilson loops) at high-β weak coupling [$\beta = \frac{6}{4\pi\alpha_0}$]
- fit results to the perturbative expansion



- 9 simulations → 9 data pts (α_V,-ln$W_{R,T}$)
- fit to the series

$$\frac{-\ln W_{R,T}}{2(R+T)\alpha_V} = c_1 + c_2\alpha_V + c_3\alpha_V^2 + \ldots$$

Alternative: High-β Simulations

- high $\beta \Rightarrow$ perturbative phase: $(La)^{-1} >> \Lambda_{QCD}$, here: $\beta \sim 9$ to 60 and $a\Lambda \sim 10^{-3}$ to 10^{-30}
- several studies:
  - Dimm, Lepage & Mackenzie [Nucl. Phys. B (Proc. Suppl.), **42**, 403 (1995)]
    1st-order mass renormalization for Wilson fermions
  - Trottier, Shakespeare, Lepage & Mackenzie [PRD, **65**, 094502 (2002)]
    3rd-order Wilson loops for the Wilson plaquette action
  - Hart, Horgan & Storoni [PRD, **70**, 034501 (2004)]
    tadpole factor in pure gauge theories

this study:

3^{rd} –order Wilson loops with dynamical staggered fermions

- unimproved: Wilson action + unimproved staggered quark action
- MILC action: 1-loop improved gauge field action + Asqtad

→ important cross-check of the PT input to the determination of α_s by the HPQCD Collaboration

Challenges: Systematic Errors

- statistical errors in Wilson loops ~ O(10^{-5}-10^{-6})
- probing 3rd-order coefficients

$$\alpha \sim 0.01 \Rightarrow \alpha^3 \sim O(10^{-6})$$

- account for all possible sources of errors
  - tunneling between Z_3 center phases
  - fitting and truncation errors
  - zero momentum modes
  - step size errors in simulation equations
  - autocorrelation
  - inaccuracy in matrix inversions

Tunneling and TBC

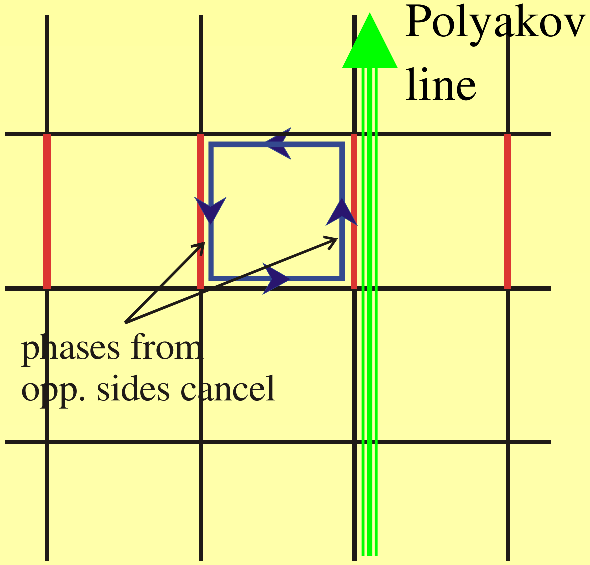
- gauge field action has a Z_3 symmetry

$$U_{x,\hat{\mu}} \to z U_{x,\hat{\mu}} \quad \text{where} \quad z \in \left\{ I, e^{i2\pi/3} I, e^{i4\pi/3} I \right\}$$

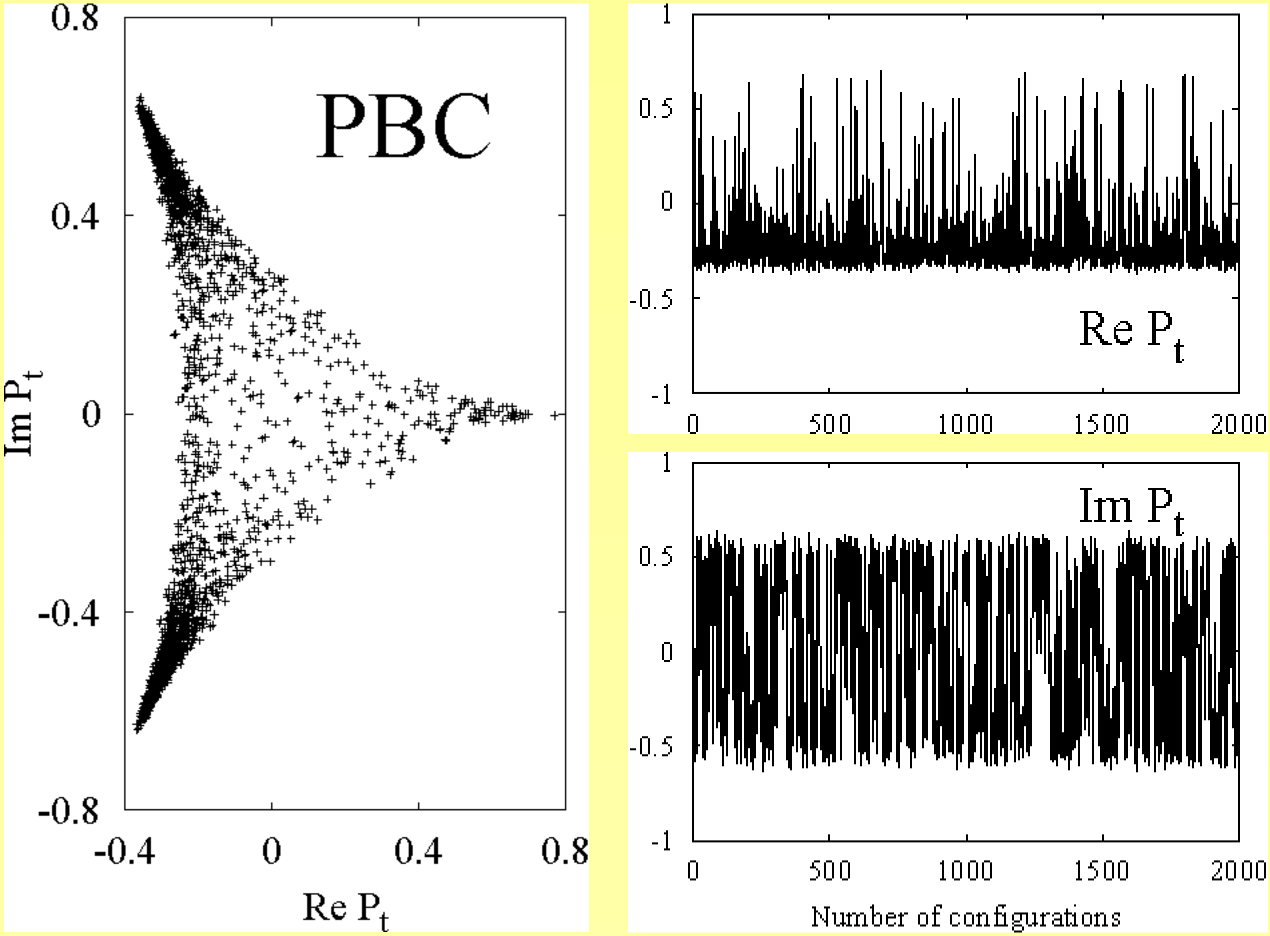
- multiple vacuum, analytic PT around one vacuum
- tunneling can occur in simulations $\to$ non-perturbative effects
- tunneling detection: Polyakov loop in one direction
- tunneling $\to$ pick up a Z_3 phase
- suppressed using TBC

$$U_{x+L\hat{\nu},\hat{\mu}} = \Omega_\nu U_{x,\hat{\mu}} \Omega_\nu^+$$

$$\psi_{x+L\hat{\nu}} = e^{i\pi/3} \Omega_\nu \psi_x \Omega_\nu^+$$

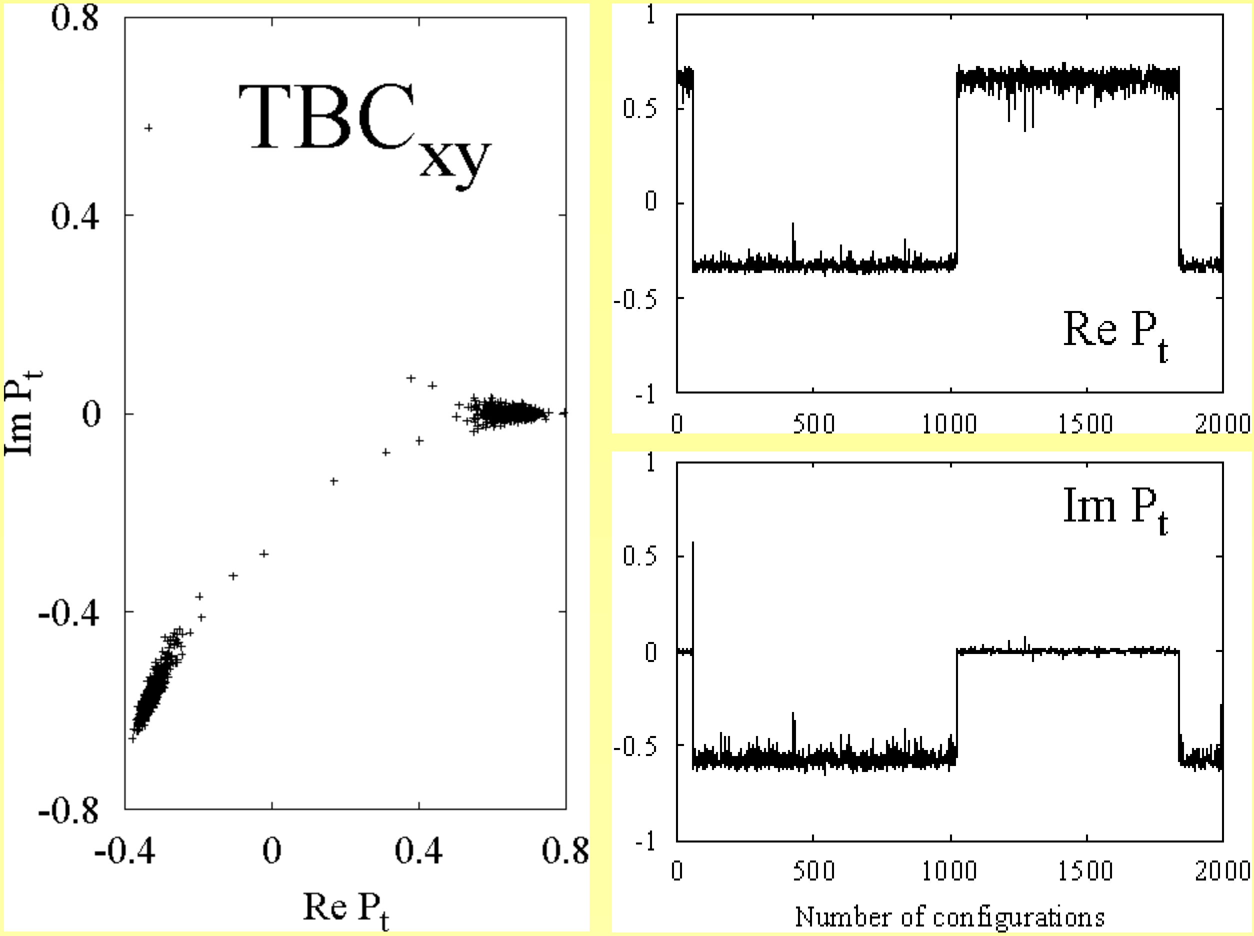


Periodic Boundary Conditions



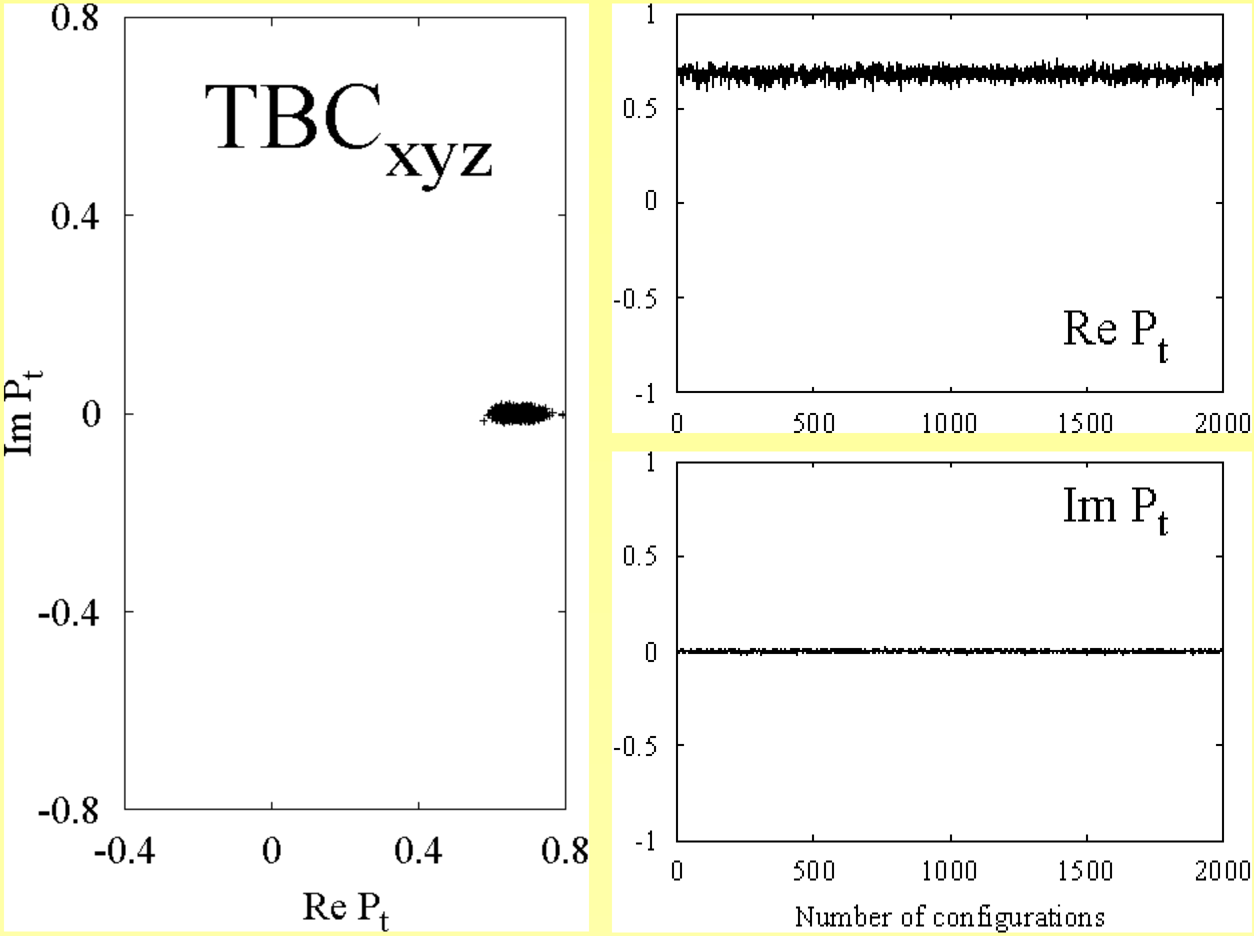
unquenched: the unimproved action, $n_f = 3$, $\beta = 16$, $V = 4^4$

Twisted Boundary Conditions (xy)



unquenched: the unimproved action, $n_f = 1$, $\beta = 16$, $V = 4^4$

Twisted Boundary Conditions (xyz)



unquenched: the unimproved action, $n_f = 1$, $\beta = 16$, $V = 4^4$

Simulation Parameters – MILC Action

- 9 values of β, $n_f = 1$, $V = 8^4$, $am = 0.1$

β	9.5	11.0	13.5	16.0	19.0	24.0	32.0	47.0	80.0
$\alpha_0 = \frac{6}{4\pi\beta}$	0.113	0.092	0.071	0.058	0.047	0.036	0.027	0.018	0.010
$\alpha_V(q^*_{1,1})$	0.127	0.101	0.076	0.061	0.050	0.038	0.027	0.018	0.010
$a\Lambda$	6×10^{-2}	2×10^{-2}	3×10^{-3}	5×10^{-4}	6×10^{-5}	1×10^{-6}	3×10^{-9}	4×10^{-14}	6×10^{-25}

- expansion parameter: the "V-scheme" $V(q) = -\frac{4}{3}\frac{4\pi\alpha_V(q)}{q^2}$
- input from analytic PT: 3rd-order expansion of the plaquette

  $\rightarrow$ connection between α_0 and α_V

Fitting – MILC Action

- fit data points with $-\frac{\ln W_{R,T}}{2(R+T)\alpha_V} = c_1 + c_2\alpha_V + c_3\alpha_V^2 + \ldots$
- constrained curve fitting $\chi^2 \to \chi^2_{\arg} \equiv \chi^2 + \sum_{n=1}^{n=N} \frac{(c_n - \bar{c}_n)^2}{\bar{\sigma}_n^2}$
- data tell us how much it can determine

vary N ($\bar{\sigma}_n = 1.0$)

c_n	$N = 4$	$N = 5$	$N = 6$
c_1	1.1499(4)	1.1499(4)	1.1499(4)
c_2	-0.641(15)	-0.641(15)	-0.641(15)
c_3	0.58(20)	0.58(20)	0.58(20)
c_4	-0.2(9)	-0.2(9)	-0.2(9)
c_5		-0.1(10)	-0.1(10)
c_6			0.0(10)
χ^2	0.53	0.53	0.53

vary $\bar{\sigma}_n$ (N = 6)

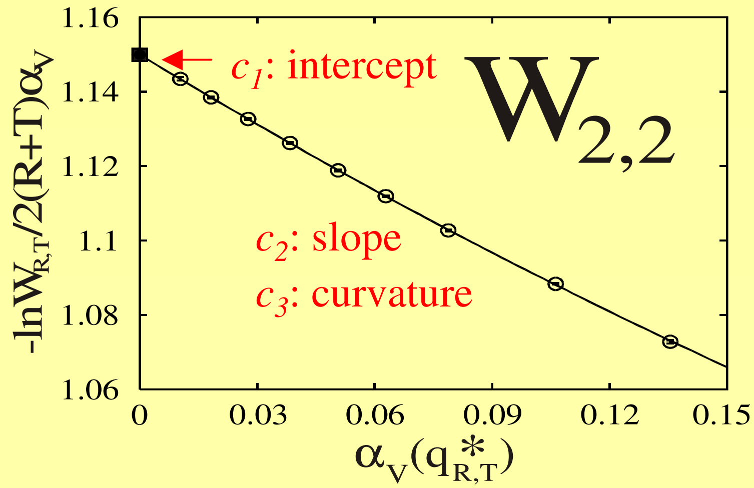
c_n	$\bar{\sigma}_n = 0.5$	$\bar{\sigma}_n = 1.0$	$\bar{\sigma}_n = 5.0$
c_1	1.1498(3)	1.1499(4)	1.1502(5)
c_2	-0.636(12)	-0.641(15)	-0.658(27)
c_3	0.51(12)	0.58(20)	0.85(43)
c_4	0.0(5)	-0.2(9)	-1.4(23)
c_5	0.0(5)	-0.1(10)	-0.8(48)
c_6	0.0(5)	0.0(10)	-0.2(50)
χ^2	1.2	0.53	0.25

2x2 loop, analytic PT: c_1 = 1.1499(0), c_2 = -0.643(2), c_3 = 0.59(9)

Results – MILC Action

- $n_f = 1$, $V = 8^4$, $am = 0.1$

	Perturbation Theory			Monte Carlo Simulations		
Loop	c_1	c_2	c_3	c_1	c_2	c_3
1x2	0.9252(0)	-0.646(0)	0.23(5)	0.9251(3)	-0.644(13)	0.20(18)
1x3	0.9845(9)	-0.595(1)	0.38(6)	0.9845(3)	-0.599(14)	0.37(19)
2x2	1.1499(0)	-0.643(2)	0.59(9)	1.1499(4)	-0.641(15)	0.58(20)
2x3	1.2341(0)	-0.595(3)	0.85(16)	1.2342(4)	-0.599(19)	0.88(26)



Results – MILC Action

- errors on c_2 and c_3 can be reduced if c_1 is fixed to the analytic value in fitting

	PT		MC	
Loop	c_2	c_3	c_2	c_3
1x2	-0.646(0)	0.23(5)	-0.649(5)	0.26(12)
1x3	-0.595(1)	0.38(6)	-0.600(6)	0.39(13)
2x2	-0.643(2)	0.59(9)	-0.642(7)	0.59(14)
2x3	-0.595(3)	0.85(16)	-0.594(7)	0.82(15)

- fix both c_1 and c_2 to the analytic values

	PT	MC
Loop	c_3	c_3
1x2	0.23(5)	0.21(4)
1x3	0.38(6)	0.28(5)
2x2	0.59(9)	0.61(6)
2x3	0.85(16)	0.84(8)

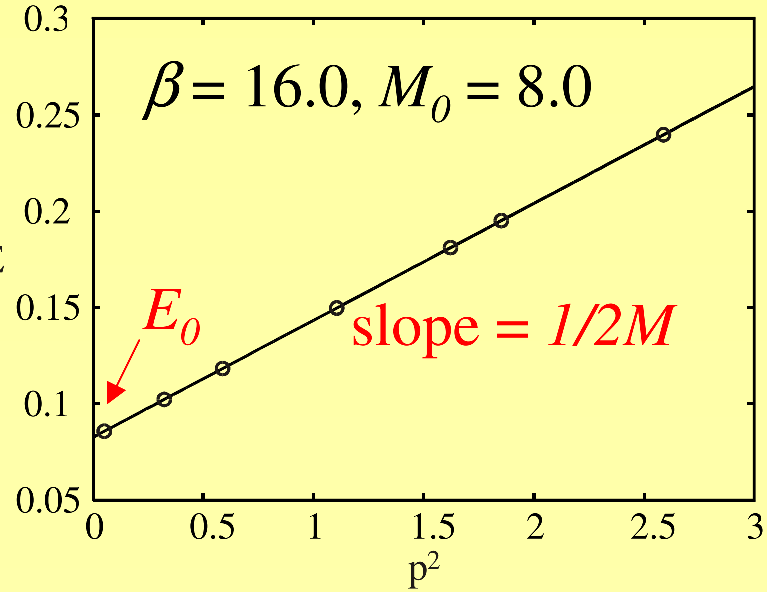
Energy Shift and Mass Renormalization in NRQCD (Preliminary)

- quenched: Wilson plaquette action, NRQCD action $H = -\frac{\Delta^2}{2M_0}$
- observable: dispersion relation $E = E_0 + \frac{p^2}{2M}$

$$E_0 = c_1 \alpha_V(q_E^*) + c_2 \alpha_V^2(q_E^*) + \ldots$$

$$M = M_0\left(1 + c_1 \alpha_V(q_M^*) + c_2 \alpha_V^2(q_M^*) + \ldots\right)$$

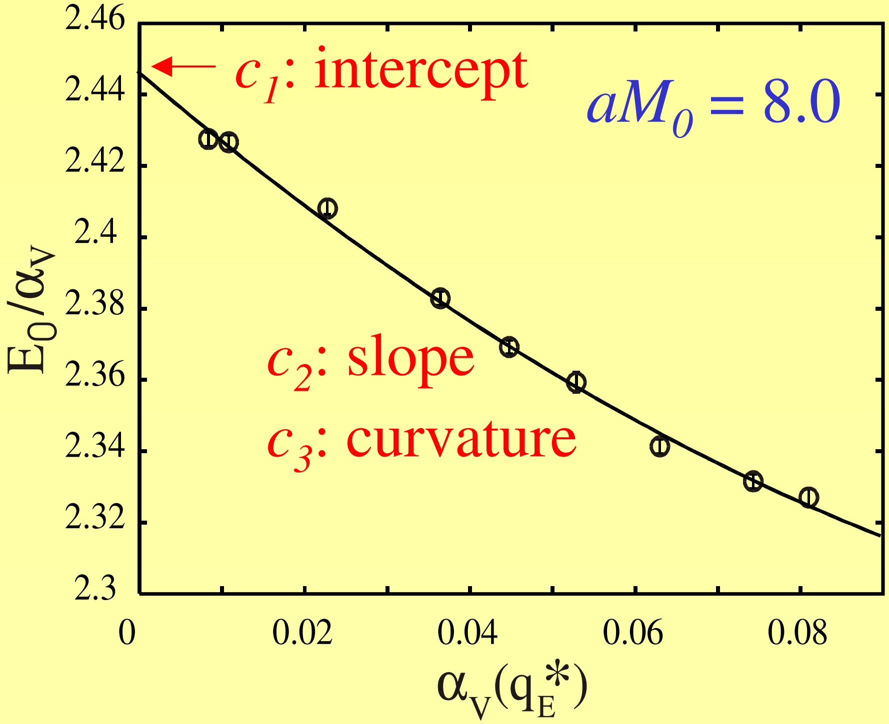
- for each β:



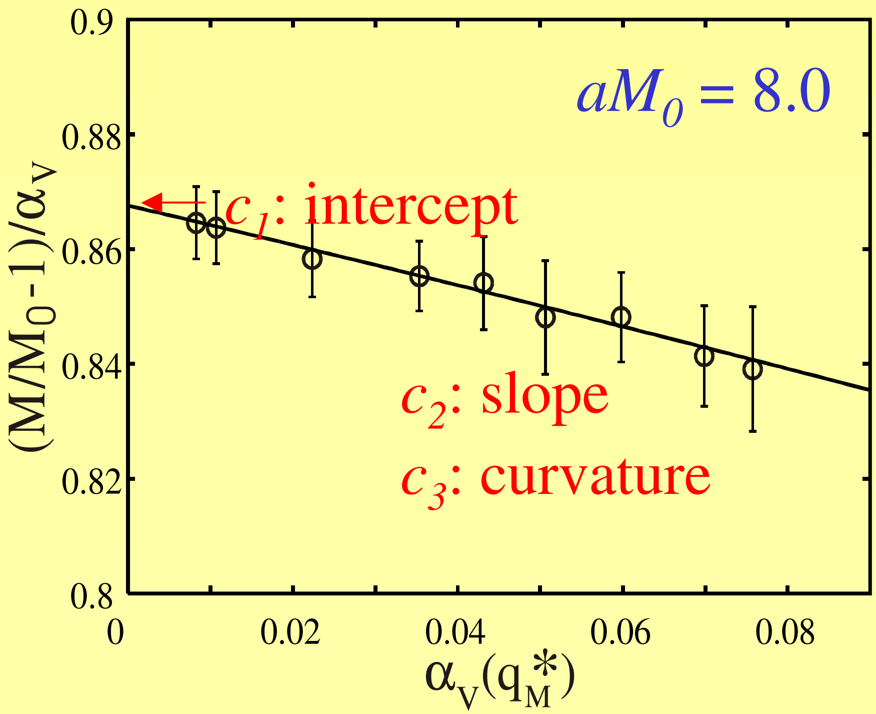
Results (Preliminary)

- simulations
  - 9 values of β: 9.0 – 60.0
  - 5 bare mass aM_0: 8.0, 6.0, 5.0, 4.0, 1.8

energy shift



mass renormalization



Results for c_1 (Preliminary)

- analytic PT done by C. Morningstar [PRD, **50**:5902 (1994)]

	c_1 for E_0		c_1 for M	
M_0a	*PT*	*MC*	*PT*	*MC*
8.0	2.450	2.447(2)	0.850	0.867(6)
6.0	2.570	2.565(5)	0.830	0.849(7)
5.0	2.655	2.648(5)	0.820	0.842(9)
4.0	2.782	2.773(6)	0.770	0.798(10)
1.8	3.511	3.569(7)	1.100	1.245(10)

Predictions for c_2 & c_3 (Preliminary)

	energy shift E_0		mass renormalization M	
M_0a	c_2	c_3	c_2	c_3
8.0	-1.98(8)	5.9(1.4)	-0.34(37)	-0.2(4.9)
6.0	-1.91(7)	6.3(1.4)	-0.33(42)	0.5(5.3)
5.0	-2.02(7)	7.3(1.4)	-0.53(45)	0.7(6.1)
4.0	-2.13(7)	7.4(1.4)	-0.91(56)	-0.28(6.7)
1.8	-2.94(8)	7.6(1.5)	-0.87(39)	2.7(4.6)

good test run

$\rightarrow$ future: full NRQCD & mNRQCD

Conclusions

- perturbative coefficients of WL in full QCD (MILC)
- important cross-check of the PT input to the determination of α_s by the HPQCD Collaboration
- can also be applied to NRQCD

higher-order coefficients & complicated actions

future: analytic PT (low order) + high-β (high order)