Lattice OPE and higer moments of structure functions and distribution amplitudes

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Jlab
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Based on

W. Detmold and C-JDL, Phys Rev D71, 054510 (2005).
W. Detmold and C-JDL, hep-lat/0507007.

Outline

- Hadronic tensor and the OPE.
- The OPE on the lattice.
- Extracting moments from lattice data.
- Chiral extrapolation and finite-volume effects.
- Conclusion.

Hadronic tensor and the OPE

$$\underbrace{W_S^{\mu\nu}(p,q) = \int d^4x \; \mathrm{e}^{iq\cdot x} \langle p,S|\,[J^\mu(x), J^\nu(0)]\,|p,S\rangle}$$

↓ Imaginary part of

$$\underbrace{T_S^{\mu\nu}(p,q) = \int d^4x \; \mathrm{e}^{iq\cdot x} \langle p,S|T\,[J^\mu(x) J^\nu(0)]\,|p,S\rangle}$$

↓ the OPE

$$T\,[J^\mu(x)J^\nu(0)] = \sum \mathcal{C}_i(x^2,\mu^2) x_{\mu_1}\ldots x_{\mu_n}\, \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu)$$

Lattice calculations

- Analytic continuation

  $\rightarrow$ Difficult to obtain $T_S^{\mu\nu}$ directly.

- Operator mixing and renormalisation

  $\rightarrow$ Difficult for high-spin operators.

The OPE on the lattice

General features

$$\underbrace{\langle p,S|T\left[J^{\mu}(x)J^{\nu}(0)\right]|p,S\rangle}_{\text{Simulation}} = \underbrace{\sum \mathcal{C}_i(x^2,\mu^2)\; x_{\mu_1}\ldots x_{\mu_n}}_{\text{Analytical calculation}} \; \underbrace{\langle p,S|\mathcal{O}_i^{\mu\nu\mu_1\cdots\mu_n}(\mu)|p,S\rangle}_{\text{Fits}}$$

- First investigated in kaon physics.

  C. Dawson *et al.*, 1998.

- Applied to structure functions.

  S. Capitani *et al.*, 1999.

Our proposal

- Simulation of $\sum_S T_S^{\mu\nu} \to$ Continuum limit.

  $\to$ No power divergence.

- Perform the OPE in Euclidean space
- Fit the matrix elements.

  $\to$ No need for analytic continuation.

  $\to$ No need for operatos matching.

The OPE on the lattice

Special features

- Take the Fourier transform.
- A fictitious valence heavy quark Ψ and current

$$J^{\mu}_{\Psi,\psi}(x) = \bar{\Psi}(x)\gamma^{\mu}\psi(x) + \bar{\psi}(x)\gamma^{\mu}\Psi(x).$$

- Study the Euclidean Compton scattering tensor

$$T^{\mu\nu}_{\Psi,\psi} = \sum_{S} \int d^4x \ \mathrm{e}^{iq\cdot x} \left\langle p, S \left| T\left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0) \right] \right| p, S \right\rangle$$

- Sum the target-mass effects.
- Compute the twist-two matrix elements.

Why a valence heavy quark?

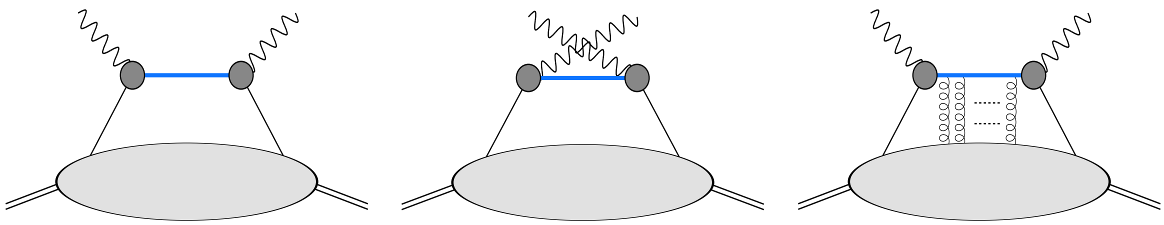
- Two large scales, q^2 and m_Ψ.

$$\Lambda_{\mathrm{QCD}} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{a}$$

- Remove many higher-twist contributions.
- No all-to-all propagator in the simulation.
- The Fourier transform is practical $\rightarrow x_4 \sim 1/m_\Psi$.

The OPE on the lattice

Some details



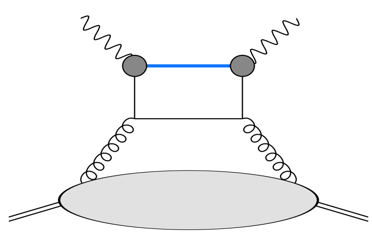
Higher-twist components

$$\frac{i(i\not{D}+\not{q})+m_\Psi}{(iD+q)^2+m_\Psi^2} = -\frac{i(i\not{D}+\not{q})+m_\Psi}{Q^2+D^2-m_\Psi^2}\sum_{n=0}^{\infty}\left(\frac{-2i\ \ q\cdot D}{Q^2+D^2-m_\Psi^2}\right)^n$$

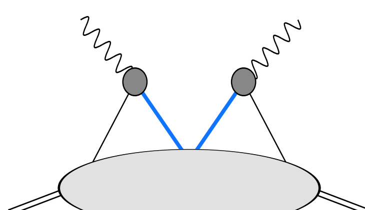
$$\frac{\Lambda_{\mathrm{QCD}}^2}{q^2+m_\Psi^2}$$

$$Q^2 = -q^2, \qquad M_\Psi = m_\psi + \alpha/2$$

$$Q^2 + D^2 - m_\Psi^2 = \tilde{Q}^2 = Q^2 - M_\Psi^2 + \alpha M_\Psi + \beta$$



No contribution in $T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}$.



Removed because Ψ is non-dynamical.

Extrating moments from data

The Euclidean Compton tensor

$$\sum_S \left\langle p,S\left|\bar{\psi}\gamma^{\{\mu_1}(iD^{\mu_2})\ldots\left(iD^{\mu_n\}}\right)-\text{tr}\right|p,S\right\rangle = A^n_\psi(\mu^2)\left[p^{\mu_1}\ldots p^{\mu_n}-\text{tr}\right]$$

$$\sum_S \left\langle p,S\left|\bar{\psi}\left(iD^{\{\mu_1}\right)\ldots\left(iD^{\mu_n\}}\right)-\text{tr}\right|p,S\right\rangle = \widehat{A}^n_\psi(\mu^2)\left[p^{\mu_1}\ldots p^{\mu_n}-\text{tr}\right]$$

$$T^{\{\mu\nu\}}_{\Psi,\psi}(p,q) = i\sum_{\substack{n=2\\ \text{even}}}^{\infty} A^n_\psi(\mu^2)\ \zeta^n\ \mathcal{F}\left[C^{(1)}_n(\eta),C^{(2)}_{n-1}(\eta),C^{(3)}_{n-2}(\eta),n,q^2,\tilde{Q}^2,\mu^2\right]$$
$$-2i\frac{M(m_\Psi-m)}{\tilde{Q}^2}\delta^{\mu\nu}\sum_{\substack{n=0\\ \text{even}}}^{\infty}\widehat{\mathcal{C}}_n\ \widehat{A}^n_\psi(\mu^2)\ \zeta^n\ C^{(1)}_n(\eta)$$

- The Gegenbauer polynomial: target-mass effects.

$$\eta = \frac{p\cdot q}{\sqrt{p^2q^2}}$$

- $\tilde{Q}^2 \sim -q^2 - M^2_\Psi$ is the large scale for the OPE.

$$\zeta = \frac{\sqrt{p^2q^2}}{\tilde{Q}^2}$$

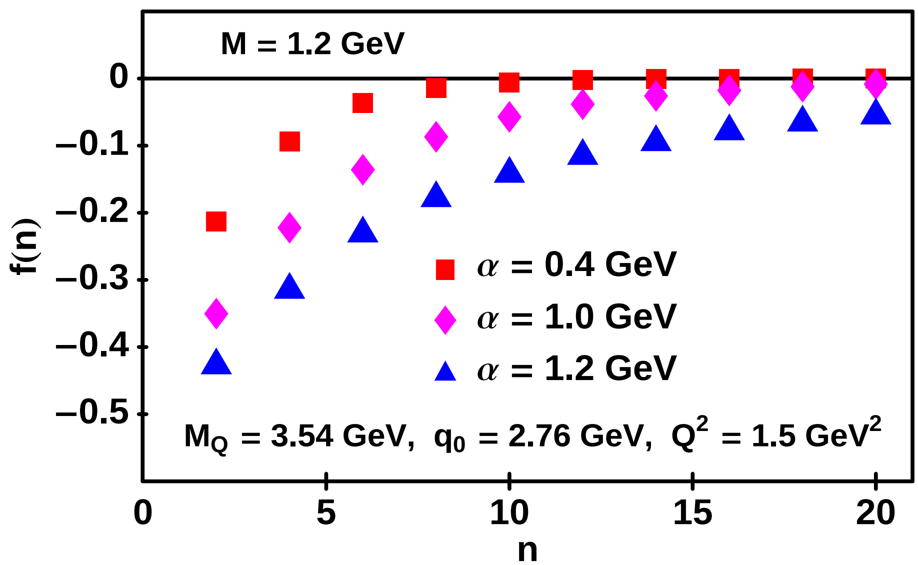
- Remove $\widehat{A}^n_\psi$ by choosing $\mu \neq \nu$.

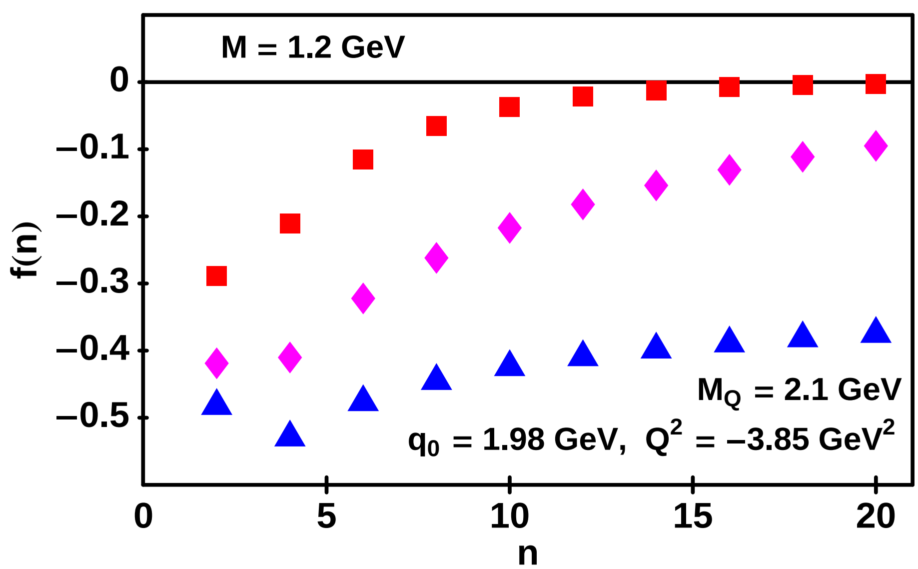
Extracting moments from data

Plots

$$T^{\{34\}}_{\Psi,\psi}(p,q) \quad = \sum_{n=2,\text{even}}^{\infty} A^n_\psi(\mu^2) f(n)$$

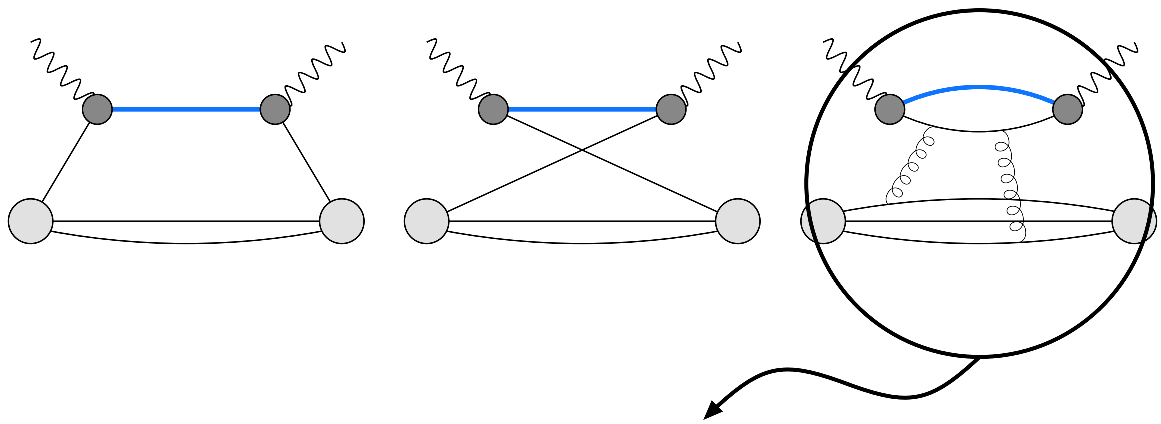
$$p = (0,0,0,iM) \quad , \quad q = (0,0,\sqrt{q_0^2 - Q^2}, iq_0)$$





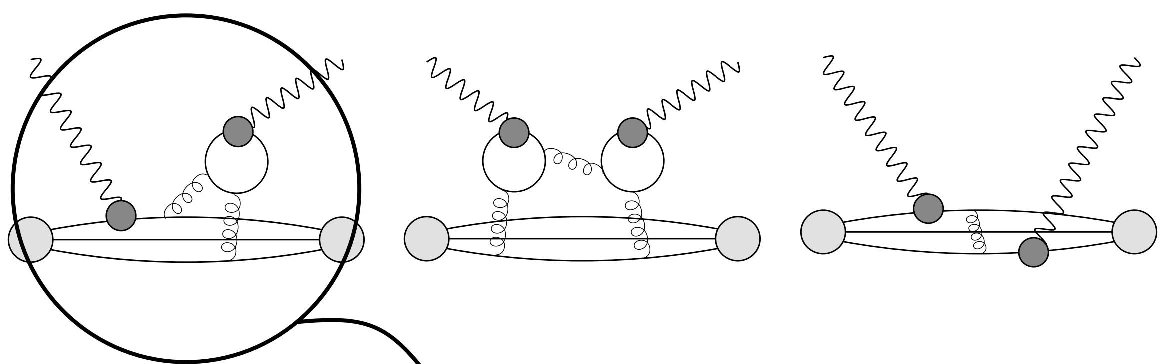
Extracting moments from data

Correlators



No contribution for $T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}$

Additional contractions if light-light current is used.



Contributes to $T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}$

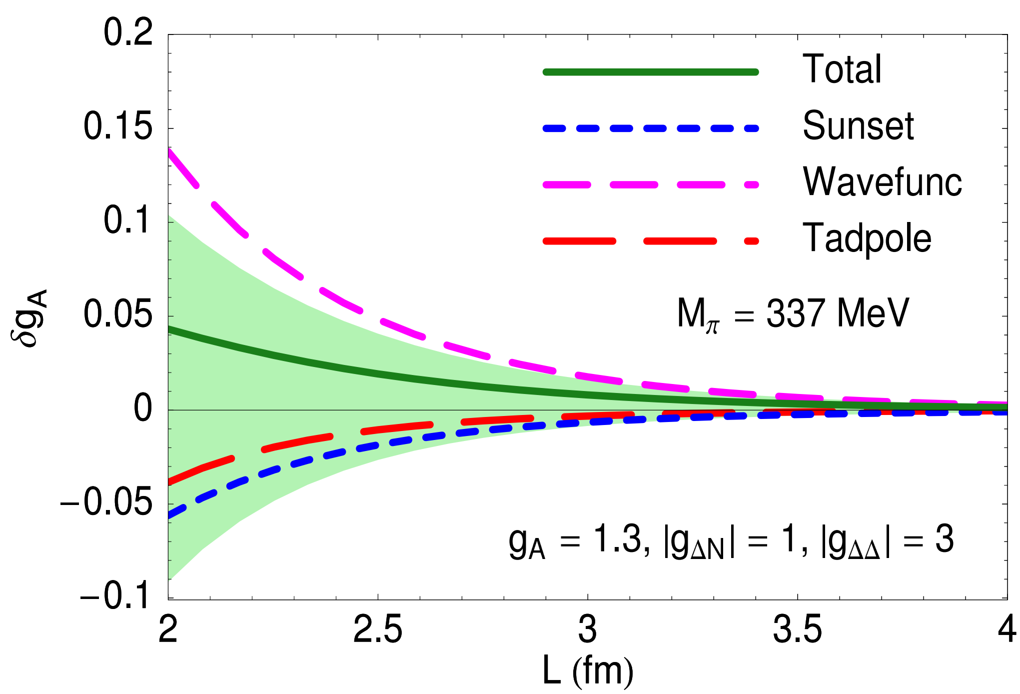
The chiral extrapolation and volume effects

W. Detmold and C-JDL, Phys Rev D71, 054510 (2005).

- Unpolarised, helicity, transversity structure functions.
- Isovector and isoscalar combinations.
- $SU(2), SU(3), SU(4|2), SU(6|3), SU(2|2)$.
- Typical FV effects

→ 5–10% for PQQCD.

→ Larger for QQQCD.



Conclusion

- Extract moments via the OPE on the lattice.
- Can be applied to other nucleon structure functions.
- Can be applied to the pion distribution amplitude

$$\left\langle \pi(p) \left| \bar{\psi}\gamma^{\{\mu_1} (iD^{\mu_2}) \ldots \left(iD^{\mu_n\}}\right) \right| 0 \right\rangle,$$

  via the OPE of the tensor

$$\left\langle \pi(p) \left| T \left[V^{\mu}_{\Psi,\psi}(x) A^{\nu}_{\Psi,\psi}(0) \right] \right| 0 \right\rangle.$$

- Volume effects in structure functions.