Excited Baryon Spectroscopy from Quenched Lattice QCD

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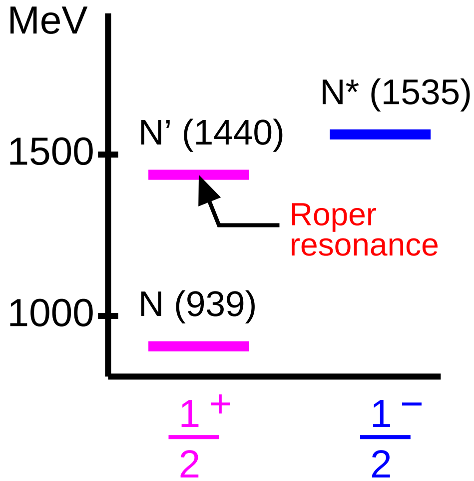
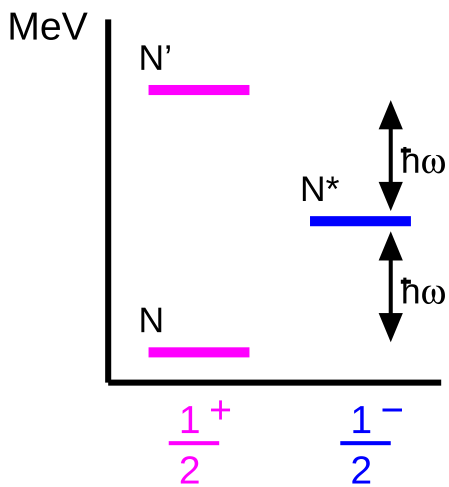
(Reference : Phys.Rev.D72,034502,2005, K.S., S.S.
Phys.Lett.B623,208,2005, K.S., S.S., T.H.)

Puzzle of the excited-baryon spectra

Nucleon spectra



Naive quark model



Experimentally, the first excited nucleon $N'(1440)$ (the Roper resonance) is in the positive-parity channel, while the second excited nucleon $N^*(1535)$ is in the negative-parity channel.

However, the harmonic oscillator quark model and the MIT bag model have difficulties in reproducing this pattern for the level order.

$$N \quad : \quad (1s)^3$$
$$N^* \quad : \quad (1s)^2\ (1p)$$
$$N' \quad : \quad (1s)^2\ (2s) \quad \text{or} \quad (1s)\ (1p)^2$$

Several attempts to reproduce this pattern have been done without decisive conclusion.

[*e.g.* Jaffe-Wilczek model, PRL**91**, 232003 (2003)
Jido-Hatsuda-Kunihiro model, PRL**84**, 3252 (2002)]

This pattern can be seen in the flavor octet Σ, Λ channel and flavor decouplet Δ channel.

Studies of the excited baryons in lattice QCD

Previous studies for the excited baryons have several numerical difficulties.

Previous studies

1. The spatial size of the lattice is small.

	La (fm)	Reference
S.Sasaki *et al.*	1.5	Phys.ReV.D**65**, 074503 (2002)
M.Gockeler *et al.*	1.5-2.2	Phys.Lett.B**532**, 63 (2002)
W.Melnitchouk *et al.*	2.0	Phys.ReV.D**67**, 114506 (2003)
J.M.Zanotti *et al.*	2.0	Phys.ReV.D**68**, 054506 (2003)
Y.Nemoto *et al.*	2.1-2.3	Phys.ReV.D**68**, 094505 (2003)
D.Guadagnoli *et al.*	1.6	Phys.Lett.B**604**, 74 (2004)
D.Brommel *et al.*	1.8,2.4	Phys.ReV.D**69**, 094513 (2004)
T.Burch *et al.*	1.8	Phys.ReV.D**70**, 054502 (2004)
N.Mathur *et al.*	3.2	Phys.Lett.B**605**, 137 (2005)

2. multi-exponential fit $\Rightarrow$ unstable

This study

1. large spatial size $(La \simeq 3.0$ fm, $a^{-1} \simeq 2.1$ GeV, etc.)
2. Maximum Entropy Method (MEM) $\Rightarrow$ stable

Parity projection (effect of boundary condition)

The nucleonic operator, $N^+ = \epsilon_{abc}[u^T C\gamma_5 d_b]u_c$, couples to the positive- and negative-parity states.
Considering the effect of the boundary condition, the nucleonic propagator $G^+(t) = \langle N^+(t)\bar{N}^+(0)\rangle$ is expressed as

$$\begin{aligned} G^+(t) &= \left[A_+\left(\frac{1+\gamma_4}{2}\right)\mathrm{e}^{-M_+ t} - A_-\left(\frac{1-\gamma_4}{2}\right)\mathrm{e}^{-M_- t}\right] \\ &\quad + b\left[A_+\left(\frac{1-\gamma_4}{2}\right)\mathrm{e}^{-M_+(T-t)} - A_-\left(\frac{1+\gamma_4}{2}\right)\mathrm{e}^{-M_-(T-t)}\right] \end{aligned}$$

($b = +1(-1)$ for periodic (anti-periodic) boundary condition)

$\Rightarrow$ The appropriate parity state cannot be extracted by only the simple projection $\mathrm{Tr}\left(\frac{1\pm\gamma_4}{2}G^+(t)\right)$ due to the effect of the b.c..

We can realize the free-boundary situation ($b = 0$) using a linear combination* of two baryonic propagators with the periodic and antiperiodic b.c..

$$G^+_{\mathrm{free}}(t) = G^+(t, b=+1) + G^+(t, b=-1)$$

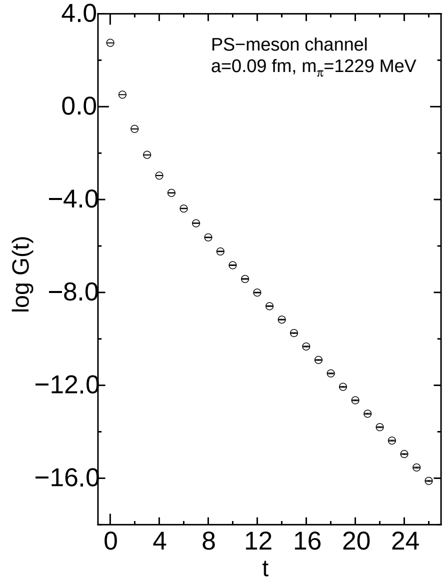
*S. Sasaki, T. Blum and S. Ohta, PR**D65** (2002) 074503

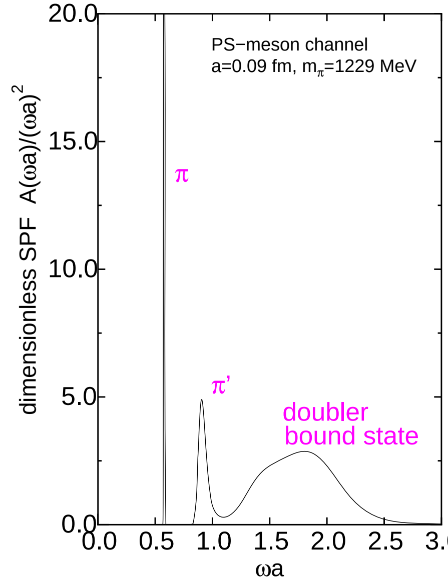
Maximum entropy method (MEM)

Spectral function $A(\omega)$: the probability density of the particle with energy ω

$$G(t) = \int d\omega\ \mathrm{e}^{-\omega t} A(\omega)$$

The MEM* is a stochastic method which reproduces a spectral function $A(\omega)$ from the hadronic propagator $G(t)$. ($P[A|GH]$ ⇐ Bayes'theorem)





It is possible to reproduce a spectral function using the MEM.

$$\left(\begin{array}{rl} \text{single-exponential fit} & : \quad t = 13 - 23 \\ \text{MEM} & : \quad t = \ \ 1 - 20 \end{array} \right)$$

*M. Asakawa, T. Hatsuda and Y. Nakahara, PPNP**46**, 459 (2001)

Finite size effect

The mass of hadron is strongly affected by the spatial size of the lattice.

(Naively, the wave function is squeezed due to the small volume,
so the kinetic energy of internal quarks increases
and the total energy of the bound state should be pushed up.)

In the quenched approx., the spatial size of a lattice is required to be $La \simeq 2.5$ fm in order to neglect the finite size effect on nucleon*.

*S.Aoki et al., PR**D50**, 486 (1994)

$\Rightarrow$ It is naively predicted that the lattice size is needed to be 3.2 fm ($\simeq 2.5$ fm $\times\, r_{\rm rms}^{N^*}/r_{\rm rms}^{N}$) in order to evaluate the mass of excited states.

(Ex.) Harmonic oscillator potential model
$$r_{\rm rms}^{N^*}/r_{\rm rms}^{N} = \sqrt{5/3} \simeq 1.3$$)

In this study, we utilize several lattices which have different sizes, and evaluate the finite size effect on the excited baryons.

Details of the simulation

Hardware : HITACHI SR8000 (KEK)
Software : Lattice Tool Kit (A.Nakamura et al.)*

Gauge Action : Wilson gauge action
Fermion Action : Wilson fermion action (quenched approx.)

β	a	N_{conf}	lattice size	La (fm)	pion mass (MeV)
5.8	0.136 fm	200	$24^3 \times 32$	3.3	0.1600, 0.1570, 0.1555, 0.1540, 0.1530
6.0	0.093 fm	580	$16^3 \times 32$	1.5	0.1550, 0.1530, 0.1515
		200	$32^3 \times 32$	3.0	0.1550, 0.1530, 0.1515, 0.1500
6.2	0.068 fm	320	$24^3 \times 48$	1.6	0.1506, 0.1497
		240	$32^3 \times 48$	2.2	0.1506, 0.1497
		210	$48^3 \times 48$	3.2	0.1520, 0.1506, 0.1497, 0.1489, 0.1480

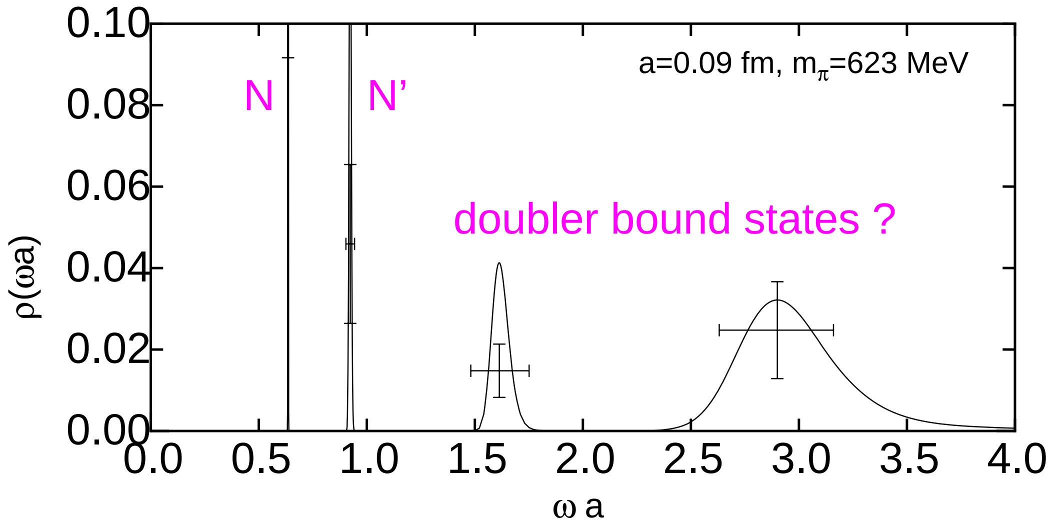
Metropolis algorithm with 20 hits at each link update

BiCGStab algorithm for the matrix inversion

Hopping parameters cover the range $m_\pi \simeq 0.6 - 1.2$ GeV.

* S.Choe et al., NPPS**106**, 1037 (2002)

Spectral function in the positive-parity nucleon channel



evaluation in the physical unit

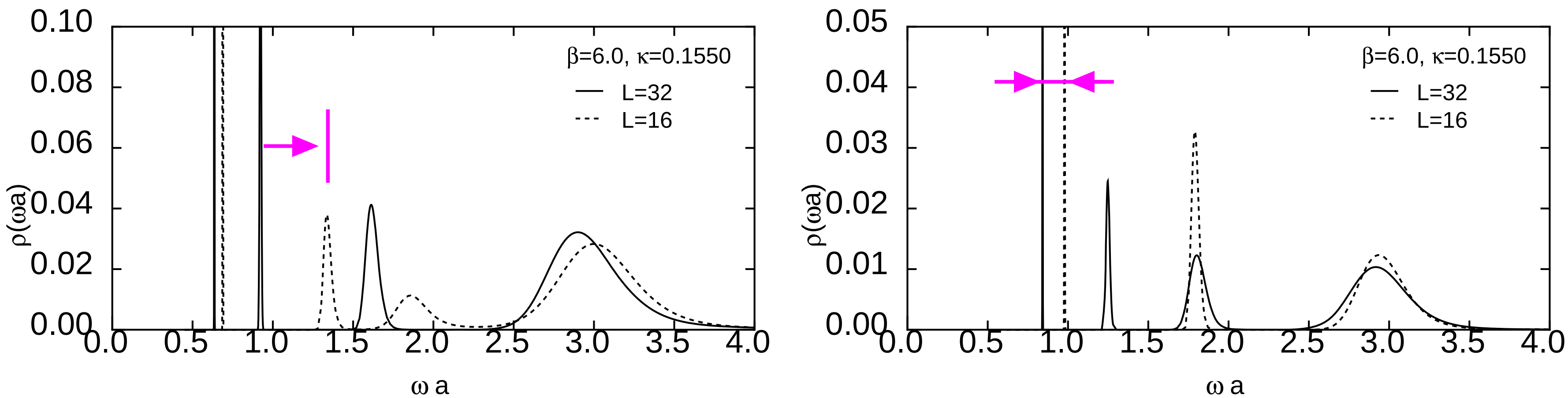
$$m_N = 970\ (44)\ \text{MeV}$$
$$m_{N'} = 1580\ (330)\ \text{MeV}$$

$a = 0.09$ fm, $La = 3.0$ fm						
	κ				chiral limit	$\chi^2/N_{\rm DF}$
	0.1500	0.1515	0.1530	0.1550		
1st peak	1.000(6)	0.898(7)	0.791(9)	0.635(14)	0.464(21)	0.33
2nd peak	1.31(6)	1.20(6)	1.09(8)	0.92(13)	0.76(16)	0.001
3rd peak	1.98(9)	1.86(9)	1.75(10)	1.61(12)	1.48(16)	0.006
4th peak	3.07(7)	3.01(6)	2.95(6)	2.90(6)	2.84(8)	0.08

The position of the first peak is consistent with the mass of the ground state evaluated by the single-exponential fit.

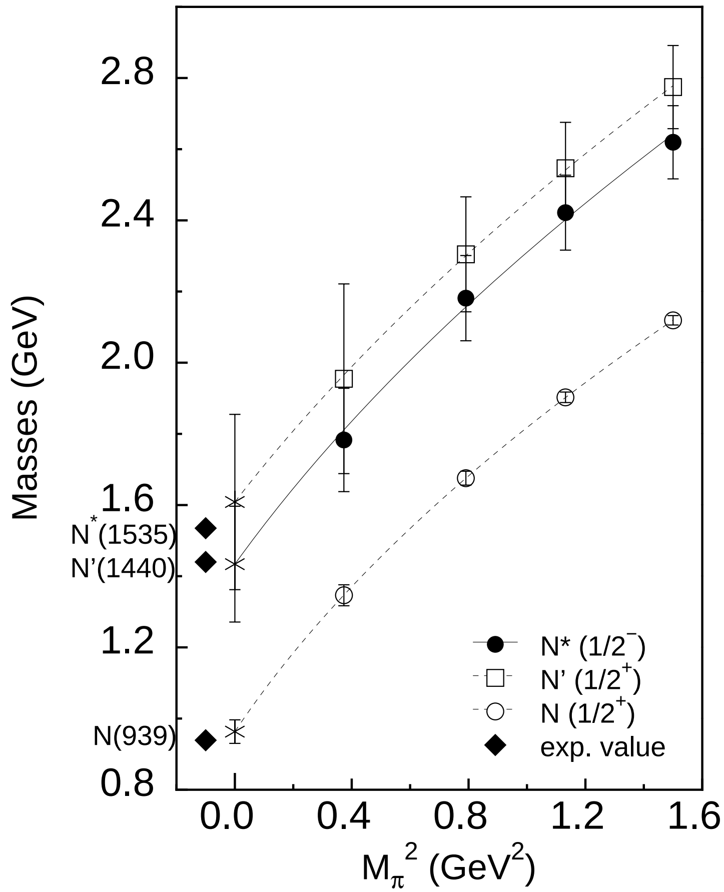
Moreover, we have the indirect evidence to identify the second peak with the Roper resonance.

Finite size effect on the spectral function



The finite size effect on the excited state cannot be neglected even in the heavy quark region where this effect on the ground state is small.
To remove this effect, we utilized the large size lattice ($La \sim 3.0$ fm).

Level order between N' and N^*



The masses of N' and N^* are almost comparable within the statistical error.

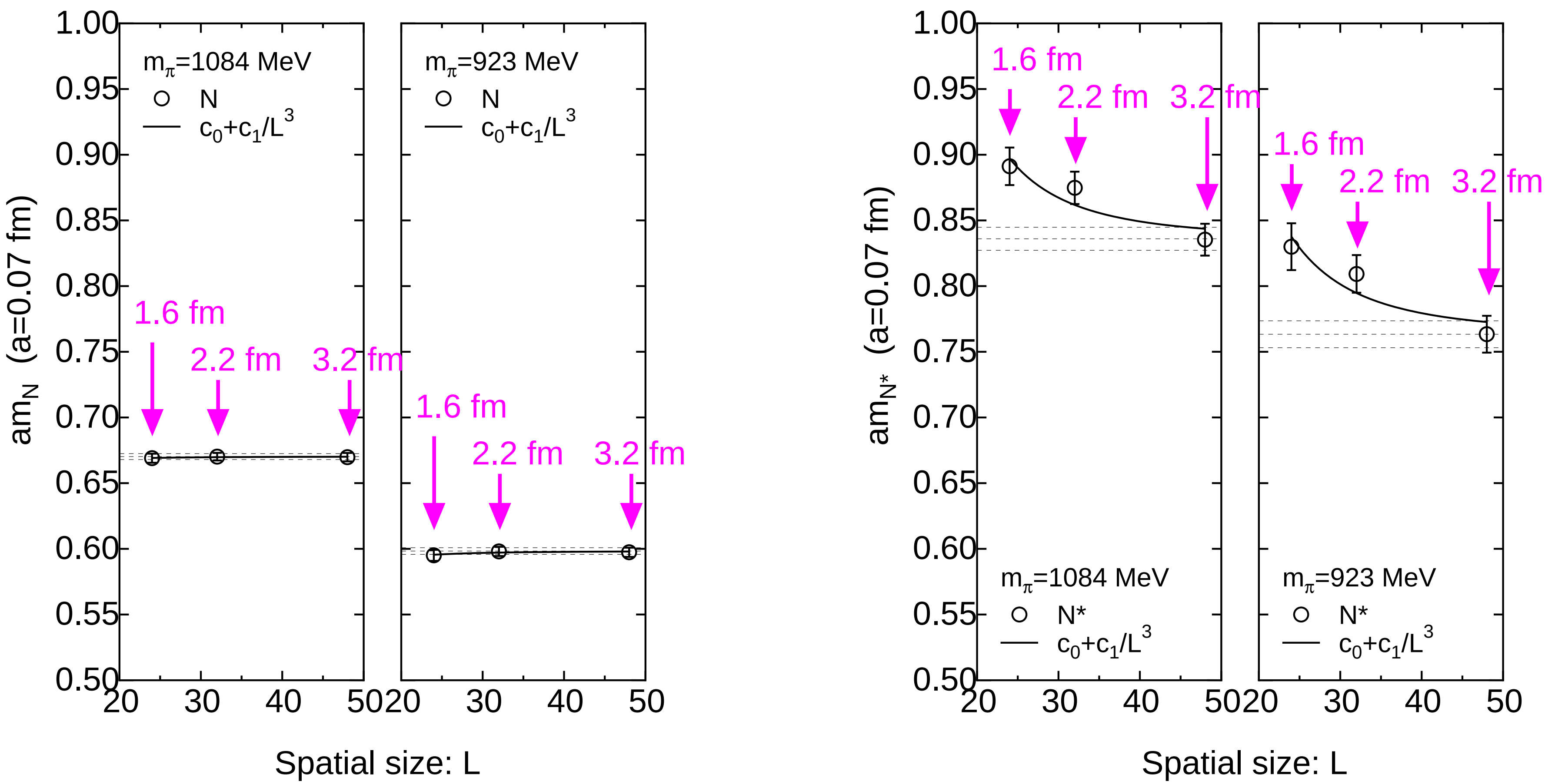
⇒ This feature is consistent with the experimental values.

⇒ On the other hands, this aspect is in contrast to the quark model prediction.

(For the chiral extrapolation,we utilize an empirical curve fit formula,

$$(aM_H)^2 = d_0 + d_2(aM_\pi)^2$$)

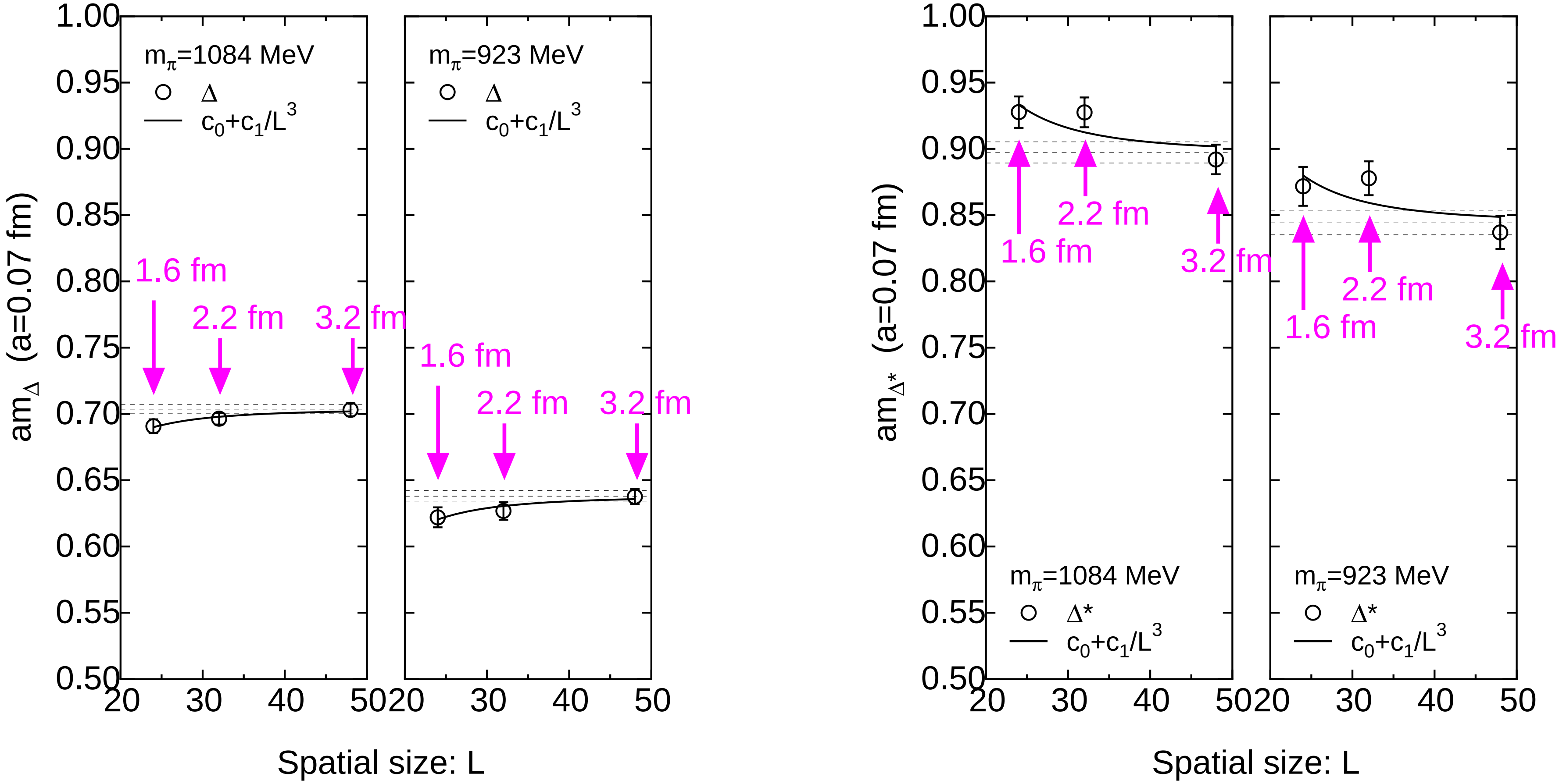
Finite size effect on nucleon and N^*



The finite size effect on the negative-parity nucleon is large even for the heavy quark region where this effect on the positive-parity nucleon can be neglected.

⇒ At least, the spatial size is required to be 3 fm to neglect this effect.

Finite size effect on $\Delta_{3/2}$ and $\Delta^*_{3/2}$



The finite size effect on these baryons is also large even for the heavy quark region. Moreover, on the positive-parity $\Delta_{3/2}$, we observe a peculiar finite-size behavior.

Brief summary of the f.s.e. on the nucleon and $\Delta_{3/2}$

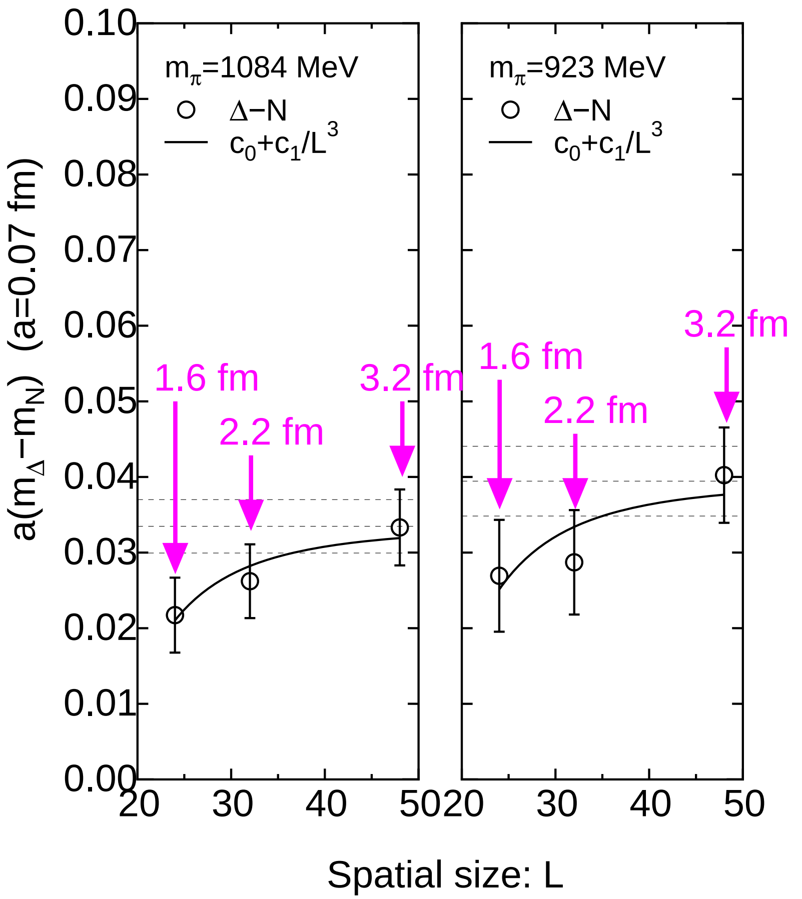
♠ The finite size effect on the excited baryons cannot be neglected even for the heavy quark region where this effect on the nucleon is small.
$\Rightarrow$ At least, the spatial size of lattice is required to be 3 fm.

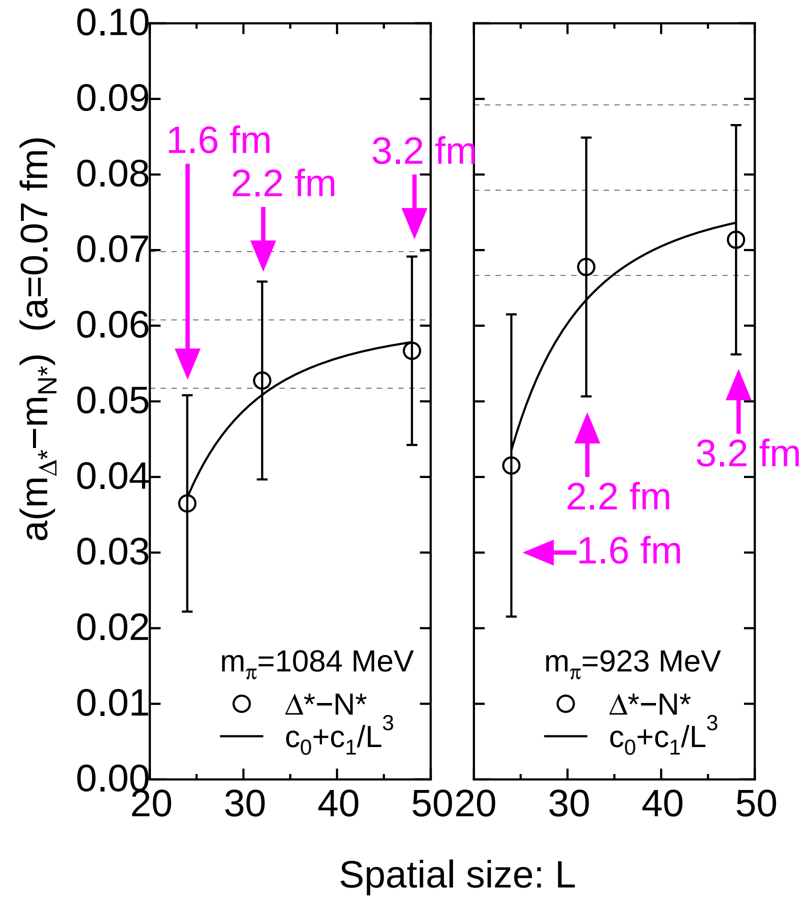
♠ Finite size behavior on the each hadrons $(\kappa = 0.1506)$

baryon	as increase L	$\sim (m_{1.6\,\mathrm{fm}} - m_\infty)/m_\infty$	$\sim (m_{3.2\,\mathrm{fm}} - m_\infty)/m_\infty$
N	$\rightarrow$	less than 1%	less than 1%
N^*	$\searrow$	11%	2%
$\Delta_{3/2}$	$\nearrow$	-3%	less than 1%
$\Delta^*_{3/2}$	$\searrow$	4%	less than 1%

· We observe a peculiar finite-size behavior on the positive-parity $\Delta_{3/2}$.
· The finite size behavior on the $\Delta^*_{3/2}$ is milder than that of N^*.

Finite size effect on hyperfine mass splittings





As the lattice sise increases, the hyperfine mass splitting also becomes large.
This feature can be seen in both the positive- and negative-parity channel.

Results on the Δ baryon spectra

baryon	(I, J^P)	our results [GeV] (r_0 input)	(M_ρ input)	physical state (status)	$SU(6) \otimes O(3)$ classification
N	$(\frac{1}{2}, \frac{1}{2}^+)$	0.968(23)	1.031(25)	$N(939)$ ****	[**56**,0^+]
N^*	$(\frac{1}{2}, \frac{1}{2}^-)$	1.555(97)	1.658(102)	$N(1535)$ S_{11} ****	[**70**,1^-]
$\Delta_{3/2}$	$(\frac{3}{2}, \frac{3}{2}^+)$	1.236(29)	1.320(31)	$\Delta(1232)$ P_{33} ****	[**56**,0^+]
$\Delta^*_{3/2}$	$(\frac{3}{2}, \frac{3}{2}^-)$	1.985(72)	2.114(75)	$\Delta(1700)$ D_{33} ****	[**70**,1^-]
$\Delta_{1/2}$	$(\frac{3}{2}, \frac{1}{2}^+)$	2.325(112)	2.478(118)	$\Delta(1910)$ P_{31} ****	[**56**,2^+] or [**70**,0^+]
$\Delta^*_{1/2}$	$(\frac{3}{2}, \frac{1}{2}^-)$	1.866(153)	1.987(165)	$\Delta(1620)$ S_{31} ****	[**70**,1^-]

Of course, the evaluated values should not be taken too seriously, since we utilize the quenched approximation and the relatively heavy quark masses ($M_\pi/M_\rho = 0.66 - 0.96$).

However, (i) the level ordering in Δ spectra ranked as four stars on the PDT is well reproduced

$$m_{\Delta_{3/2}} < m_{\Delta^*_{1/2}} \lesssim m_{\Delta^*_{3/2}} < m_{\Delta_{1/2}},$$

and (ii) $\Delta(1750)$ ($I = 3/2$ and $J^P = 3/2^+$, one star) cannot be seen in our data.

Summary 1

We performed systematic study for the mass spectra of the excited baryons from the quenched lattice QCD simulation.

- ♠ spin projection
- ♠ parity projection
- ♠ maximum entropy method (MEM)
- ♠ finite size effect
- ♠ chiral limit $(m_q \to 0)$

$\Rightarrow$ $N - N' - N^*$ spectra, $\Delta\ (J^P = 3/2^\pm,\ 1/2^\pm)$

(i) Spectral function of the nucleon

The MEM analysis enables us to separate the ground state, excited state.

(ii) Finite size effect

The finite size effect on the excited baryons cannot be neglected even for the heavy quark region where this effect on the nucleon is small.
$\Rightarrow$ At least, the spatial size of lattice is required to be 3 fm.

Summary 2

(iii) $N - N' - N^*$ spectra

$N' - N^*$ are almost comparable within statistical error.

⇒ Our result is consistent with the experimental values.

⇒ This aspect is in contrast to the prediction of quark model.

(iv) Δ baryon spectra

On our largest lattice ($La \simeq 3.2$ fm), our results reproduce several features seen in Particle Data Table.

· The level ordering in Δ spectra ranked as four stars on the PDT is well reproduced

$$m_{\Delta_{3/2}} < m_{\Delta^*_{1/2}} \lesssim m_{\Delta^*_{3/2}} < m_{\Delta_{1/2}},$$

· $\Delta(1750)$ ($I = 3/2$ and $J^P = 3/2^+$, one star) cannot be seen in our data.

Backup slide

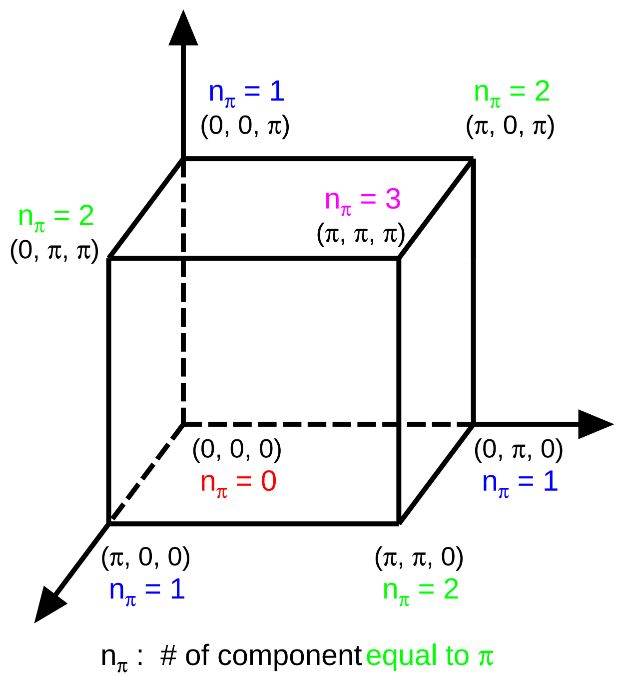
Fermion doubler

Dispersion relation for the free Wilson fermion :

$$\left[am + 2\sum_{\mu=1}^{4} \sin^2\left(\frac{p_\mu a}{2}\right)\right]^2 + \sin^2(p_\mu a) = 0$$

Without the Wilson term, in addtion to the physical solusion $pa = p^0 a$, there exist fifteen doubler solutions $pa = p^0 a + q_\pi$, where

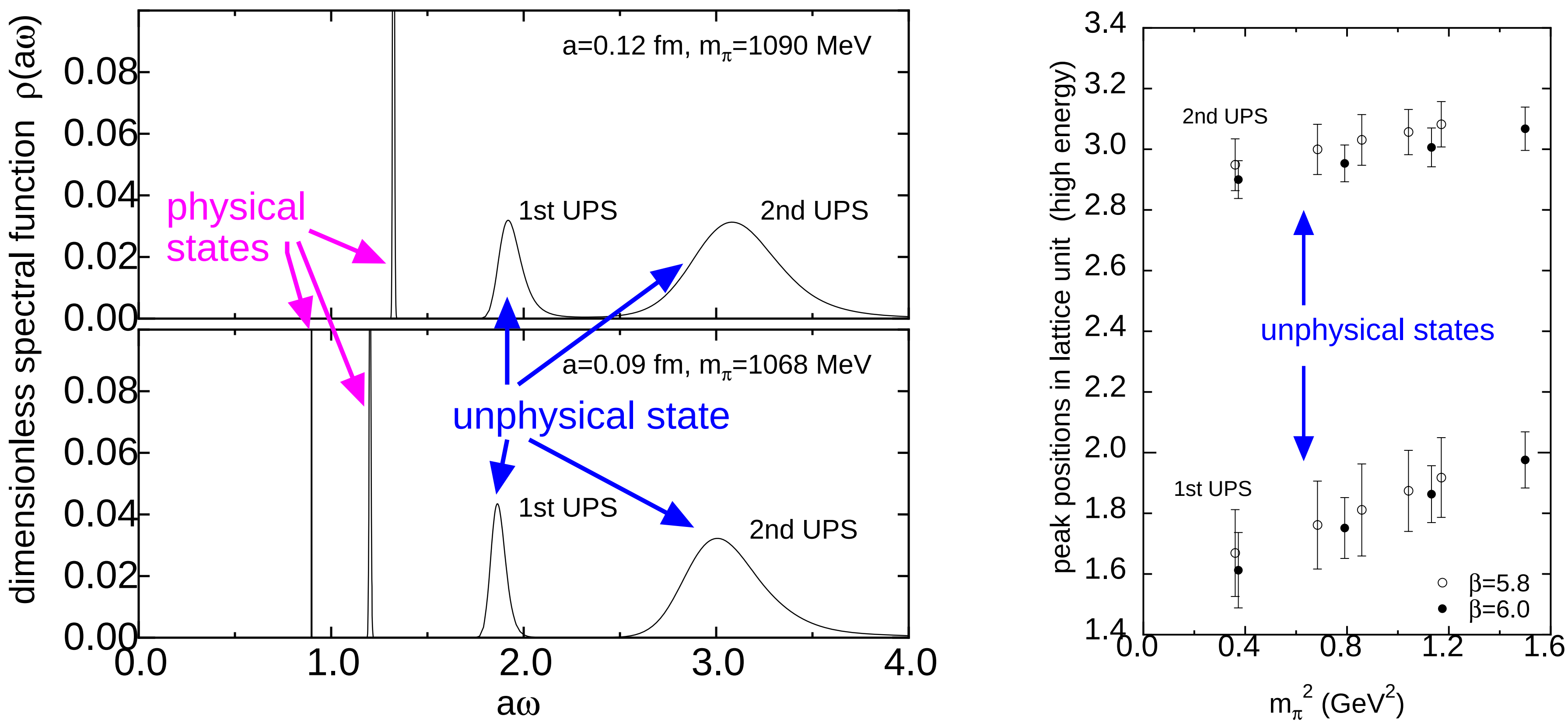
$$q_\pi = (\epsilon_1 \pi, \epsilon_2 \pi, \epsilon_3 \pi, \epsilon_4 \pi) \qquad (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 = 0 \text{ or } 1)$$



With the Wilson term, the doubler solution decouples to the physical state in the continuum limit

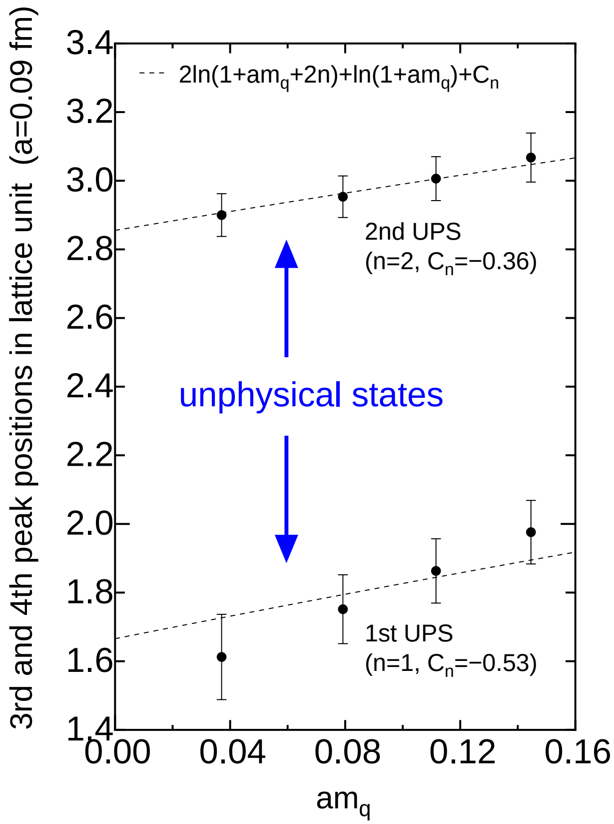
$$\begin{aligned} M &= \pm a^{-1} \ln(1 + am + 2n_\pi) \\ &\simeq \pm \left(m + \frac{2n_\pi}{a}\right) \quad (a \to 0). \end{aligned}$$

Bound state of doublers 1



In the lattice unit, the peak position in high energy region is almost insensitive to the change of the lattice spacing (coupling β).
In other words, in the physical unit, these peak positions diverge in the continuum limit and decouple to the physical states.

Bound state of doublers 2



These positions are consistent with the mass formula obtained on the assumption that these peaks represent the bound states of doublers.

$$am = 2\ln(1 + am_q + 2n) + \ln(1 + am_q) + C$$

doubler mass quark mass B.E.

The structure such as two quark and a doubler is not allowed due to the total momentum conservation.

Parity projection (couple to $+/-$ parity state)

Under the parity transformation ($\mathcal{P}q(\vec{x},t)\mathcal{P}^\dagger = +\gamma_4 q(\vec{x},t)$), the intrinsic parity of the local composite field $\mathcal{O}_B^{(\eta)}(x)$ is defined as

$$\mathcal{P}\mathcal{O}_B^{(\eta)}(\vec{x},t)\mathcal{P}^\dagger = \eta\ \gamma_4 \mathcal{O}_B^{(\eta)}(-\vec{x},t) \quad (\eta = \pm).$$

(Ex. $\mathcal{O}_N^{(+)} = \epsilon_{abc}[u_a^T C\gamma_5 d_b]u_c$, $\mathcal{O}_N^{(-)} = \epsilon_{abc}[u_a^T C\gamma_5 d_b]\gamma_5 u_c$).
Here, these operators satisfy the relation, $\mathcal{O}_B^{(+)}(x) = \gamma_5 \mathcal{O}_B^{(-)}(x)$.

Since $G^+(t) = -\gamma_5 G^-(t)\gamma_5$ is satisfied (where $G^\pm(t) = \langle 0|T\{\mathcal{O}_B^{(\pm)}(t)\bar{\mathcal{O}}_B^{(\pm)}(0)\}|0\rangle$), the baryonic operator couples to both the positive and negative parity state. Considering only the ground state in both parity channels, the baryonic propagator ($t > 0$) can be expressed as

$$\begin{aligned} G^+(t) &= A_+\left(\frac{1+\gamma_4}{2}\right)\mathrm{e}^{-M_+t} - A_-\left(\frac{1-\gamma_4}{2}\right)\mathrm{e}^{-M_-t} \\ G^-(t) &= A_-\left(\frac{1+\gamma_4}{2}\right)\mathrm{e}^{-M_-t} - A_+\left(\frac{1-\gamma_4}{2}\right)\mathrm{e}^{-M_+t} \end{aligned}$$

Spin projection

The composite operator of Δ baryon, $\Delta = \varepsilon_{abc}\left[u_a^T C\gamma_\mu u_b\right] u_c$, couples to both the spin-3/2 state ($\Delta(1232)$) and the spin-1/2 state ($\Delta(1910)$) ($1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$).

Using the spin-projection operator*

$$
\begin{aligned}
D(\hat{P}_{\frac{3}{2}})_{i\alpha,j\beta} &= \delta_{ij}\delta_{\alpha\beta} - \frac{1}{3}(\gamma_i\gamma_j)_{\alpha\beta}, \\
D(\hat{P}_{\frac{1}{2}})_{i\alpha,j\beta} &= \frac{1}{3}(\gamma_i\gamma_j)_{\alpha\beta},
\end{aligned}
$$

we can write the propagator of Δ as

$$
G^{\Delta}_{i\alpha,j\beta}(t) = G^{\Delta}_{3/2}\, D(\hat{P}_{\frac{3}{2}})_{i\alpha,j\beta} + G^{\Delta}_{1/2}\, D(\hat{P}_{\frac{1}{2}})_{i\alpha,j\beta}.
$$

*A. Aurilia and H. Umezawa, PR **182** (1969) 1682, *etc.*

Free-boundary condition in the quark level

$$G_B(t) = \sum_{\vec{x}} \mathrm{tr}_c\{S(x,0)\cdot S(x,0)\cdot S(x,0)\}$$

$$S(x,0) = \frac{1}{2}(S_P(x,0) + S_{AP}(x,0))$$

$$S_P(\vec{x},t;0) = S_P(\vec{x},t+T;0),\ \ S_{AP}(\vec{x},t;0) = -S_{AP}(\vec{x},t+T;0)$$

$$G_B(t) = \frac{1}{2}\left\{G_B^{\mathrm{p.b.c}}(t) + G_B^{\mathrm{a.p.b.c}}(t)\right\}$$

$$G_B^{\mathrm{p.b.c.}}(t) = \frac{1}{4}\sum_{\vec{x}} \mathrm{tr}_c\{S_{\mathrm{P}}\cdot S_{\mathrm{P}}\cdot S_{\mathrm{P}} + S_{\mathrm{P}}\cdot S_{\mathrm{AP}}\cdot S_{\mathrm{AP}} + S_{\mathrm{AP}}\cdot S_{\mathrm{P}}\cdot S_{\mathrm{AP}} + S_{\mathrm{AP}}\cdot S_{\mathrm{AP}}\cdot S_{\mathrm{P}}\}$$

$$G_B^{\mathrm{a.p.b.c.}}(t) = \frac{1}{4}\sum_{\vec{x}} \mathrm{tr}_c\{S_{\mathrm{AP}}\cdot S_{\mathrm{P}}\cdot S_{\mathrm{P}} + S_{\mathrm{P}}\cdot S_{\mathrm{AP}}\cdot S_{\mathrm{P}} + S_{\mathrm{P}}\cdot S_{\mathrm{P}}\cdot S_{\mathrm{AP}} + S_{\mathrm{AP}}\cdot S_{\mathrm{AP}}\cdot S_{\mathrm{AP}}\}$$

$$G_B^{\mathrm{p.b.c.}}(t) = G_B^{\mathrm{p.b.c.}}(t+T),\ \ G_B^{\mathrm{a.p.b.c.}}(t) = -G_B^{\mathrm{a.p.b.c.}}(t+T)$$

The construction of the free-boundary condition in quark level automatically realizes the free-boundary situation in the hadron level.