Kaon and pion form factors in two-flavor QCD

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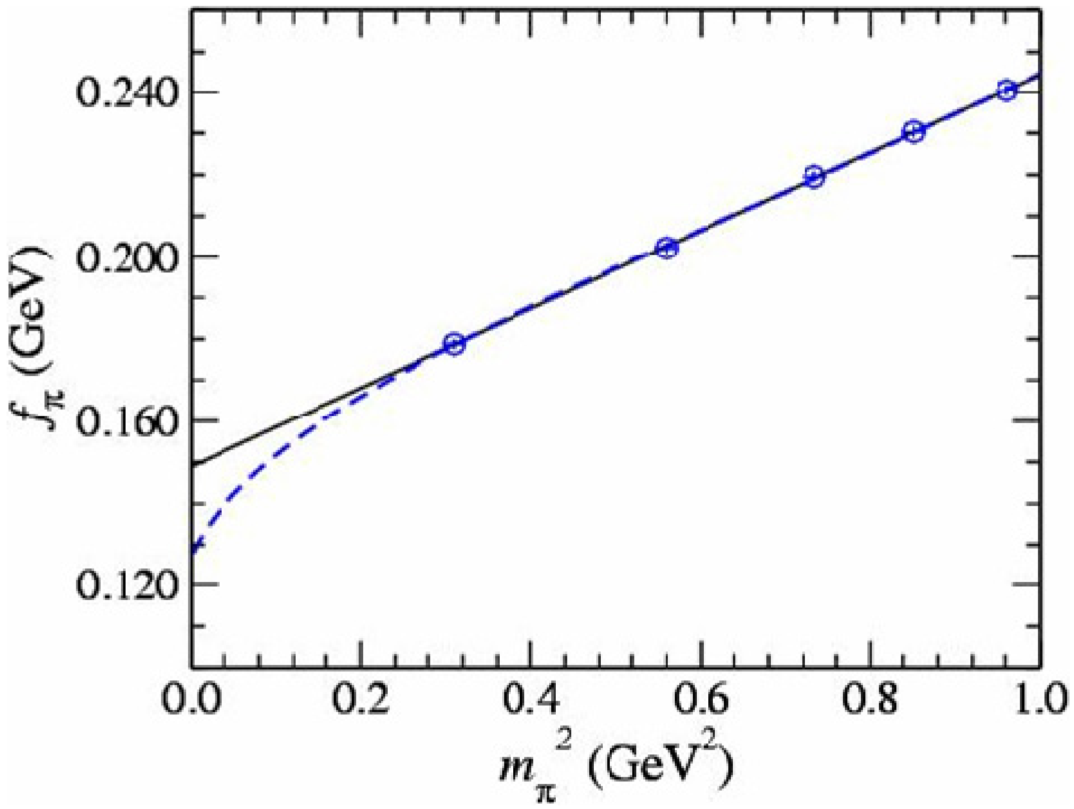
Pion and kaon form factors

- Phenomenological importance
  - Kaon form factor $f_+(0)$ is a theoretical input for the determination of $|V_{us}|$ via K_{l3} decay, reliable estimate from the first principle of QCD is highly desirable
- Proving ground for precise lattice calculation
  - form factor is the second simplest quantity (the first is decay constant), necessary step to much more complicated quantities (i.e. $K \to \pi\pi$, …)
  - precise experimental data available
  - theoretically well-understood using ChPT
  - comparison between lattice data and experimental value / ChPT prediction

JLQCD Nf=2 configurations

- Non-perturbatively *O(a)*-improved Wilson fermion + plaquette gauge
- 20^3x48, β=5.2, a^{-1}~0.1 fm
- 5 quark masses correspond to π/ρ=0.6-0.8 (m_π=550-1000 MeV)
- 1,200 configs separated by 10 HMC trajectories
- jackknife error estimate with bin size 10



• light hadron spectrum, decay constant, quark masses
• B meson decay constant and B parameters

Kaon form factors: method and results

Kaon form factor: definitions

K_{l3} decay form factors

$$\langle \pi(p')|V_\mu|K(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $f_+(0)$: a theoretical input for the determination of $|V_{us}|$, a few percent accuracy is needed
- f_- contribution is proportional to m_l^2

Scalar form factor

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$$

Another convention

$$\xi(q^2) = f_-(q^2)/f_+(q^2)$$

Previous estimates of $f_+(0)$

ChPT analysis by Leutwyler-Roos (1984)

$$f_+(0) = 1 + f_2 + f_4 = 1 - 0.023 - 0.016(8) = 0.961(8)$$

- standard value in the phenomenological analysis
- leading correction is determined unambiguously
- next-to-leading order correction is estimated from a model of the wave function of the pseudoscalar meson

first quenched result by Becirevic et al. (2005)

$$f_+(0) = 1 + f_2 + f_4^q = 1 - 0.023 + \left(-0.017 \pm 0.005_{\mathrm{stat}} \pm 0.007_{\mathrm{syst}}\right) = 0.960(9)$$

- non-perturbatively O(a)-improved Wilson and plaquette gauge, 24^3x56, β=6.2
- double ratio method is used

several groups have been carrying out the unquenched study

Lattice calculation: method

three step calculation

$$f_+(0) = f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\frac{m_K - m_\pi}{m_K + m_\pi}\right] \times \frac{f_+(0)\left[1+\xi(0)\frac{m_K - m_\pi}{m_K + m_\pi}\right]}{f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\frac{m_K - m_\pi}{m_K + m_\pi}\right]} \times \frac{1}{\left[1+\xi(0)\frac{m_K - E_\pi}{m_K + E_\pi}\right]}$$

1. determine $f_0(q_{max}{}^2)$
2. interpolation to $q^2=0$
3. subtract unnecessary contribution from $\xi(0)$

double ratio of correlation functions is used in each step

→ renormalization factors and bulk of statistical errors cancel

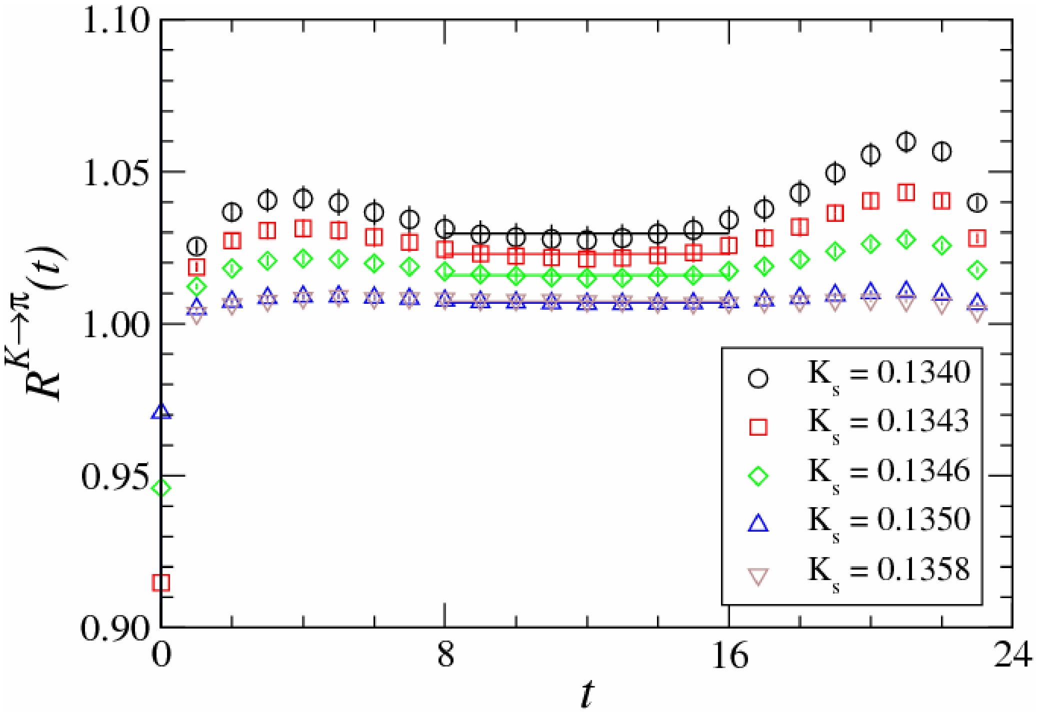
double ratio I: determine $f_0(q_{max}^2)$

$$R_1(t) = \frac{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})\,C_{KV_4\pi}(t,T/2;\vec{0},\vec{0})}{C_{\pi V_4 \pi}(t,T/2;\vec{0},\vec{0})\,C_{KV_4 K}(t,T/2;\vec{0},\vec{0})} \rightarrow \frac{\langle \pi(0)|V_4|K(0)\rangle\langle K(0)|V_4|\pi(0)\rangle}{\langle \pi(0)|V_4|\pi(0)\rangle\langle K(0)|V_4|K(0)\rangle} = \left[\frac{m_K + m_\pi}{2\sqrt{m_K m_\pi}} f_0(q_{\max}^2)\right]^2$$

three-point function

$$C_{KV_\mu \pi}(t_x, t_y; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle \pi(t_y,\vec{y}) V_\mu(t_x,\vec{x}) K(0)\rangle\, e^{+i\vec{q}\cdot\vec{x}} e^{-\vec{p}\cdot\vec{y}}$$

- Double ratio used before for semileptonic *B* decay by the Fermilab group
- measures SU(3) breaking at $q_{\max}^2 = (m_K - m_\pi)^2$
- larger deviation from 1 for larger mass differences

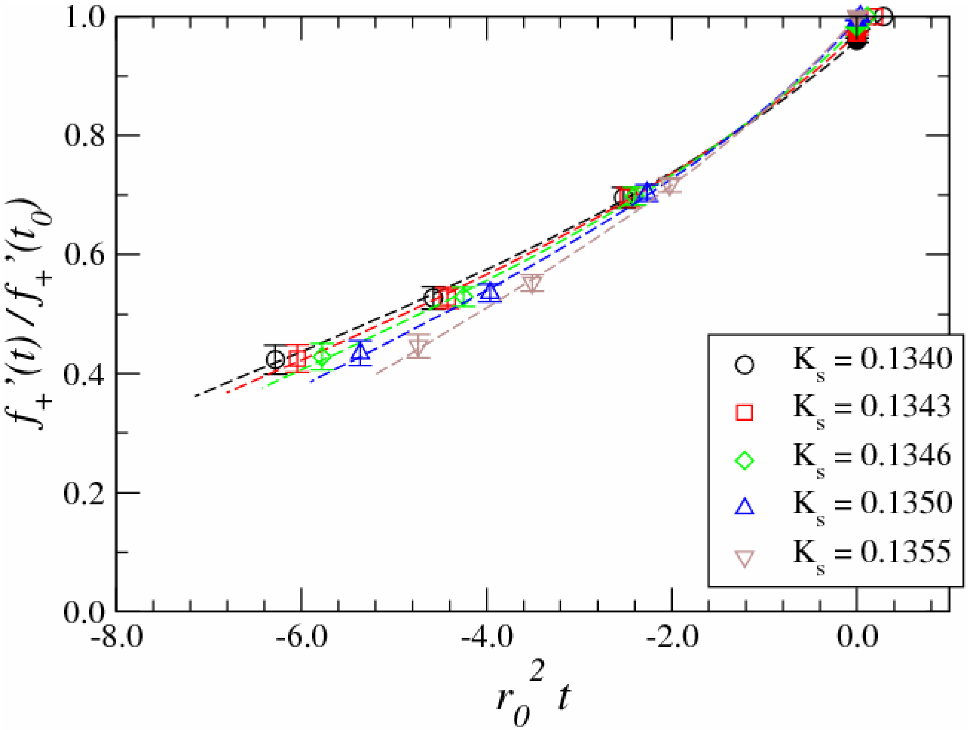


K_{sea}=0.1355 (lightest)

double ratio II: interpolate to q^2=0

$$R_2(t;\vec{p}) = \frac{\dfrac{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{0},\vec{0})}}{\dfrac{C_{\pi\pi}(t;\vec{p})}{C_{\pi\pi}(t;\vec{0})}} \to \frac{\dfrac{\langle \pi(p)|V_4|K(0)\rangle}{\langle \pi(0)|V_4|K(0)\rangle}}{\dfrac{\langle \pi(p)|P|0\rangle}{\langle \pi(0)|P|0\rangle}} = \frac{m_K+E_\pi}{m_K+m_\pi}\,\frac{f_+(q^2)\left[1+\xi(q^2)\dfrac{m_K-E_\pi}{m_K+E_\pi}\right]}{f_+(q^2_{\max})\left[1+\xi(q^2_{\max})\dfrac{m_K-m_\pi}{m_K+m_\pi}\right]}$$

- 2pt. function in the denominator is to cancel the energy mismatch
- exactly 1 in zero-recoil case
- interpolation with a quadratic function

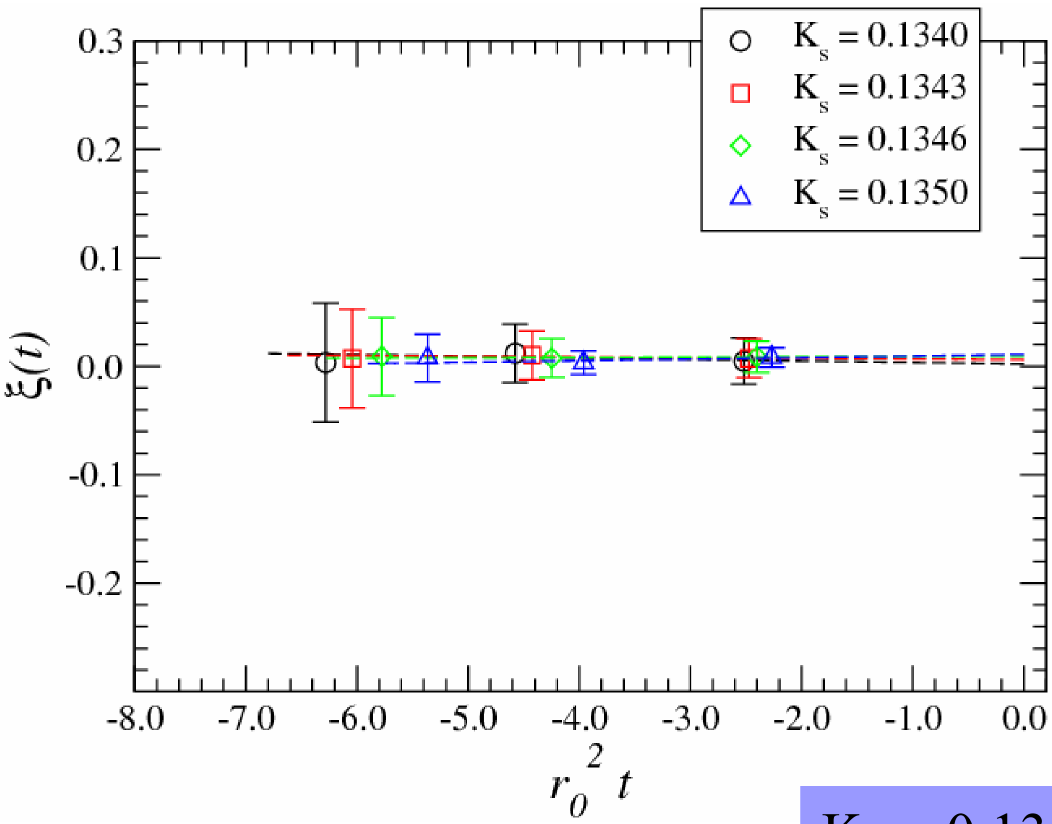


$$t = q^2 = (m_K - E_\pi)^2 - |\vec{p}|^2, \quad \left(|\vec{p}| = \frac{2\pi}{20}, \frac{2\pi}{20}\sqrt{2}, \frac{2\pi}{20}\sqrt{3}\right)$$

K_{sea}=0.1355 (lightest)

double ratio III: subtract ξ(0) contribution

$$R_3(t;\vec{p}) = \frac{\dfrac{C_{\pi V_i K}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 K}(t,T/2;\vec{p},\vec{p})}}{\dfrac{C_{\pi V_i \pi}(t,T/2;\vec{p},\vec{p})}{C_{\pi V_4 \pi}(t,T/2;\vec{p},\vec{p})}} \to \frac{\dfrac{\langle \pi(p)|V_k|K(0)\rangle}{\langle \pi(p)|V_4|K(0)\rangle}}{\dfrac{\langle \pi(p)|V_k|\pi(0)\rangle}{\langle \pi(p)|V_4|\pi(0)\rangle}} = \frac{1-\xi(q^2)}{\dfrac{m_K+E_K}{m_\pi+E_\pi}+\xi(q^2)\dfrac{m_K-E_K}{m_\pi+E_\pi}}$$



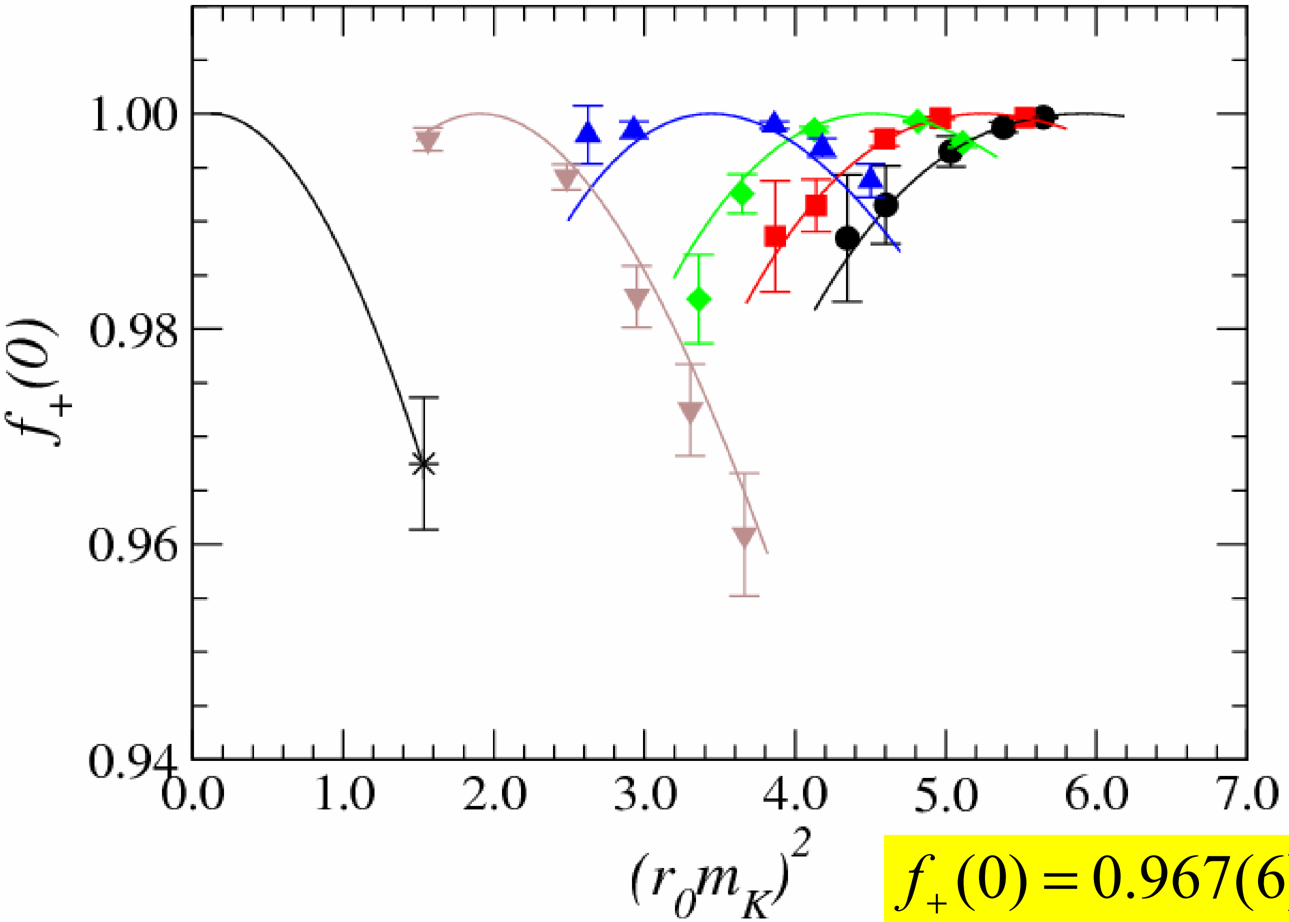
K_{sea}=0.1355 (lightest)

- q^2 dependence is very small and seems to be independent of the strange quark mass
- extrapolation to q^2=0 is done by assuming linear dependence

chiral extrapolation (quad.)

fit the data with simple quadratic function

$$f_+(0) = 1 - \left(c_0 + c_1\left[(r_0 m_K)^2 + (r_0 m_\pi)^2\right]\right)\left[(r_0 m_K)^2 - (r_0 m_\pi)^2\right]^2$$



$f_+(0) = 0.967(6)$ (preliminary)

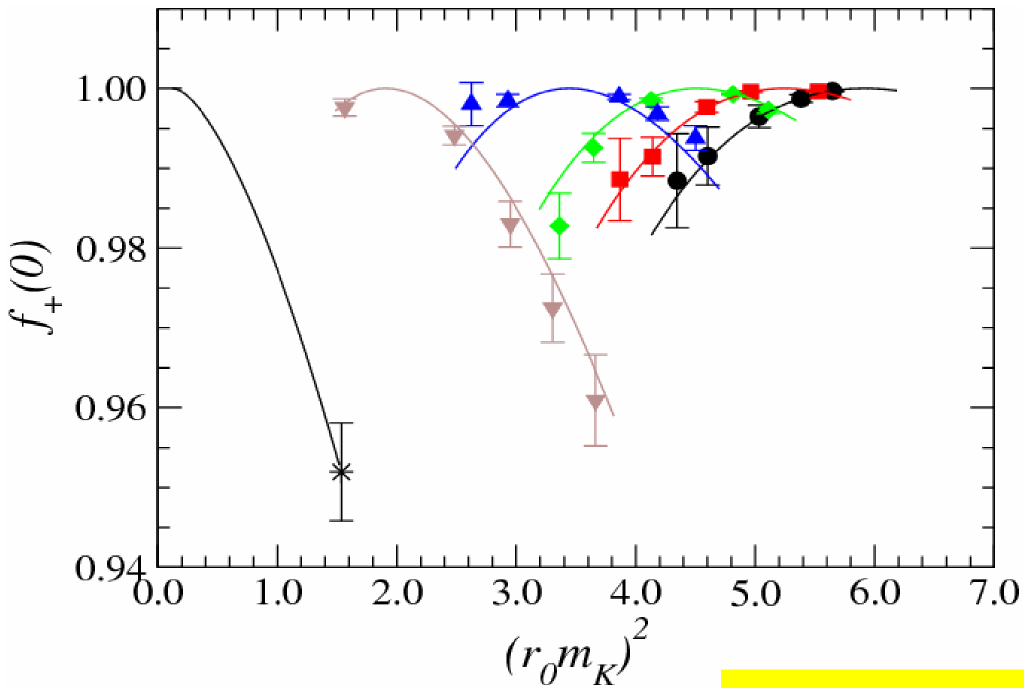
chiral extrapolation (ChPT)

fit with one-loop ChPT plus a quadratic function

$$f_+(0) = 1 + \frac{3}{2} H_{K\pi}(0) + \frac{3}{2} H_{K\eta}(0) - \left(c_0 + c_1 \left[(r_0 m_K)^2 + (r_0 m_\pi)^2\right]\right)\left[(r_0 m_K)^2 - (r_0 m_\pi)^2\right]^2$$

$$H_{PQ}(0) = -\frac{1}{128\pi^2 f^2}\left[m_P^2 + m_Q^2 + \frac{2 m_P^2 m_Q^2}{m_P^2 - m_Q^2} \ln \frac{m_Q^2}{m_P^2}\right]$$

: unambiguously determined

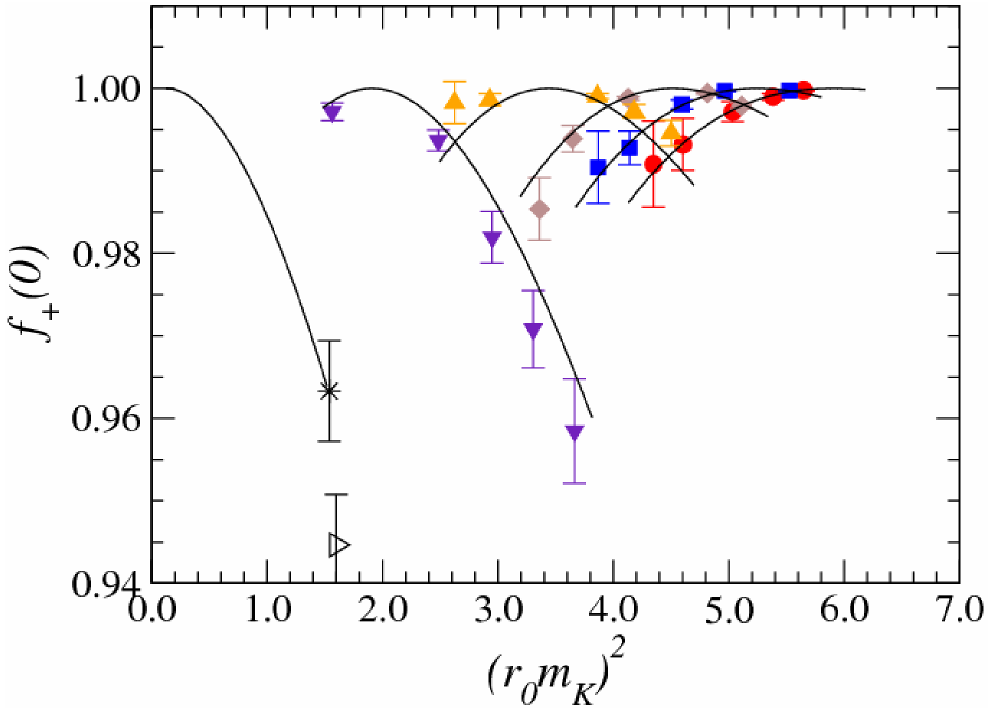


$$f_+(0) = 0.952(6) \text{ (preliminary)}$$

chiral extrapolation (pqChPT)

partially quenched ChPT formula by Becirevic et al. (2005)

$$f_2^{pq} = -\frac{2m_K^2 + m_\pi^2}{32\pi^2 f^2} - \frac{3m_K^2 m_\pi^2 \ln\frac{m_\pi^2}{m_K^2}}{64\pi^2 f^2 (m_K^2 - m_\pi^2)} + \frac{m_K^2 (4m_K^2 - m_\pi^2) \ln\left(2 - \frac{m_\pi^2}{m_K^2}\right)}{64\pi^2 f^2 (m_K^2 - m_\pi^2)}$$



$$f_+(0) = f_+^{pq}(0) - f_2^{pq} + f_2 = 0.945(6) \text{ (preliminary)}$$

chiral extrapolation (summary)

- the chiral logarithm is significant only in the region where $m_\pi^2 << m_K^2$, while the data region $½ < m_\pi^2/m_K^2 < 2$ is well described by the quadratic form

Leutwyler-Roos (1984)	0.961(8)
Becirevic et al. (2005)	0.960(9)
This work (quad.)	0.967(6)
This work (ChPT + quad.)	0.952(6)
This work (qpChPT + quad.)	0.945(6)

Vector charge radius

- charge radius is a slope of the form factor near $q^2=0$

$$f_+^{K\pi}(q^2) = 1 + \frac{1}{6}\left\langle r^2 \right\rangle_V^{K\pi} q^2 + \cdots$$

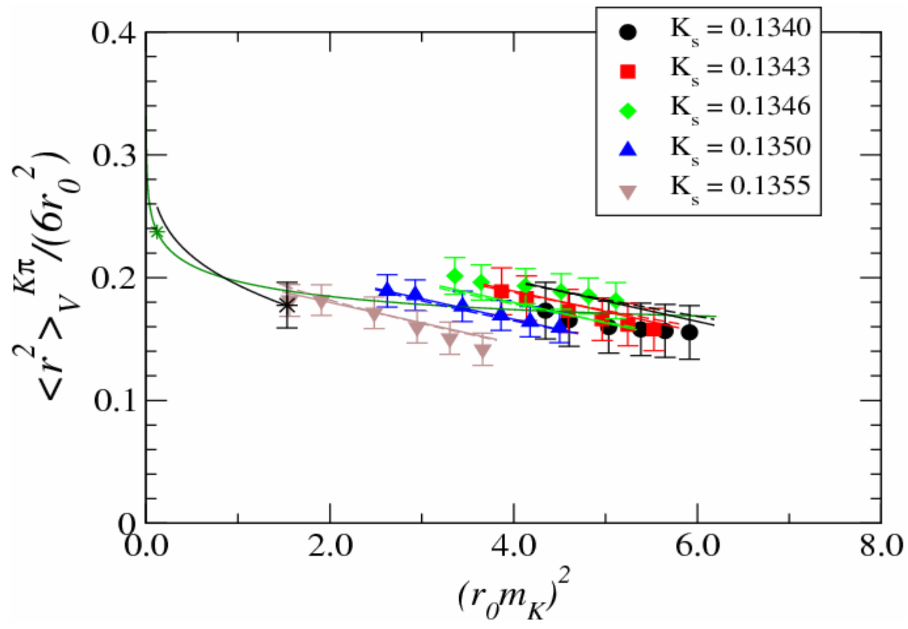
- one-loop ChPT predicts

$$\left\langle r^2 \right\rangle_V^{K\pi} = \left\langle r^2 \right\rangle_V^{\pi} - \frac{1}{64\pi^2 f^2}\left[3h_1\left(\frac{m_\pi^2}{m_K^2}\right) + 3h_1\left(\frac{m_\eta^2}{m_K^2}\right) + \frac{5}{2}\ln\frac{m_K^2}{m_\pi^2} + \frac{5}{2}\ln\frac{m_\eta^2}{m_K^2} - 6\right]$$

$$\left\langle r^2 \right\rangle_V^{\pi} = \frac{12L_9}{f^2} - \frac{1}{32\pi^2 f^2}\left[2\ln\frac{m_\pi^2}{\mu^2} + \ln\frac{m_K^2}{\mu^2} + 3\right]$$

- we fit the data with ChPT + linear

$$\left\langle r^2 \right\rangle_V^{K\pi} = 0.26(3)\ \text{fm}^2\ \text{(preliminary)}$$



cf) the exp. value

$$\left\langle r^2 \right\rangle_V^{K\pi} = 0.331(8)\ \text{fm}^2$$

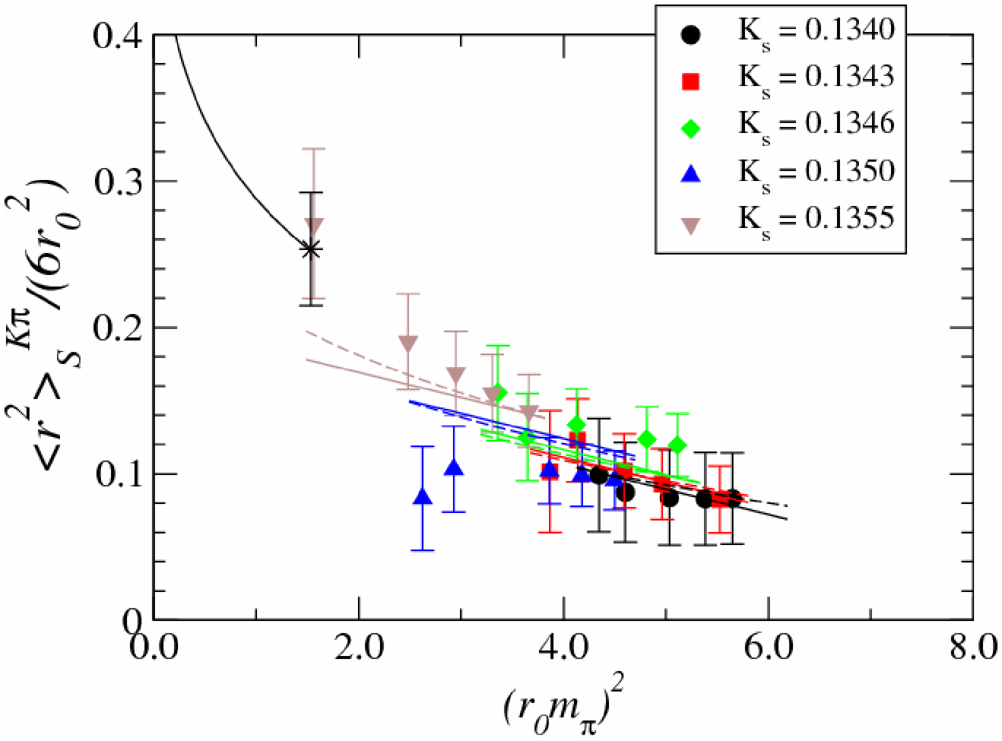
Scalar charge radius

- The same analysis can be applied to the scalar charge radius

$\left\langle r^2 \right\rangle_S^{K\pi} = 0.37(6)\ \mathrm{fm}^2$ (preliminary)

- overshoots the exp. value (PDG2004)

$\left\langle r^2 \right\rangle_S^{K\pi} = 0.21(3)\ \mathrm{fm}^2$



Pion form factors and charge radii

Pion form factor

- Pion electromagnetic form factor

$$\left\langle \pi(p') \middle| J^{\mu}_{em} \middle| \pi(p) \right\rangle = G_{\pi}(q^2)(p+p')^{\mu}$$

  - normalized as $G_{\pi}(0)=1$ by charge conservation
- scalar form factor

$$\left\langle \pi(p') \middle| \bar{q}q \middle| \pi(p) \right\rangle = G_S(q^2)$$

- these are calculated from double ratio II, which is used to study q^2 dependence

$$R_2(t;\vec{p}) = \frac{\dfrac{C_{\pi V_4 \pi}(t, T/2; \vec{p}, \vec{p})}{C_{\pi V_4 \pi}(t, T/2; \vec{0}, \vec{0})}}{\dfrac{C_{\pi\pi}(t;\vec{p})}{C_{\pi\pi}(t;\vec{0})}} \rightarrow \frac{m_{\pi} + E_{\pi}}{2m_{\pi}} \frac{G_{\pi}(q^2)}{G_{\pi}(0)}$$

Vector charge radius

- charge radius is a slope of the form factor near $q^2=0$

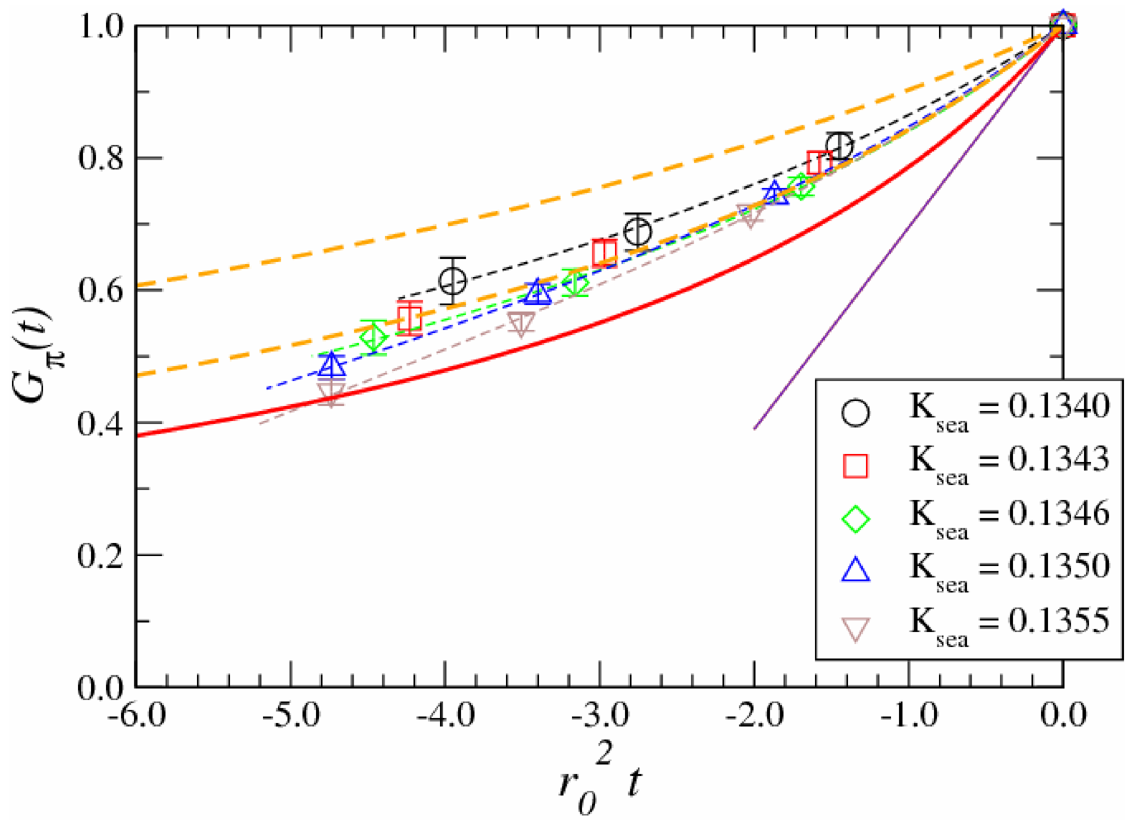
$$G_\pi(q^2) = 1 + \frac{1}{6}\left\langle r^2 \right\rangle_V^\pi q^2 + \cdots$$

- exp. data are well described by VMD
- two different fit forms
  - free pole

$$G_\pi(q^2) = \frac{1}{1 - c_0(r_0^2 q^2)} + c_1(r_0^2 q^2)^2$$

  - measured pole

$$G_\pi(q^2) = \frac{1}{1 - (r_0^2 q^2)/(r_0^2 m_V^2)} + d_0(r_0^2 q^2)$$



Both can describe the data well

Vector charge radius: chiral extrapolation

- one-loop ChPT predicts the chiral logarithm

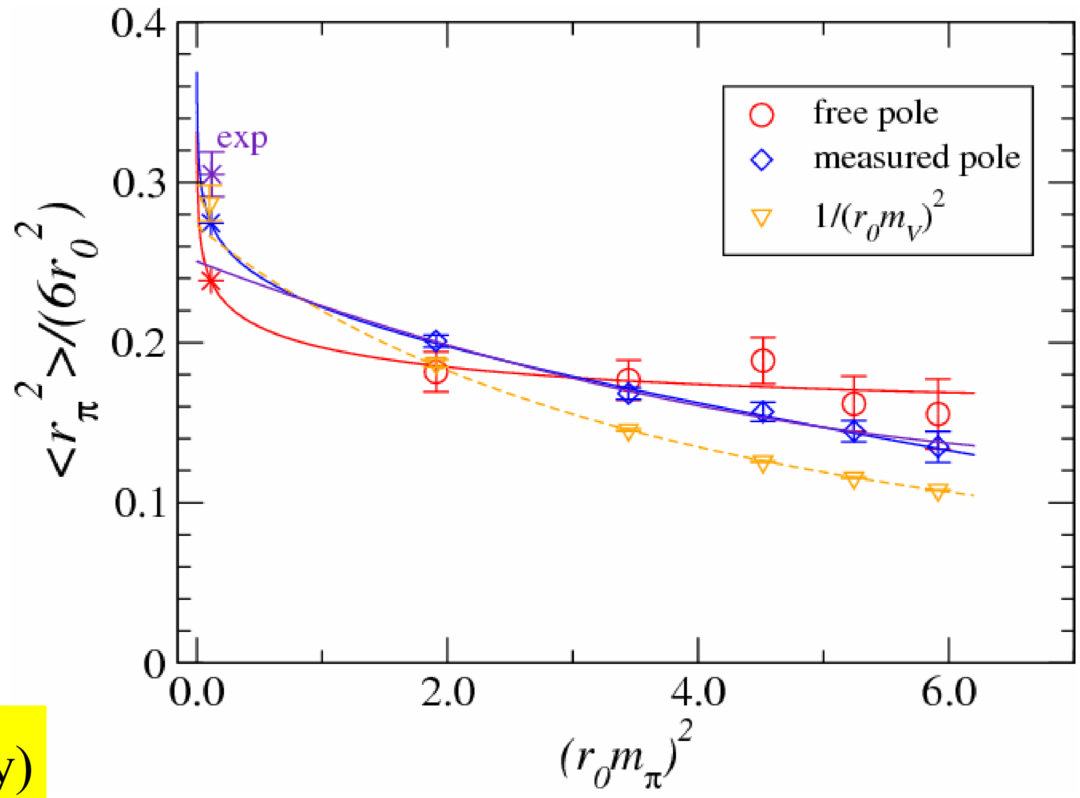
$$\left\langle r^2 \right\rangle_V^\pi = \frac{12 L_9^r}{f^2} - \frac{1}{(4\pi f)^2}\left[\ln\frac{m_\pi^2}{\Lambda} + \frac{3}{2}\right]$$

- fit the data using ChPT + quadratic

$$\left\langle r^2 \right\rangle_V^\pi = 0.396(10)\ \mathrm{fm}^2\ (\text{prelimina ry})$$

- significantly smaller than the experimental value

$$\left\langle r^2 \right\rangle_V^\pi = 0.452(11)\ \mathrm{fm}^2\ (\text{PDG2004})$$



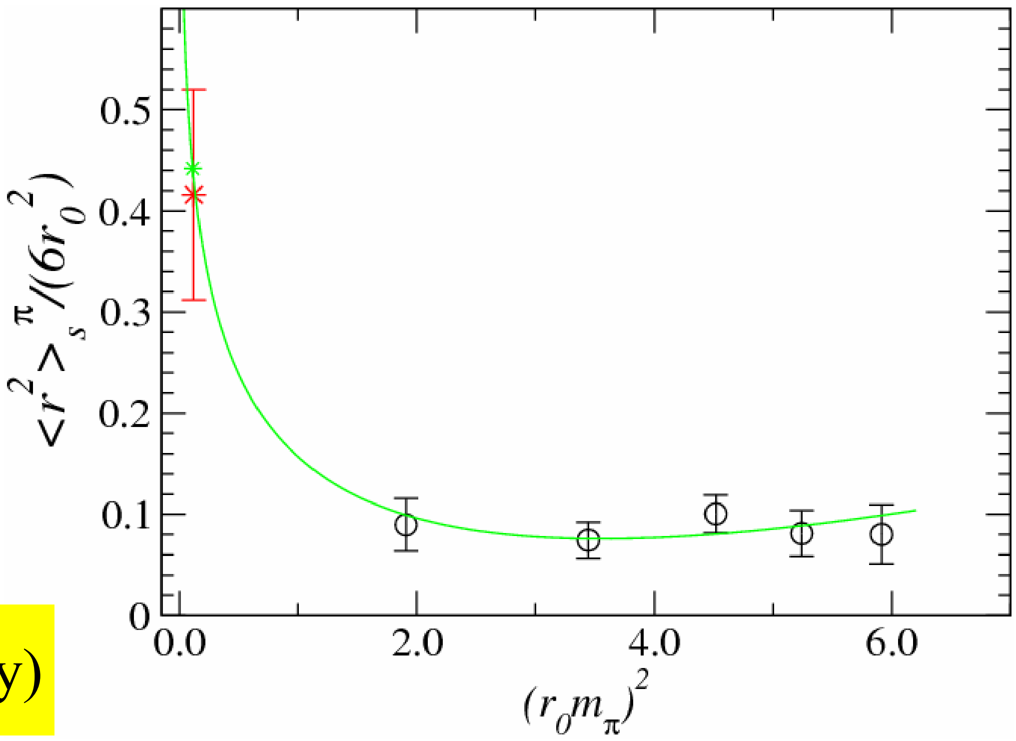
Scalar charge radius: chiral extrapolation

- for the scalar charge radius, chiral logarithm is rather stronger

$$\left\langle r^2 \right\rangle_S^\pi = \frac{12L_9^r}{f^2} - \frac{15}{2}\frac{1}{(4\pi f)^2}\left[\ln\frac{m_\pi^2}{\Lambda} + \frac{3}{2}\right]$$

- fit the data using ChPT + quadratic

$$\left\langle r^2 \right\rangle_S^\pi = 0.60(15)\ \mathrm{fm}^2\ \text{(prelimina ry)}$$



- consistent with the experimental value $\left\langle r^2 \right\rangle_S^\pi = 0.61(4)\ \mathrm{fm}^2$ (PDG2004)
- However, sensitive to the details of the fit function

Summary

- a two-flavor QCD calculation of kaon and pion form factors is presented
- form factors can be calculated with good precision using double ratio method
- systematic errors should be under control in the future study
- to make contact ChPT, much lighter sea quarks will be necessary