# Nucleon Form Factors from $N_\mathrm{f} = 2$ Clover Fermions

QCDSF Collaboration

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# Introduction

- Electromagnetic form factors ☞ hints at internal structure of nucleon
- Recent interest triggered by JLAB results for

$$\mu^{(p)} G_e^{(p)}(q^2)/G_m^{(p)}(q^2)$$

- We compute $G_e$ and $G_m$ from

$$\begin{aligned} G_e(q^2) &= F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2) \\ G_m(q^2) &= F_1(q^2) + F_2(q^2) \end{aligned}$$

- Key issue: $q^2$ scaling

# Report on Work in Progress

- The form factors are calculated on dynamical configurations with $N_\mathrm{f} = 2$ O(a)-improved Wilson fermions

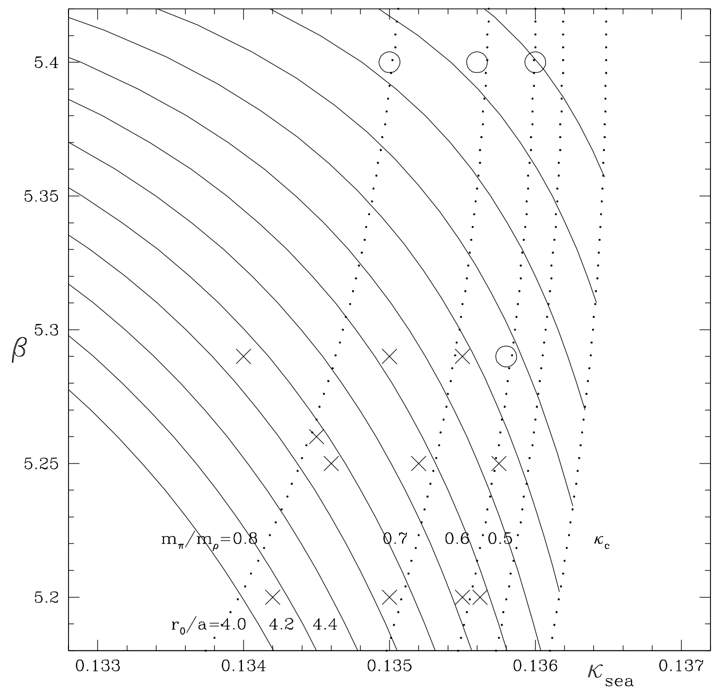
- Renormalisation factors are determined non-perturbatively
  ☞ Reduction of discretisation effects

- Results are for various lattice spacings available
  ☞ Check for discretisation effects

- Simulations cover larger range of sea quark masses and some partially quenched results are available
  ☞ Investigation of quark mass dependence
  ☞ Check for unquenching effects

# Simulation Details

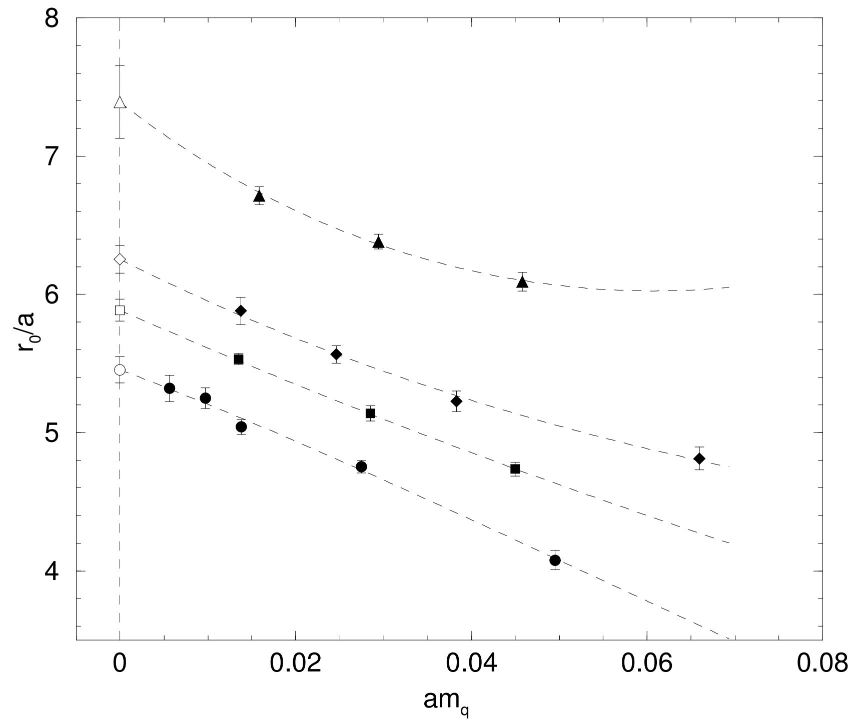
Configurations with $N_{\mathrm{f}} = 2$ O(a)-improved dynamical quarks generated by UKQCD+QCDSF:




$$m_{\mathrm{PS,sea}} = 590, ..., 1170 \text{ MeV} \qquad a = 0.07, ..., 0.11 \text{ fm}$$
$$m_{\mathrm{PS,val}} = 470, ..., 1140 \text{ MeV} \qquad V = 1.4, ..., 2.0 \text{ fm}$$

# Scale Definition

- $r_0$ can be determined with good precision on the lattice
  → Good for scaling lattice results
- Experimental value less well known
  → Use nucleon mass for conversion into physical units
  → $r_0 = 0.467$ fm



# Nucleon Form Factors

$$\langle p', s' | J^\mu | p, s \rangle = \overline{\psi}(p', s') \left[ \gamma_\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2(q^2) \right] \psi(p, s)$$

- Momentum transfer is defined as $q = p' - p$

- We will consider

  Proton form factors: $\frac{2}{3}\overline{u}\gamma^\mu u - \frac{1}{3}\overline{d}\gamma^\mu d$

  Isovector form factors: $\overline{u}\gamma^\mu u - \overline{d}\gamma^\mu d$ $\quad\rightarrow$ Disconnected terms cancel

# Matrix Elements on the Lattice

$$R(t,\tau,\vec{p}\,',\vec{p}) = \frac{C_3(t,\tau,\vec{p}\,',\vec{p})}{C_2(t,\vec{p}\,')} \times \left[\frac{C_2(\tau,\vec{p}\,')C_2(t,\vec{p}\,')C_2(t-\tau,\vec{p})}{C_2(\tau,\vec{p})C_2(t,\vec{p})C_2(t-\tau,\vec{p}\,')}\right]^{1/2}$$

where

$$C_2(t,\vec{p}) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p})\bar{B}_\beta(0,\vec{p})\rangle$$

and

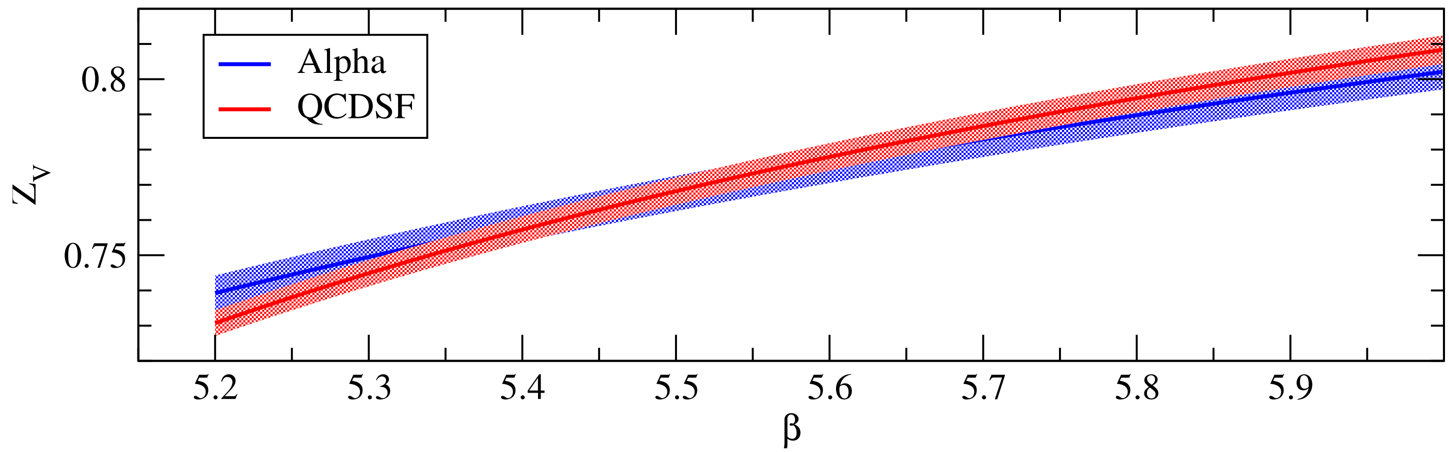
$$C_3(t,\tau,\vec{p}\,',\vec{p}) = \sum_{\alpha\beta} \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}\,')\mathcal{O}(\tau)\bar{B}_\beta(0,\vec{p})\rangle$$

We use the local vector current: $\overline{\psi}(x)\,\gamma_\mu\,\psi(x)$

# Renormalisation and Improvement

$$V_\mu = Z_V(1 + b_V a m_q)\left[\bar{\psi}\gamma_\mu\psi + \mathrm{i}c_V a\partial_\lambda(\bar{\psi}\sigma_{\mu\lambda}\psi)\right]$$

- Demand same behaviour for conserved and local vector current
  ➔ non-perturbative determination of $Z_V$ and $b_V$




- $c_V$ known only perturbatively ➔ neglected here

# Momenta and Polarisations

- 3 initial state momentum:

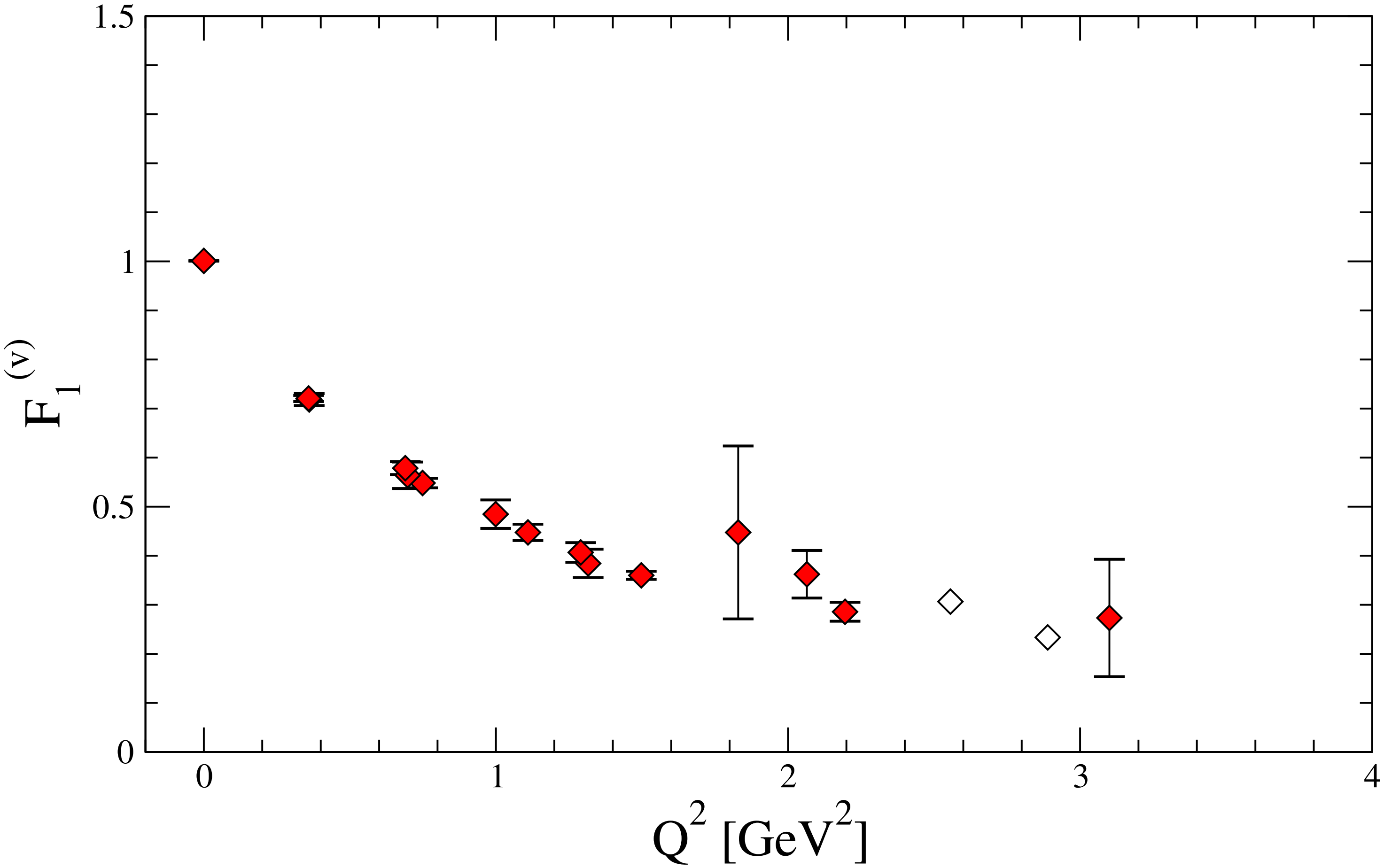
$$\frac{L}{2\pi}\vec{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

- 3 choices for polarisations:

$$\begin{aligned} \Gamma &= \frac{1}{2}(1+\gamma_4) \\ \Gamma &= \frac{1}{2}(1+\gamma_4)\mathrm{i}\gamma_5\gamma_1 \\ \Gamma &= \frac{1}{2}(1+\gamma_4)\mathrm{i}\gamma_5\gamma_2 \end{aligned}$$

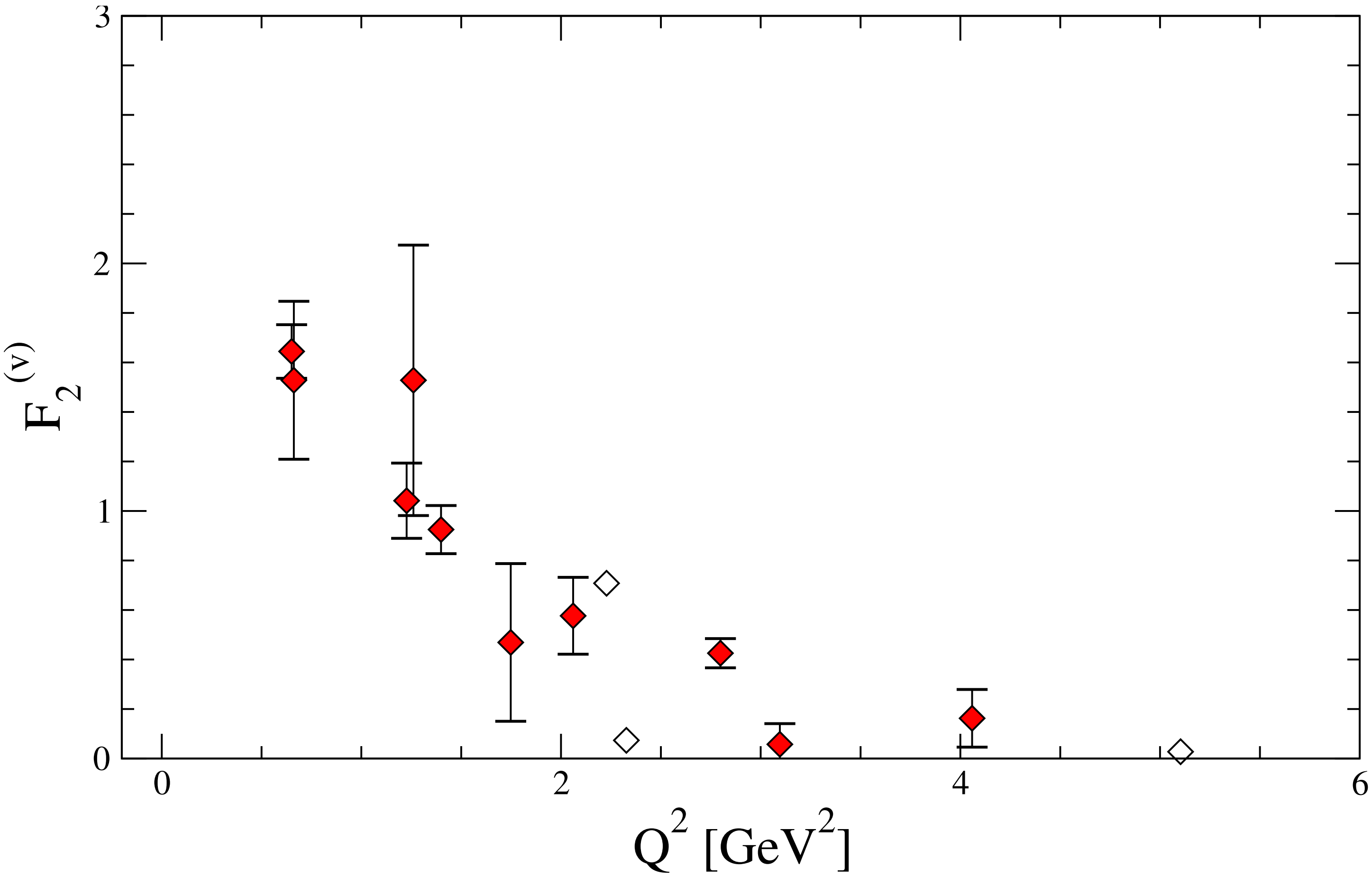
- 17 different choices of $\vec{q} = \vec{p}\,' - \vec{p}$

# Example for some nice data ...




$\beta = 5.25$, $\kappa_{\rm sea} = 0.13575$, $V = 24^3 \times 48$

# Example for some less nice data ...




$\beta = 5.20$, $\kappa_{\text{sea}} = 0.13550$, $V = 16^3 \times 32$

## $q^2$ Scaling of $F_1$ and $F_2$

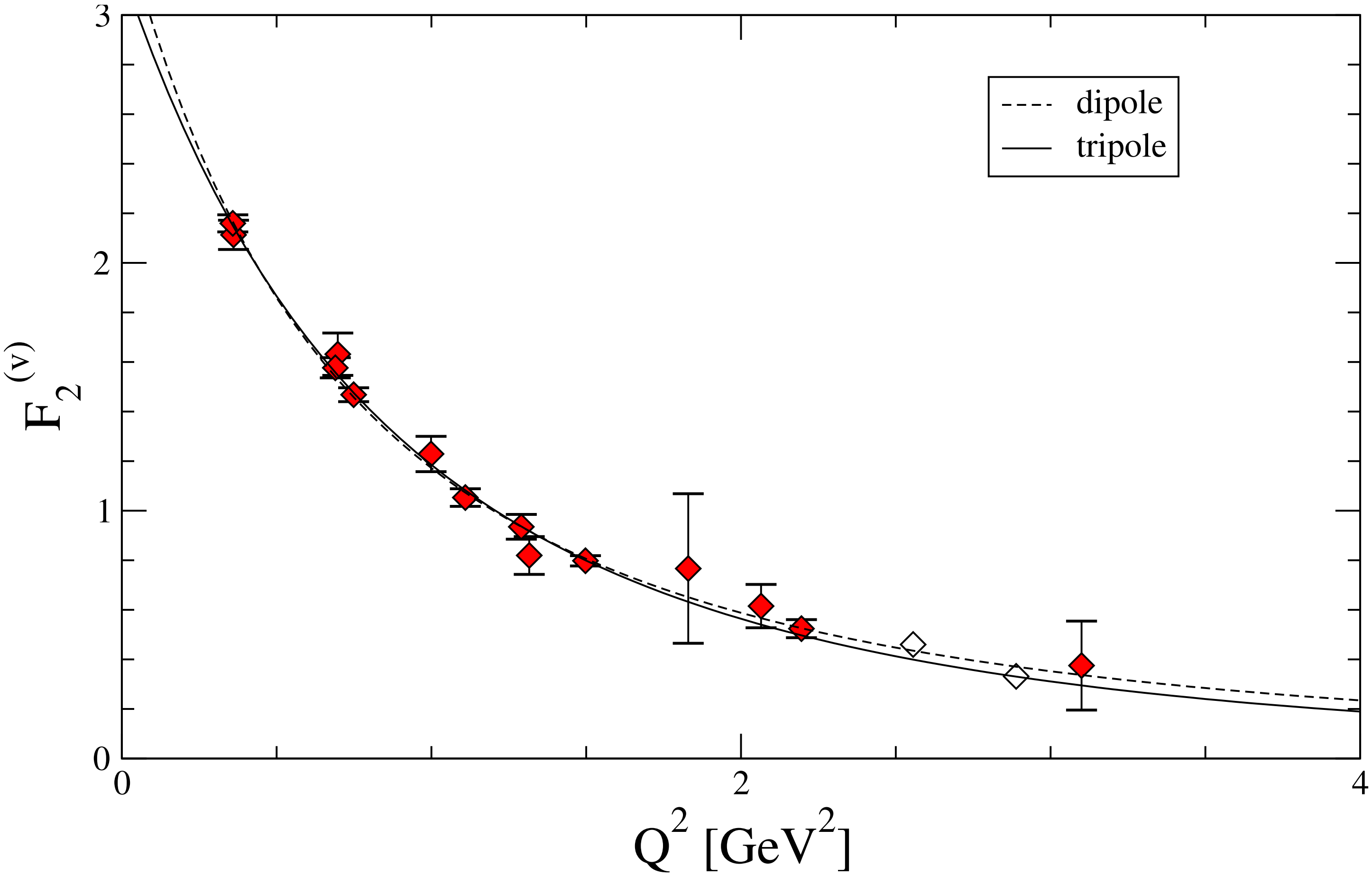
Naive expectation from dimensional counting:

$$F_1 \propto \frac{1}{Q^4}$$
$$F_2 \propto \frac{1}{Q^6}$$

and therefore

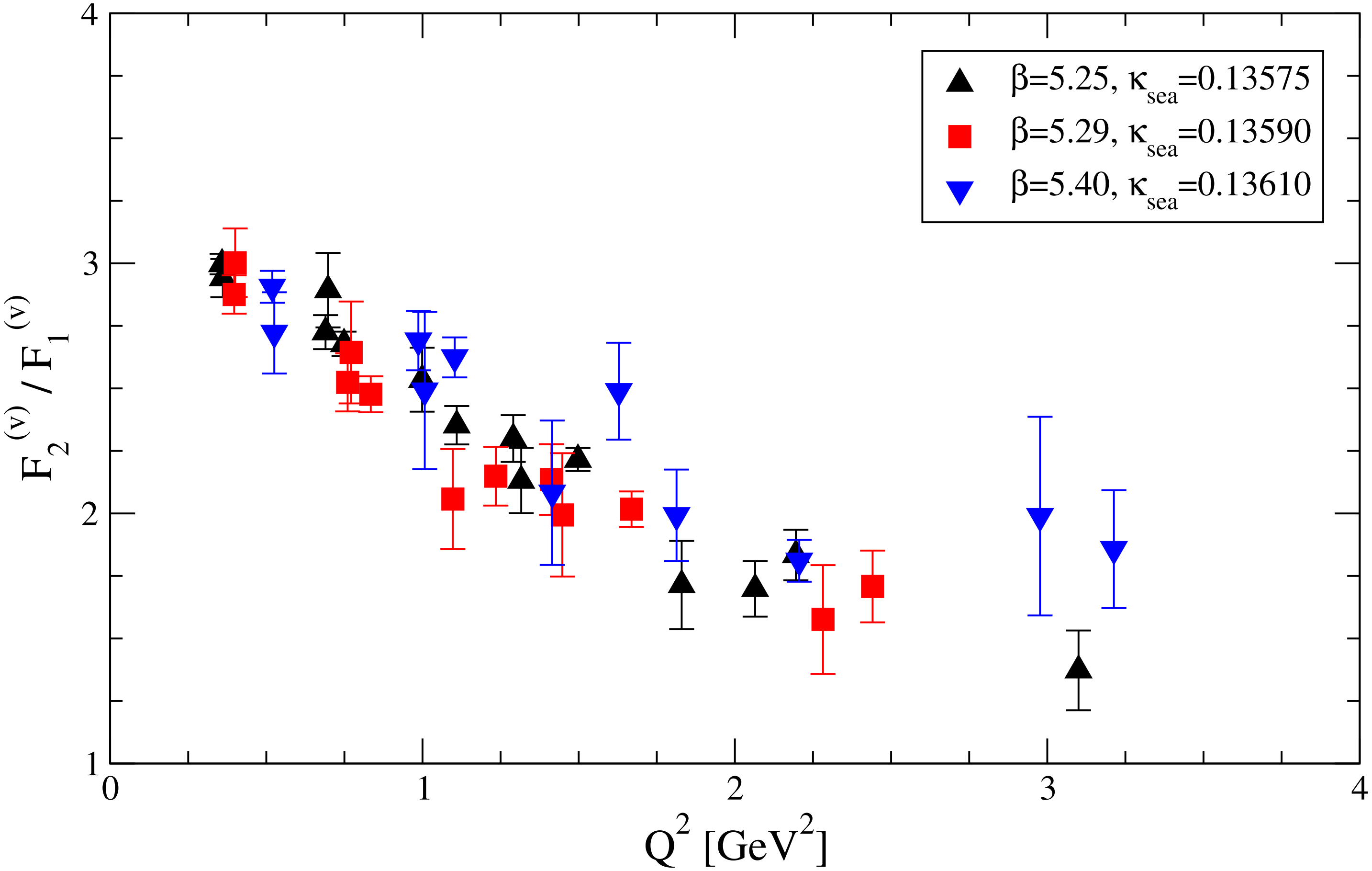
$$Q^2 \frac{F_2}{F_1} \propto \text{const}$$

# Test Scaling of $F_2^{(v)}$




$\beta = 5.25$, $\kappa_{\text{sea}} = 0.13575$, $V = 24^3 \times 48$

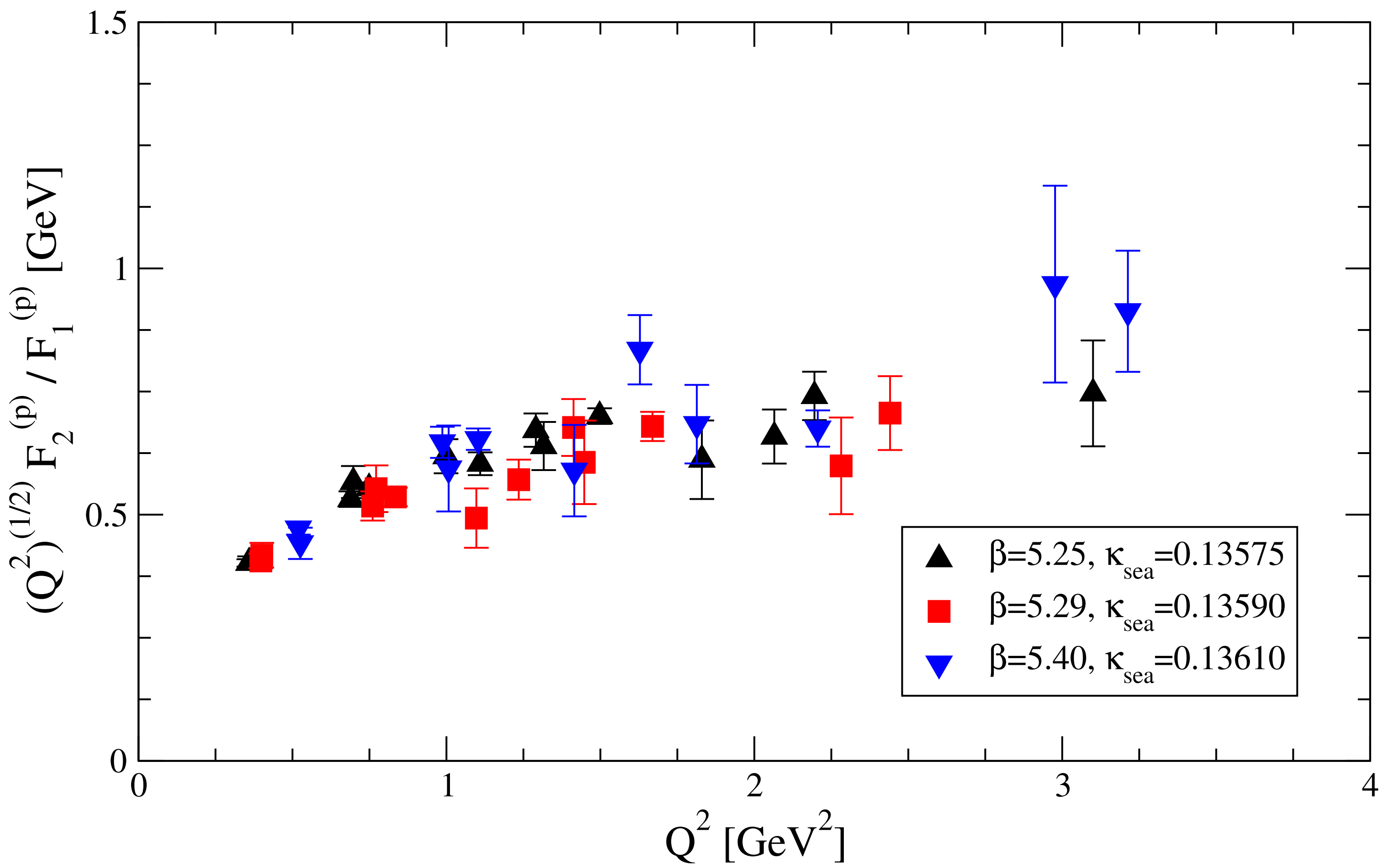
# Test Scaling of $F_2^{(v)}/F_1^{(v)}$




$m_{\rm PS} \approx 600$ MeV, $a = 0.084, \ldots, 0.070$ fm

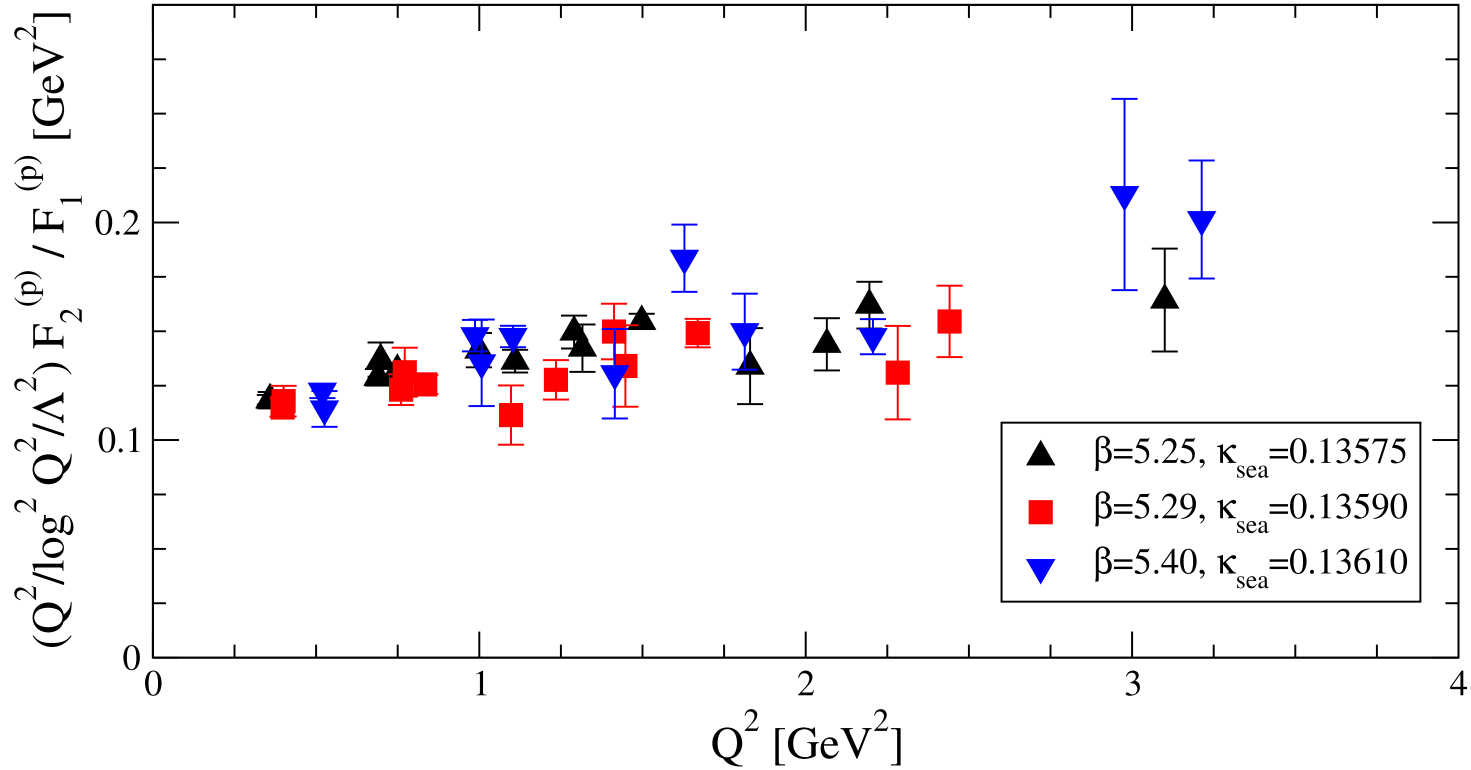
# Test Scaling of $F_2^{(p)}/F_1^{(p)}$ (1)

Experimental data suggest $1/\sqrt{Q^2}$ scaling:



# Test Scaling of $F_2^{(p)}/F_1^{(p)}$ (2)

Perturbative QCD: asymptotic scaling $\sim \log(Q^2/\Lambda^2)/Q^2$ [Belitsky et al., 2003]




$\Lambda = 200$ MeV, $m_{\rm PS} \approx 600$ MeV

# Scaling Ansatz

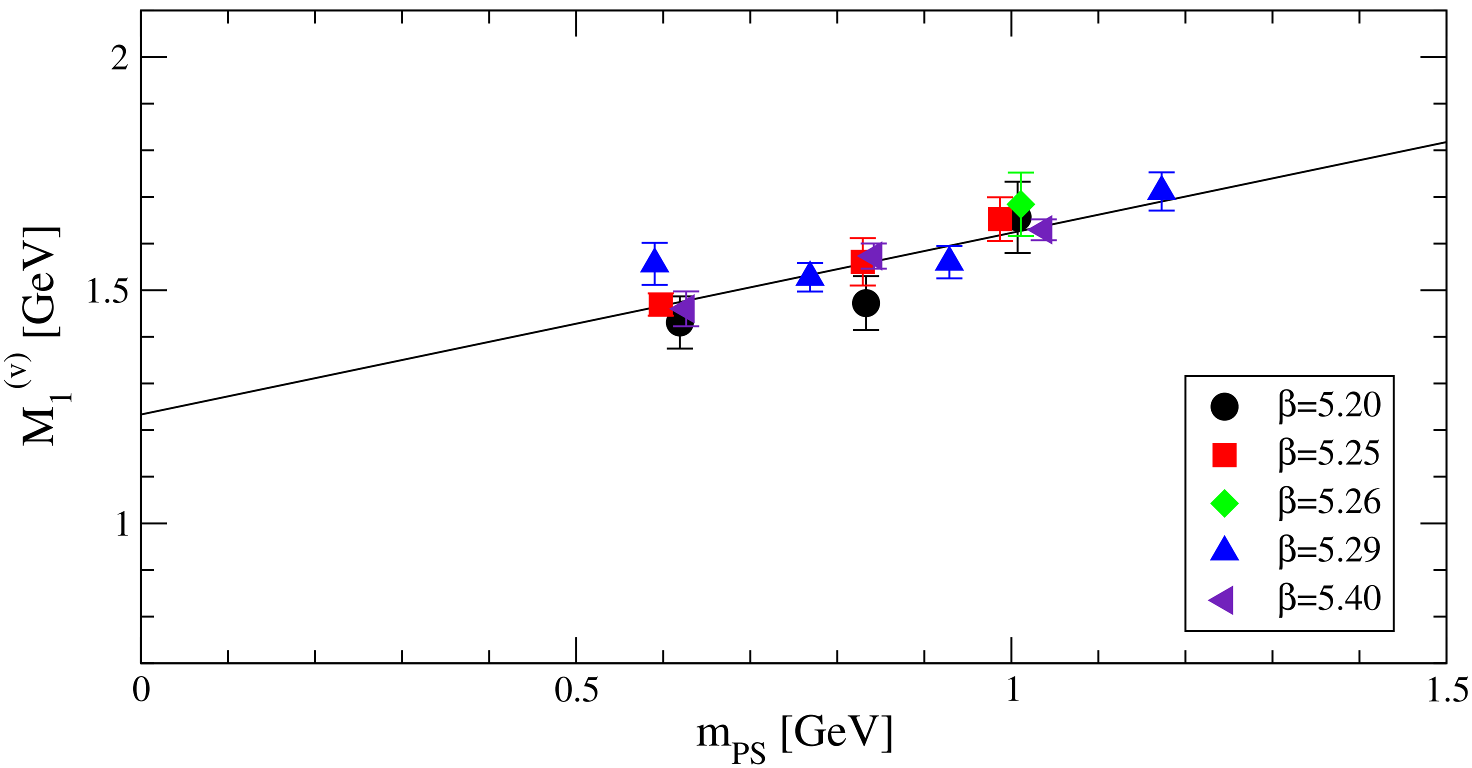
- Dipole for $F_1$:

$$F_1(q^2) = \frac{A_1}{(1 - q^2/M_1^2)^2}$$

- Tripole for $F_2$:

$$F_2(q^2) = \frac{A_2}{(1 - q^2/M_2^2)^3}$$

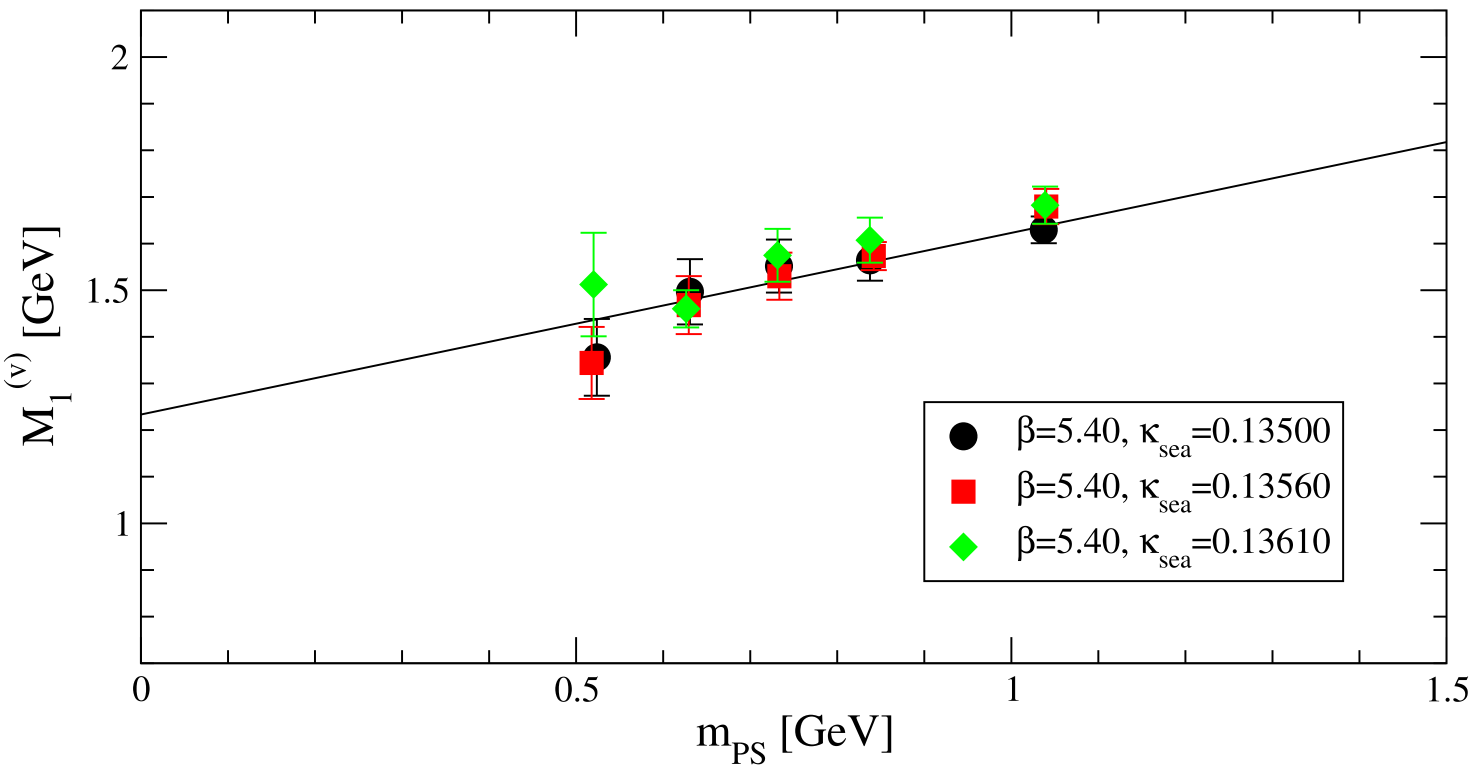
## $F_1$ Dipole Masses




☞ Discretisation errors small

☞ Extrapolation linear in $m_{\mathrm{PS}}$

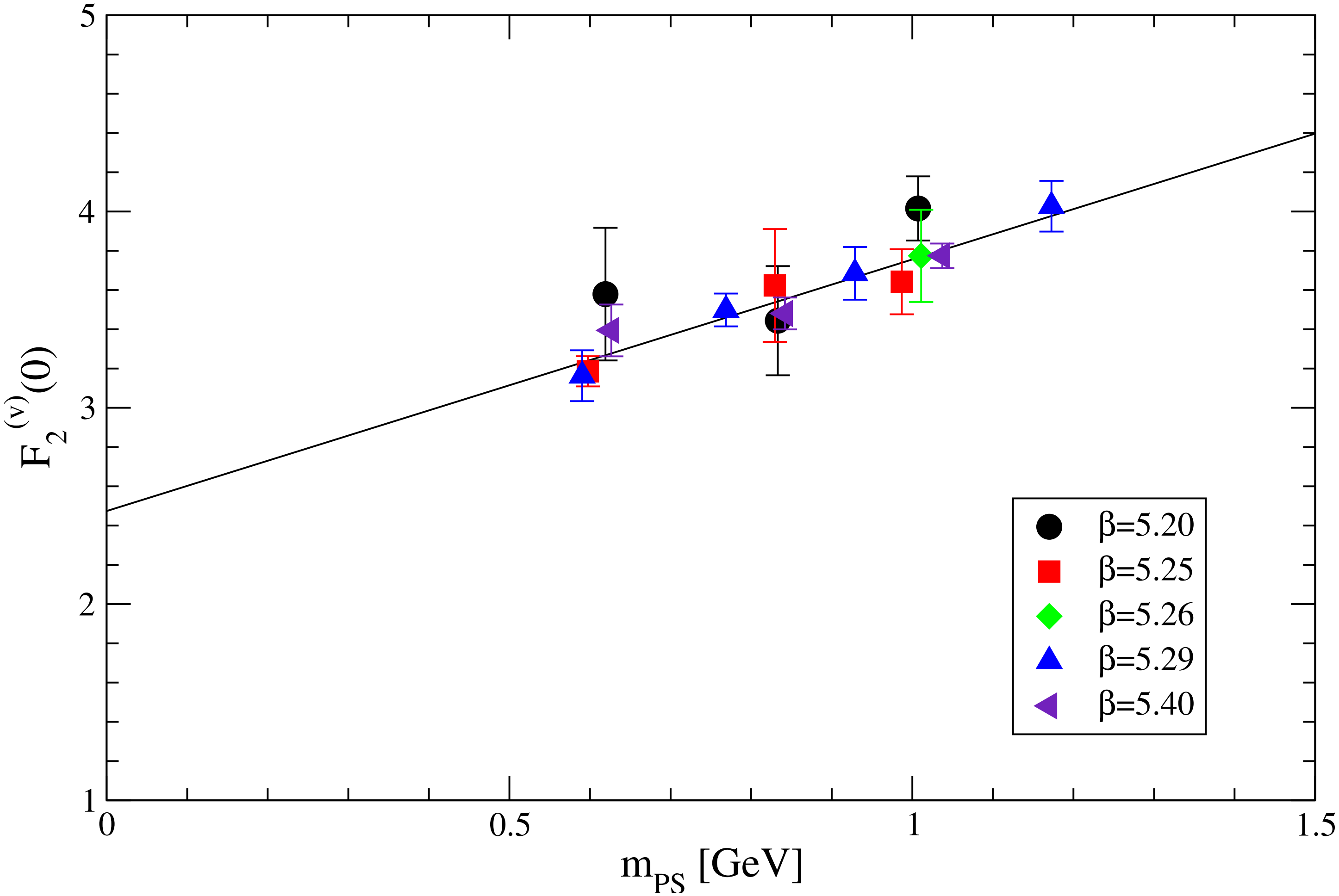
# Unquenching Effects



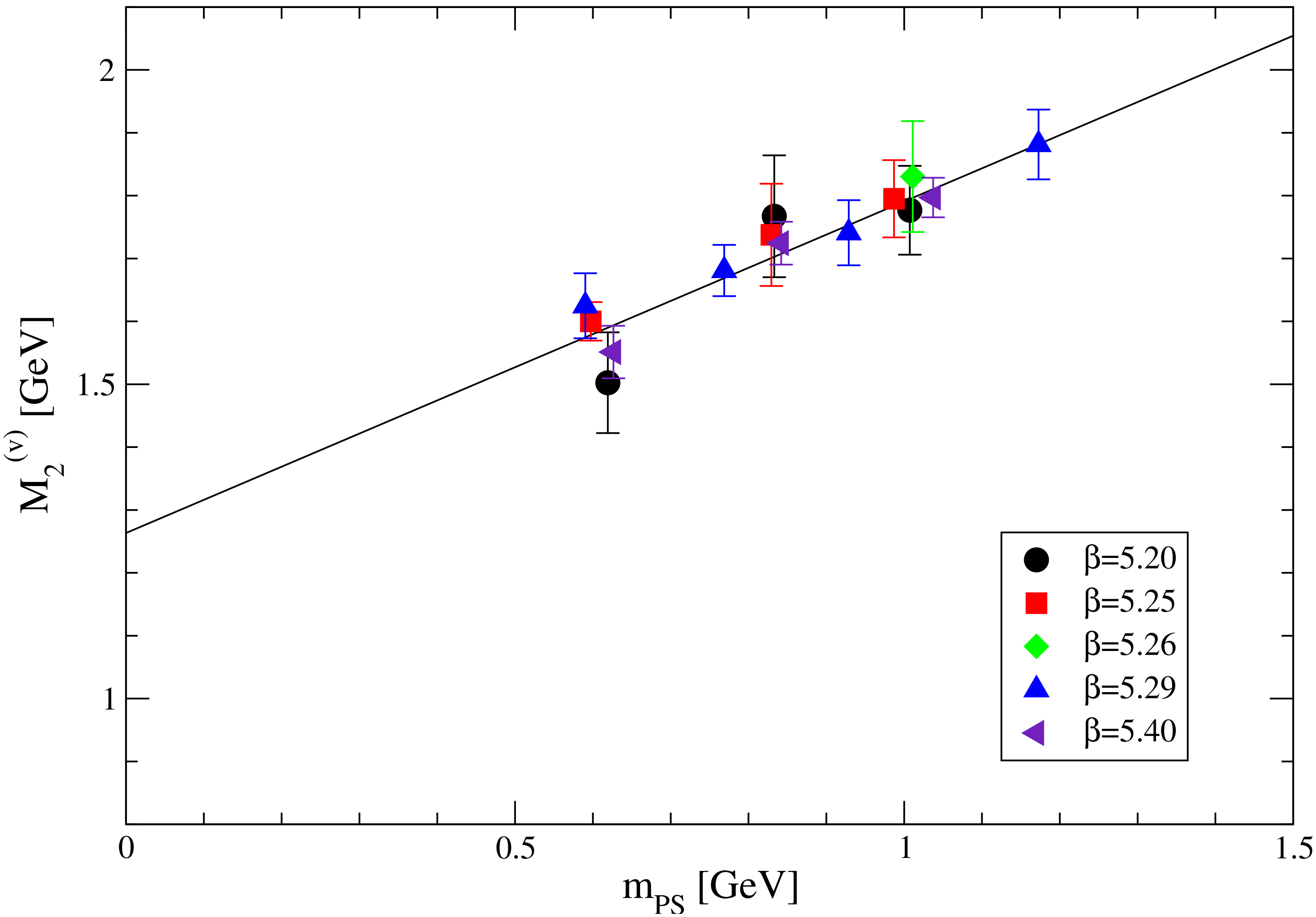

☞ Line from fit to unquenched data

☞ No significant unquenching effects

# $F_2^{(v)}$ Tripole Ansatz (1)



## $F_2^{(v)}$ Tripole Ansatz (2)



# Form Factor Radii and Magnetic Moment

Definitions:

- Form factor radii $r_i$:

$$F_i(q^2) = F_i(0)\left[1 + \frac{1}{6}\,\mathbf{r_i^2} q^2 + \mathcal{O}(q^4)\right]$$

- Magnetic moment $\mu$ / anomalous magnetic moment $\kappa$:

$$\mu = 1 + \kappa = G_m(0)$$

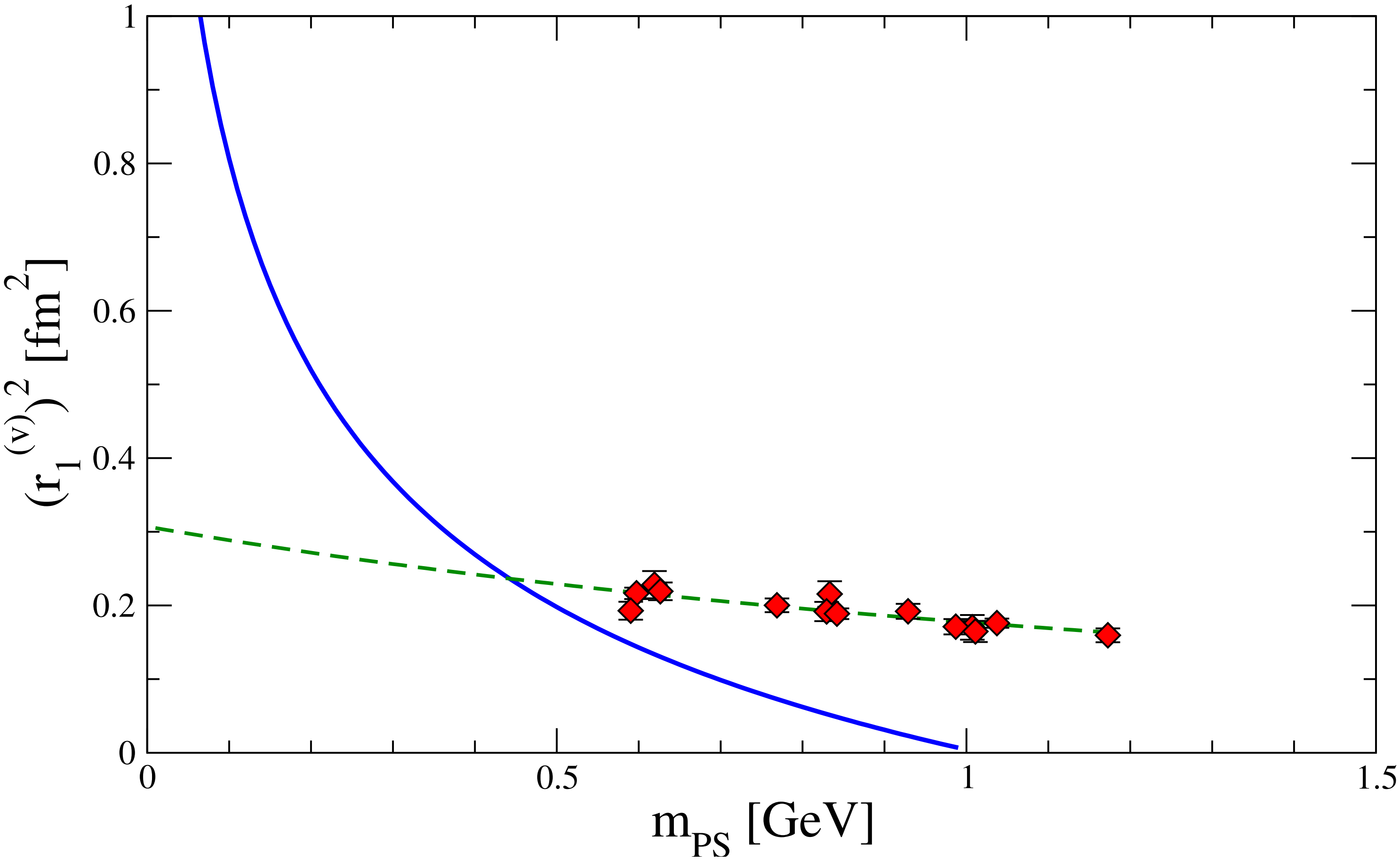
# ChEFT Result for $[r_1^{(v)}]^2$

[Hemmert and Weise, 2002; QCDSF 2003]

$$\left(r_1^{(v)}\right)^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7{g_A}^2 + \left(10g_A^2 + 2\right)\log\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]\right\}$$

$$+\frac{{c_A}^2}{54\pi^2 F_\pi^2}\left\{26+30\log\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]+30\frac{\Delta}{\sqrt{\Delta^2 - m_{\mathrm{PS}}^2}}\log\left[\frac{\Delta}{m_{\mathrm{PS}}} + \sqrt{\frac{\Delta^2}{m_{\mathrm{PS}}^2} - 1}\right]\right\}.$$

# $[r_1^{(v)}]^2$: Comparison ChEFT vs. Lattice



## ChEFT Result for $[r_2^{(v)}]^2$

$$\left(r_2^{(v)}\right)^2 = \frac{{g_A}^2 M_N}{8{F_\pi}^2 \kappa^{(v)}(m_{\mathrm{PS}})\pi m_{\mathrm{PS}}} +$$

$$\frac{{c_A}^2 M_N}{9F_\pi^2 \kappa^{(v)}(m_{\mathrm{PS}})\pi^2\sqrt{\Delta^2 - m_\pi^2}} \log\left[\frac{\Delta}{m_{\mathrm{PS}}} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right] + \frac{24 M_N}{\kappa^{(v)}(m_{\mathrm{PS}})} B_{c2}\,.$$

# ChEFT Result for $\kappa^{(v)}$

$$\kappa^{(v)}(m_{\rm PS}) = \kappa^{(v)0} - \frac{{g_A}^2\, m_{\rm PS} M_N}{4\pi {F_\pi}^2} +$$

$$\frac{2{c_A}^2 \Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1-\frac{m_{\rm PS}^2}{\Delta^2}}\log R(m_{\rm PS}) + \log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$

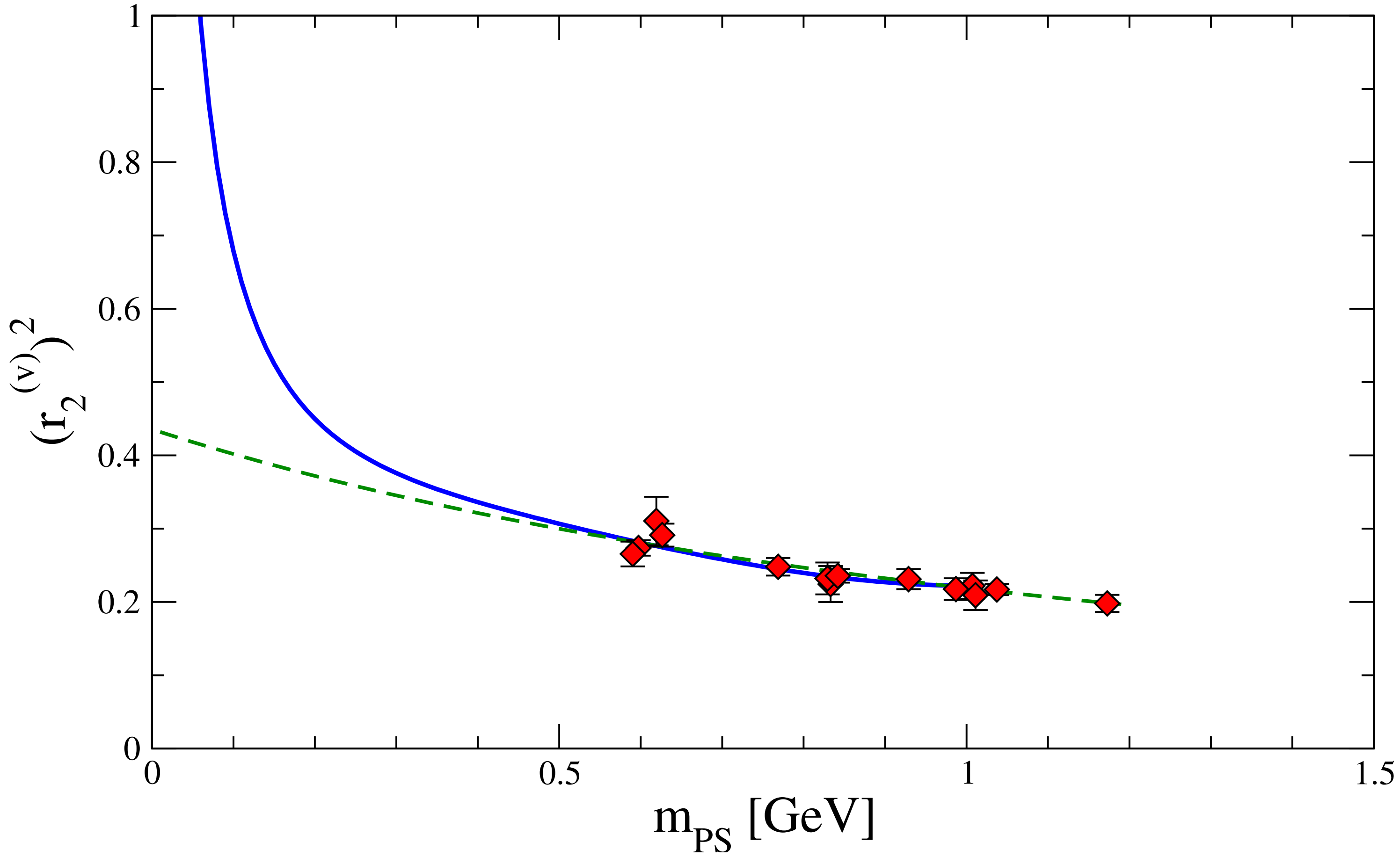
$$- 8E_1^{(r)}(\lambda) M_N m_{\rm PS}^2 + \frac{4c_A c_V g_A M_N m_{\rm PS}^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_{\rm PS}^3}{27\pi F_\pi^2 \Delta}$$

$$-\frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1-\frac{m_{\rm PS}^2}{\Delta^2}\right)^{3/2}\log R(m_{\rm PS}) + \left(1-\frac{3m_{\rm PS}^2}{2\Delta^2}\right)\log\left[\frac{m_{\rm PS}}{2\Delta}\right]\right\}$$

where $R(m) = \frac{\Delta}{m} + \sqrt{\frac{\Delta^2}{m^2} - 1}$

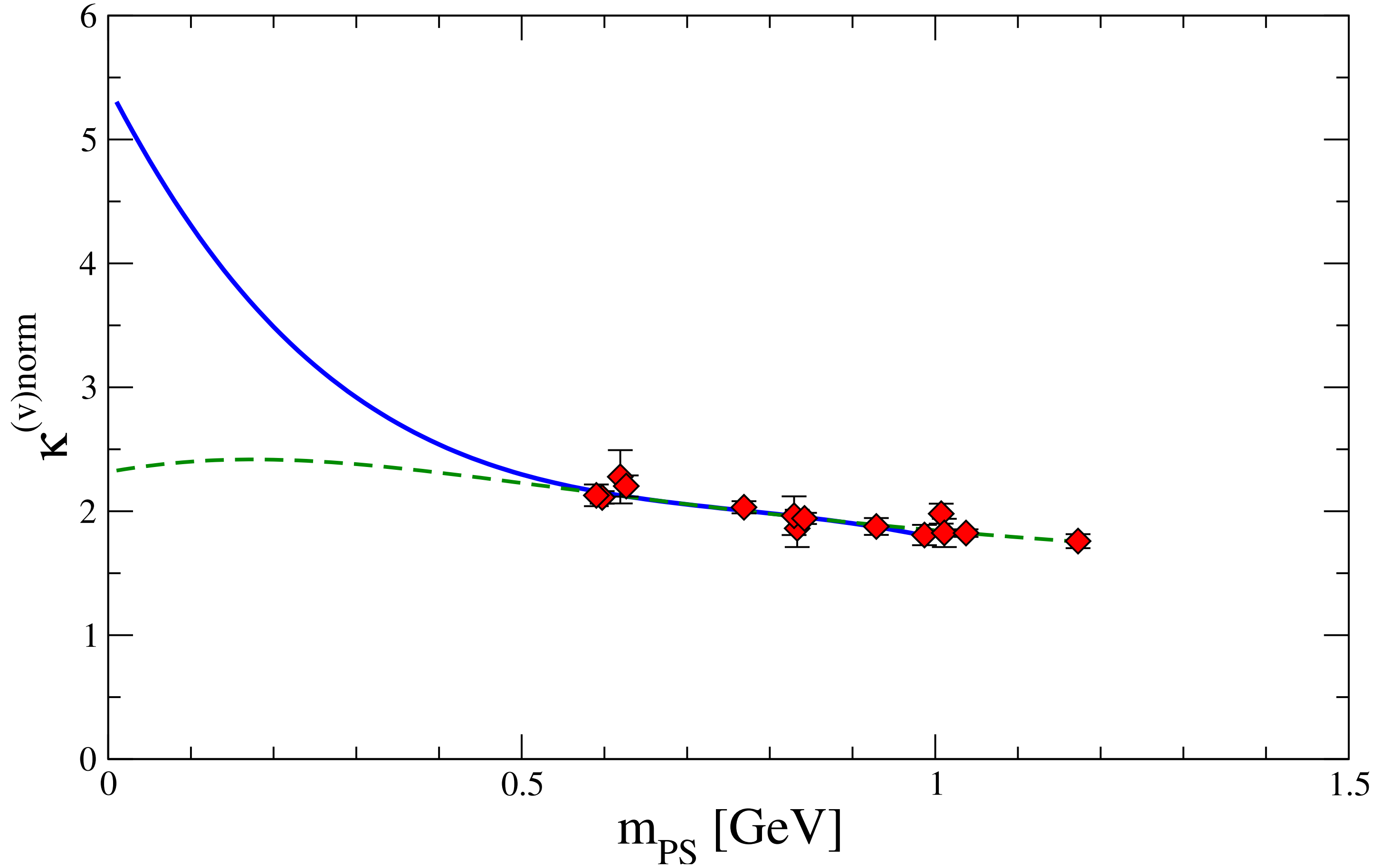
## $[r_2^{(v)}]^2$: Comparison ChEFT vs. Lattice

☞ Joined fit to $[r_2^{(v)}]^2$ and $\kappa^{(v)}$:



# $\kappa^{(v)}$: Comparison ChEFT vs. Lattice

$$\kappa^{(v)\mathrm{norm}} = \kappa^{(v)}\, m_{\mathrm{N}}(m_\pi)/m_{\mathrm{N}}(m_{\mathrm{PS}})$$

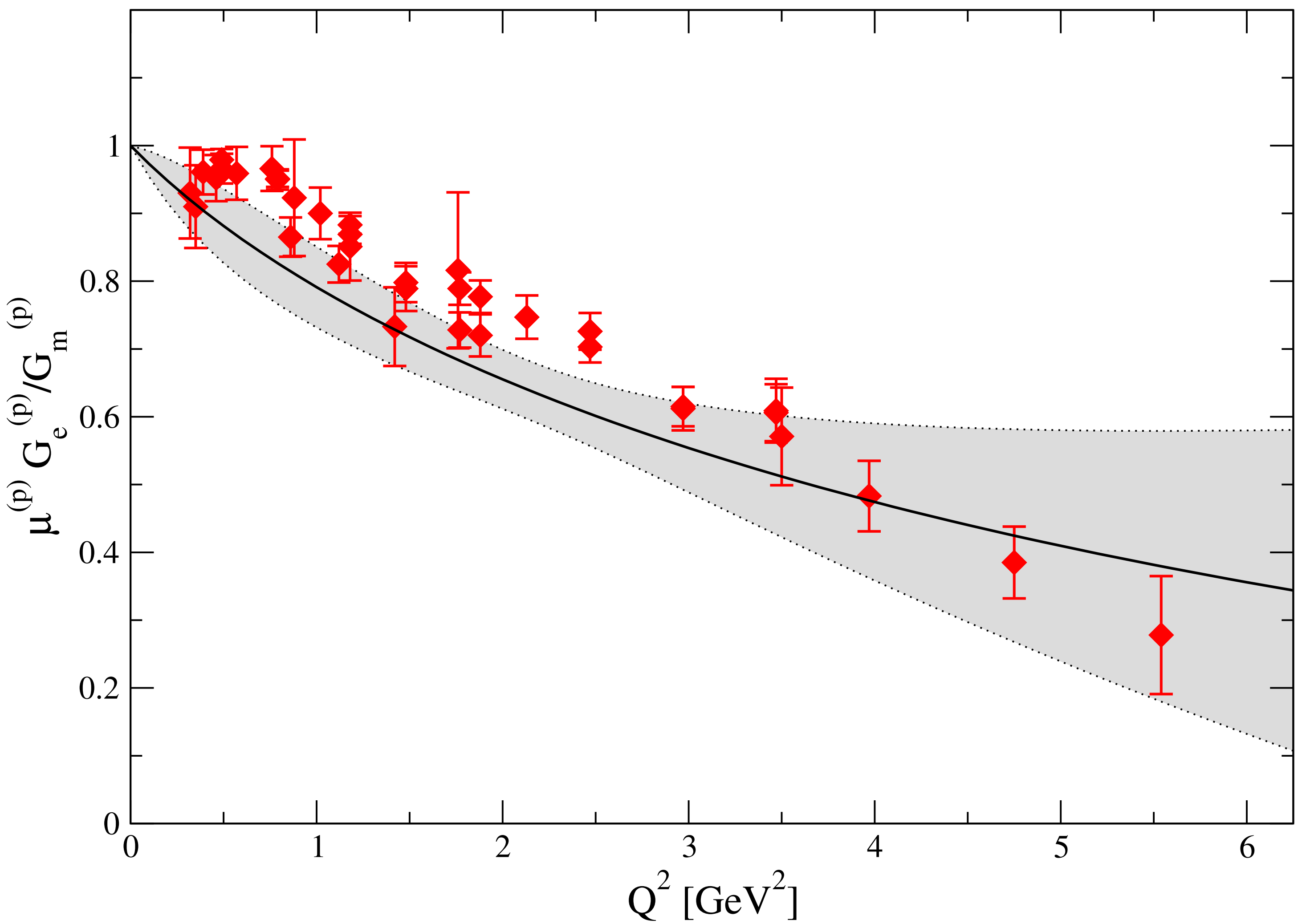


# Calculation of $\mu^{(p)} G_e^{(p)}(q^2) / G_m^{(p)}(q^2)$

- Assume dipole (tripole) scaling to fit $\frac{2}{3}u - \frac{1}{3}d$ data for $F_1$ ($F_2$)
- Perform (naive) chiral extrapolation of $M_1$, $F_2(0)$ and $M_2$
- Calculate $\mu^{(p)}$, $G_e^{(p)}(q^2)$ and $G_m^{(p)}(q^2)$ in the chiral limit using:

$$
\begin{aligned}
G_e(q^2) &= F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2) \\
G_m(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$

# Comparison with JLAB Data



# Conclusions

- ❑ We presented initial results for the electromagnetic form factors from full QCD on the lattice using O(a)-improved Wilson fermions
- ❑ With current data it is possible to
  - Explore quark mass dependency
  - Check for unquenching effects
  - Check for discretisation effects
- ❑ We find good agreement with experimental data assuming $F_1$ ($F_2$) scaling as a dipole (tripole)
- ❑ Comparison with ChEFT raises questions concerning the chiral extrapolation ☞ more work needs to be done