Twisting Form Factors out of Chiral Perturbation Theory

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Outline

- (Partially) Twisted Boundary conditions
- Finite volume effects in χPT
- Single particle matrix elements

BT PLB617
T. Mehen & BT PRD72
work in progress

Note: Multiparticle

NN Bedaque, Bedaque & Chen

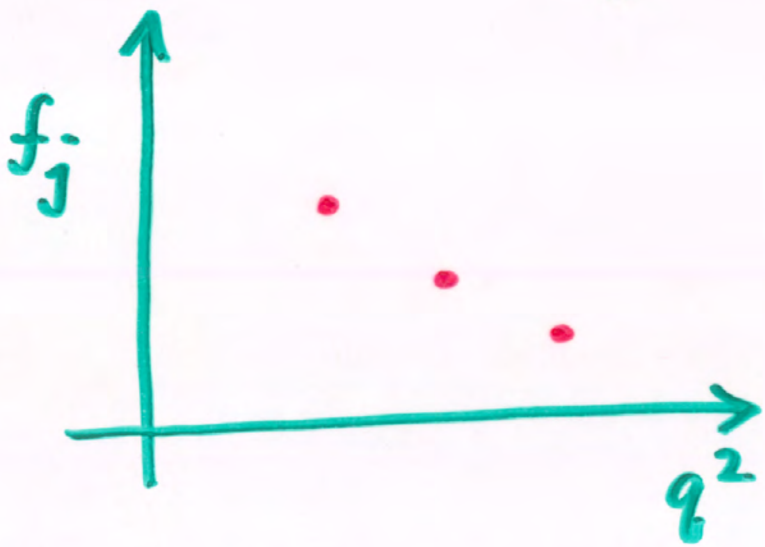
$K \to \pi\pi$ Sachrajda & Villadoro

Periodic Boundary Conditions:

$$\phi(x_i + L) = \phi(x_i)$$

- lattice momenta $p_i = \frac{2\pi}{L} n_i$
- $L = 24a$, $a^{-1} \sim 2\ \mathrm{GeV} \Rightarrow \frac{2\pi}{L} \sim .5\ \mathrm{GeV}$

$$\langle H'(p') | \mathcal{O} | H(p) \rangle \sim \sum_j \alpha_j f_j(q^2)$$



$\alpha_j \propto q$

- Extrapolation, χPT m_q, q
- $L \Rightarrow$ gauge configurations
- Background fields ...

Twisted Boundary Conditions:

$$\phi(x_i + L) = \mathcal{U}\,\phi(x_i)$$

- $\mathcal{U}\mathcal{U}^\dagger = 1$, global symmetry of action
- e.g. $\mathcal{U} = e^{i\theta_i^a T^a}$, $T^a \in$ Cartan $U(N_f)$ (valence)

$$p_i = \frac{2\pi}{L} n_i + \frac{\theta_i}{L}$$

$$\phi(x) = e^{i\vec{\theta}^a \cdot \vec{x}\, T^a / L}\, \hat{\phi}(x)$$

$\phi(x)$: twisted

$\hat{\phi}(x)$: periodic, coupled to uniform gauge field

$$\sum_x \bar{\psi}(\not{D} + m)\psi = \sum_x \bar{\hat{\psi}}(\hat{\not{D}} + m)\hat{\psi}$$

single valued

$$\hat{D}_\mu = D_\mu + i B_\mu$$

$$B_\mu = \left(\frac{\vec{\theta}^a}{L} T^a, 0\right)$$

Kinematic Effects (χPT)

de Divitiis
Petronzio
Tantalo

- $\check{D}_\mu \tilde{\Sigma}_i = D_\mu \tilde{\Sigma}_i + i\left[B_\mu, \tilde{\Sigma}_i\right]$

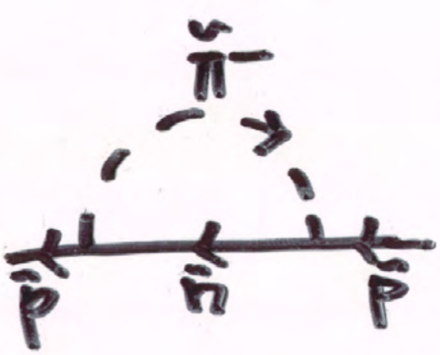
$$E_{\pi^+} = \sqrt{m_\pi^2 + \vec{B}_{\pi^+}^{\,2}} \;, \quad B_{\pi^+} = B_u - B_d$$

- $\check{D}_\mu \tilde{B}^{ijk} = D_\mu \tilde{B}^{ijk} + i\left(B_\mu^i + B_\mu^j + B_\mu^k\right)\tilde{B}^{ijk}$

$$E_p = M_p + \frac{\vec{B}_p^{\,2}}{2M_p} \;, \quad B_p = 2B_u + B_d$$

Dynamic Effects: Finite Volume

$$M_p = M_p(\vec{B}) \sim M_0 + \alpha\, m_q + \beta\, m_q^{3/2}$$



$$\sim \int d^4k \, \frac{\left[(k+B_\pi)\cdot S\right]^2}{\left[(k+B_\pi)^2 + m_\pi^2\right](k+B_\pi)\cdot v}$$

$$\int_k \longrightarrow \int_{k-B_\pi} \quad \text{vs.} \quad \sum_{\vec{k} = \frac{2\pi\vec{n}}{L}}$$

- Flavor symm. breaking in FV.

Twisting Form Factors out of Lattice QCD

- nucleon axial current

- $\check{J}_5^{+\mu} = \bar{\check{u}}\, \gamma_\mu \gamma_5\, \check{d}$

Kinematics:

$$C(t,t') = \sum_{\vec{x},\vec{x}'} \langle 0 | \check{P}(\vec{x},t)\, \check{J}_5^{+\mu}(\vec{x}',t')\, \bar{\check{N}}(\vec{0},0) | 0 \rangle$$

propagate in background field

$$= \left\langle P_{\vec{B}_p}(t)\, J_5^{+\mu}(t')\, \bar{N}_{\vec{B}_N}(0) \right\rangle$$

- $\vec{q} = \vec{B}_p - \vec{B}_N = \vec{B}_{\pi^+}$

@ zero lattice momentum

Nucleon Axial Current : Dynamical

- FV corrections
- PQ, PT, HBχPT
- Form Factors q^2-dependence

$+ \; 1 \text{ loop} = 0$

$+$ recoil $1/M_0$, two loops

$$\langle \tilde{p}(0) | J_5^{+\mu} | \tilde{n}(0) \rangle$$

$$= \bar{u}(0) \Big\{ 2 S^\mu \Big[G_A(0) - \frac{1}{6} \langle r_A^2 \rangle \vec{B}_{\pi^+}^{\,2} \Big]$$

$$+ \vec{B}_{\pi^+}^\mu \, \vec{B}_{\pi^+} \cdot \vec{S} \Big[\frac{g_A}{\vec{B}_{\pi^+}^{\,2} + m_\pi^2} + \frac{1}{3} \langle r_A^2 \rangle \Big] \Big\} u(0)$$

Nucleon Vector Current

$$\langle p' | J^\mu_{em} | p \rangle = \bar{u}(p') \left[f_1(q^2) \gamma^\mu + \frac{[\sigma^\mu, \sigma^\nu]}{M} q_\nu f_2(q^2) \right] u(p)$$

- μ, $\langle r_E^2 \rangle$, ... q^2-dependence from χPT
- need flavor change to induce momentum transfer

- $m_u = m_d \Rightarrow$ vector current conservation J^μ

$$\Rightarrow \langle p(p') | J^\mu_{em} | p(p) \rangle - \langle n(p') | J^\mu_{em} | n(p) \rangle$$

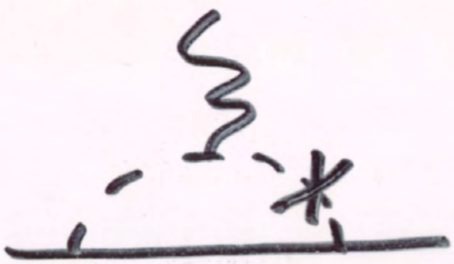
$$= \langle p(p') | J^{+\mu} | n(p) \rangle$$

- $\mu_{iso} = \mu_p - \mu_n$
- $\langle r^2_{iso} \rangle = \langle r_p^2 \rangle - \langle r_n^2 \rangle$

can be extracted @ zero lattice momentum

Nucleon Vector Current: Finite Volume

- pion cloud of nucleon sensitive to boundary conditions
- flavor changing diagrams more sensitive to volume than EM



PQ - hairpins

- But cancel in sum (CVC)
- new contributions

$V_\infty = 0$

$FV(B=0) = 0$

$FV(B \neq 0) \neq 0$

- FV corrections ~ without twisting
- Exponentially suppressed for large volumes

Twisting in the ε-regime

- $m_\phi L \ll 1$
- strongly coupled mode

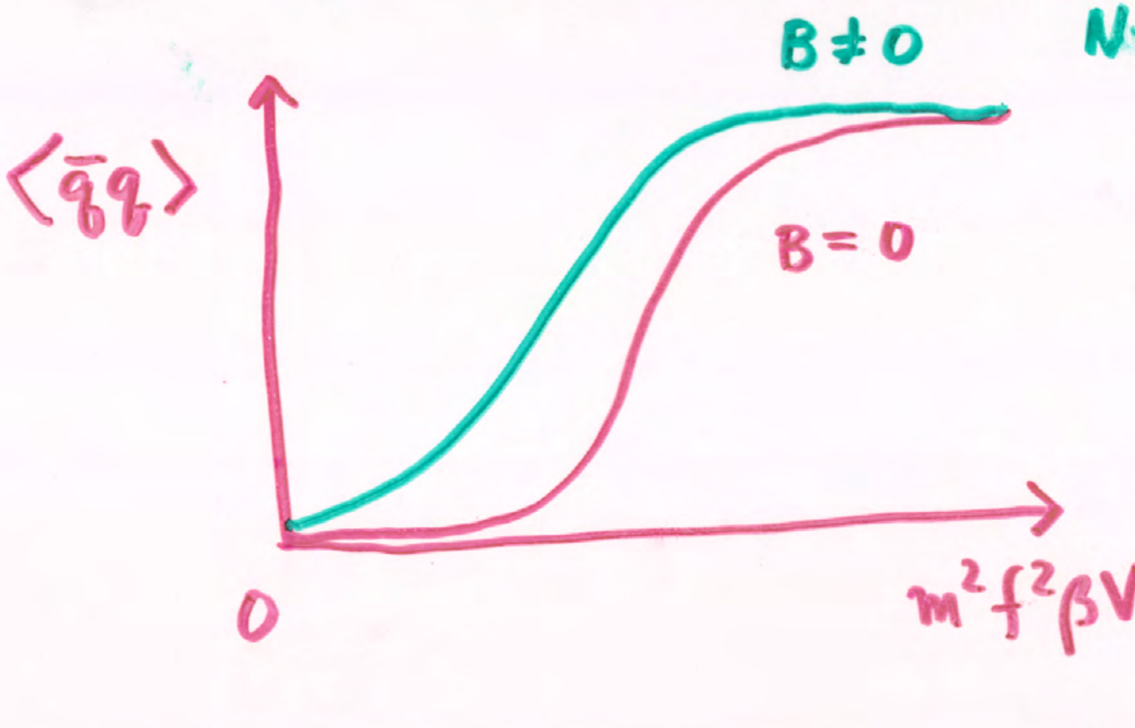
  $n_\mu = (\vec{0}, 0) \qquad \langle \phi \phi \rangle \sim \frac{1}{m_\phi^2}$

- effective mass from twisting

  $$\langle \phi \phi \rangle \sim \frac{1}{\vec{B}_\phi^2 + m_\phi^2}$$

- acts to explicitly break χ-sym

  $$SU(N)_L \otimes SU(N)_R \longrightarrow \underbrace{U(1) \otimes \ldots \otimes U(1)}_{N-1}$$



Summary

- Matrix elements of flavor changing operators accessed at continuous momentum transfer
- Pions light & see flavor dependent interactions at boundary
- χEFTs needed to address finite volume corrections
- combined chiral & momentum extrapolation constrained