Physics with QCDOC

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JLAB
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1. The effects of m_{res}
2. Choosing a gauge action - m_{res} versus β_R/β_P at fixed a
3. Choosing simulation parameters
4. Some first physics results for the chiral limit and B_K

RBRC-BNL-CU (RBC) Collaboration, July 2005

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Domain Wall Fermion Operator

- Introduce extra dimension, labeled by s

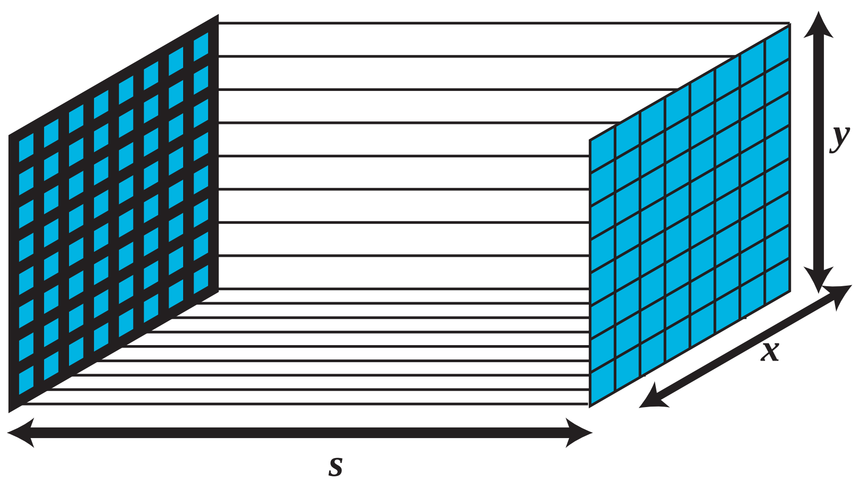
$$D_{x,s;x',s'} = \delta_{s,s'} D^{\parallel}_{x,x'} + \delta_{x,x'} D^{\perp}_{s,s'}$$

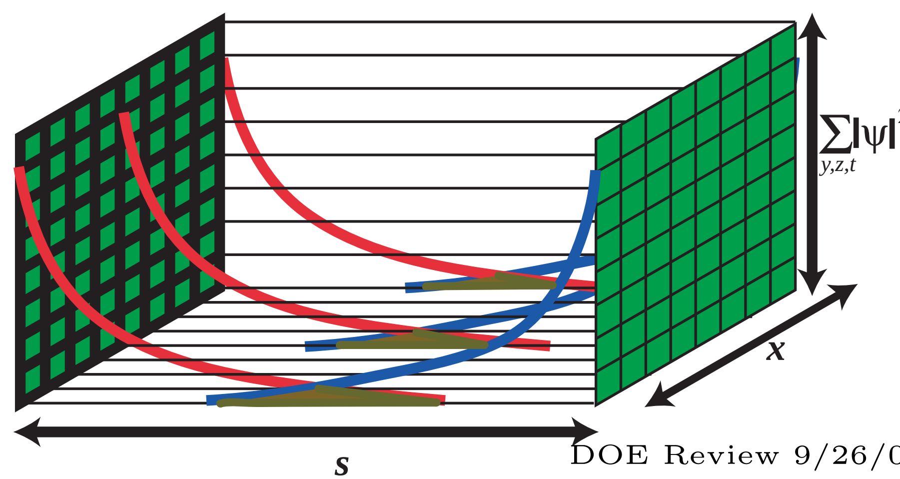
- $D^{\parallel}_{x,x'}$ is a Wilson Dirac operator with an opposite sign for the mass term.

$$D^{\parallel}_{x,x'} = \frac{1}{2}\sum_{\mu=1}^{4}\left[(1-\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},x'} + (1+\gamma_\mu)U^{\dagger}_{x',\mu}\delta_{x-\hat{\mu},x'}\right] + (M_5-4)\delta_{x,x'}$$

- $D^{\perp}_{s,s'}$ couples points in fifth dimension, distinguishing left and right handed fermions

$$\frac{1}{2}\left[(1-\gamma_5)\delta_{s+1,s'}+(1+\gamma_5)\delta_{s-1,s'}-2\delta_{s,s'}\right]-\frac{m_f}{2}\left[(1-\gamma_5)\delta_{s,L_s-1}\delta_{0,s'}+(1+\gamma_5)\delta_{s,0}\delta_{L_s-1,s'}\right]$$





Residual Chiral Symmetry Breaking for DWF

- Consider introducing in action a $SU(N_f)$ matrix Ω through term at $l \equiv L_s/2$

$$-\sum_x \left\{ \overline{\Psi}_{x,l-1} P_L \left(\Omega^\dagger - 1\right) \Psi_{x,l} + \overline{\Psi}_{x,l} P_R \left(\Omega - 1\right) \Psi_{x,l-1} \right\} \qquad \Omega \rightarrow U_R \, \Omega \, U_L^\dagger$$

- Conventional DWF recovered by $\Omega \rightarrow 1$
- QCD chiral Lagrangian $\mathcal{L}^{(2)}_{\rm QCD}$, with $\Sigma \equiv \exp\left[2i\phi^a t^a/f\right]$ and mass matrix M is:

$$\frac{f^2}{8} {\rm Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + v \, {\rm Tr}\left[M\Sigma + (M\Sigma)^\dagger\right] + v' \, {\rm Tr}\left[\Omega\Sigma + (\Omega\Sigma)^\dagger\right] + v'' \, {\rm Tr}\left[\Omega M^\dagger + \Omega^\dagger M\right]$$

- For modes bound to walls of fifth dimension, Ω enters Green's functions as

$$\Omega e^{-\alpha L_s} \quad \Rightarrow \quad v', v'' \sim e^{-\alpha L_s}$$

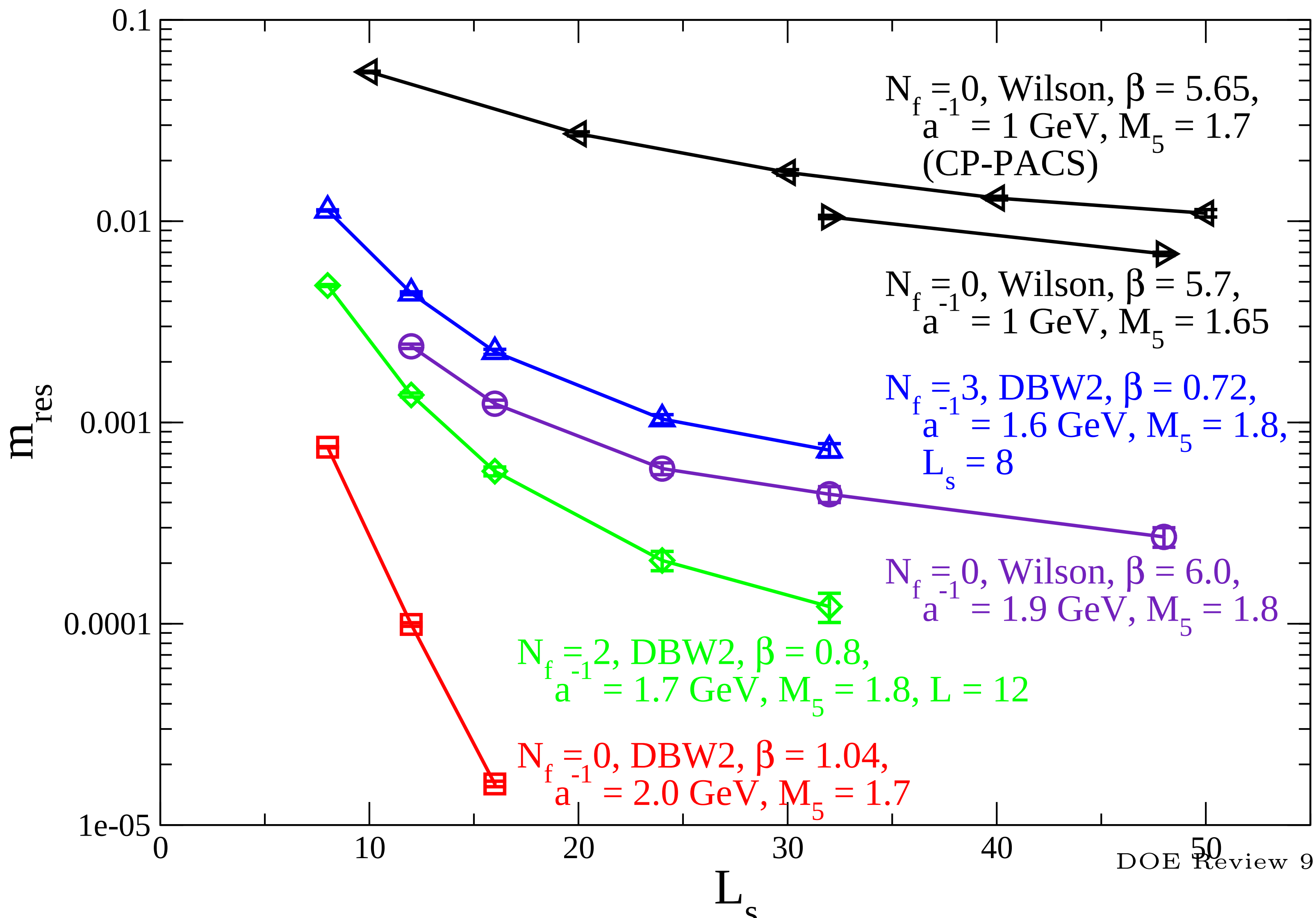
- Chiral condensate from differentiating w.r.t. mass, m_π^2 from expanding Σ

$$-\langle \overline{q} q \rangle (m_f = 0, L_s) \sim v + v'' \qquad v = \frac{f^2 m^2_{\pi^+}}{4(m_u + m_d + 2 m_{\rm res})} \quad m_{\rm res} \equiv v'/v$$

$m_{\rm res}$ **versus** L_s **for** $N_f = 0, 2$ **and 3**

Compare gauge actions composed of plaquette and rectangle terms.

$$S_g = (\beta/3)\left\{(1-8c_1)\sum \mathrm{Re}(\mathrm{Tr}U_P) + c_1 \sum \mathrm{Re}(\mathrm{Tr}U_R)\right\}$$



DWF QCD, 2+1 and 3 flavors, $V = 16^3 \times 32$

3 flavors, "R" algorithm

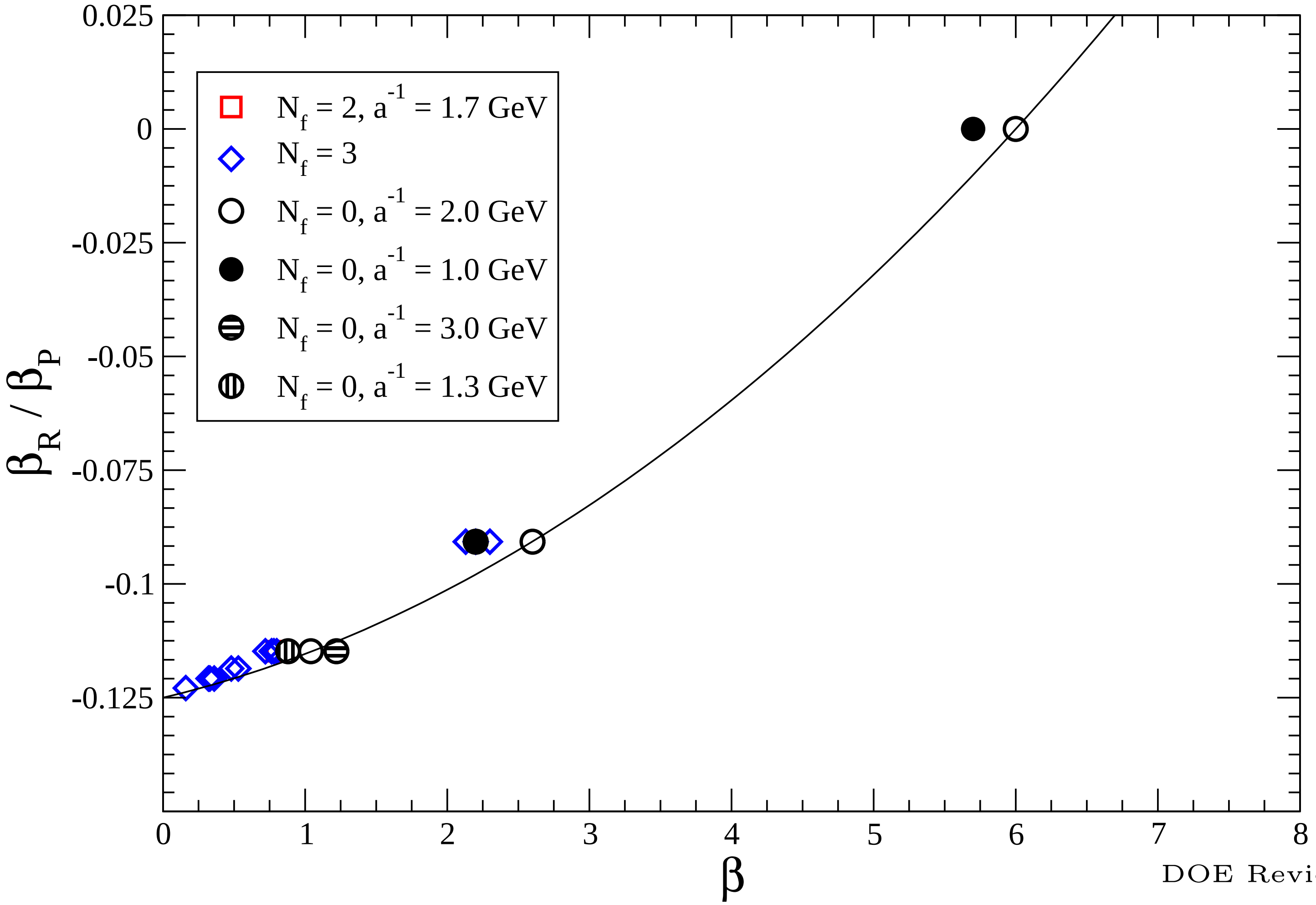
β	c_1	m_l	m_h
0.80	-1.4069	0.04	0.04
0.72	-1.4069	0.02	0.02
0.72	-1.4069	0.04	0.04
0.48	-2.3	0.04	0.04
0.32	-3.57	0.04	0.04
0.16	-7.47	0.04	0.04
0.53	-2.3	0.04	0.04
0.36	-3.57	0.04	0.04
0.33	-3.57	0.04	0.04

2+1 flavors, RHMC

β	c_1	m_l	m_h
0.72	-1.4069	0.01	0.04
0.72	-1.4069	0.02	0.04
0.72	-1.4069	0.04	0.04
0.764	-1.4069	0.02	0.04
0.764	-1.4069	0.04	0.04
0.78	-1.4069	0.02	0.04
0.78	-1.4069	0.04	0.04
2.13	-0.331	0.02	0.04
2.13	-0.331	0.04	0.04
2.2	-0.331	0.02	0.04
2.2	-0.331	0.04	0.04
2.3	-0.331	0.04	0.04

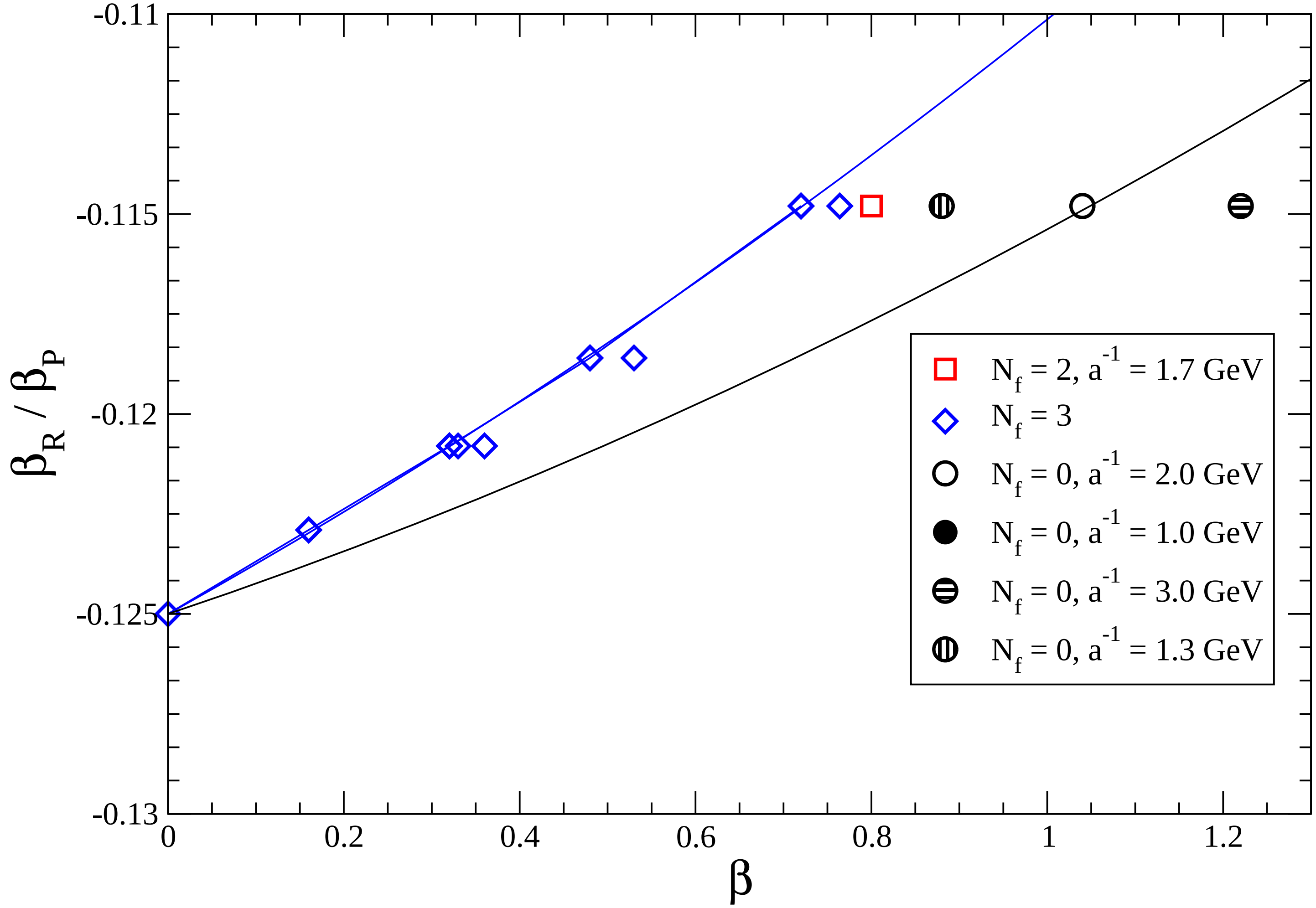
Full QCD with Plaquette plus Rectangle Actions

- Need lines of constant lattice spacing, a, to compare $m_{\rm res}$ at fixed a.
- $N_f = 0$, $a^{-1} = 2$ GeV points nicely fit by $\beta_R/\beta_P = -0.125 + a_1\beta + a_2\beta^2$.
- Used this form to propose $N_f = 3$ DBW2 values.

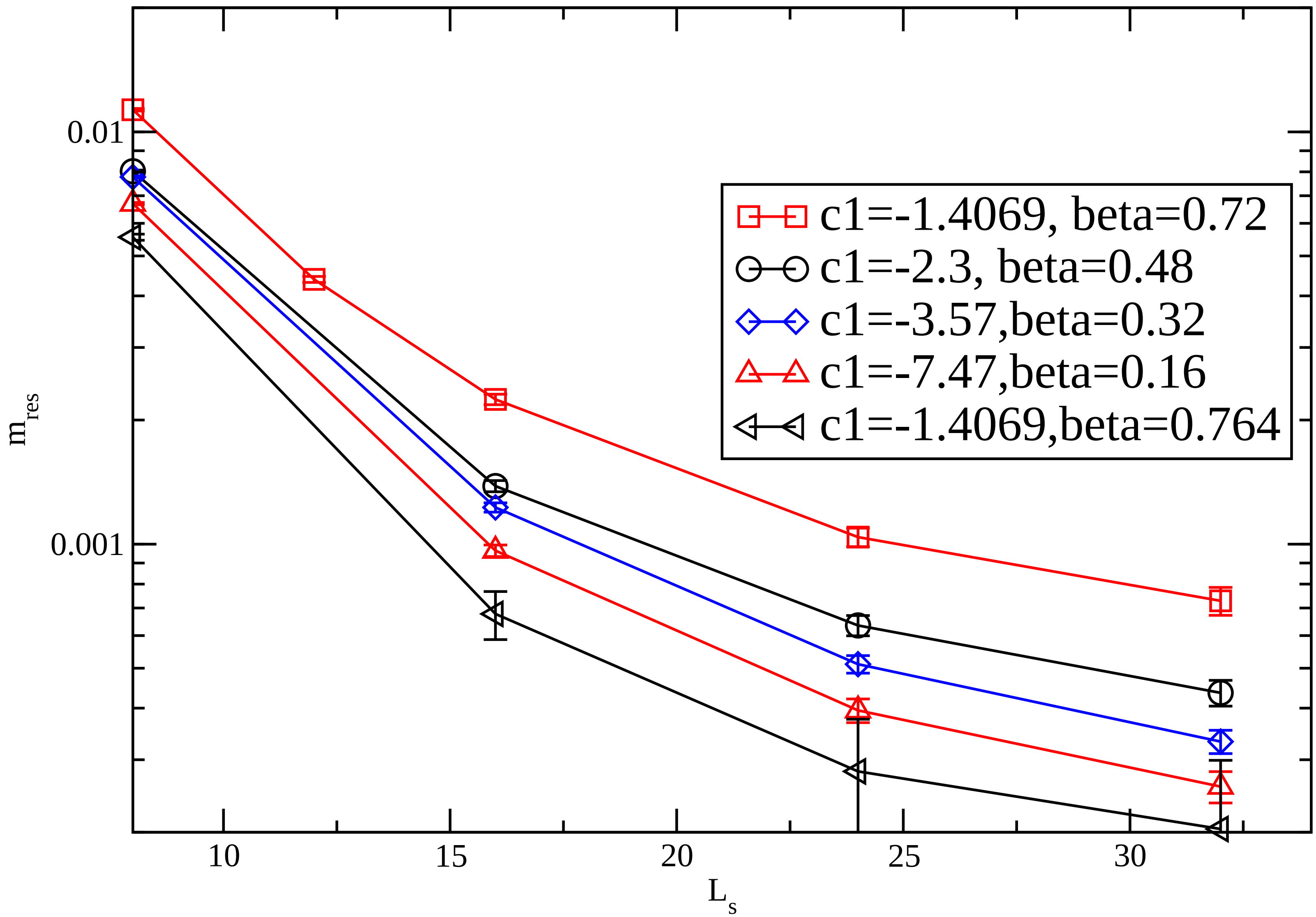


Choosing $N_f = 3$ Simulations with large $|\beta_R/\beta_P|$

- Predict 3 (β, c_1) pairs that should have same lattice spacing.
- $m_{\rm res}$ varies rapdily with lattice spacing, requiring a very similar.

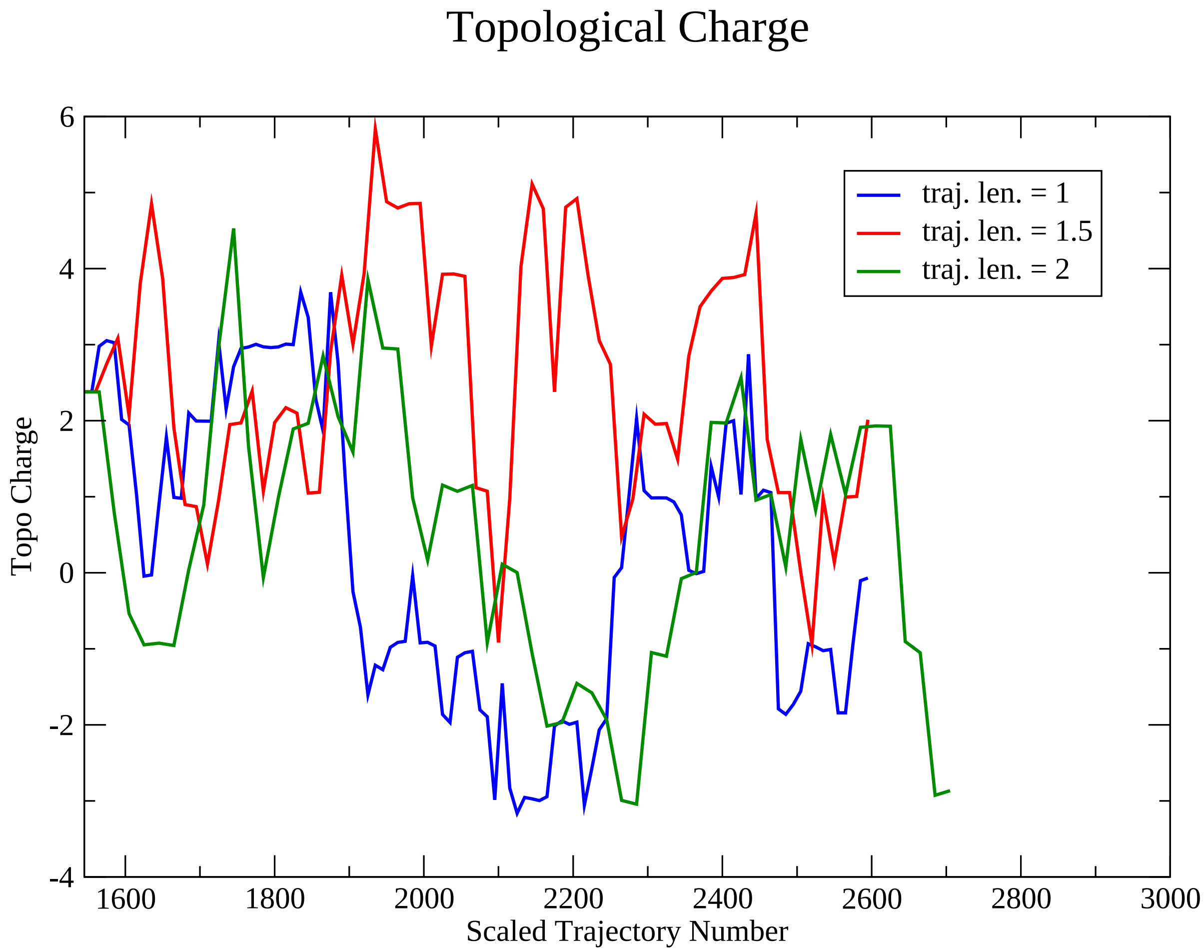


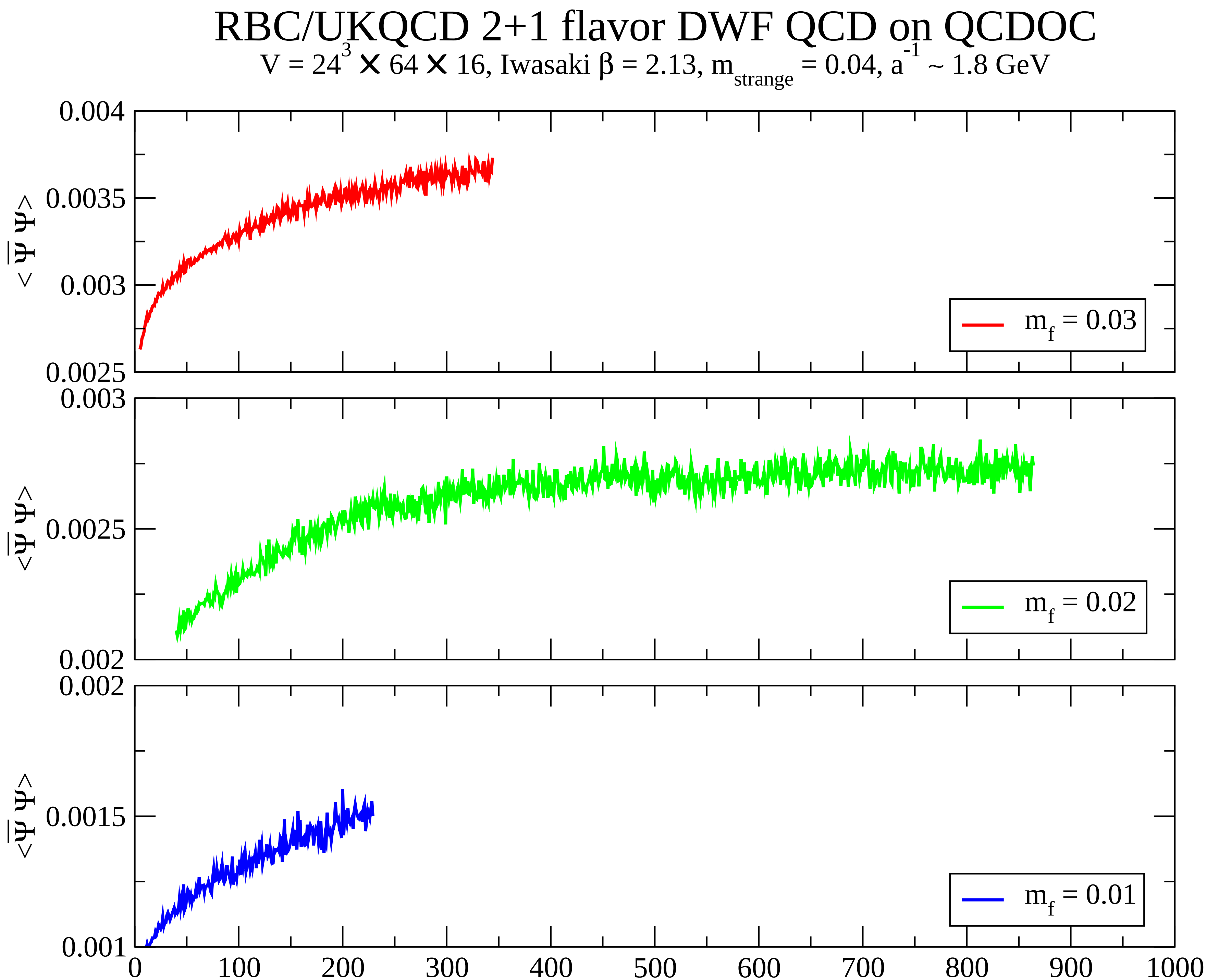
$m_{\rm res}$ for Plaquette plus Rectangle Actions



Evolution of $N_f = 3$ Lattices with $m_{\mathrm{dyn}} \approx m_{\mathrm{strange}}$

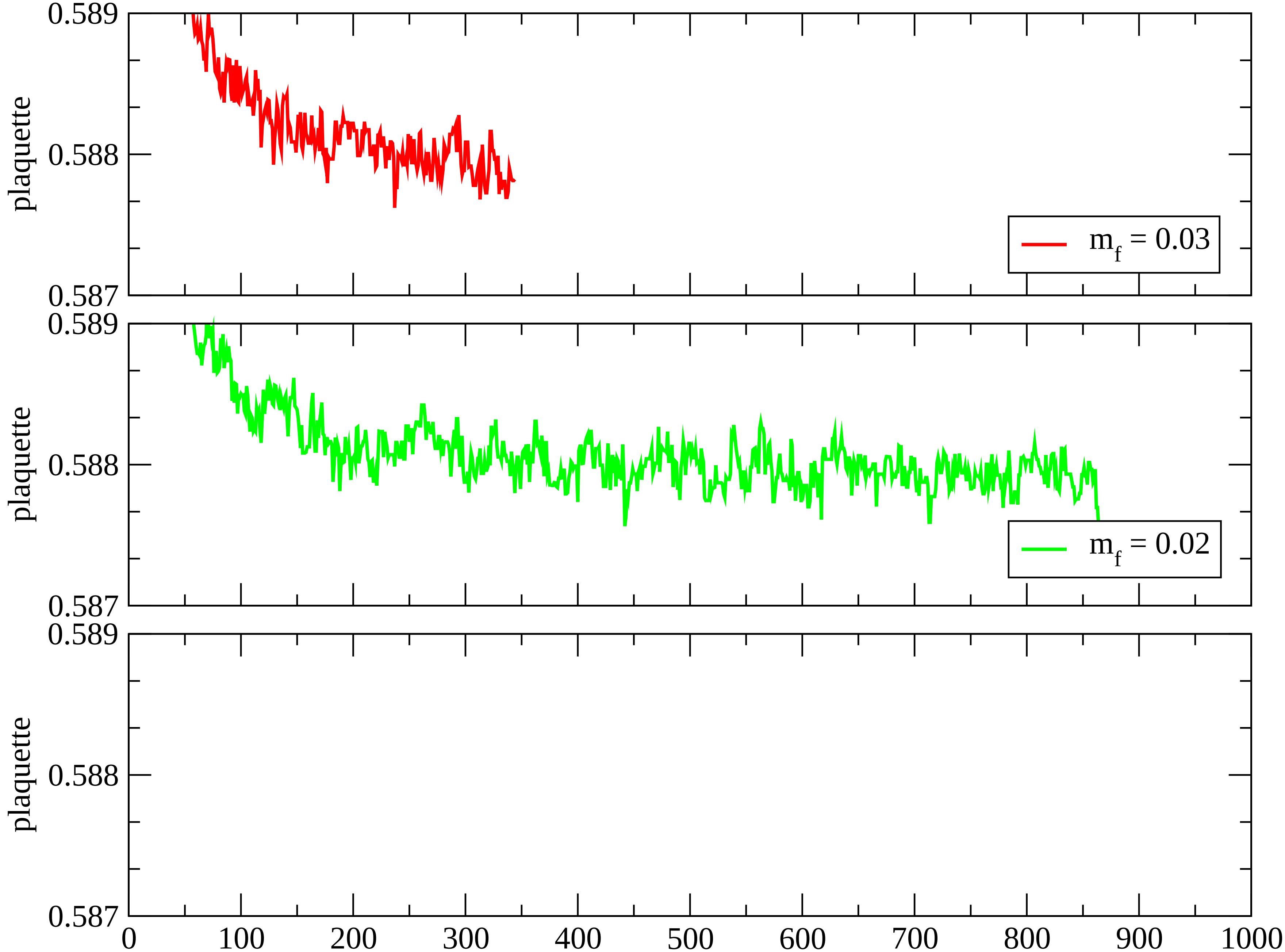
- Check correlation time versus length of trajectory with RHMC.
- 10 days of running on 3, 1024 node QCDOC racks (Meifeng Lin).




RBC/UKQCD 2+1 flavor DWF QCD on QCDOC
V = 24^3 × 64 × 16, Iwasaki β = 2.13, m_strange = 0.04, a^-1 ~ 1.8 GeV
<Ψ̄ Ψ>
0.004
0.0035
0.003
0.0025
m_f = 0.03
0.003
0.0025
0.002
m_f = 0.02
0.002
0.0015
0.001
m_f = 0.01
0
100
200
300
400
500
600
700
800
900
1000

RBC/UKQCD 2+1 flavor DWF QCD on QCDOC

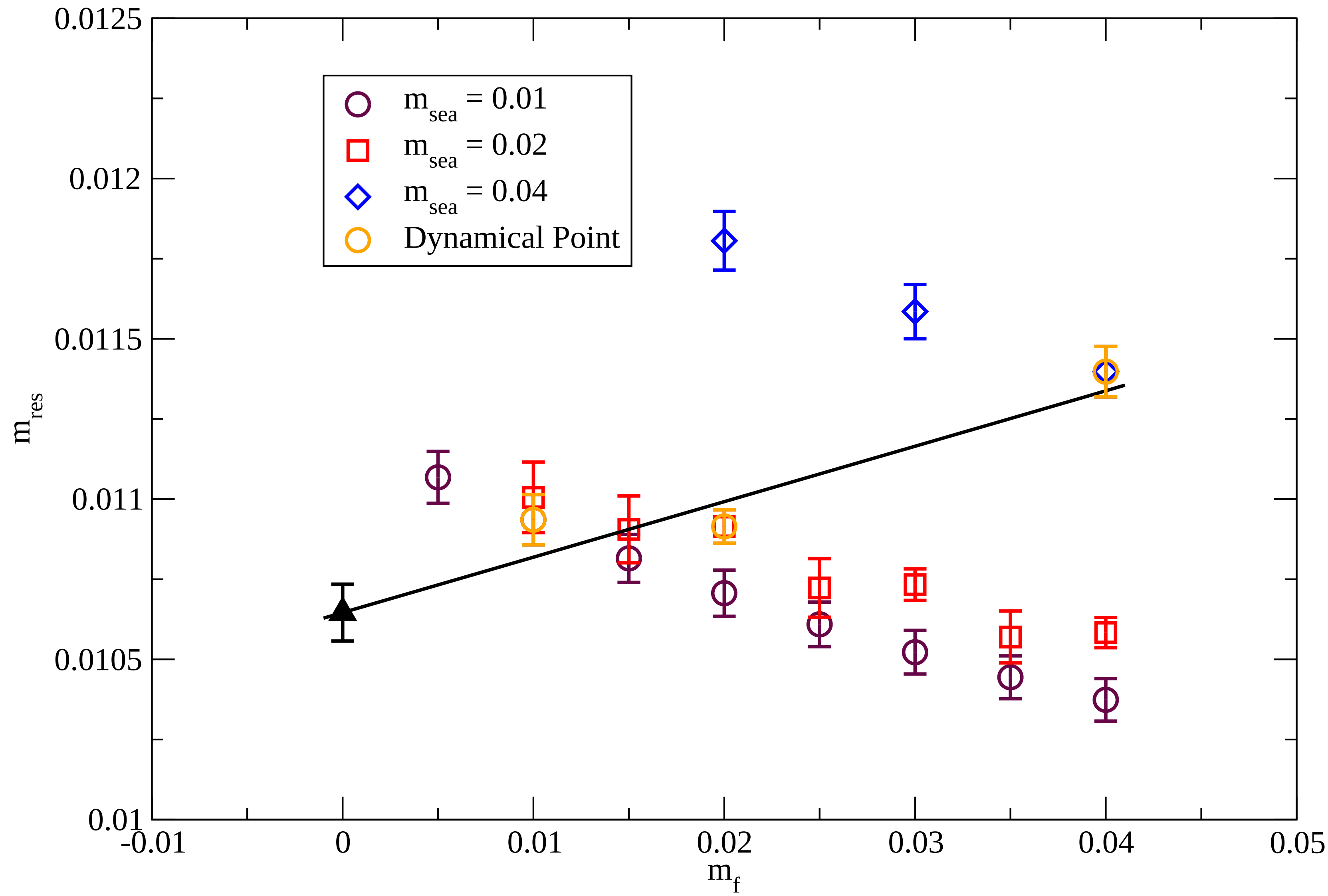
$V = 24^3 \times 64 \times 16$, Iwasaki $\beta = 2.13$, $m_{strange} = 0.04$, $a^{-1} \sim 1.8$ GeV



QCD with 2+1 Flavors of Dynamical DWF

- 3 values of m_l: 0.01, 0.02, 0.04 and $m_{\text{strange}} = 0.04$
- $m_{\text{res}} = 0.0106(1)$ with $L_s = 8$
- $16^3 \times 32 \times 8$
- $a^{-1} = 1.6(1)$ GeV, (2 fermi spatial volume
- Exact algorithm - RHMC (Rational Hybrid Monte Carlo) of Clark and Kennedy
- 6,000 trajectories for $m_l = 0.01$ and 0.02
- Discuss chiral limit of m_π and f_π
- Preliminary measurement of 2+1 flavor B_K.

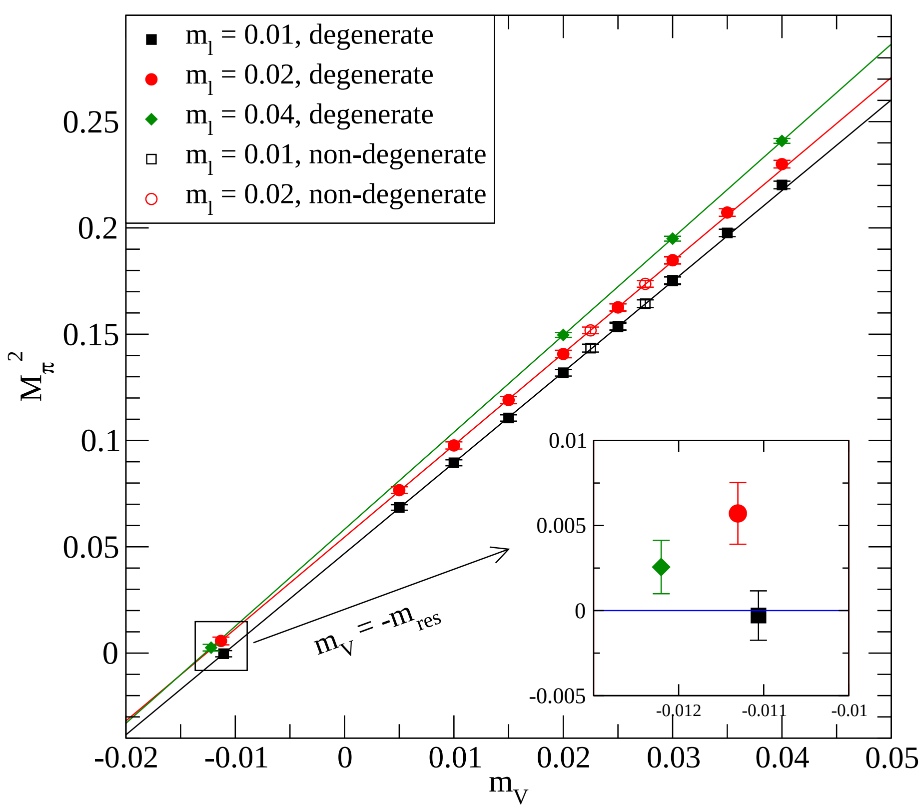
m_{res} for 2+1 flavors at $a^{-1} = 1.6$ GeV

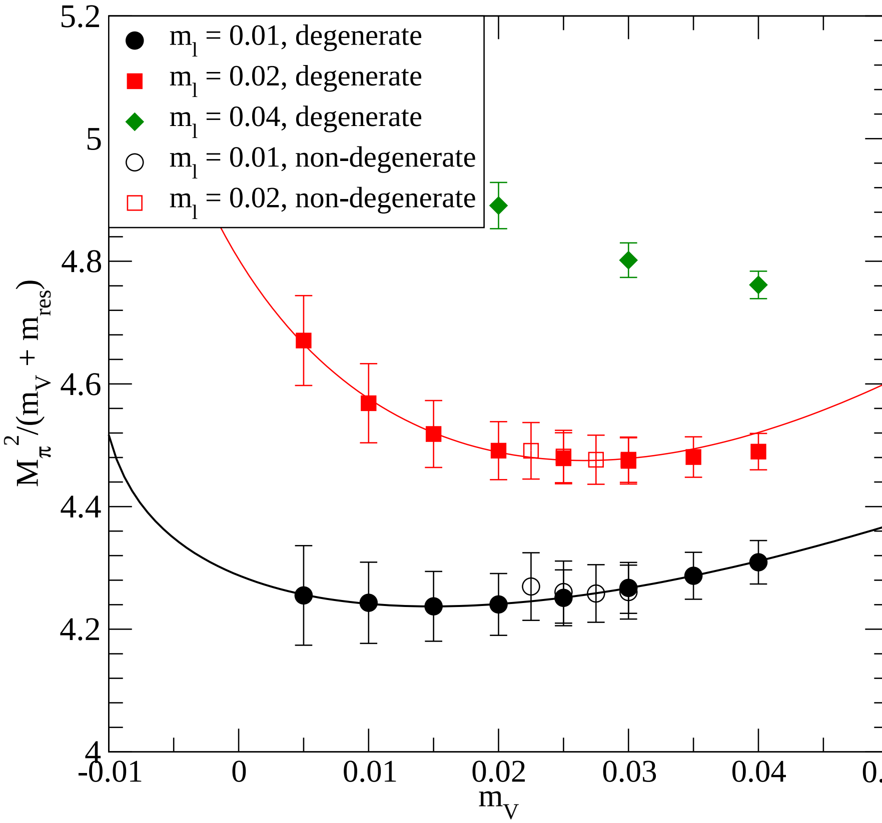


$N_f = 2+1$ Chiral Extrapolation for m_π

- Continuum PQχPT known to NNLO (Bijnens). Try NLO fits to current data

$$
\begin{aligned}
M_\pi^2 \quad = \quad & \chi_V \Big\{ 1 + \frac{16N}{f^2}(2L_6 - L_4)\bar{\chi} + \frac{16}{f^2}(2L_8 - L_5)\chi_V \\
& + \frac{1}{8f^2\pi^2 N}\Big[\frac{2\chi_V - \chi_l - \chi_s}{\chi_V - \chi_\eta}\chi_V \log\chi_V - \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_V \log\chi_V \\
& + \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{\chi_V - \chi_\eta}(1 + \log\chi_V) + \frac{(\chi_\eta - \chi_l)(\chi_\eta - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_\eta \log\chi_\eta\Big]\Big\}
\end{aligned}
$$

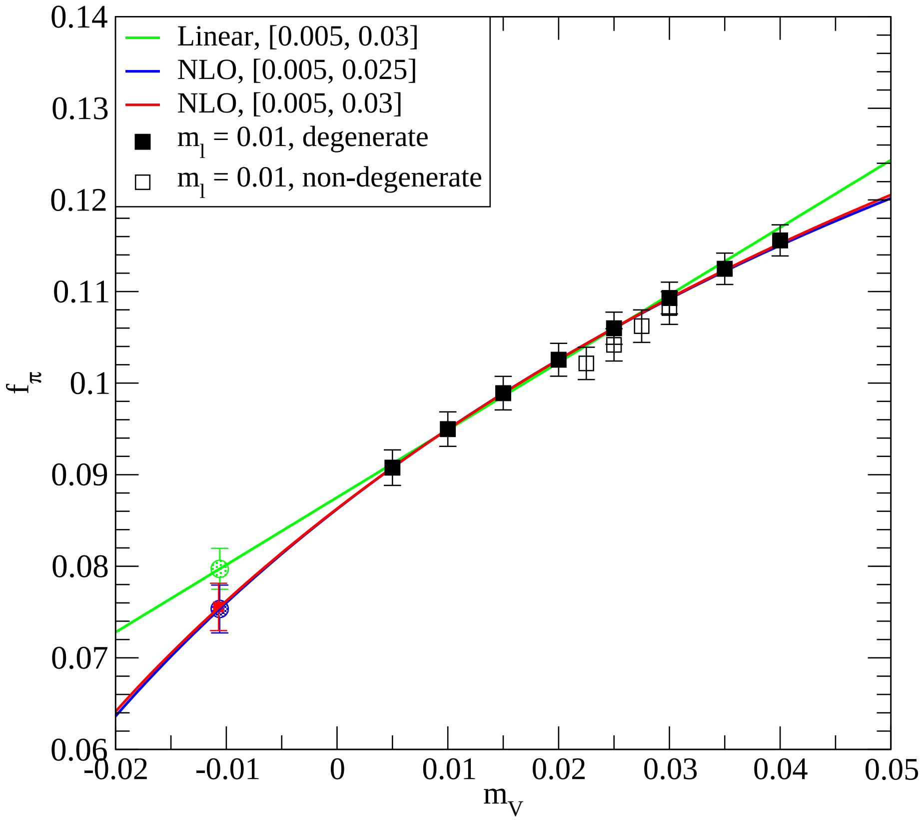


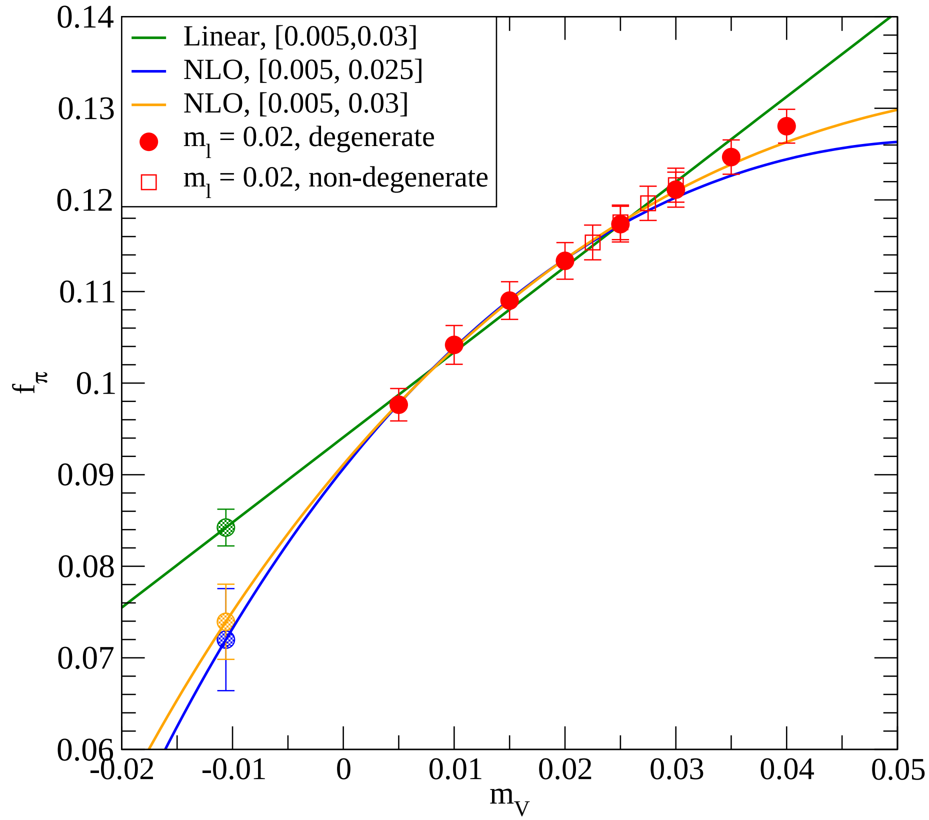


$N_f = 2+1$ Chiral Extrapolation for f_π

- Try NLO fits to current data

$$f_\pi = f\left\{1 + \frac{8}{f^2}(NL_4\bar{\chi} + L_5\chi_V)\right\}$$
$$- f\left\{\frac{1}{16\pi^2 f^2}\left[(\chi_V+\chi_l)\log\frac{\chi_V+\chi_l}{2} + \frac{\chi_V+\chi_s}{2}\log\frac{\chi_V+\chi_s}{2}\right]\right\}$$

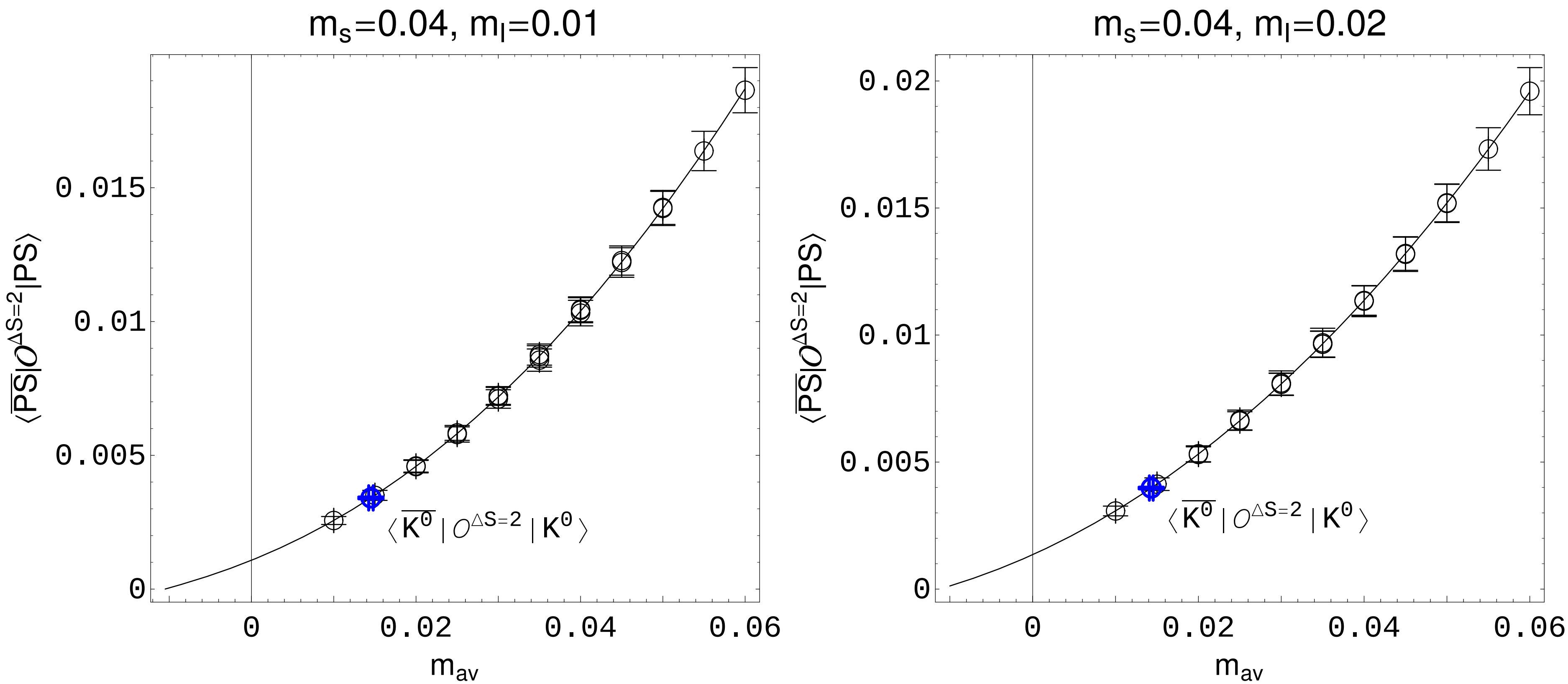




$N_f = 2+1$ Chiral Extrapolation for (27,1) Operator

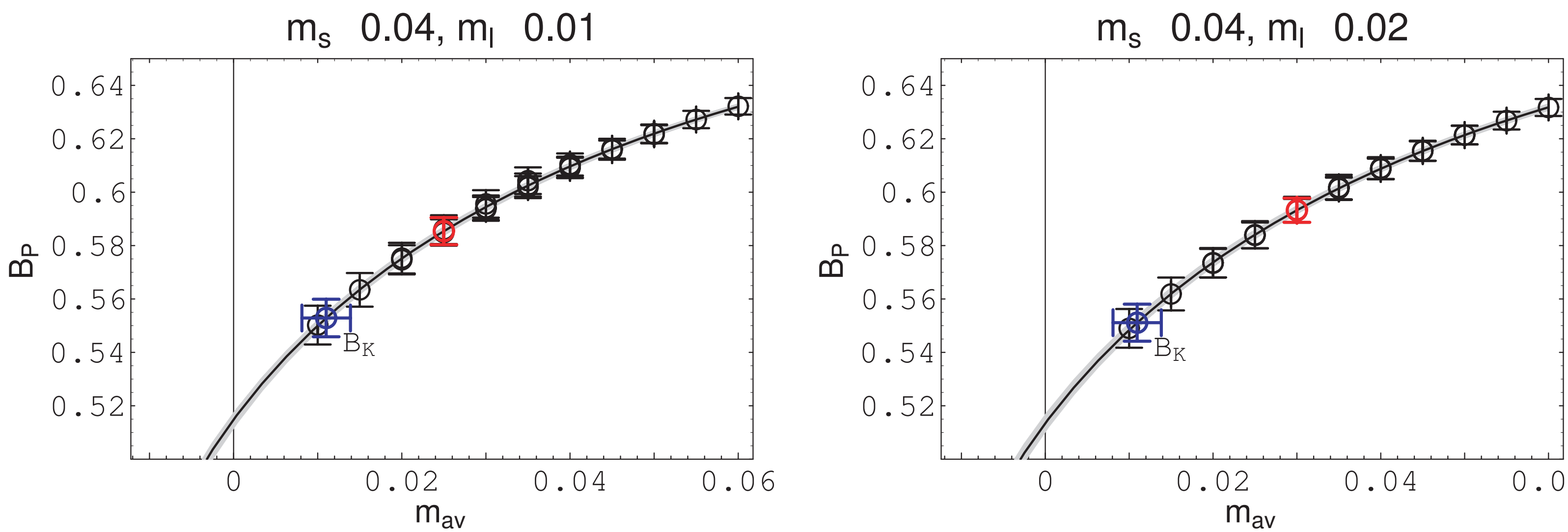
- Chiral log fit, with free coefficient, that must vanish at $m_f = -m_{\rm res}$.
- Good description of data - large $m_{\rm res}$ ($L_s = 8$) apparently not a problem
- Coarse lattices with $L_s = 16$ or larger will be better.

$\langle \overline{PS} | \mathcal{O}^{\Delta S=2} | PS \rangle$



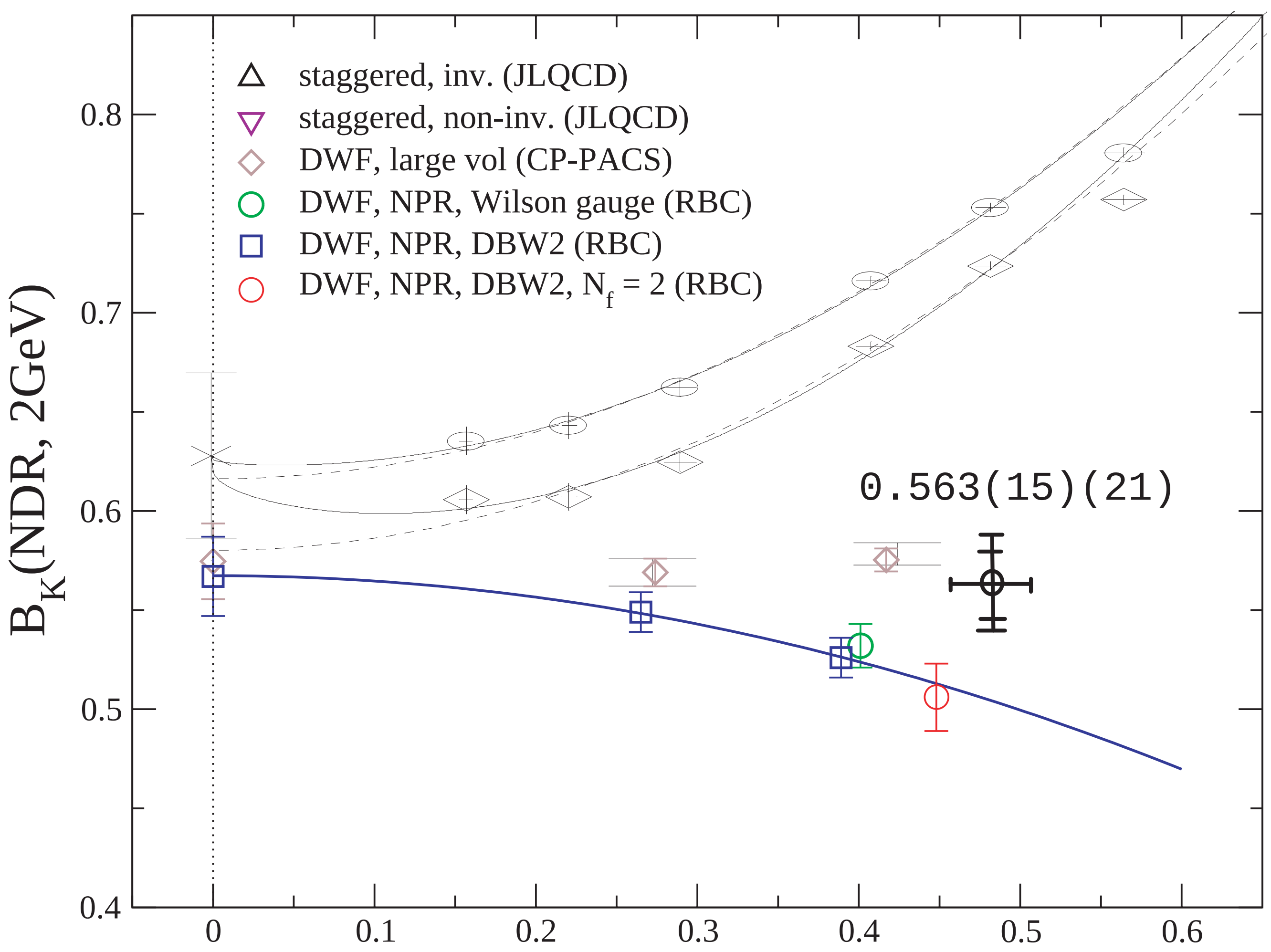
$B_{\rm PS}$ **for** $N_f = 2 + 1$

- 2+1 flavor continuum PQχPT recently calculated to NLO (Sharpe and van der Meer)
- Use simple chiral parameterization for current data
- Extrapolate to light dynamical quark limit

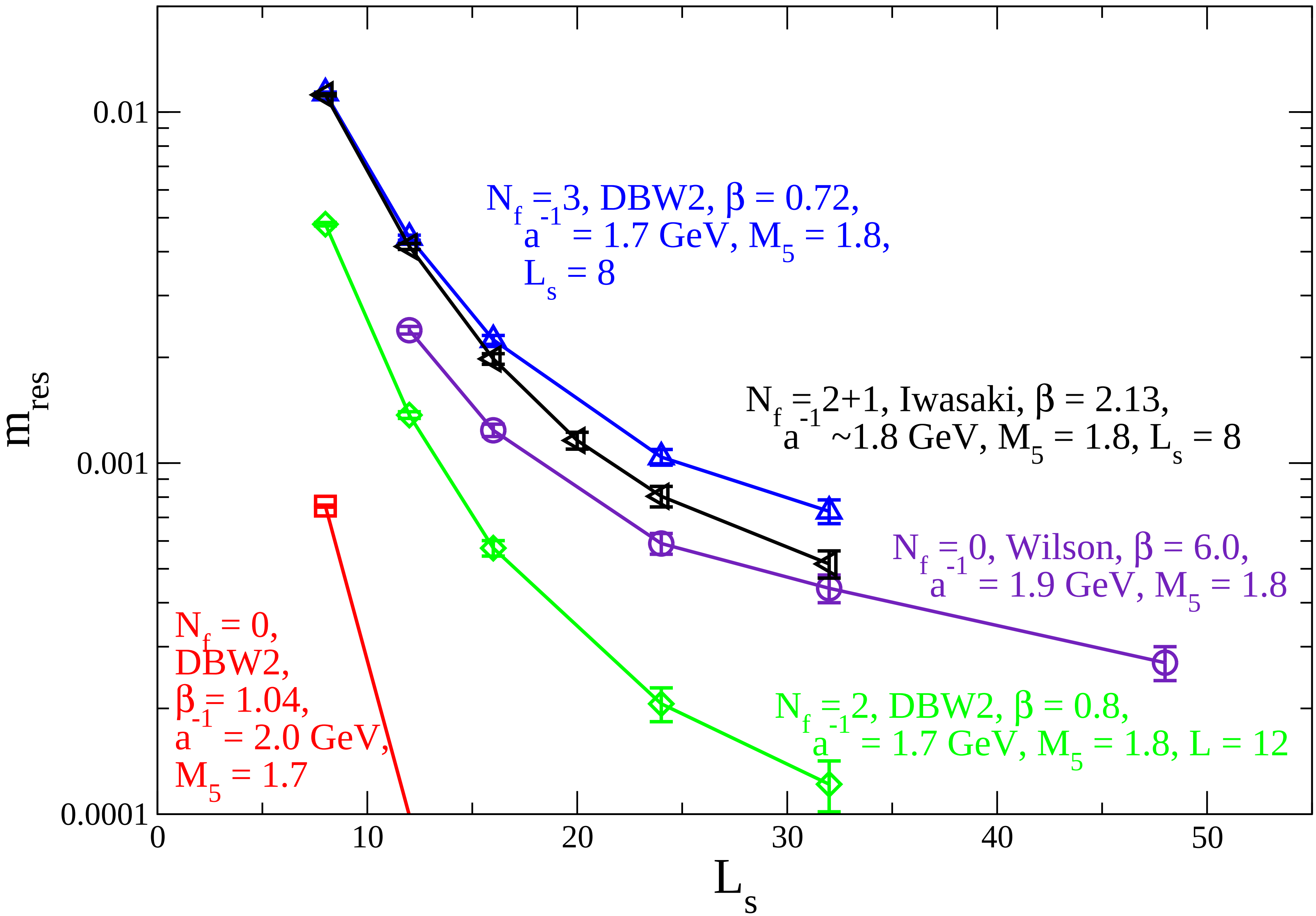


The Kaon B Parameter, $B_K^{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV})$

	quenched $a \to 0$			dyn. $a^{-1} = 1.7$ GeV
PDG	JLQCD (stag)	CP-PACS (DWF)	RBC (DWF)	RBC (2f DWF)
0.65 ± 0.15	0.628 ± 0.042	0.575 ± 0.019	0.563 ± 0.021	0.492 ± 0.018
			$\pm 39 \pm 30$	



Revisiting $m_{\rm res}$ versus L_s for $N_f = 0, 2$ and 3



Conclusions and Outlook

- QCDOC provides powerful resource for Lattice QCD.
- 2+1 flavor dynamical DWF simulations with the RHMC algorithm underway by RBC/UKQCD. DWF working.
- Controlling m_{res}, having reasonable topological tunneling rate and running for different a values possible.
- Iwasaki gauge action with $m_{\mathrm{light}} = 3/4$, $1/2$ and $1/4$ of m_{strange} underway.