Neutron electric dipole moment from lattice QCD

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Introduction

- Both CKM matrix phase and QCD vacuum effects contribute to CP violation parameter.

$$\bar{\theta} = \arg\det M_{\mathrm{CKM}} + \theta_{\mathrm{QCD}}$$

$$L_{\mathrm{QCD}}^{\mathrm{CP\ odd}} = -\frac{i\theta_{\mathrm{QCD}}}{32\pi^2}\int d^4x \frac{1}{4}\varepsilon_{\alpha\beta\mu\nu}F_{\mu\nu}F_{\alpha\beta}$$

- **Neutron electric dipole moment (NEDM)**

$$|d_n| < 6.3\times 10^{-26}\ \mathrm{e\cdot cm}\ \ (90\%\ \mathrm{CL})$$ **Harris, et al. (1999)**

Some model estimations give

$$|d_n|/\bar{\theta} \approx O(10^{-15})\ \mathrm{e\cdot cm} \rightarrow \bar{\theta} \sim 10^{-10}$$

This small bound is theoretically unnatural.

(i.e. fine tune is needed) ➡ "Strong CP problem"

Lattice calculation

Reliable and accurate estimation from first principles of QCD

$\Longrightarrow$ precise determination of $\bar{\theta}$

Previous works

- Aoki, Gocksch, PRL63 (1989) 1125
- Berruto, Blum, Orginos, Soni, NP Proc. Suppl. 140 (2005) 411
- **Shintani, et al. PRD72, 014504 (2005)**

Contents

NEDM from form factors

Shintani et al. (2005)

- **Nucleon electromagnetic form factors**

$$\left\langle N^{\theta}(p_{out}) \left| J_{\mu}^{\mathrm{EM}}(q) \right| N^{\theta}(p_{in}) \right\rangle$$

$$= \bar{u}^{\theta}(p_{out}) \left[\underline{F_1 \gamma_\mu + \frac{F_2}{2m_N} q_\nu \sigma_{\mu\nu}} \right.$$

CP even

$$\left. + i\theta \left(\underline{\frac{F_3}{2m_N} q_\nu \sigma_{\mu\nu} \gamma_5 + (q_\mu \not{q} - \gamma_\mu q^2) \gamma_5 F_A} \right) + O(\theta^2) \right] u^{\theta}(p_{in})$$

CP odd

with momentum transfer $q = p_{\mathrm{out}} - p_{\mathrm{in}}$

- **Electric dipole moment**

$$d_n = \lim_{q^2 \to 0} \frac{e}{2m_N} F_3(q^2)$$

We have to do the momentum extrapolation.

Extraction of form factors (1)

- **3-point function in θ vacuum**

<u>*Path integral formalism*</u>

$$\sum_{\vec{x},\vec{y}} \left\langle N(\vec{x},t) J_\mu^{\mathrm{EM}}(\vec{y},\tau) \overline{N}(0) \right\rangle_\theta = \sum_{\vec{x},\vec{y}} \left\langle N(\vec{x},t) J_\mu^{\mathrm{EM}}(\vec{y},\tau) \overline{N}(0) \right\rangle + i\theta \sum_{\vec{x},\vec{y}} \left\langle N(\vec{x},t) J_\mu^{\mathrm{EM}}(\vec{y},\tau) \overline{N}(0) Q \right\rangle + O(\theta^2)$$

<u>*Operator formalism*</u>

$$\sum_{\vec{x},\vec{y}} \left\langle N(\vec{x},t) J_\mu^{\mathrm{EM}}(\vec{y},\tau) \overline{N}(0) \right\rangle_\theta$$

$$\underset{\substack{t-\tau\to\infty \\ \tau\to\infty}}{\approx} \left| Z_N^\theta \right|^2 \sum_{s',s} u^\theta(p',s') \overline{u}^\theta(p',s') \left[F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{2m_N} q_\nu \sigma_{\mu\nu} + i\theta \left(\frac{F_3(q^2)}{2m_N} \gamma_5 \gamma_\nu \sigma_{\mu\nu} + F_A(q^2) \left(\not{q} q_\mu - q^2 \gamma_\mu \right) \gamma_5 \right) \right] u^\theta(p,s) \overline{u}^\theta(p,s) e^{-E'^\theta_N (t-\tau)} e^{-E^\theta_N \tau}$$

Nucleon spinor structure in θ vacuum

CP non-invariant phase factor $e^{if(\theta)\gamma_5}$ can appear into the mass term.

$$\sum_s u^\theta(p,s)\overline{u}^\theta(p,s) = \frac{-ip\cdot\gamma + m_N^\theta\, e^{if(\theta)\gamma_5}}{2E_N^\theta}$$

- **2-point function**

Path integral formalism

$$\left\langle N_\alpha(t)\overline{N}_\beta(0)\right\rangle_\theta = \left\langle N_\alpha(t)\overline{N}_\beta(0)\right\rangle + i\theta\left\langle N_\alpha(t)\overline{N}_\beta(0)Q\right\rangle + O(\theta^2)$$

Operator formalism

$$\left\langle N_\alpha(t)\overline{N}_\beta(0)\right\rangle_\theta = \left|Z_N^\theta\right|^2 e^{-E_N t}\left[\frac{-ip\cdot\gamma + m_N^\theta e^{if(\theta)\gamma_5}}{2E_N^\theta}\right]_{\alpha\beta}$$

Nucleon propagator in each order of θ

We consider

$$m_N^\theta = m_N + O(\theta^2), \quad \left|Z_N^\theta\right|^2 = \left|Z_N\right|^2 + O(\theta^2),$$

$$f(\theta) = f^1\theta + O(\theta^2)$$

- θ^0 *term*

$$\left\langle N(t)\overline{N}(0)\right\rangle = \left|Z_N\right|^2 \frac{-ip\cdot\gamma + m_N}{2E_N} e^{-E_N t}$$

- θ^1 *term*

$$\left\langle N(t)\overline{N}(0)Q\right\rangle = \frac{m_N}{2E_N}\left|Z_N\right|^2 f^1 \gamma_5 e^{-E_N t}$$

$\boxed{f^1 \text{ is determined by order } \theta^1 \text{ term.}}$

Extraction of form factors (2)

- θ^0 term

$$\sum_{\vec{x},\vec{y}}\left\langle N(\vec{x},t)J_\mu^{\mathrm{EM}}(\vec{y},\tau)\bar{N}(0)\right\rangle$$

$$=\left|Z_N\right|^2 e^{-E'_N(t-\tau)-E_N\tau}\frac{-ip'\cdot\gamma+m_N}{2E'_N}\left[F_1(q^2)\gamma_\mu+\frac{F_2(q^2)}{2m_N}\gamma_\nu\sigma_{\mu\nu}\right]\frac{-ip\cdot\gamma+m_N}{2E_N}$$

- θ^1 term

$$\sum_{\vec{x},\vec{y}}\left\langle N(\vec{x},t)J_\mu^{\mathrm{EM}}(\vec{y},\tau)\bar{N}(0)Q\right\rangle=\left|Z_N\right|^2 e^{-E'_N(t-\tau)-E_N\tau}$$

$$\times\Bigg[\frac{-ip'\cdot\gamma+m_N}{2E'_N}\Bigg(\overbrace{\frac{q_\nu}{2m_N}\sigma_{\mu\nu}\gamma_5F_3+\frac{q^2\gamma_\mu-(q\cdot\gamma)q_\mu}{4m_N^2}\gamma_5F_A}^{\text{CP odd form factors}}\Bigg)\frac{-ip\cdot\gamma+m_N}{2E_N}$$

$$\left.\begin{aligned}&+\frac{-ip'\cdot\gamma+m_N}{2E'_N}W_\mu^{\mathrm{even}}(q^2)\times\gamma_5\frac{f^1m_N}{2E_N}\\&+\gamma_5\frac{f^1m_N}{2E'_N}\times W_\mu^{\mathrm{even}}(q^2)\frac{-ip\cdot\gamma+m_N}{2E_N}\Bigg]\end{aligned}\right\}\longleftarrow$$

CP odd form factors

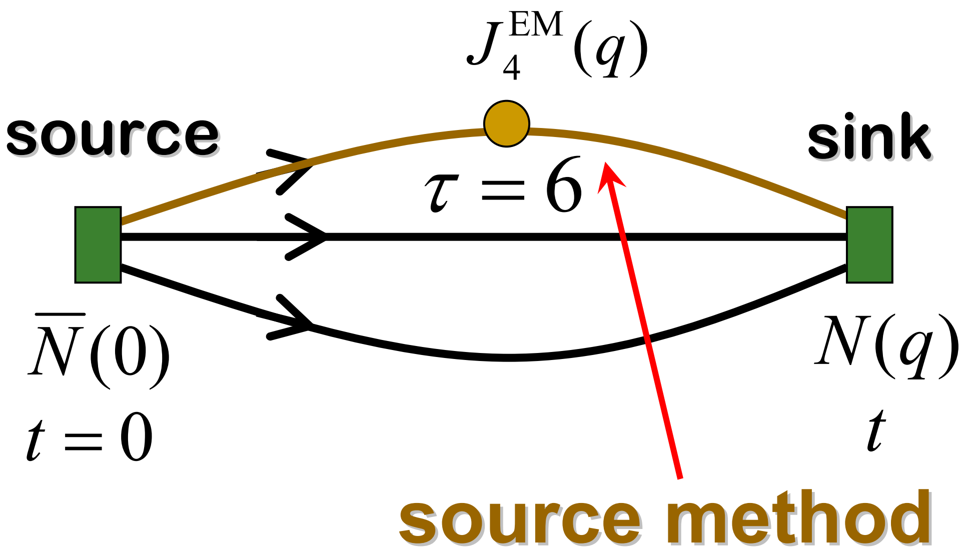
CP even form factors

× CP odd phase factor

Mixing term

Numerical results

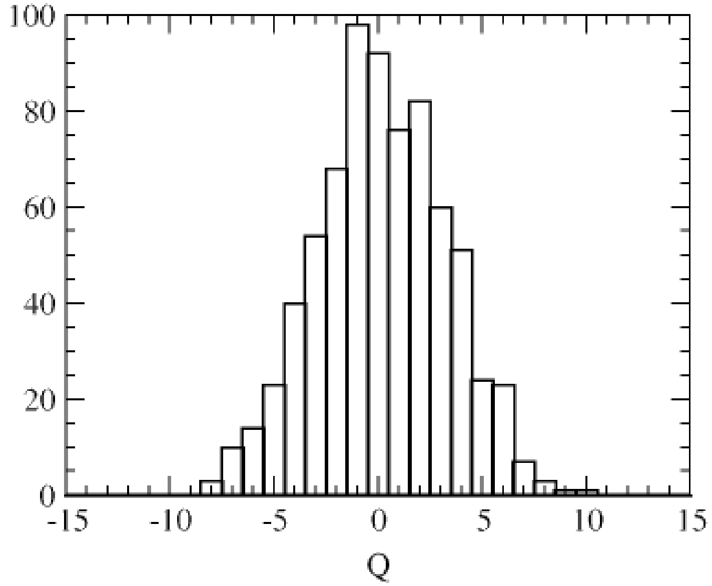
- **Lattice parameters**
  - quenched approximation, RG Iwasaki gauge action $\beta = 2.6$
  - #configuration=730.
  - Lattice size: $16^3 \times 32$ Lattice spacing: $a^{-1} \cong 2\,\mathrm{GeV}$
  - Domain wall fermion $N_s = 16$, $m_f = 0.03$, $m_\pi / m_\rho \cong 0.62$

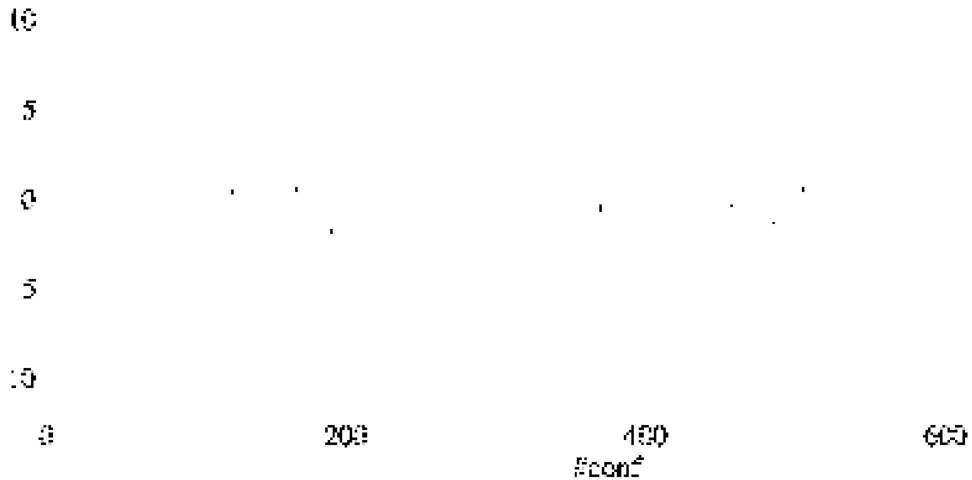


- Nucleon momentum:
  $\vec{q} = (\pi/8,0,0), (0,\pi/8,0), (0,0,\pi/8)$
- J_4^{EM} is conserved vector current. $Z = 1$

Topological charge

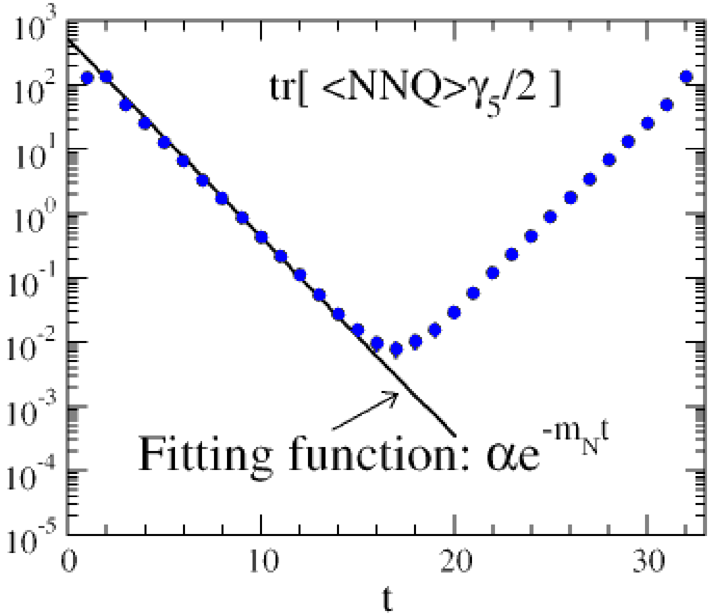
- Bosonic definition with O(a^2) improvement
- Measurements in 20 cooling steps config.

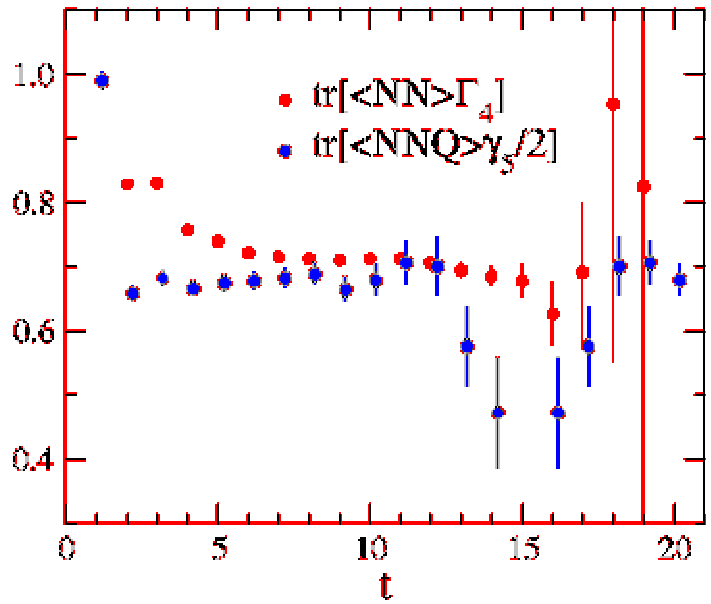




2-point function with Q

$$\langle N(t)\overline{N}(0)Q\rangle \propto \gamma_5 f^1 e^{-E_N t}$$





We have expected time dependence for $\langle N(t)\overline{N}(0)Q\rangle$

These results are consistent with our formulation.

Extraction of CP odd form factor F_3

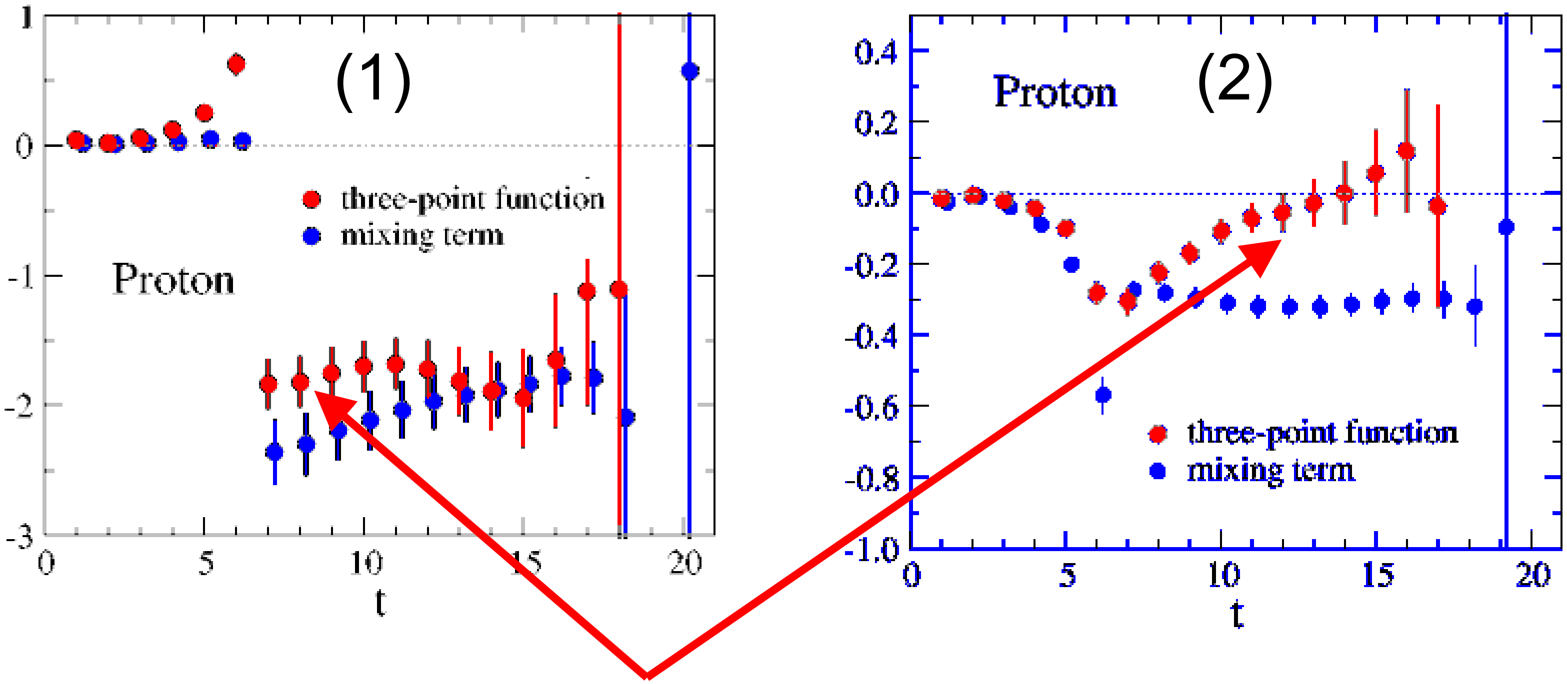
Two independent projections

- $$\mathrm{tr}\left[\left\langle N(t) J_4^{\mathrm{EM}}(\tau)\overline{N}(0) Q\right\rangle \frac{1+\gamma_4}{2}\gamma_5\right] = \left|Z_N\right|^2 e^{-E_N(t-\tau)-m_N\tau}$$
$$\times\left(\frac{\vec{q}^{\,2}}{2E_N m_N}F_3 + \left[\frac{E_N+m_N}{2E_N}F_1 + \frac{\vec{q}^{\,2}}{4m_N E_N}F_2\right] f^1\right) \quad (1)$$

- $$\mathrm{tr}\left[\left\langle N(t) J_4^{\mathrm{EM}}(\tau)\overline{N}(0) Q\right\rangle \frac{1+\gamma_4}{2} i\gamma_5\gamma_i\right] = \left|Z_N\right|^2 e^{-E_N(t-\tau)-m_N\tau}$$
$$\times\left(-\frac{E_N+m_N}{2E_N m_N} q_i F_3 + \left[-\frac{q_i}{2E_N}F_1 - \frac{q_i(E_N+3m_N)}{4m_N E_N}F_2\right] f^1\right) \quad (2)$$

Extraction of CP odd form factor F_3

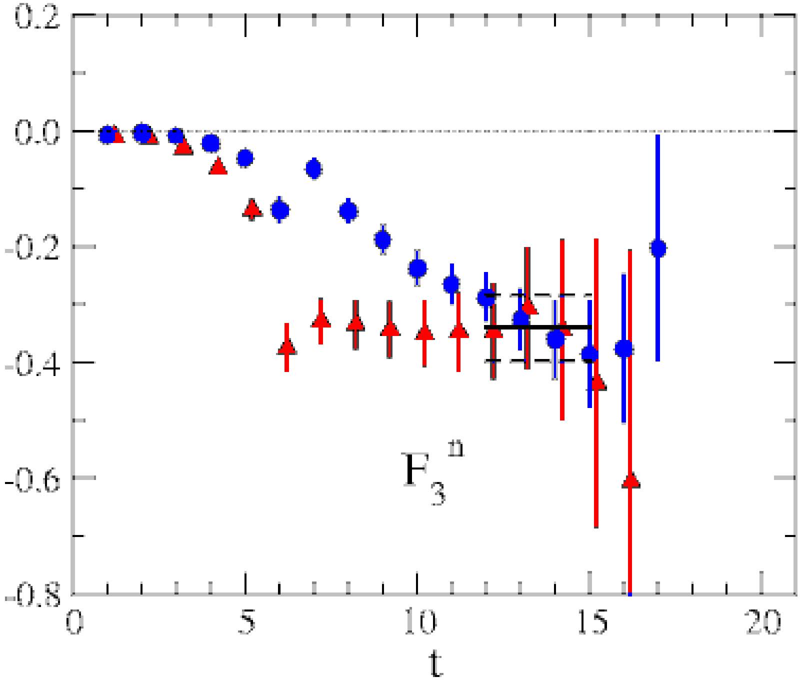
Two independent projections

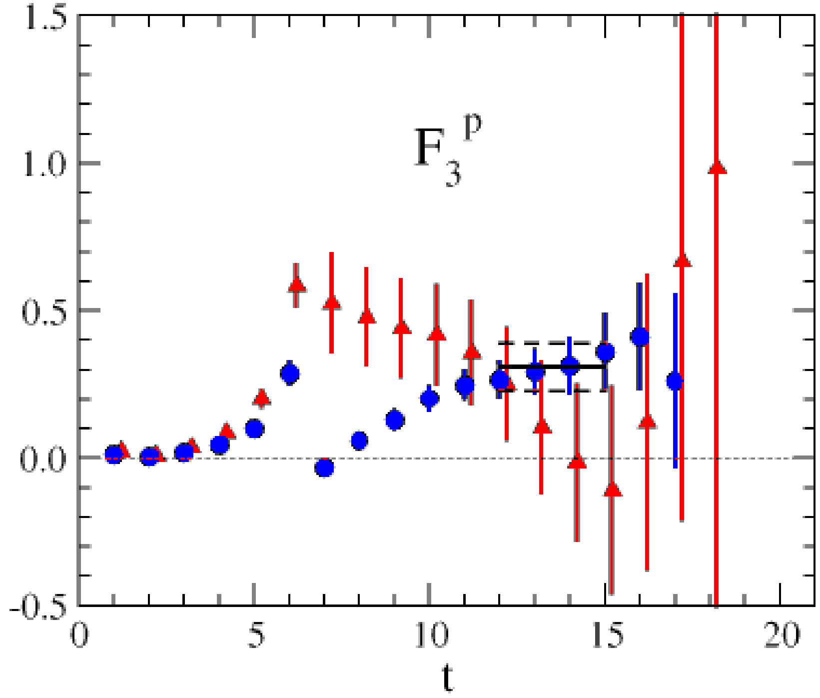


3-point function is different between two projections.

$\Rightarrow$ If we do not consider mixing term, F_3 can not obtain from 3-point function.

EDM form factor





$$\frac{F_3(q^2)}{2m_N} = \begin{cases} -0.0239\,(40)\, e\cdot\mathrm{fm} & \text{for neutron} \\ 0.0218\,(58)\, e\cdot\mathrm{fm} & \text{for proton} \end{cases}$$

Before the systematic study …

To obtain the physical value of EDM, there are various extrapolations :

- $q^2 \rightarrow 0$ ➡ $\boxed{\text{Changing } q^2 \text{ is not so easy.}}$
- $m_q \rightarrow m_{phys}$
- $a \rightarrow 0$

In order to avoid momentum extrapolation we try another idea for EDM calculation.

NEDM from energy shift

- Definition

Spin dependent energy difference in static and uniformed electric field $\vec{E}$, and CP-odd vacuum angle θ :

$$m^{\uparrow}_{\mathrm{N}^{\theta}}(E) - m^{\downarrow}_{\mathrm{N}^{\theta}}(E) = d^{\theta}_{n}\vec{\hat{s}} \cdot \vec{E} = \theta d_{n}\vec{\hat{s}} \cdot \vec{E} + O(\theta^{2})$$

$m^{\uparrow,\downarrow}_{\mathrm{N}^{\theta}}$: spin up or down nucleon energy on θ vacuum

$\vec{\hat{s}}$: spin direction

Method on the lattice

Real electric field is included in link variables

$$U_i(x) \to e^{q_e E_i t} U_i(x),\ \ i = 1,2,3$$

We can choose arbitrary value for E

but periodicity is broken in time direction. $\Rightarrow E << 1$

In $\vec{E} = (0,0,E)$

$$R(E,\theta;t) = \frac{\langle N_1 \overline{N}_1(E,t)\rangle_\theta}{\langle N_2 \overline{N}_2(E,t)\rangle_\theta} = Z \exp[-d_n^\theta E t] + \cdots$$

where

$$\langle N_s \overline{N}_s(E,t)\rangle_\theta = \sum_Q \langle 0|N_s(t)\overline{N}_s(0)|0\rangle_Q (E)\, e^{i\theta Q}$$

Sampling of topological charge is important !

Numerical results for energy shift

- **Lattice parameters**
  - quenched approximation, #configuration = 1000
  - Lattice size : $16^3 \times 32$, RG Iwasaki : $\beta = 2.6$
  - Domain-wall quark, $N_5 = 16,\ M = 1.8,\ m_q = 0.12$
  - Nucleon mass :

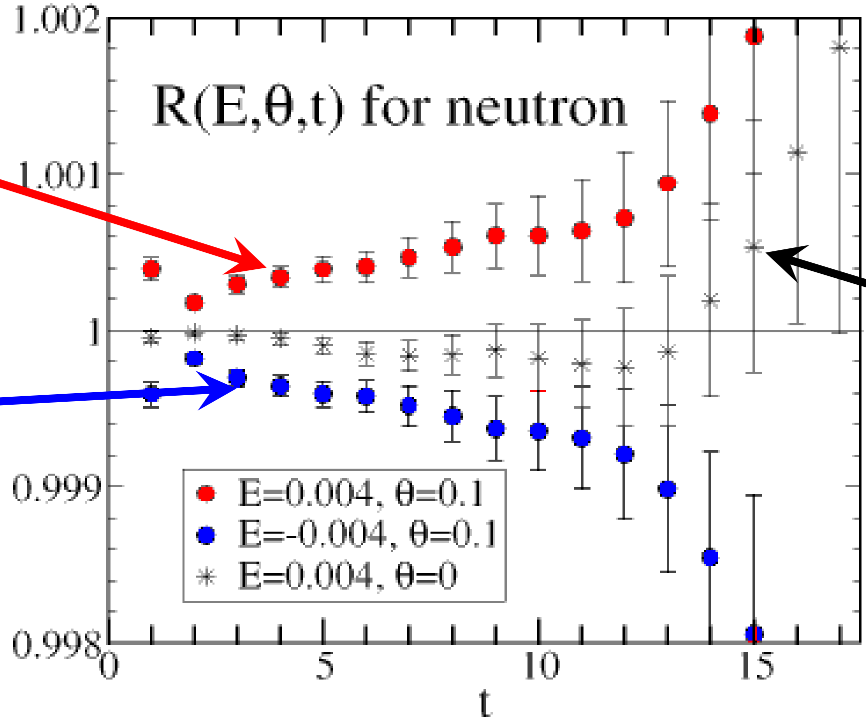
$$m_N a = 1.1 \sim 1.2 \ (\approx 2.2\,\mathrm{GeV})$$

    corresponding to $m_\pi / m_\rho \approx 0.85$
  - Electric field : $a^2 E = 0.004$

Results of $R(E, \theta, t)$

E=0.004
$\theta = 0.1$

E=-0.004
$\theta = 0.1$



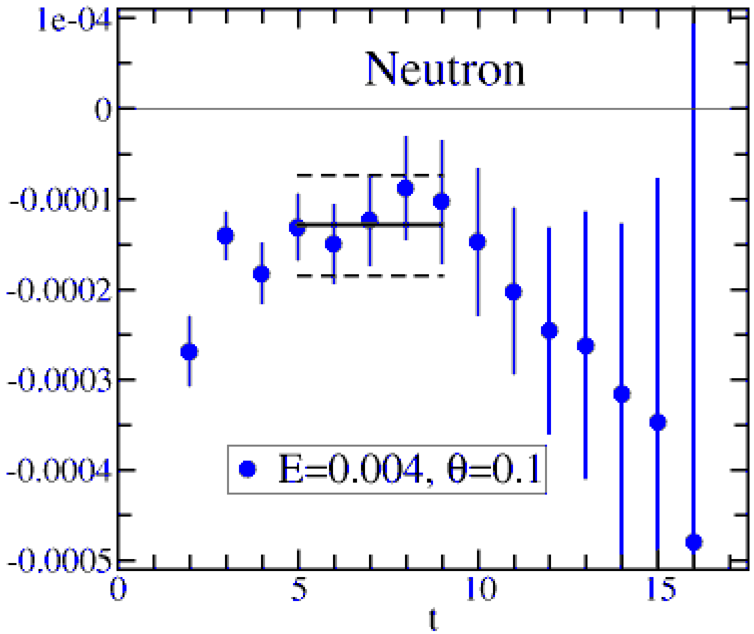
E=0.004
$\theta = 0$

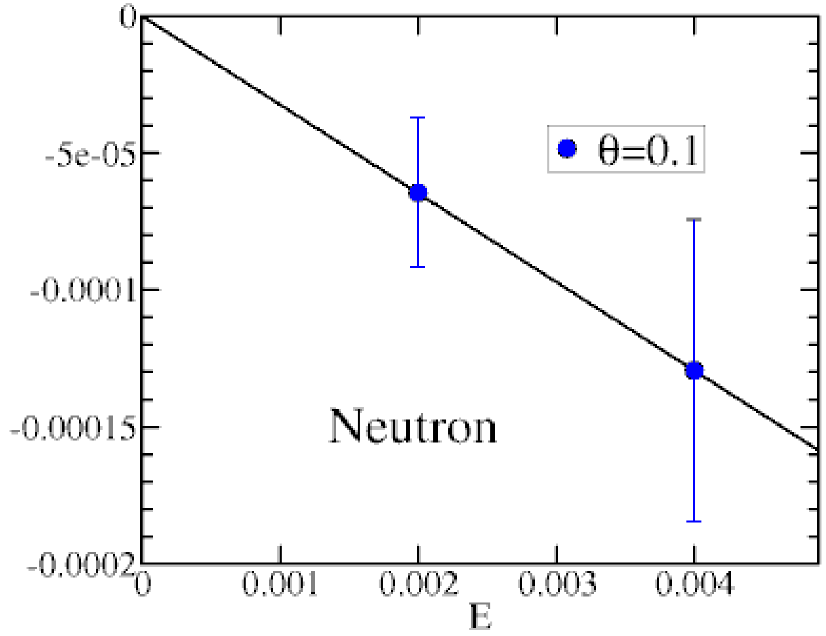
- There is no signal with $\theta = 0$
- Expected E dependence :

$$R(E, \theta; t) = Z \exp[-2 d_n^{\theta} E t] + \cdots$$

Results of $R(E,\theta,t)$

$$\ln\left[\frac{R(E,\theta,t)}{R(E,\theta,t+1)}\right] = d_n^\theta E$$

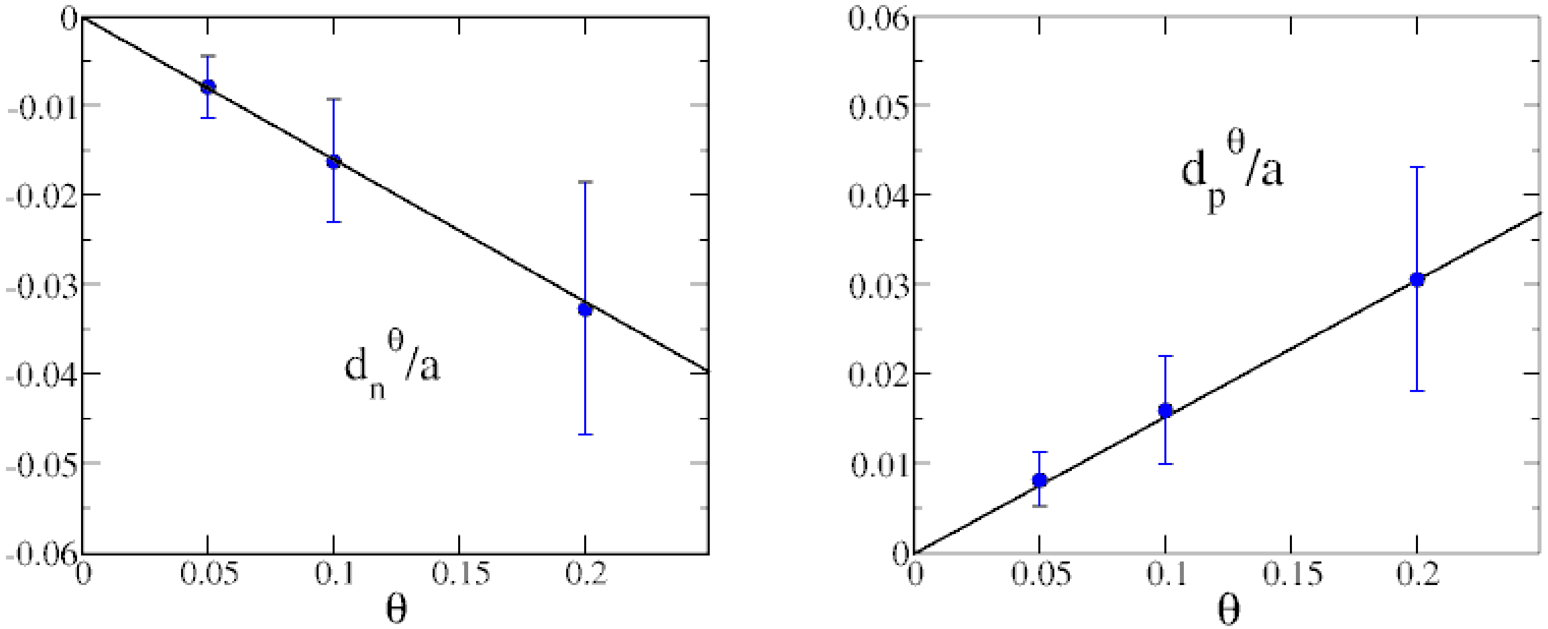




- plateau for $5 \le t \le 9$
- linearity for E

NEDM signal in this method.

θ dependence of d^θ (preliminary)



d^θ has a good linear behavior in term of θ

EDM results:

$$\begin{cases} d_n^\theta / \theta = -0.0189(60)\,\mathrm{e}\cdot\mathrm{fm} \\ d_p^\theta / \theta = 0.0192(72)\,\mathrm{e}\cdot\mathrm{fm} \end{cases}$$

$$\left(\begin{array}{l} F_3(q^2 \cong 0.58\mathrm{GeV}^2)/2m_N = \\ -0.0239\,(40)\,e\cdot\mathrm{fm} \quad \text{for neutron} \\ \;\;\,0.0218\,(58)\,e\cdot\mathrm{fm} \quad \text{for proton} \end{array} \right)$$

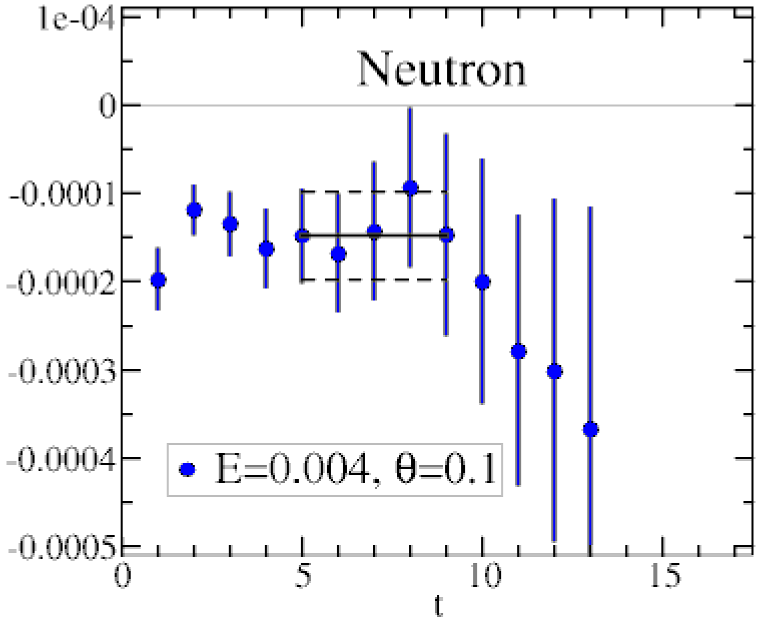
Comparison of clover and DW

In order to apply this calculation to dynamical configuration generated by CP-PACS, we try to do with clover fermion with similar nucleon mass parameters

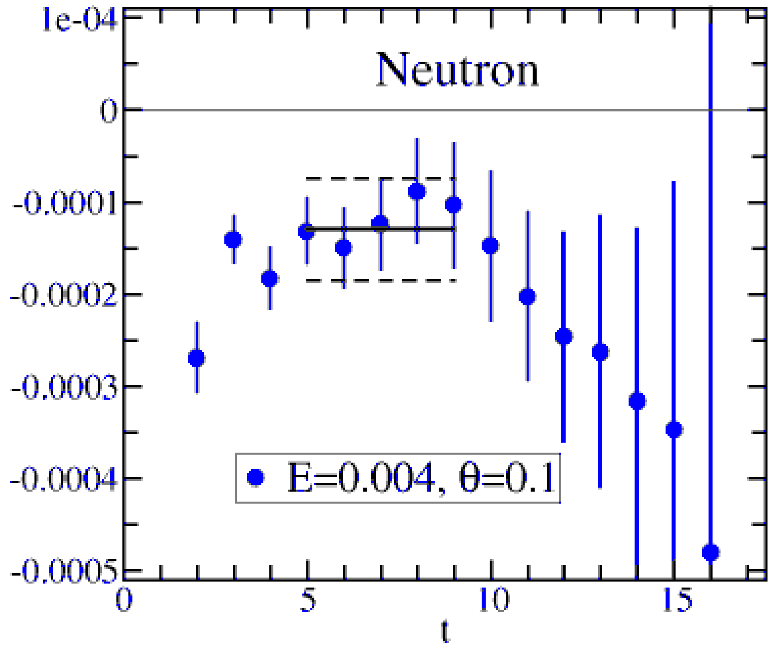
$$\kappa = 0.1320 \ \rightarrow m_N a \approx 1.1$$

on the same gauge confgs.

Comparison of clover and DW



Clover fermion



DW fermion

This method is also successful in clover fermion.

$\Longrightarrow$ chirality of fermion is not so important for this method.

The systematic study of this method

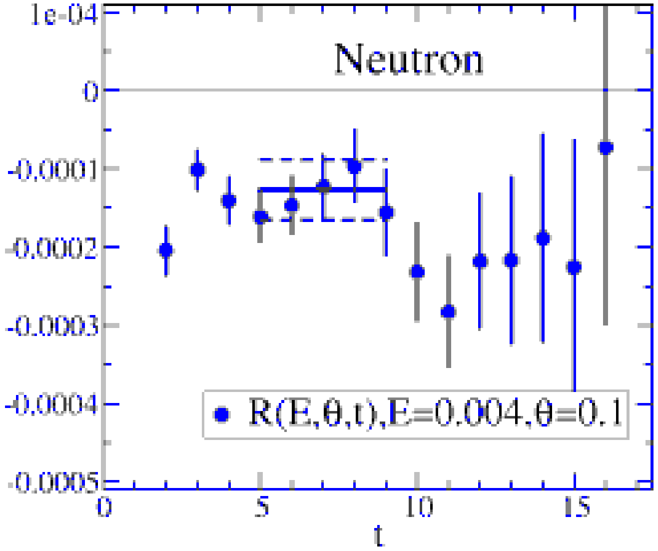
- By using clover fermion the computational cost is reasonable for the study of
  - reduction of finite size effects

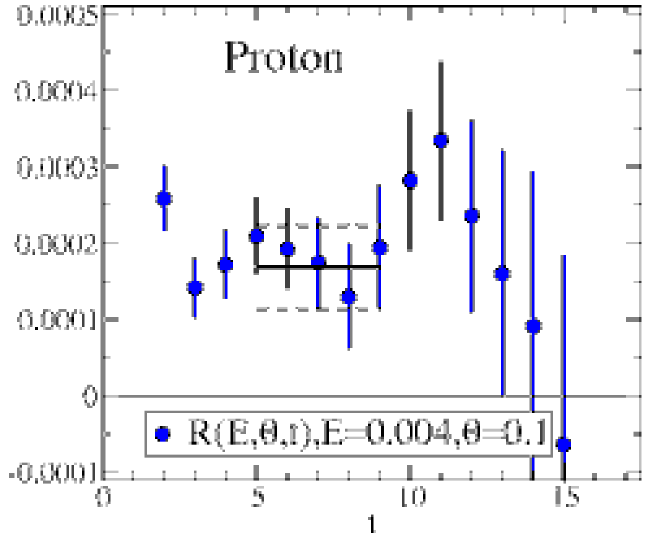
    $\Longrightarrow$ large lattice size
  - mass dependence of EDM

    $\Longrightarrow$ small quark mass

Lattice size $24^3 \times 32$, clover fermion

- **K=0.1320** $(m_{PS}/m_V \cong 0.85)$ **, #configs.=2000**





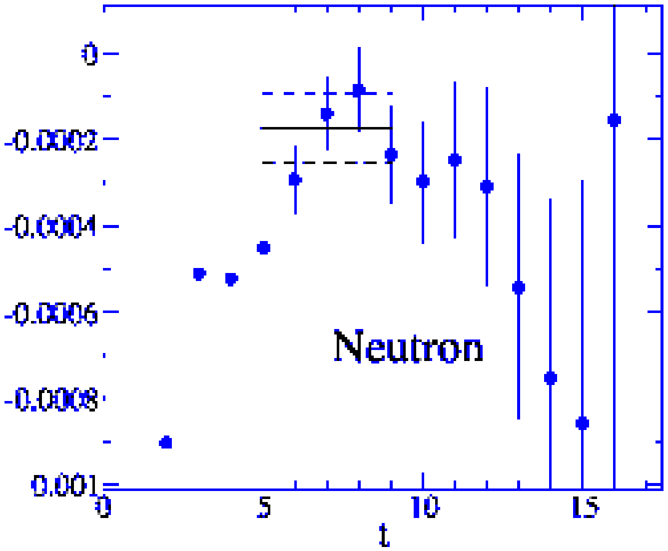
Results of EDM:

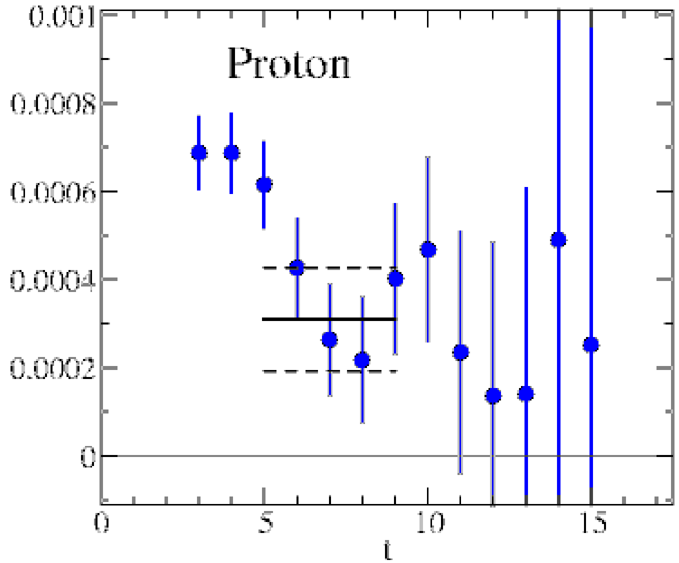
$$\begin{cases} d_n^\theta/\theta = -0.0190(57)\,\mathrm{e\cdot fm} \\ d_p^\theta/\theta = 0.0254(82)\,\mathrm{e\cdot fm} \end{cases}$$

$16^3 \times 32$ lattice:

$$d_n^\theta/\theta = -0.0224(78)\,\mathrm{e\cdot fm}$$
$$d_p^\theta/\theta = 0.0236(91)\,\mathrm{e\cdot fm}$$

K=0.1340 $(m_{PS}/m_V \cong 0.72)$, #configs.=2000

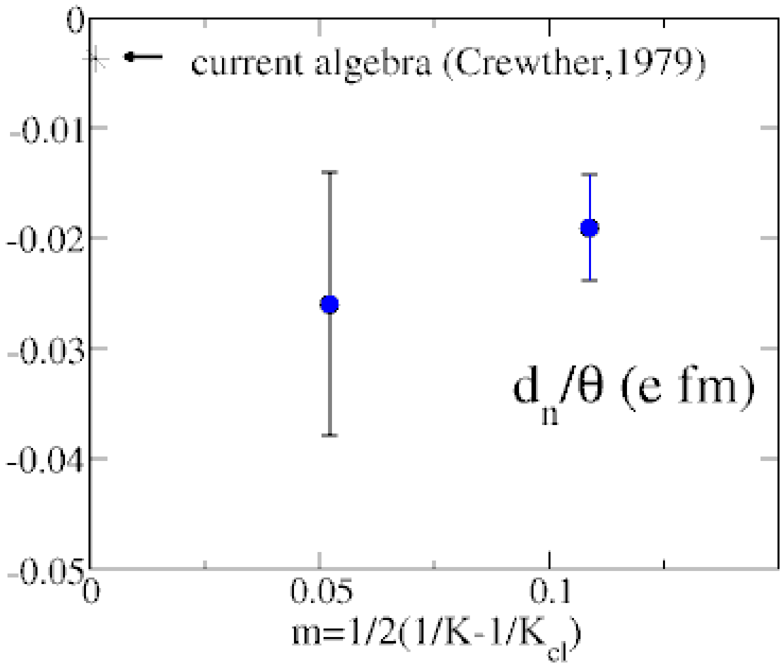


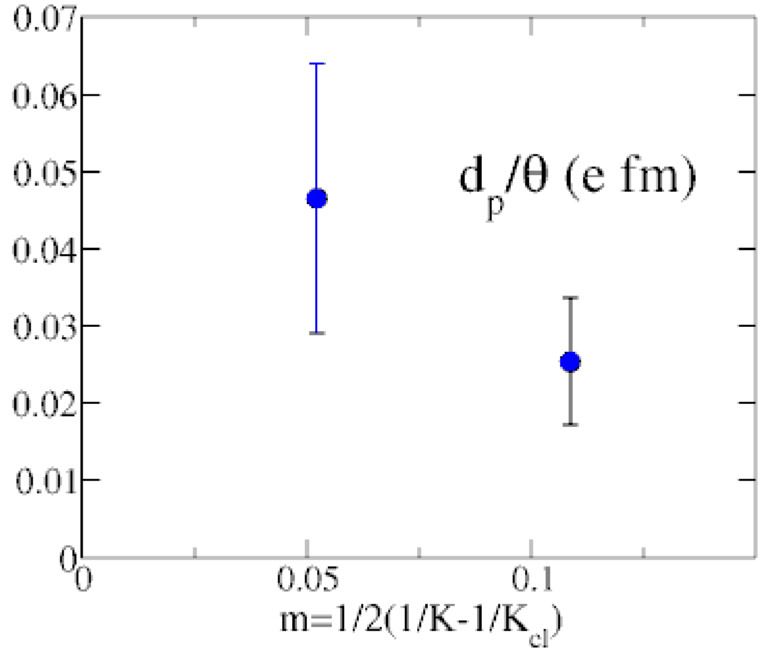


Results of EDM:

$$\begin{cases} d_n^\theta / \theta = -0.026(12)\,\mathrm{e}\cdot\mathrm{fm} \\ d_p^\theta / \theta = 0.047(17)\,\mathrm{e}\cdot\mathrm{fm} \end{cases}$$

Quark mass dependence





These results show that EDM does not vanish in chiral limit. ➡ Quenched effects

The result in light quark mass is still large error. Then we need more statistics for a determination of mass dependence.

Summary

- We obtain EDM form factor at $q^2 \cong 0.58\,\mathrm{GeV}^2$ as

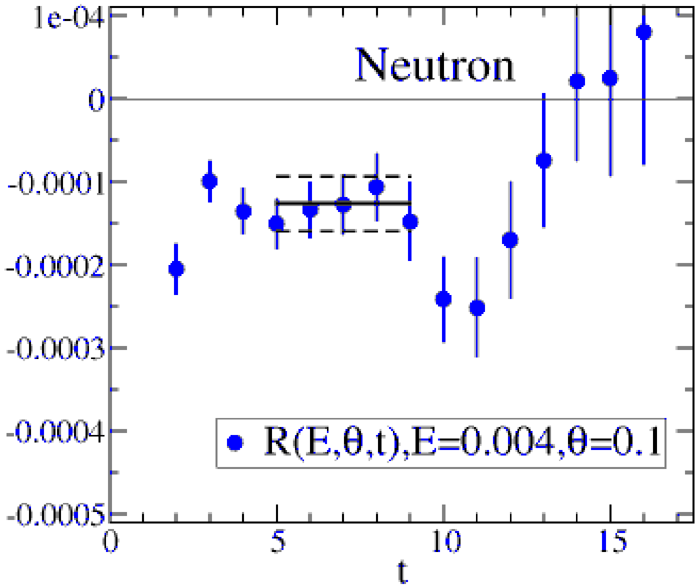
$$\frac{F_3(q^2)}{2m_N} = \begin{cases} -0.0239\,(40)\,e\cdot\mathrm{fm} & \text{for neutron} \\ 0.0218\,(58)\,e\cdot\mathrm{fm} & \text{for proton} \end{cases}$$

  However, this method requires several extrapolations to obtain physical EDM.
- Another idea to extract EDM from energy difference in the constant electric field.
  - We can obtain NEDM signals in this method.
  - This method works well in both domain-wall and clover
  - EDM does not vanish in chiral limit, but the error is still large.

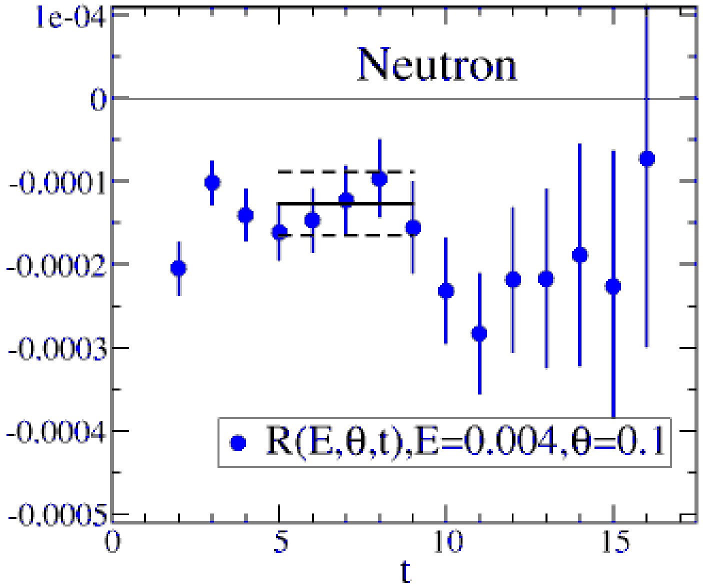
Future works

- Reduction of error with average of all direction of $\vec{E}$

  (e.g. K=0.1320 results)



Average of x,y,z



z-direction only

$$\text{Av.}: d_N = -0.0189(50)\,\mathrm{e}\cdot\mathrm{fm},\ \ (\text{z only}: -0.0190(57))$$

- Calculation on 2 and 2+1 flavor configs. of CP-PACS