Two dimensional $\mathcal{N} = (2, 2)$ super Yang-Mills theory on the lattice via dimensional reduction

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§1 Introduction

- Super Yang-Mills on lattice
  - Orbifolding and Deconstruction Kaplan et al.
  - Nilpotent SUSY Sugino, Catterall
  - Non-commutable differential operator Kawamoto et al.

    $\Rightarrow$ OK for 2D theory

$\Downarrow$ Technical difficulties

- Complex determinant?
- Fermion zero mode?
- Complex gauge field?
- Unusual gauge action

- Fine tuning approach

  Curci and Veneziano, Montvay et al, Nishimura, $\cdots$

  - Fine tuning is simple for 4D $\mathcal{N}=1$ SYM action
  - SUSY point $=$ chiral symmetric point
  - $U(1)_R$ is anomalous
  - Fine tuning is not simple for operators

- Fine tuning is simple for 2D SYM
  - Theory is super renormalizable
  - UV divergence could appear at one loop
  - Fine tuning can be performed perturbatively
  - Usual gauge and fermion action
  - Fermion determinant is real or positive

§2 Continuum 2 dimensional $\mathcal{N} = (2,2)$ SYM

- Dimensional reduction from 4D $\mathcal{N} = 1$ $SU(N_c)$ SYM

$$S = \int \mathrm{d}^4 x \left\{ \frac{1}{4} F^a_{MN} F^a_{MN} + \frac{1}{2} \lambda^{aT} C \Gamma_M D_M \lambda^a \right\}$$

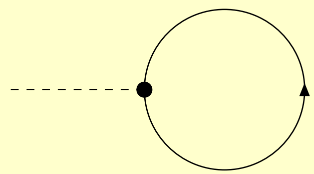
$M, N = 0 \sim 3$, C: charge conjugation

$\Downarrow$ Dimensional reduction

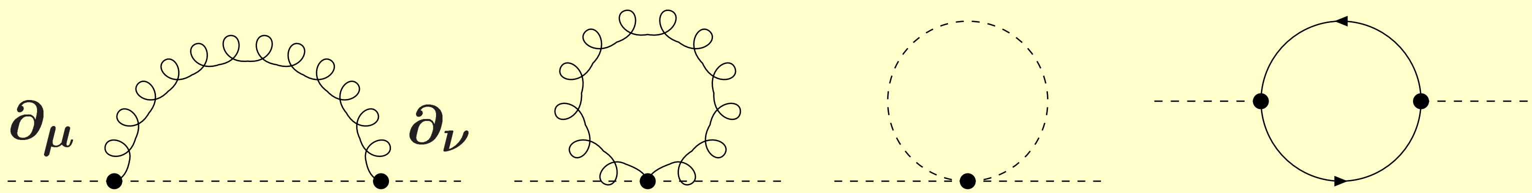
$$\begin{aligned} S \;=\; \int \mathrm{d}^2 x \{ & \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} D_\mu \phi^a D_\mu \phi^a + \frac{1}{2} D_\mu \varphi^a D_\mu \varphi^a \\ & + \frac{1}{2} g_0^2 f_{abc} f_{ade} \varphi^b \phi^c \varphi^d \phi^e + \overline{\psi}^a \gamma_\mu D_\mu \psi^a \\ & - i g_0 f_{abc} \overline{\psi}^a (\phi^b + i \gamma_5 \varphi^b) \psi^c \} \end{aligned}$$

g_0 : mass dimension, $\phi = A_3$, $\varphi = A_2$

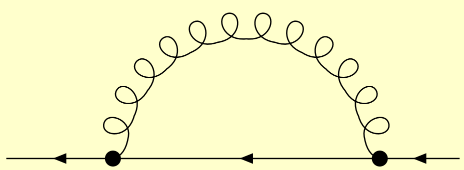
- Perturbative expansion



$\Rightarrow$ **Vanishes for non-singlet gauge group**



$\Rightarrow$ **Log divergence**



$\Rightarrow$ **Finite**

- No divergence for higher loops $l \geq 2$
- Fine tuning is needed for scalar mass term

- Effective potential analysis
  - Contribution to scalar mass term

$$\phi^a(x) = \phi^a + \frac{1}{L}\sum e^{ipx}\tilde{\phi}^a(p),\ \varphi^a(x) = \varphi^a + \frac{1}{L}\sum e^{ipx}\tilde{\varphi}^a(p),$$
$$A^a_\mu(x) = \frac{1}{L}\sum e^{ipx}\tilde{A}^a_\mu(p),$$
$$\psi^a(x) = \frac{1}{L}\sum e^{ipx}\tilde{\psi}^a(p), \quad \overline{\psi}^a(x) = \frac{1}{L}\sum e^{ipx}\tilde{\overline{\psi}}^a(p),$$

  - Gaussian integral for tilde fields
    - Contribution from non-zero mode

$$V_B = \frac{1}{L^2}\{\sum_{p\neq 0}\frac{1}{p^2} - \sum_{p\neq 0}\frac{1}{p^2}\}g_0^2 N_c(\phi^a\phi^a + \varphi^a\varphi^a) = 0$$

$\Rightarrow$ Criterion for fine tuning on lattice

- Cannot treat zero mode along flat direction

$$[\phi, \varphi]^a = i f_{abc} \varphi^b \phi^c = 0$$

  - Integral is not Gaussian

- Gaussian evaluation gives non-zero result

$$\begin{aligned} V_0 &= \frac{1}{L^2}\{\mathrm{tr}\log\{\Phi^2 + \Psi^2\} \\ &\quad + \frac{1}{2}\mathrm{tr}\log\begin{pmatrix} \Psi^2 & -2\Psi\Phi + \Phi\Psi \\ -2\Phi\Psi + \Psi\Phi & \Phi^2 \end{pmatrix} \\ &\quad - \mathrm{tr}\log\{\Phi^2 + \Psi^2 + i(\Psi\Phi - \Phi\Psi)\}\} \end{aligned}$$

$$(\Phi)_{ab} = g_0 f_{acb} \phi^c, \quad (\Psi)_{ab} = g_0 f_{acb} \varphi^c,$$

§3 SYM on lattice

- 4D $\mathcal{N}=1$ $SU(N_c)$ SYM action

$$S[U,\lambda] = S_{\mathrm{G}}[U] + S_{\mathrm{F}}[U,\lambda] + S_{\mathrm{counter}}[U]$$
$$S_{\mathrm{G}}[U] = \frac{1}{a^2 g_0^2}\sum_x \sum_{M,N} \mathrm{Re}\ \mathrm{tr}\,(1 - P_{MN}(x))$$
$$S_{\mathrm{F}}[U,\lambda] = -a^2 \sum_x \mathrm{tr}\{\lambda(x) C D_{\mathrm{w}} \lambda(x)\}$$

- Plaquette

$$P_{MN}(x) = U_M(x) U_N(x+\hat{M}) U_M(x+\hat{N})^{-1} U_N(x)^{-1}$$

- Covariant derivative for adjoint fermion

$$\nabla_M \lambda(x) = \frac{1}{a}\left\{U_M(x)\lambda(x+\hat{M})U_M(x)^{-1} - \lambda(x)\right\}$$
$$\nabla^\dagger_M \lambda(x) = \frac{1}{a}\left\{\lambda(x) - U_M(x-\hat{M})^{-1}\lambda(x-\hat{M})U_M(x-\hat{M})\right\}$$

- Dimensional reduction by boundary condition

$$U_N(x+a\hat{M}) = U_N(x),\ \lambda(x+a\hat{M}) = \lambda(x) \text{ for } M = 2,\ 3$$
$$U_N(x+L\hat{M}) = U_N(x),\ \lambda(x+L\hat{M}) = \lambda(x) \text{ for } M = 0,\ 1$$

- Scalar fields on the link variable

$$U_3(x) = \exp\{ag_0\phi^a(x)T^a\}, \quad U_2(x) = \exp\{ag_0\varphi^a(x)T^a\}.$$

$\Rightarrow$ Flat direction is compact

- Discrete $SO(2)$ symmetry

$$U_2(x) \to U_3(x)^{-1},\ U_3(x) \to U_2(x),\ \lambda(x) \to \exp\left\{-\frac{\pi}{4}\Sigma_{23}\right\}\lambda(x)$$

- $SO(2)$ symmetric scalar mass term

$$\begin{aligned} S_{\text{counter}}[U] &= -\mathcal{C}N_c\sum_x\Big(\text{tr}\{U_3(x)+U_3(x)^{-1}-2\} \\ &\quad +\text{tr}\{U_2(x)+U_2(x)^{-1}-2\}\Big) \\ &\to \frac{1}{2}g_0^2\mathcal{C}N_c\int d^2x\,(\phi^a\phi^a+\varphi^a\varphi^a) \end{aligned}$$

- **Perturbative expansion**
  - **Change the variable**

$$U_\mu(x) = \exp\{ag_0 A_\mu^a(x) T^a\}$$
$$U_3(x) = \exp\{ag_0 \phi^a(x) T^a\}, \qquad U_2(x) = \exp\{ag_0 \varphi^a(x) T^a\}$$

  - **Gauge fixing**

$$S_{\mathrm{gf}}[U] = -a^2 \sum_x \sum_{\mu,\nu=0}^{1} \lambda_0 \mathrm{tr}\{\partial_\mu^* A_\mu(x) \partial_\nu^* A_\nu(x)\},$$

  - **Measure term**

$$S_{\mathrm{measure}} = -\sum_x \sum_M \frac{1}{2} \mathrm{tr}\ln\left\{2\frac{\cosh(a\mathcal{A}_M(x)) - 1}{a\mathcal{A}_M(x)}\right\}$$
$$= a^2 \sum_x \left\{\frac{1}{24} g_0^2 N_c [\phi^a(x)\phi^a(x) + \varphi^a(x)\varphi^a(x)] + \cdots\right\}$$

- Effective potential for scalar fields

$$\phi^a(x) = \phi^a + \frac{1}{L}\sum e^{ipx}\tilde{\phi}^a(p),\ \varphi^a(x) = \varphi^a + \frac{1}{L}\sum e^{ipx}\tilde{\varphi}^a(p),$$

$$A_\mu^a(x) = \frac{1}{L}\sum e^{ipx}\tilde{A}_\mu^a(p), \quad \lambda^a(x) = \frac{1}{L}\sum e^{ipx}\tilde{\lambda}^a(p),$$

  - UV divergence only appear in scalar mass term

- Requirement
  - Non-zero mode contribution to the mass term should vanish

$$\left\{\frac{1}{2}\mathcal{C} + \frac{1}{L^2}\sum_{p\neq 0}\left(\frac{1}{\hat{p}^2} - \frac{1+\frac{1}{2}a^2\hat{p}^2}{\mathring{p}^2 + \frac{a^2}{4}(\hat{p}^2)^2}\right)\right\} g_0^2 N_c \phi^a\phi^a = 0$$

$$\mathring{p}_\mu = \frac{1}{a}\sin(ap_\mu), \qquad \hat{p}_\mu = \frac{2}{a}\sin\left(\frac{1}{2}ap_\mu\right),$$

$$\Rightarrow \mathcal{C} = 0.65948255(8)$$

  - Zero mode contribution reproduce continuum result

- **Using overlap Dirac operator**

$$S_{\mathrm{F}}[U,\lambda] = -a^2 \sum_x \mathrm{tr}\{\lambda(x) C D \lambda(x)\},$$

$$D = \frac{1}{a}\{1 - A(A^\dagger A)^{-1/2}\}, \qquad A = 1 - aD_{\mathrm{w}},$$

- **Counter term is evaluated with effective potential**

  $\Rightarrow \mathcal{C} = -0.28891909(1)$

- **Virtue of overlap Dirac operator**
  - **Fermion determinant is real positive (Neuberger)**
  - $U(1)_R$ **symmetry**
    - **Broken by measure with $\mathcal{O}(a)$ artifact**

§4 What to measure?

- SUSY WT identity

- Mass gap, which is predicted to be zero
  - light-cone quantization (Matsumura et al)
  - ’t Hooft anomaly matching condition (Witten)

  $\Rightarrow$ Low energy effective theory is CFT

- Property of the CFT

- Phase of the theory

§5 Conclusion

- SYM on lattice by fine tuning
  - 2D SYM is super-renormalizable

    $\Rightarrow$ Fine tuning is possible perturbatively
    - One loop calculation is enough

- Counter term is needed for scalar mass term
  - Evaluated coefficient of counter term at one loop
  - Counter term is constant in lattice spacing
    - g_0 does not run

- Continuum limit is taken by trivial scaling of ag_0

§6 Future works

Numerical simulation

- **Higher dimension (Elliott and Moore)**
- **Higher SUSY**