# Hadron structure with domain wall fermions - I

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# LHPC collaborators (SF project)

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- A. Pochinsky (MIT)
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- D. Richards (Jlab)
- W. Schroers (DESY)

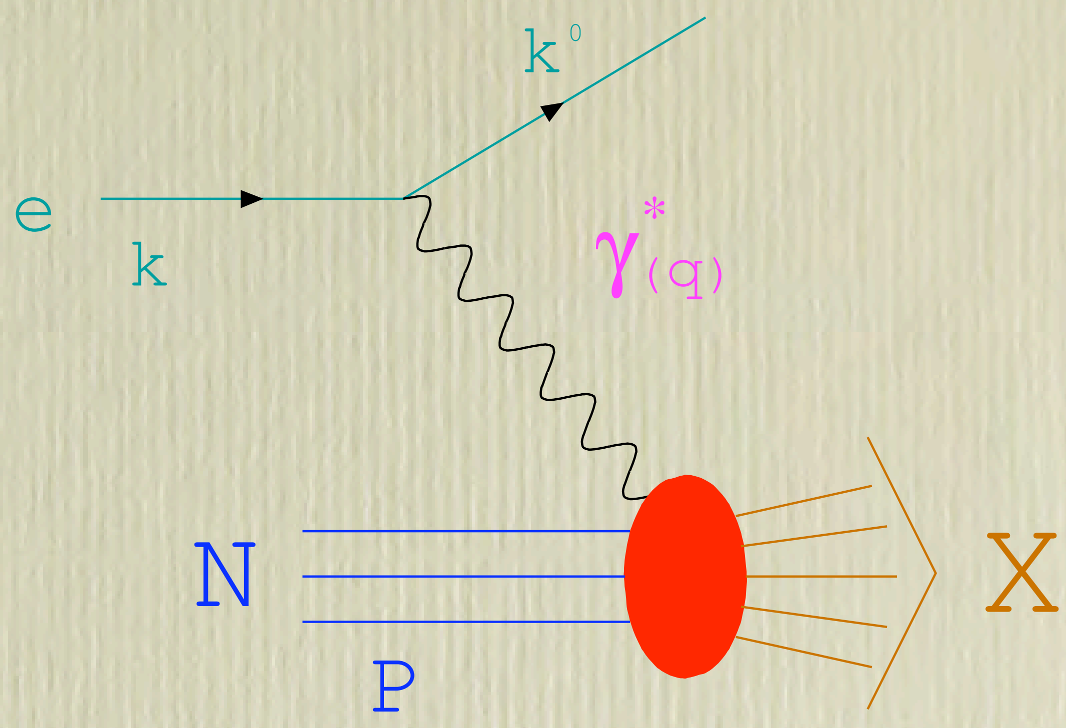
# Summary

- The LHPC program
- Details of the calculation
  - The hybrid scheme
  - Domain wall fermion checks
- Calculating diquark binding energy
- Results and discussion

# Moments of Structure Functions

$$2\int_0^1 dx x^{n-1} F_1(x,Q^2) \longrightarrow \langle x^n \rangle_q$$

$$2\int_0^1 dx x^{n} g_1(x,Q^2) \longrightarrow \langle x^n \rangle_{\Delta q}$$



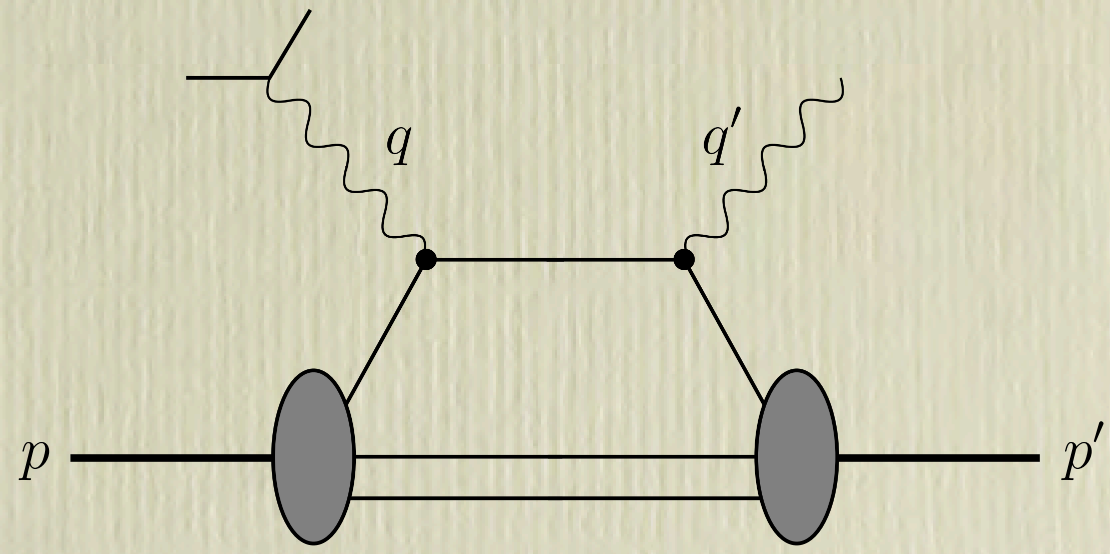

- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$ Nucleon matrix elements of local operators $\langle P,S|\mathcal{O}|P,S\rangle$

$$\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}} = \overline{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

$$\mathcal{O}^{5q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \overline{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_5\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

# GPDs and Form Factors

Deeply virtual Compton scattering:




Euclidean Matrix elements:

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle$$
$$\sim \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$
$$\to A_{ni}(t), B_{ni}(t), C_n(t)$$

# Realistic Calculations

- 2+1 Dynamical flavors
  - 2 light (up down) 1 heavy (strange)
  - charm bottom top (treated in HQET as extrernal)
- Light quark masses $m_\pi < 400\text{MeV}$
  - Chiral extrapolations
  - Finite volume corrections
  - Numerical algorithm slows down (algorithm scaling $\sim \frac{1}{m_q^{2.5}}$)
- Continuum extrapolations
  - compute at several lattice spacings (algorithm scaling $\sim \frac{1}{a^7}$)

# The LHPC program

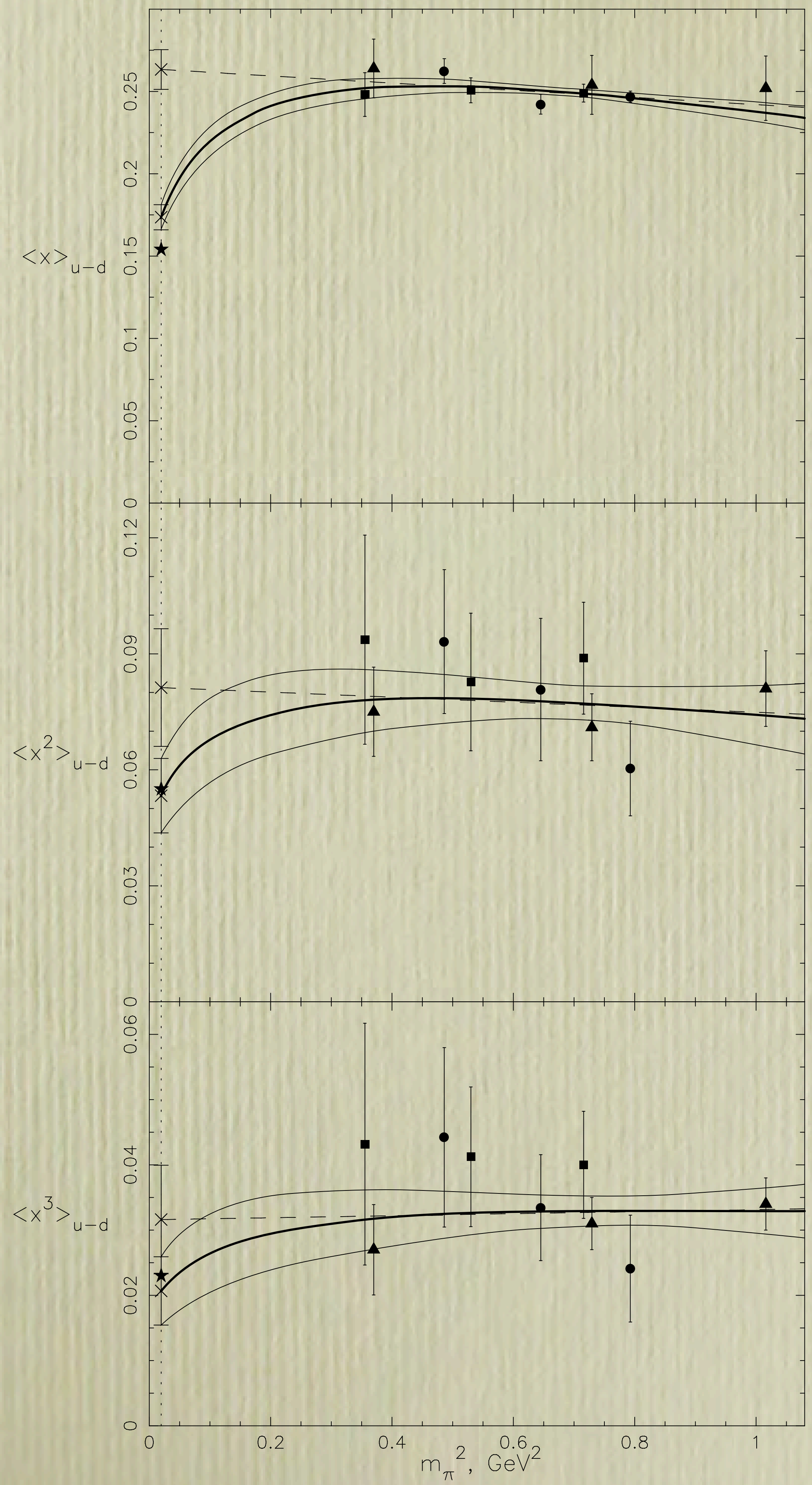
- Domain wall fermions for valence (with hyp smeared links)
  - Chiral symmetry
  - Ward Identities

- Kogut-Susskind 2+1 Dynamical flavors
  - Improved KS action (Asqtad: $O(a^4, g^2 a^2)$) [KO, Sugar, Toussaint '99]
  - MILC has generated lattices: Ready to milk the MILC

- Light quark masses: Lightest pion $m_\pi$ ~ 250MeV

- Volumes: 2.6 to 3.2 fm

- Future: Continuum extrapolation
  - MILC lattice spacings: a=0.125fm, 0.09fm
  - a=0.06fm in 1 - 2 years




LHPC-SESAM:

diamonds - quenched,

squares - dynamical

QCDSF:

quenched - triangles

[hep-lat/0201021]

$$\langle x\rangle_{u-d} \sim a_1\Big[1 - \frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln\Big(\frac{m_\pi^2}{m_\pi^2+\mu^2}\Big)\Big] + b_1 m_\pi^2.$$

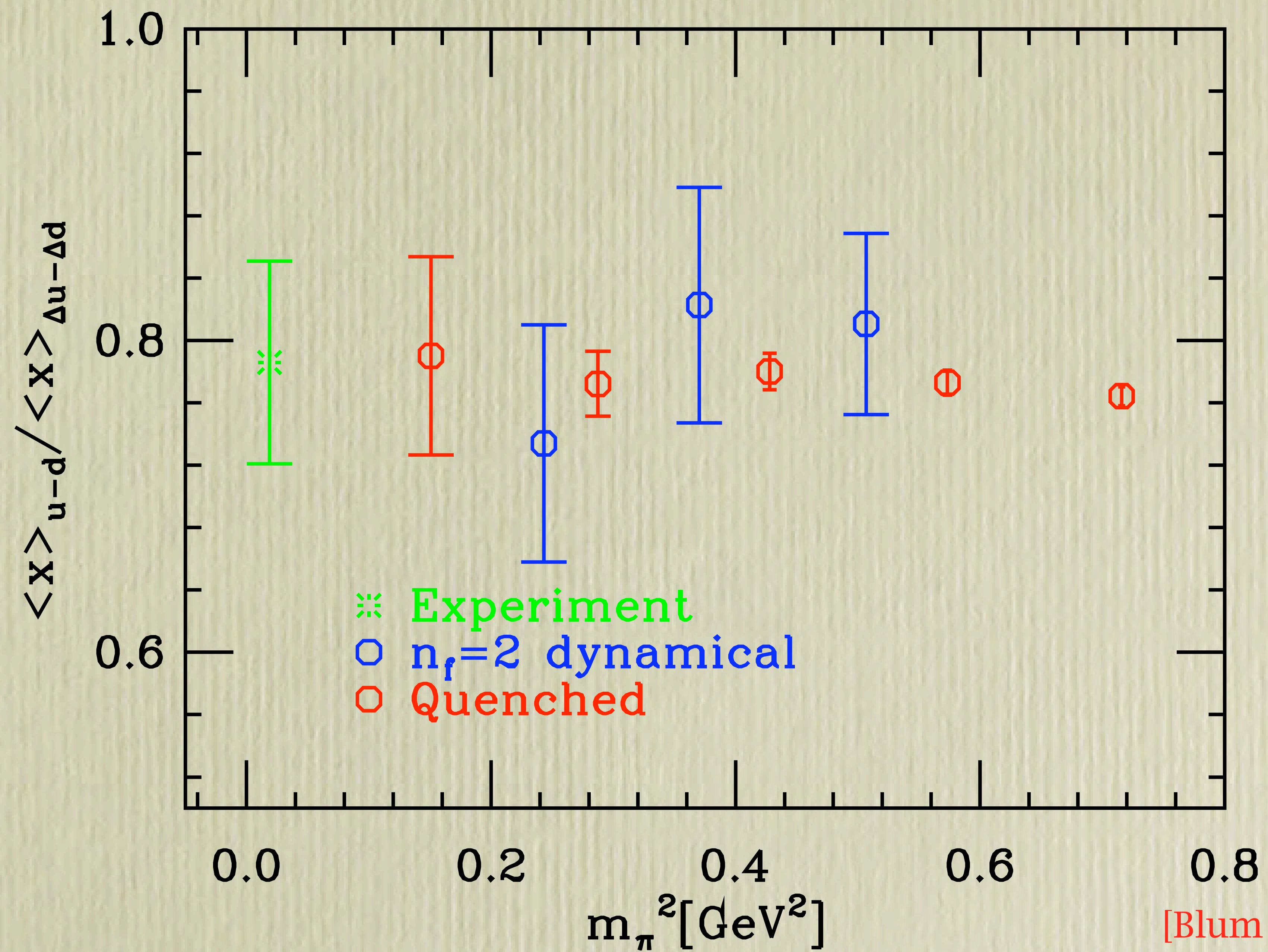
Where $\mu = 550 MeV$

The log coefficient is valid for full QCD

[Detmold et.al. Phys.Rev.D87 2001]

# Ratio of first moments

(polarized and unpolarized)




[Blum KO Ohta '05]

- No curvature observed down to **400MeV** pions (Quenched)
- Renormalization constant cancels in the ratio for DWF
- Ratio agrees with experimental expectations

# The DWF quark masses

- Domain wall fermions for valence (hyp smeared links)
  - We tune the DWF quark mass to the staggered Goldstone pion
- Baer et.al.: tune to the taste singlet for $m_\pi$
- Not clear it helps for other quantities ( ex. $f_\pi$ )
- Unitarity is restored in the continuum in any case

# Checks of the scheme

- The residual mass
- Dependence on L5 [W. Schroers LAT04]
- Locality of the action
- The iso-vector scalar correlator

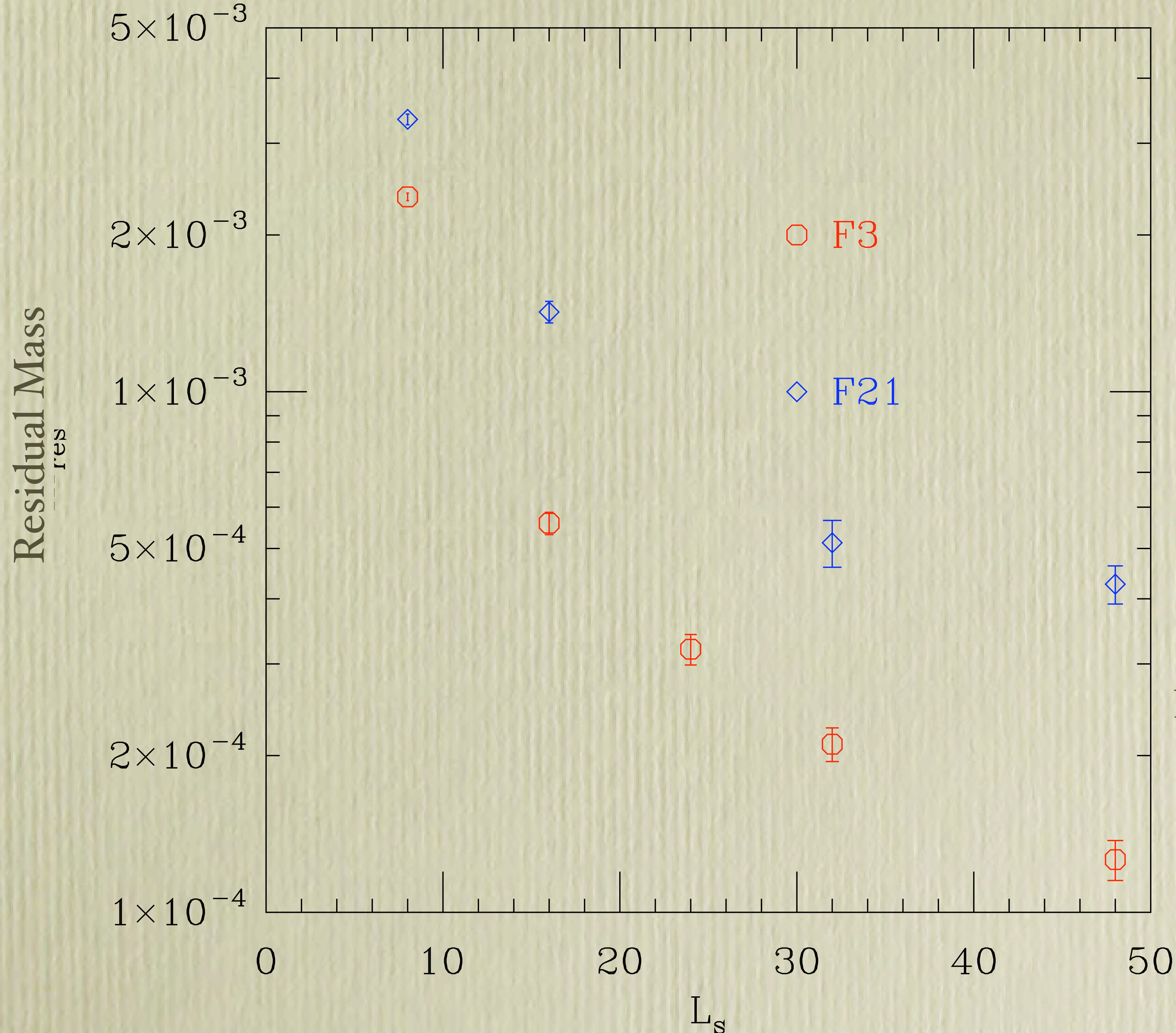
# Chiral symmetry breaking

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x)\mathcal{O}\rangle = 2\, m_f\, \langle J^a_5(x)\mathcal{O}\rangle + 2\langle J^a_{5q}(x)\mathcal{O}\rangle + i\langle \delta^a_x \mathcal{O}\rangle$$

- The size of $\langle J^a_{5q}(x)\mathcal{O}\rangle$ measures chiral symmetry breaking
- Let's use for the operator $\mathcal{O} = J^a_5(0)$
- Assume at long distances $J^a_{5q} \sim J^a_5$
- The proportionality constant is the residual mass

$$M_{\rm res} = \left.\frac{\sum_{x,y}\langle J^a_{5q}(y,t) J^a_5(x,0)\rangle}{\sum_{x,y}\langle J^a_5(y,t) J^a_5(x,0)\rangle}\right|_{t\geq t_{min}}$$

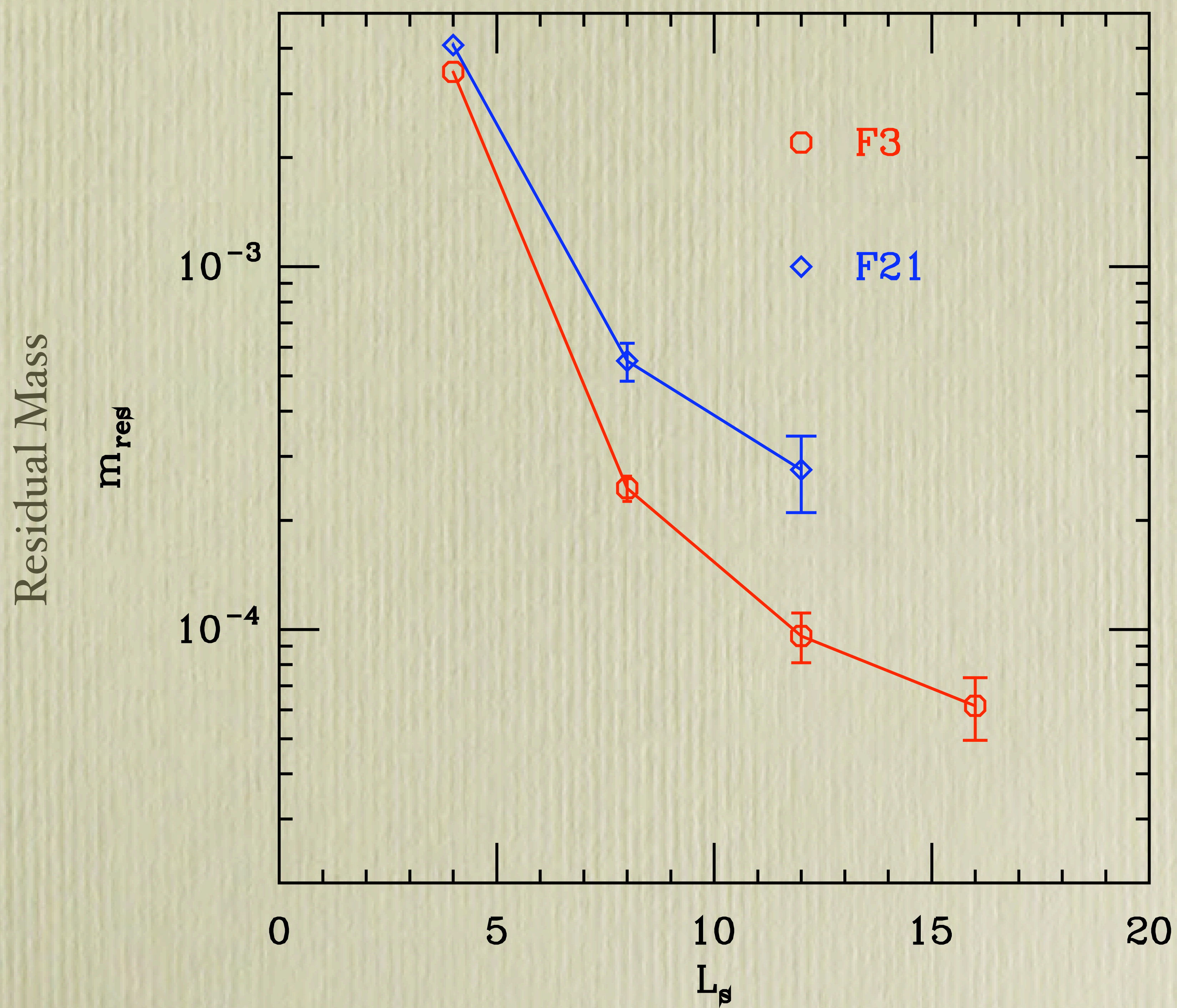
# Residual Mass vs Ls




a=0.125fm

At Ls = 16:
1MeV < $m_{res}$ <2.5MeV

# Residual Mass vs Ls




a=0.09fm

At Ls = 12:
0.2MeV<$m_{res}$ <0.7MeV

# The 4D effective operator

With a little algebra we get

$$\mathcal{P}^{-1}\frac{1}{D_{dwf}(1)}D_{dwf}(m)\mathcal{P} = \begin{bmatrix} D_{ov}(m) & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 1 & 0 & 0 & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+2}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ -(1-m)\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ -(1-m)T^{L_s/2-1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} P_- & P_+ & \cdots & 0 \\ 0 & P_- & P_+\cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_+ \\ P_+ & 0 & \cdots & P_- \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -T^{-L_s+1}M_+ & 1 & 0 & 0 & \cdots \\ -T^{-L_s+2}M_+ & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -T^{-1}M_+ & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$M_- = P_- - m\,P_+ \qquad M_+ = P_+ - m\,P_-$$

$$T^{-1} = \frac{1+H_T}{1-H_T} \qquad H_T = \gamma_5 D$$

$$D_{ov}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathcal{E}_{L_s}[\gamma_5 D(M_5)]$$

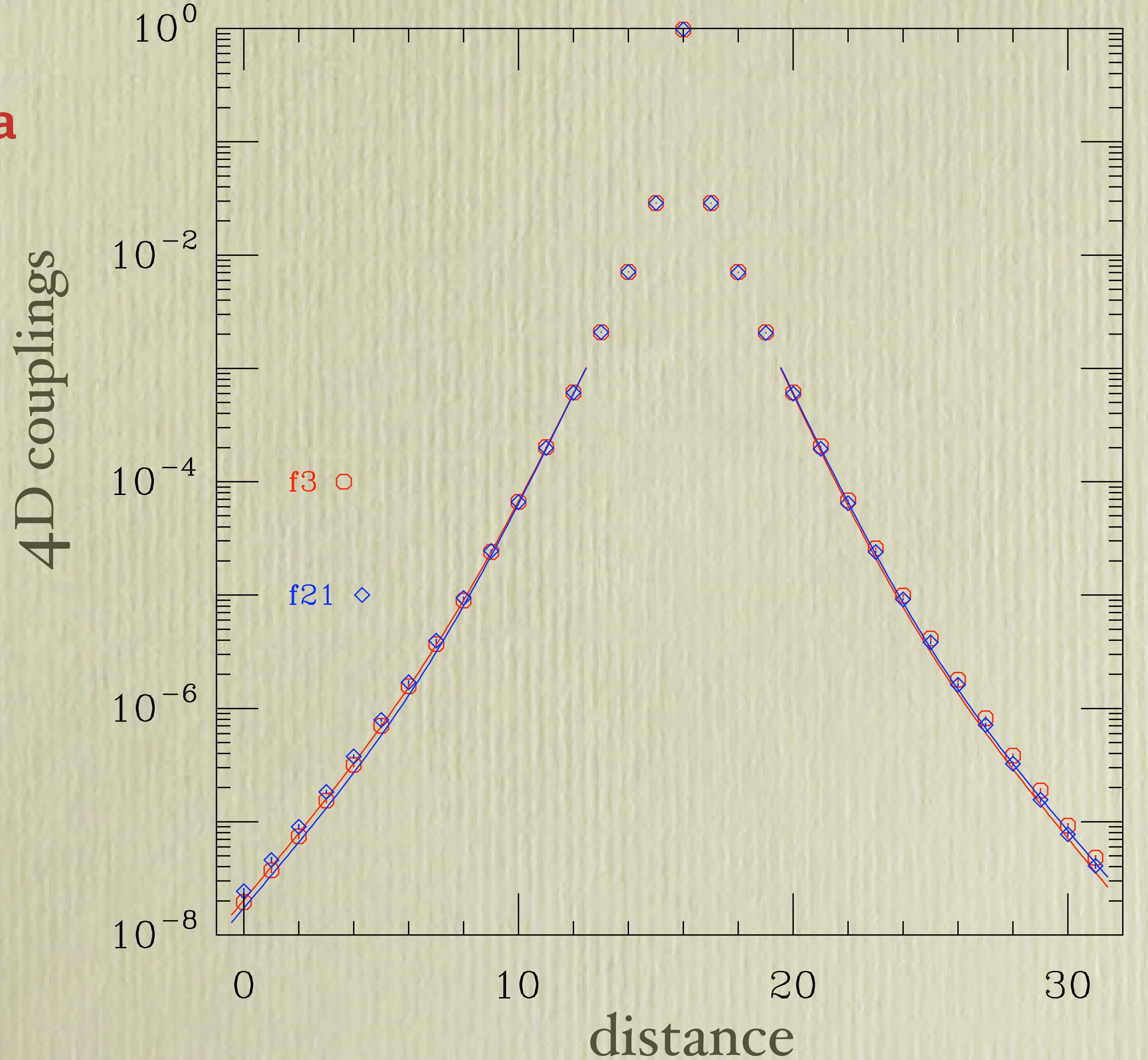
$$\varepsilon_{L_s} = \frac{T^{-L_s}-1}{T^{-L_s}+1} = \frac{(1+H_T)^{L_s}-(1-H_T)^{L_s}}{(1+H_T)^{L_s}+(1-H_T)^{L_s}}$$

$$D = (b_5+c_5)\frac{D_w}{2+(b_5-c_5)D_w} = \alpha\frac{D_w}{2+a_5 D_w}$$

- Overlap: $\alpha=2$, $a_5=0$ (Borici)
- DWF: $\alpha=1$, $a_5=1$ (Shamir)

# Locality of the 4D action

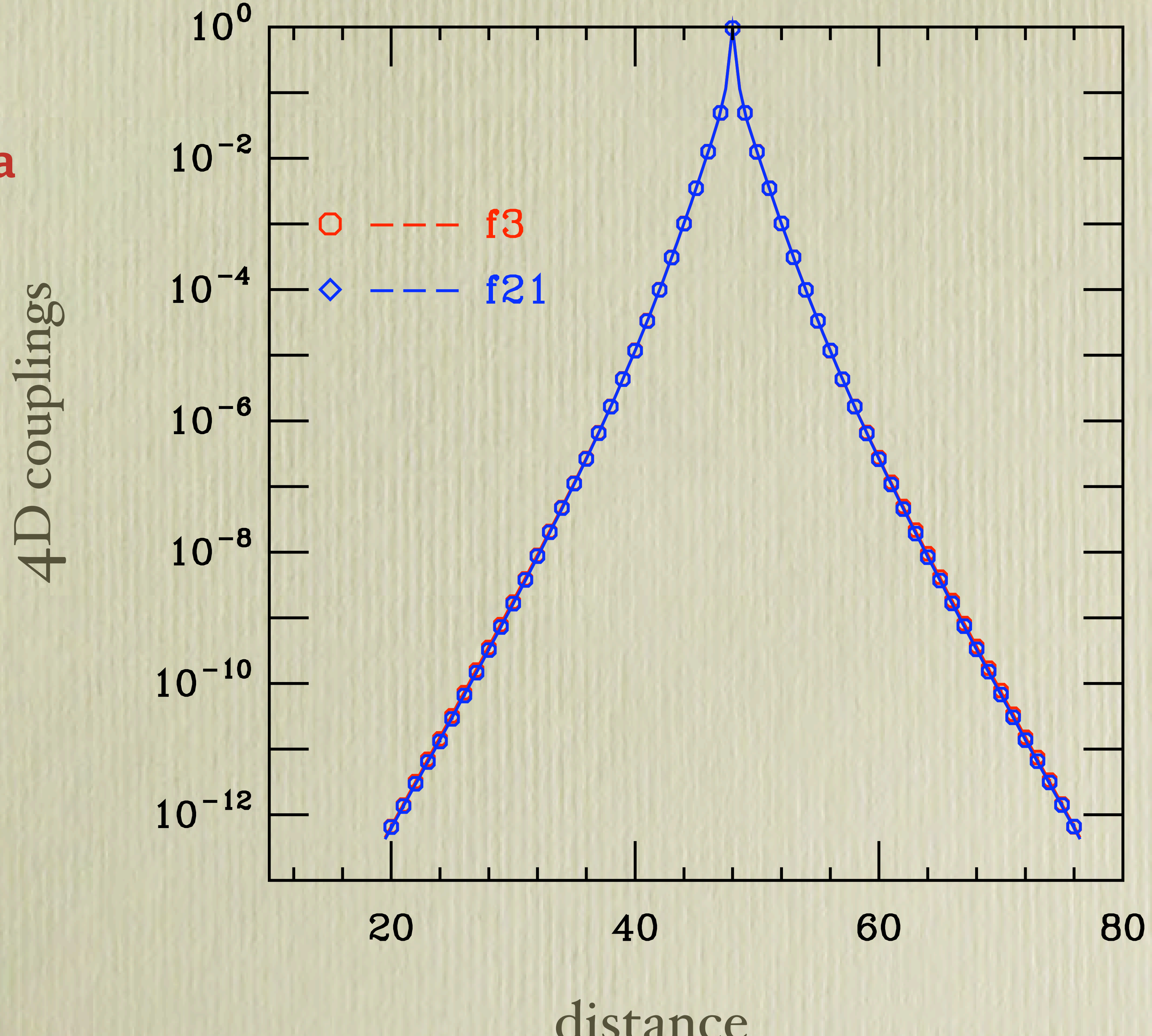
Localization: **~1.5a**




a=0.125fm

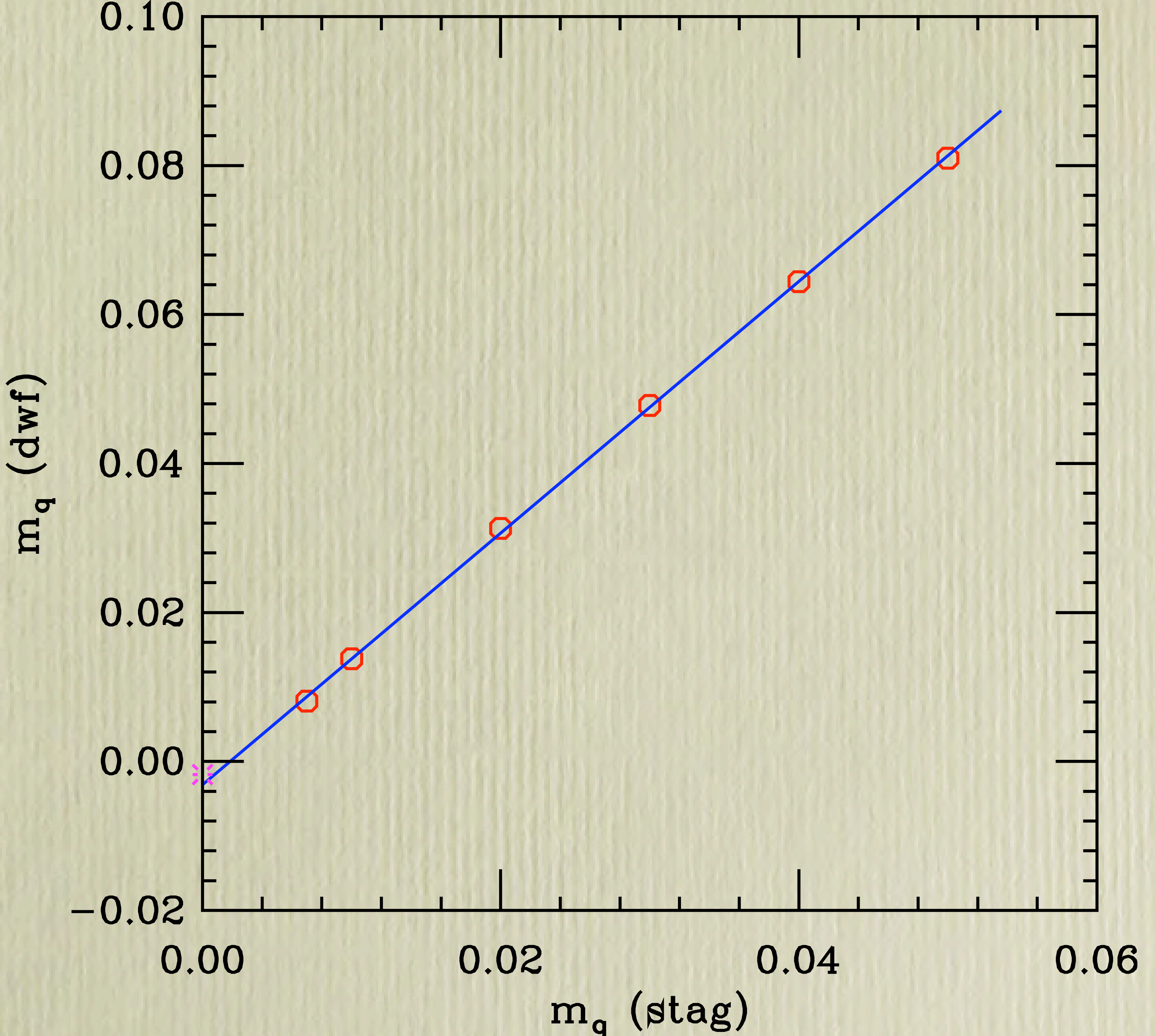
# Locality of the 4D action

Localization: **~1.3a**



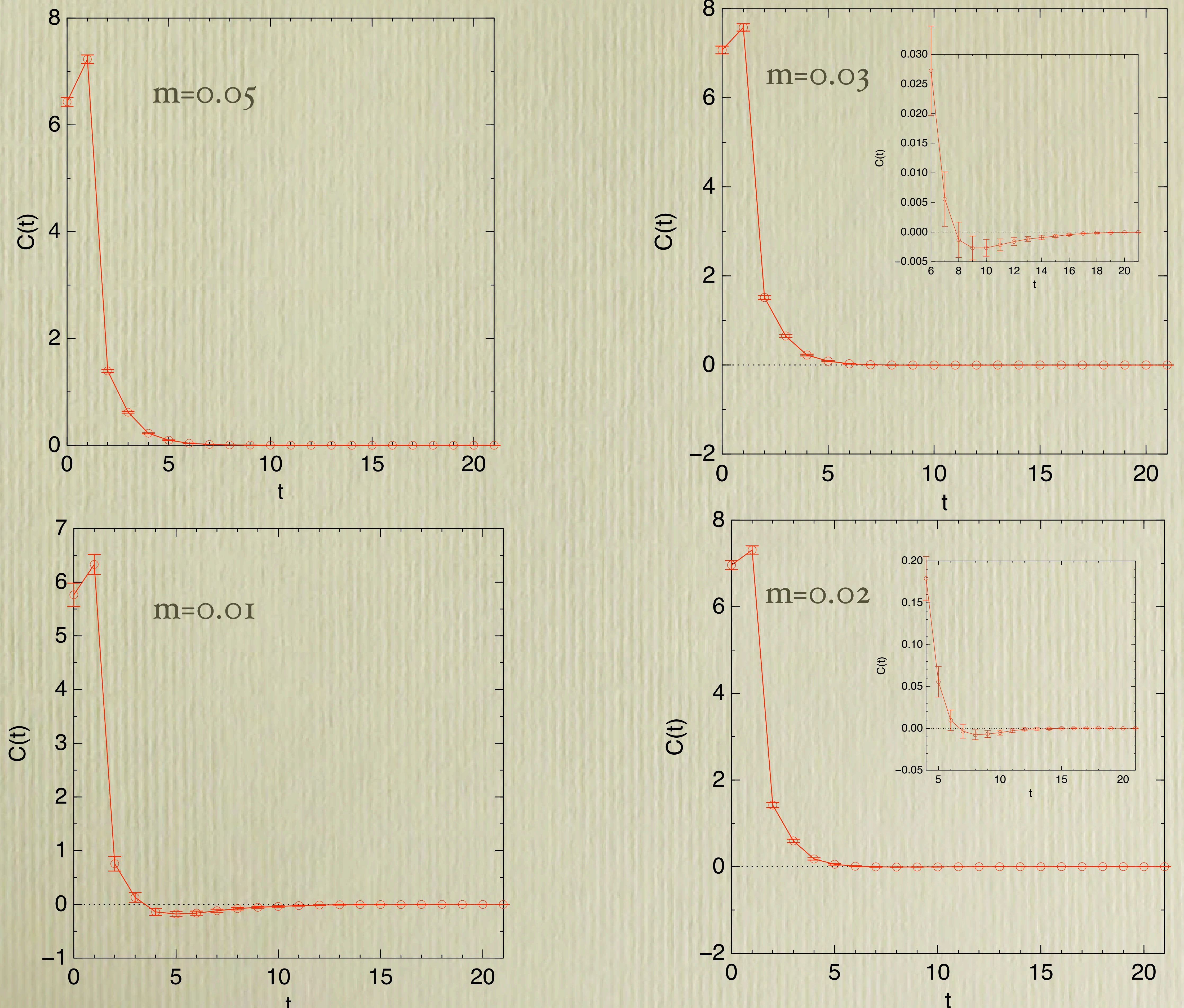

a=0.09fm

# The DWF quark masses




a=0.125fm

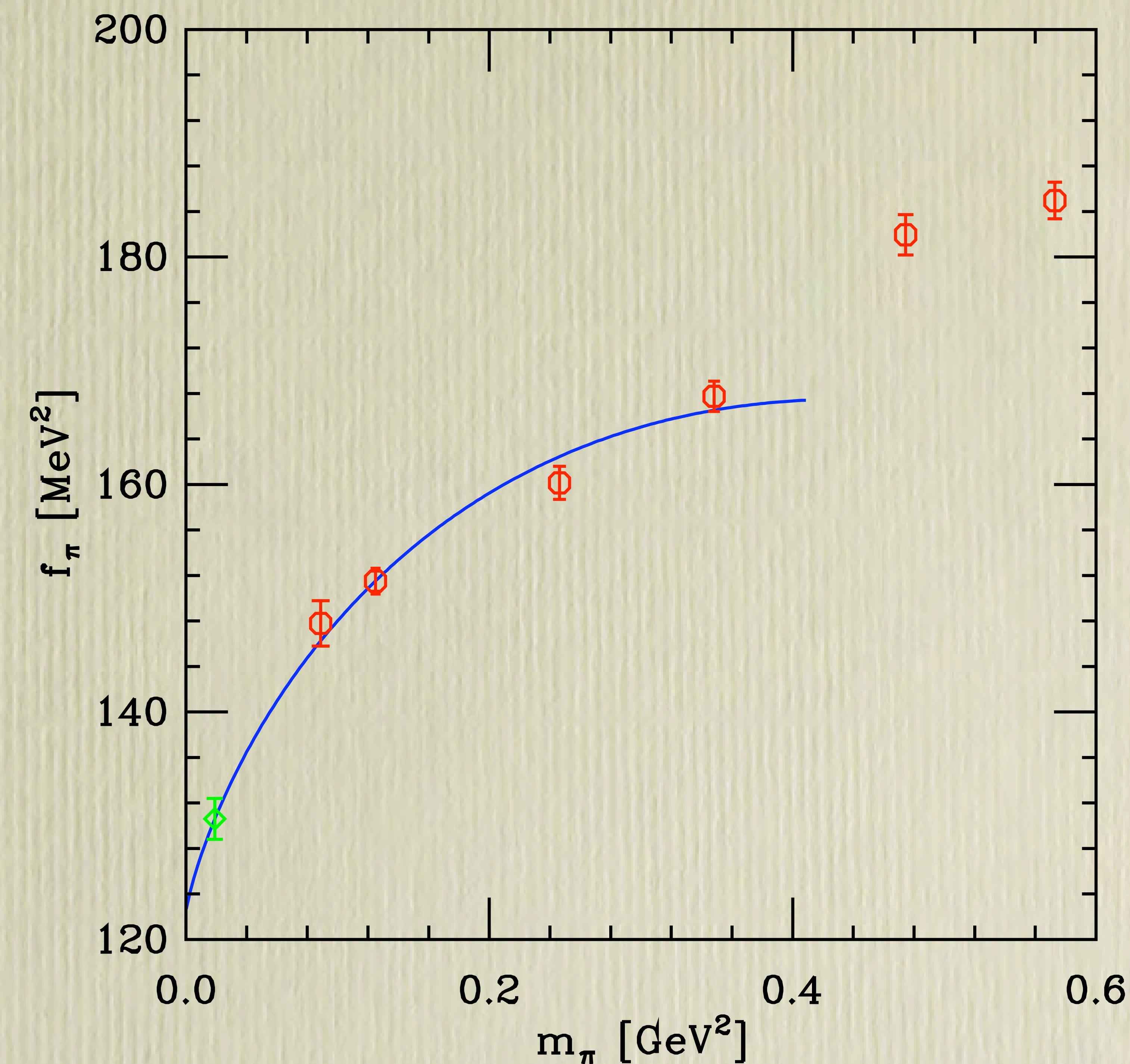
# IsoVector scalar correlator




χPT calculation: Prelovsek LAT 05

# Pion decay constant

- Fit the lower 4 points
- Scale used a = 0.125 fm
- One loop χPT extrapolation: 130.6(1.8)MeV
- Systematic error:
  - chiral extr. 3 MeV
  - 2% from scale setting
- $\chi^2$/d.o.f. ~ 2
- Need mixed χPT of Baer et.al.



# Diquarks

The diquark: made out of two quarks

$$\mathbf{3} \times \mathbf{3} \rightarrow \mathbf{6} + \bar{\mathbf{3}}$$

diquark

- Anti-fundamental channel attractive
- One gluon exchange
- ‘tHooft interaction

# Diquark Properties

- Scalar diquark most attractive channel (”good”)

$$q_f^a C\gamma_5 q_{f'}^b \epsilon_{cab}\epsilon^{ff'}$$

- Spin triplet flavor symmetric (”bad”)

$$q_f^a C\gamma_\mu q_{f'}^b \epsilon_{cab}$$

- Spin interaction: Stronger for light quarks

# Questions

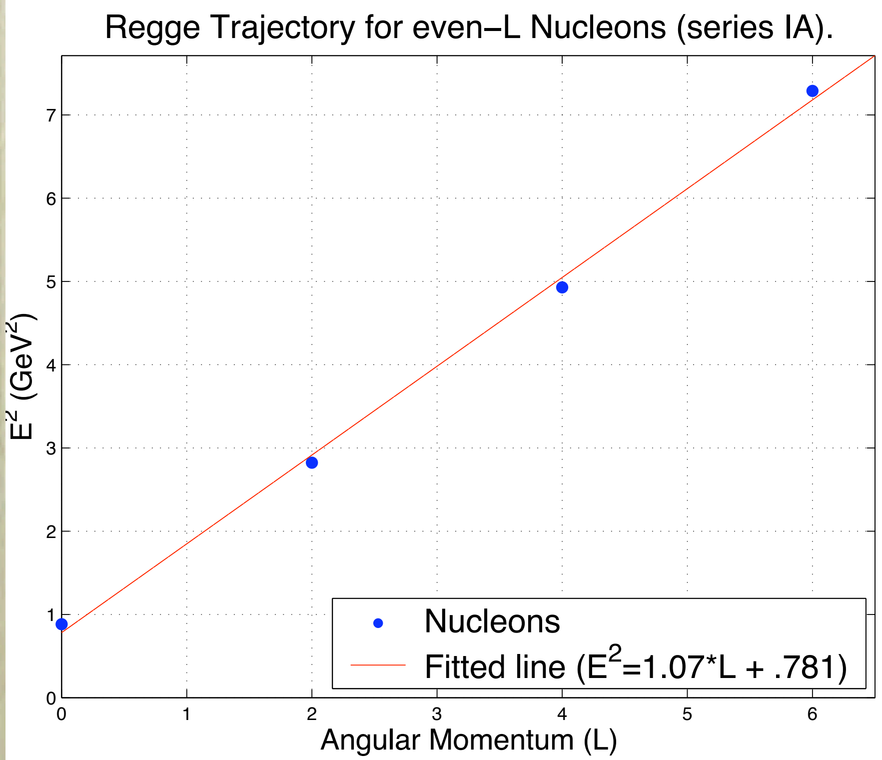
- Weak coupling arguments / instanton model
- Expect to be valid of finite density
- Color supperconductivity {Alford/Wilczek/Rajakopal}
- Do we have any non-perturbative information?
- Any evidence in the QCD spectrum?
  {Jaffe/Salem/Wilczek}
- What is the binding energy non-perturbatively?

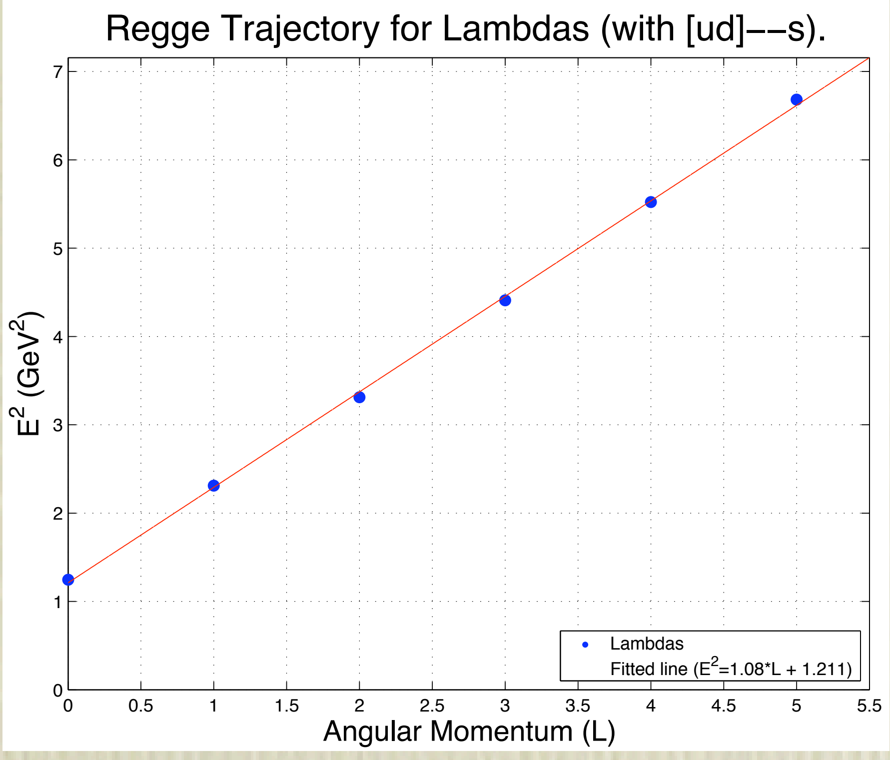
# Diquarks in QCD

- Pentaquarks [Jaffe/Salem/Wilczek]
- Color supercoductivity [Alford/Wilczek/Rajakopal]
- QCD spectrum [Salem/Wilczek]
- The $\Delta$ - N, $\Lambda$ - $\Sigma^*$ mass splittings
- The $\Delta I=1/2$ rule in Kaon decays [Stech - Neurbert]

# Diquarks in Hadrons

- Baryon: closely bound diquark connected with a flux tube to the quark [Salem/Wilczek]
- Good and bad diquarks: $0^+$ is energetically favored [Jaffe/Wilczek]
- QCD spectrum implies ~250MeV diquark binding energy splitting [Wilczek]



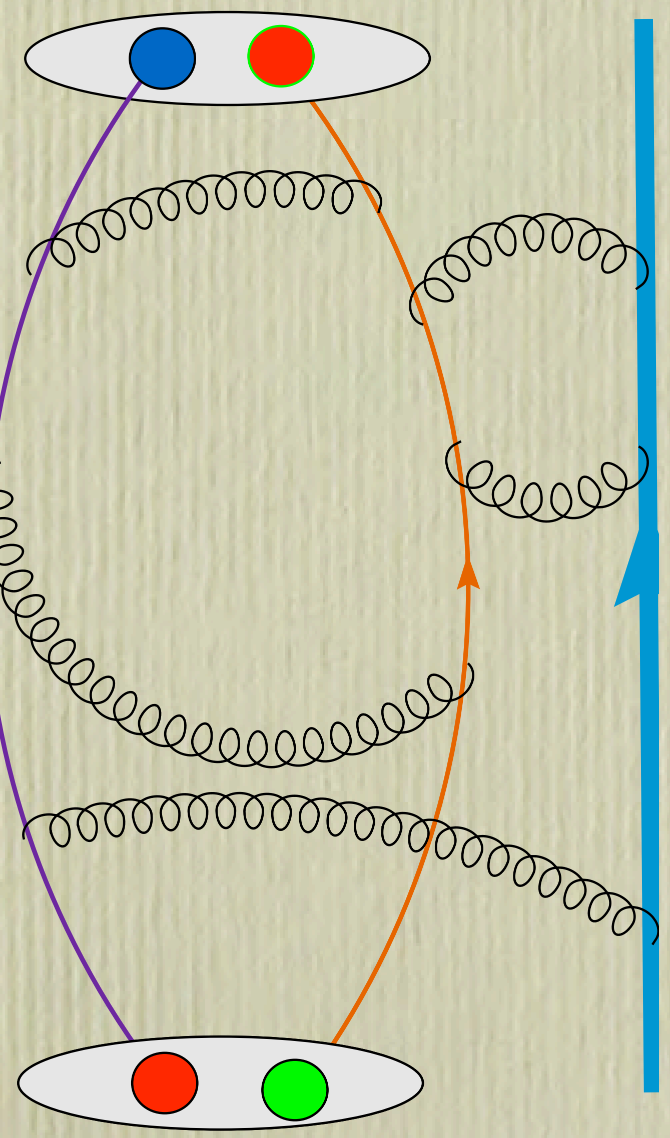



# Diquarks in Lattice QCD

- Study the spectrum
  - The quark mass is tunable parameter
  - Artificial calculations to probe specific properties such as binding energy
- Major issue: the diquark is a color source!
  - Can study it embedded in color singlet objects

# The calculation

- Compute the binding energy difference of good and bad diquarks
- Mass splitting of baryons (Δ-N , Λ - Σ ... )
- Light baryons: spin interactions present problem
- One heavy quark attached to the diquark ($\Lambda_Q$-$\Sigma_Q$)
- $\Lambda_b$ mass is known while the $\Sigma_b$ mass is not
- On the lattice can use an infinitely heavy quark

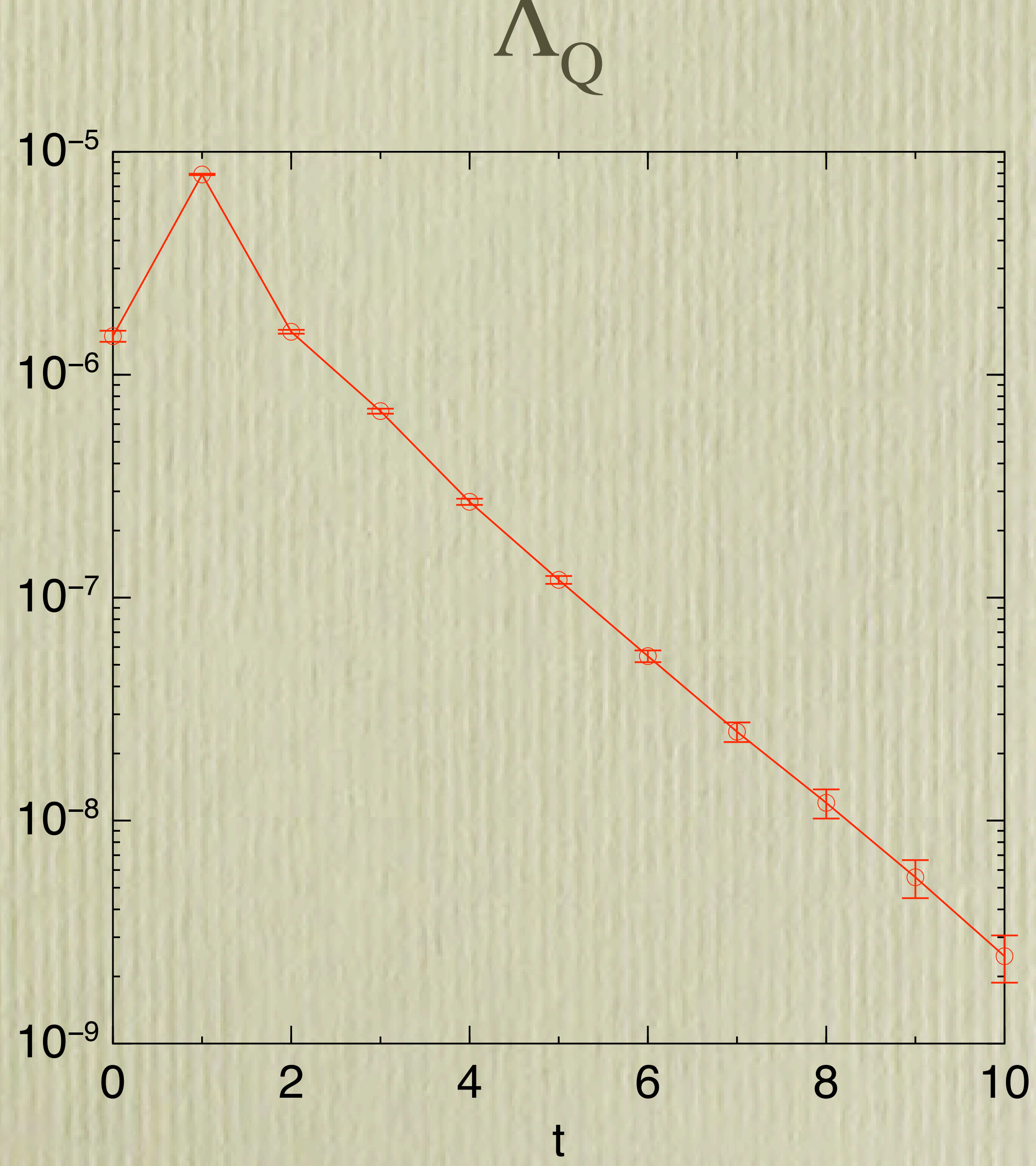


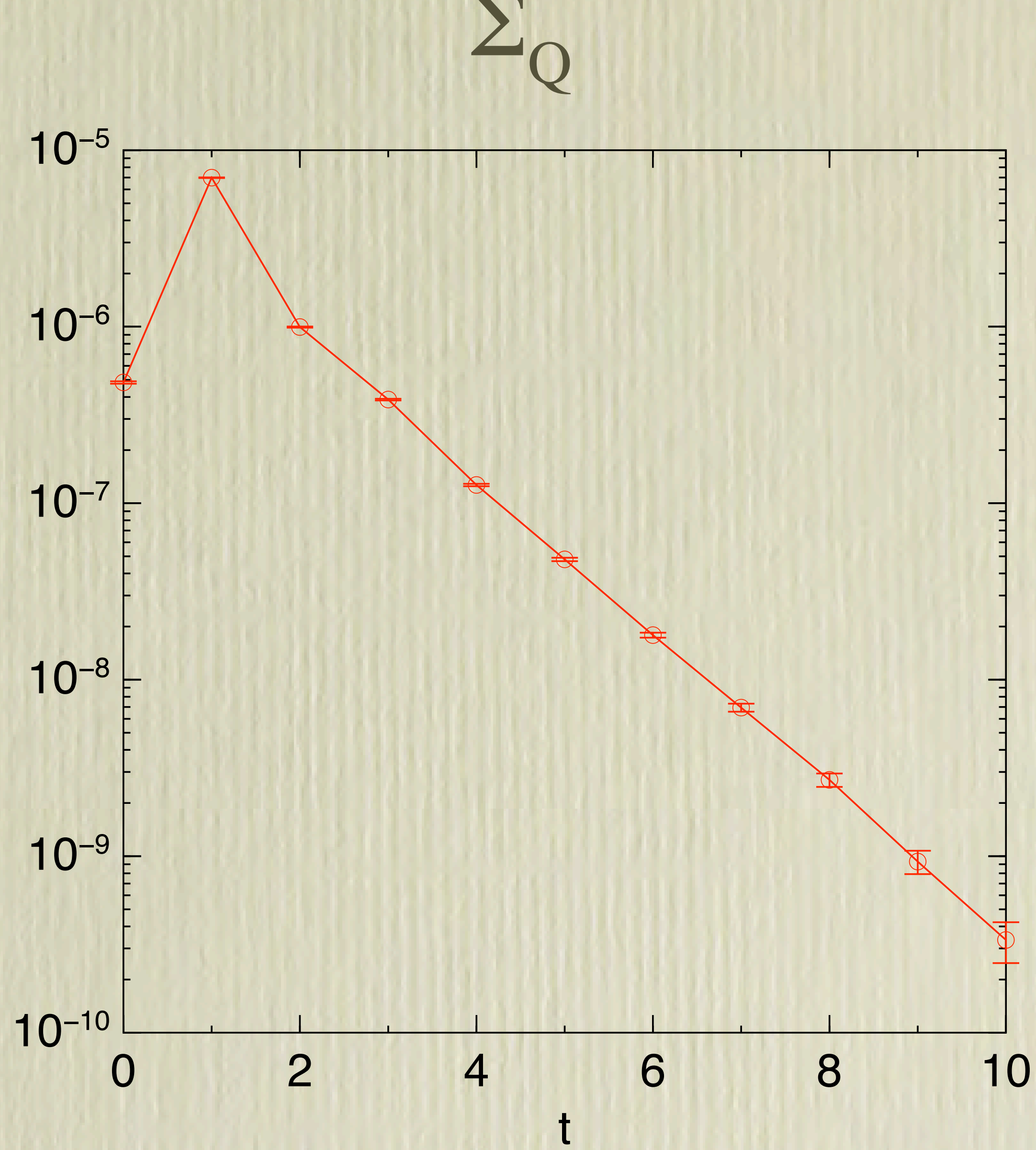
Compute the $\Sigma_Q - \Lambda_Q$ mass splitting

$\bar{u} C \gamma_\mu d$ $\bar{u} C \gamma_5 d$

with M. Savage

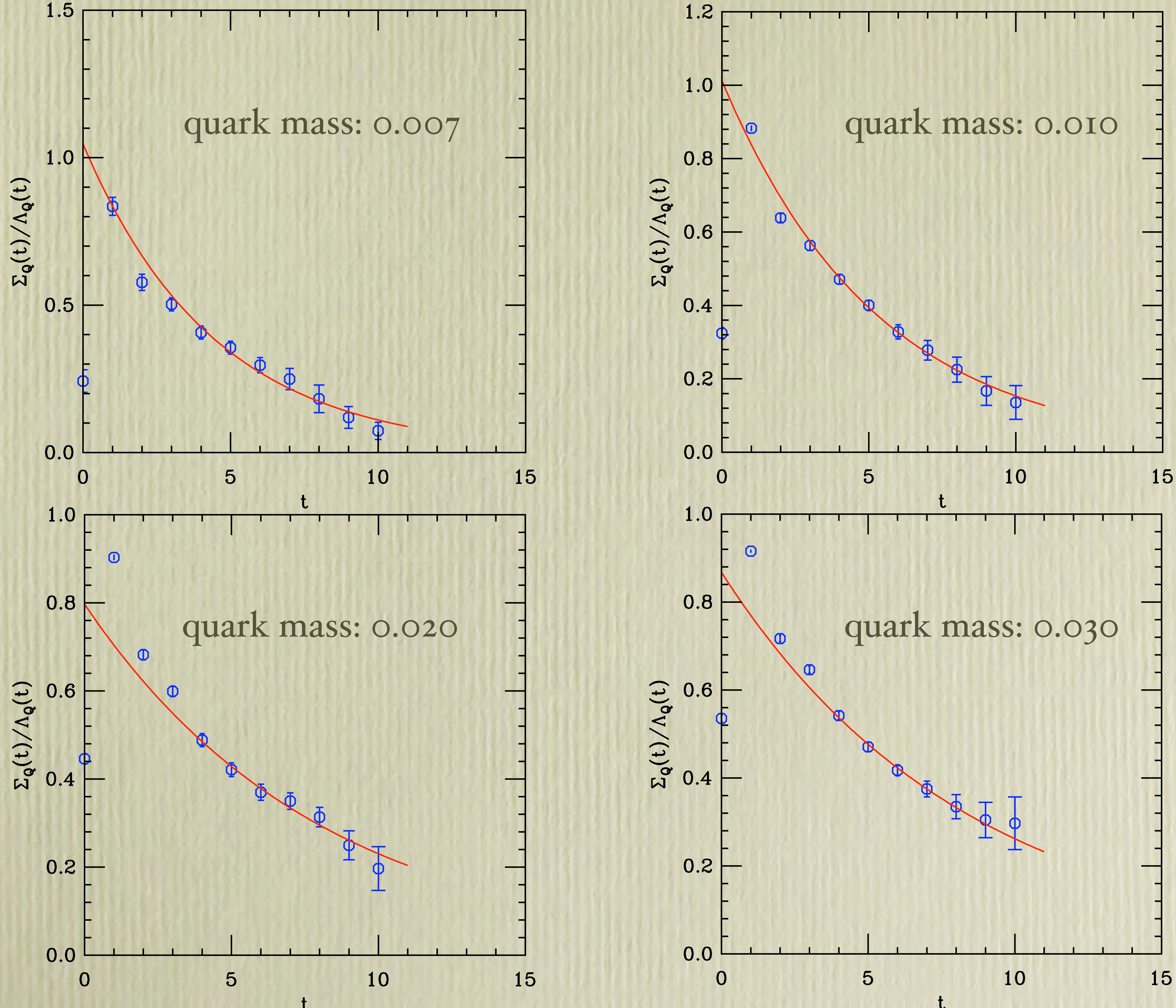
# Lattice Correlators



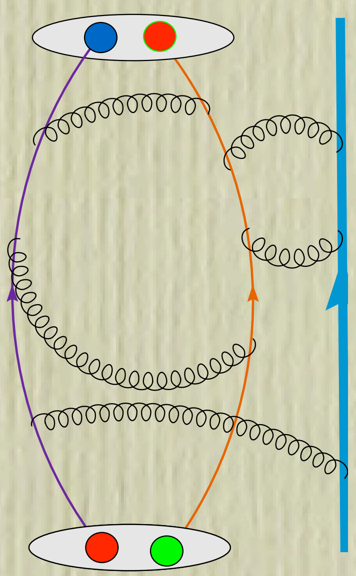




staggered quark mass: 0.010

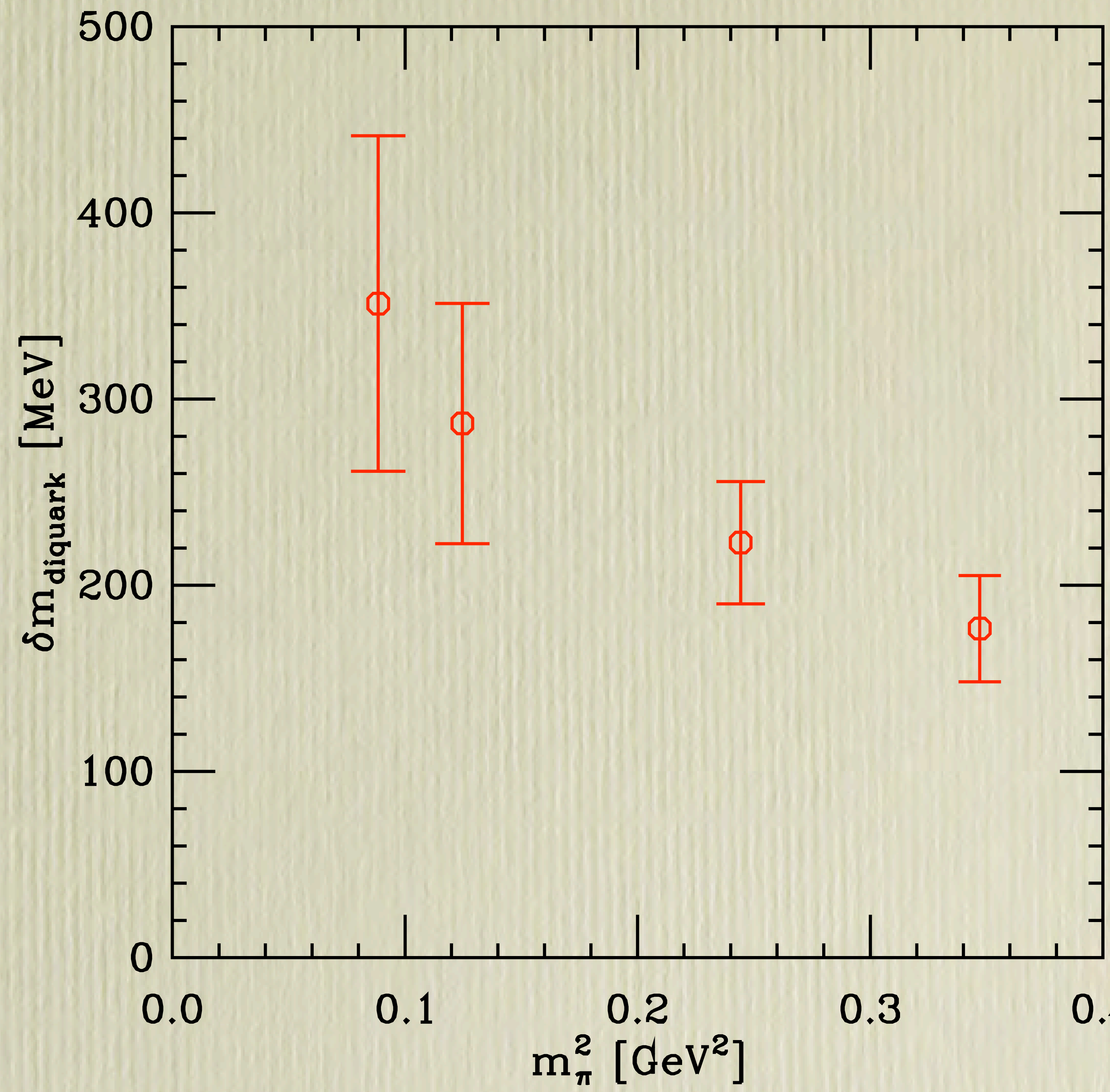
# Correlator ratio



# Diquark binding energy



$\bar{u}C\gamma_5 d$ $\bar{u}C\gamma_\mu d$




$\Sigma_Q - \Lambda_Q$ mass splitting

$\Sigma_c - \Lambda_c$ mass splitting 215MeV

$\Sigma(3/2^+) - \Lambda$ mass splitting 268MeV

$\Delta(3/2^+) - N$ mass splitting 292MeV

Hess et.al '98: diquark mass splitting 100MeV

# Conclusions

- Can measure diquark binding energy splitting
- Linear extrapolation ~ 360(70)MeV
- Result is large  compared to QCD scale
- Splitting increases with quark mass
- Future: increase precision to address systematics