Non-perturbative determination of the Fermilab parameters

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Outline

- Review of the action.
- Proposal.
- Simulation details.
- Preliminary result and improvement.
- Conclusion and outlook.

Relativistic Heavy Quark Action (RHQ) - Review

- The general relativistic heavy quark action:

$$S = \sum_n \bar{\psi}_n (D_{RHQ} + m_0)\psi_n$$

$$D_{RHQ} + m_0 = m_0 + \gamma_0 D_0 + \zeta \vec{\gamma}\cdot\vec{D} - r_t a D_0^2 - r_s \sum_i D_i^2$$

$$+ \sum_i \frac{i}{2} c_E \sigma_{0i} F_{0i} + \sum_{i,j} \frac{i}{4} c_B \sigma_{ij} F_{ij}$$

El-Khadra et. al. (hep-lat/9604004)

S. Aoki et. al. (hep-lat/0107009)

- 6 parameters : m_0, ζ, c_B, c_E, r_t and r_s.
- $r_t = 1$ (redundant)
- $r_s = \zeta$ (Fermilab action)
- m_0, c_B, c_E and ζ to be tuned.

Fermilab vs Tsukuba action

- Parameters:

action	$\gamma_0 D_0$	$\vec{\gamma}\cdot\vec{D}$	D_0^2	$\sum_i D_i^2$	$\sigma_{0i}F_{0i}$	$\sigma_{ij}F_{ij}$
Fermilab	1.0	ζ	1.0	$1.0\times\zeta$	$c_E\zeta$	$c_B\zeta$
Tsukuba	1.0	ν	1.0	r_s	c_E	c_B

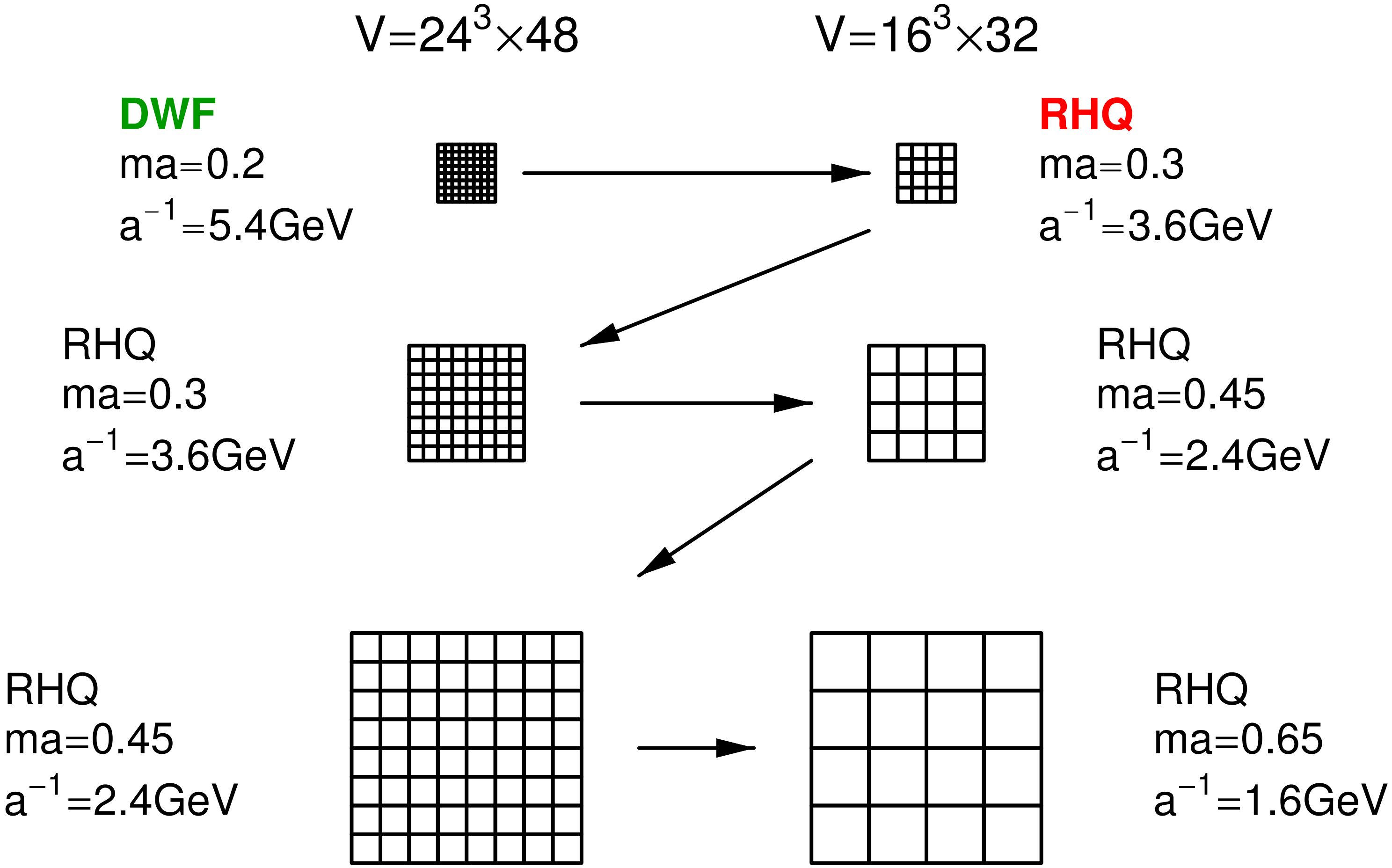
- Possible connection?? Consider an spectrum-conserved O(a) field transformation:

$$\psi \to [1 + a\delta\vec{\gamma}\vec{D}]\psi$$
$$\bar{\psi} \to \bar{\psi}[1 - a\delta\vec{\gamma}\overleftarrow{D}].$$

- In the continuum form, $\delta = \frac{-r_s-\zeta}{2(ma-\zeta)}$ seems do the trick.
- On the lattice, they are slightly different because of the discritization for D_μ^2 and $D_\mu D_\mu$.
- No effect on the particle spectrum.
- The r_s could be determined from the spinor structure of the nucleon propagator.

NP determined Fermilab coefficients

We propose an idea of matching on-shell quantities through an approach similar to the step-scaling technique by *Sommer et. al. hep-lat/0310035*.



Matched Quenched Configurations

- Hardware: QCDOC 512-node machine at frequency 420 Hz, reproducibility checked.
- Wilson gauge action with heatbath algorithm.
- Choices of β:

$$\ln(a/r_0) = d_0 + d_1(\beta - 6) + d_2(\beta - 6)^2 + d_3(\beta - 6)^3$$

  with $d_0 = -1.6804$, $d_1 = -1.7339$, $d_2 = 0.7849$, and $d_3 = -0.4428$ for $5.7 \leq \beta \leq 6.92$. *ALPHA(hep-lat/0108008)*
- Choose $\beta = 6.638$ for $a^{-1} = 5.4$ GeV lattice and $\beta = 6.351$ for $a^{-1} = 3.6$ GeV one.

Static quark potential

- The static quark potential

$$V(\vec{r},t) = \log\left[\frac{\langle W(\vec{r},t)\rangle}{\langle W(\vec{r},t+1)\rangle}\right] = C - \frac{\alpha}{R} + \sigma R$$

where R is the size of the Wilson loop.

- The lattice spacing is determined from

$$R^2 F(R)|_{R=r_0} = 1.65$$

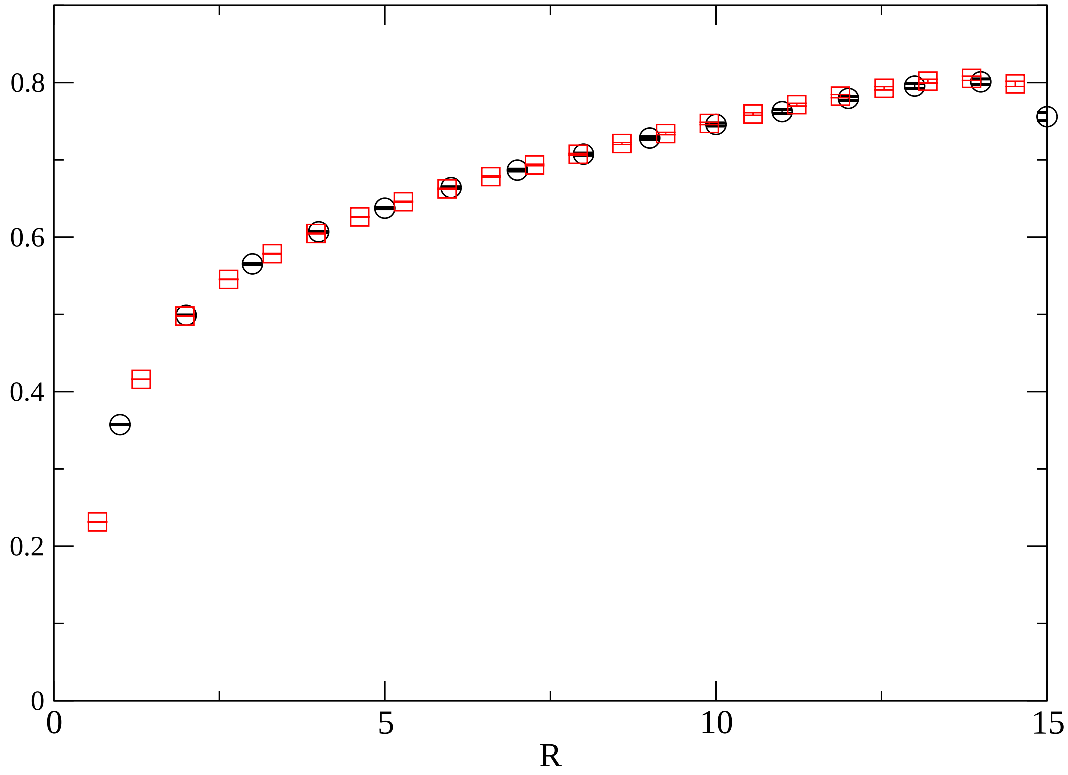
for $\beta < 6.57$ and

$$R^2 F(R)|_{R=r_1} = 0.65$$

for $\beta > 6.57$ where $\frac{r_1}{r_0} = 0.5133(24)$.

Lattice spacing ratio: λ

- Alternative check without fixing different scales of r_0 and r_1.
- Fit $\lambda(= a_2/a_1)$ to match potentials:



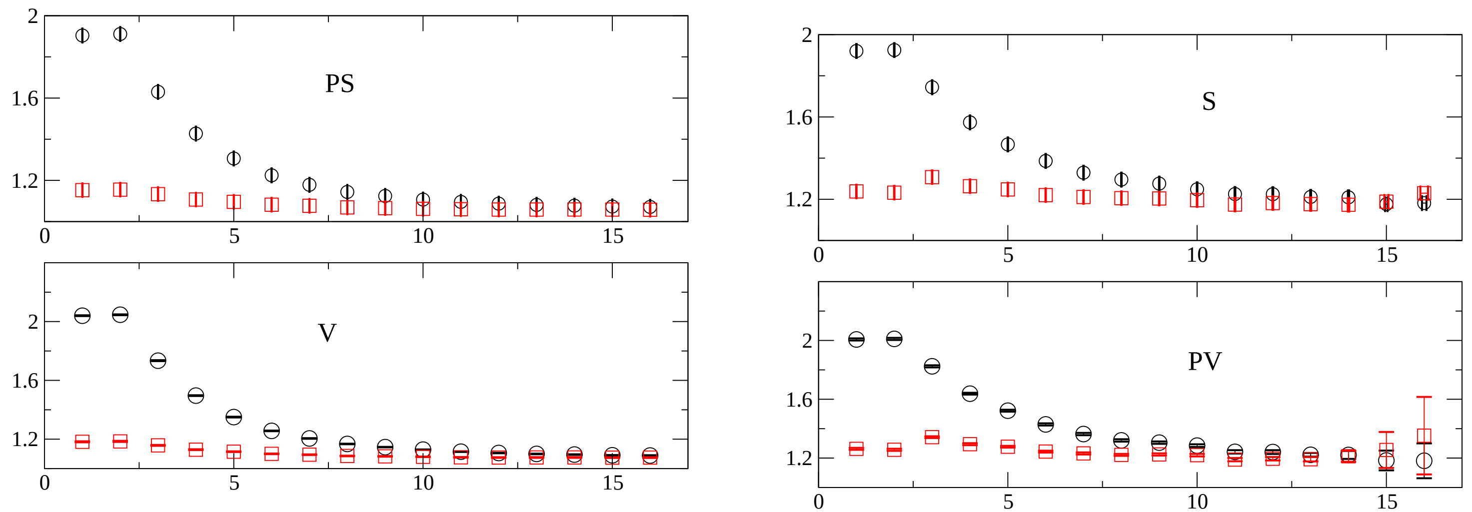
- $\lambda = 1.51(2)$.

Heavy Quark Source Smearing

- Coulomb gauge-fixed hydrogenic source smearing

$$\Psi_{gnd}(r) = e^{-r/r_0}$$
$$\Psi_{ex}(r) = (1 - r/2r_0)e^{-r/2r_0}$$

- Effective Mass Plot



Fixed Parameters/Quantities

- Parameters:

Label	β	V	S_L	m_L	S_H	m_H	a^{-1}(SQ pot.)
Fine	6.638	$24^3 \times 48$	DWF	0.02	DWF	0.2	5.4 (GeV)
Coarse	6.351	$16^3 \times 32$	DWF	0.03	RHQ	Unknowns	3.6 (GeV)

with fixed L=0.9 fm, and $L_s = 12$, $M_5 = 1.5$ for DWF actions.

- Results:
  - Fine

	m_{PS}	m_V	m_1/m_2	m_{PV}	m_S
light-light	0.175(3)	0.233(5)	–	–	–
heavy-light	0.467(2)	0.485(3)	–	–	–
heavy-heavy	0.716(1)	0.728(1)	1.02(2)	0.810(5)	0.799(4)

  - Coarse

	m_{PS}	m_V	c^2
light-light	0.259(6)	0.328(10)	N/A

- 100 configurations.

Strategy

- Linear combinations of 7 on-shell small volume physical masses, including $\vec{p} \neq 0$, denoted as $\{(Y^i_{coarse})_a\}_{1\leq a\leq 7}$:

  $\{\frac{1}{4}(M_{PS}+3M_V)^{hh,i}, (M_{PS}-M_V)^{hh,i}, (M_{PV}-M_S)^{hh,i}, \frac{1}{4}(M_{PV}+3M_S)^{hh,i}, (m_1/m_2)^{hh,i}, \frac{1}{4}(M_{PS}+3M_V)^{hl,i}, (M_{PS}-M_V)^{hh,i}\}$

- Analysis: Fit

  $$Y^i_{coarse} = A + J \cdot X^i_{RHQ}$$

  A: d-dimensional vector
  J: $d \times 4$ matrix
  $\{(X_{RHQ})^i_a\}_{1\leq a\leq 4} = \{m^i_0, c^i_B, c^i_E, \zeta^i\}$

  with individual weight of $(\frac{Y_{fine}-Y^i_{coarse}}{Y_{fine}})^2$

- Minimizing

  $$\chi^2_{fine} = \sum_{n=1}^{d} \frac{|(J\cdot X_{RHQ} + A - Y_{fine})_n|^2}{\sigma^2_{fine,n}}.$$

  to solve the new set of parameters X_{RHQ}.

Preliminary Result

- Selections of shortest distance in data space from the fine lattice data:

m_0	c_B	c_E	ζ
0.023	1.609	1.438	1.044
0.0426	1.609	1.438	1.044
0.0328	1.511	1.538	1.036
0.0274	1.709	1.307	1.039
0.023	1.511	1.438	1.036
0.09	1.713	1.578	1.025
0.08	1.707	1.576	1.023
0.0426	1.550	1.438	1.007

- $X^{out} = \{$ 0.04(3), 1.70(22), 1.22(16), 1.047(27)$\}$

Unique?

Category data into 2 separate regions of c_B:

m_0	c_B	c_E	ζ
0.00	1.552	1.458	1.013
0.07	1.547	1.424	1.001
0.0426	1.550	1.438	1.007
0.0426	1.550	1.438	1.100
0.0426	1.550	1.438	0.900
0.0426	1.550	1.438	1.007
0.0	1.552	1.438	1.013
0.0328	1.511	1.438	1.036
0.0230	1.511	1.438	1.036
Output			
0.05(4)	1.73(17)	1.43(2)	1.05(1)

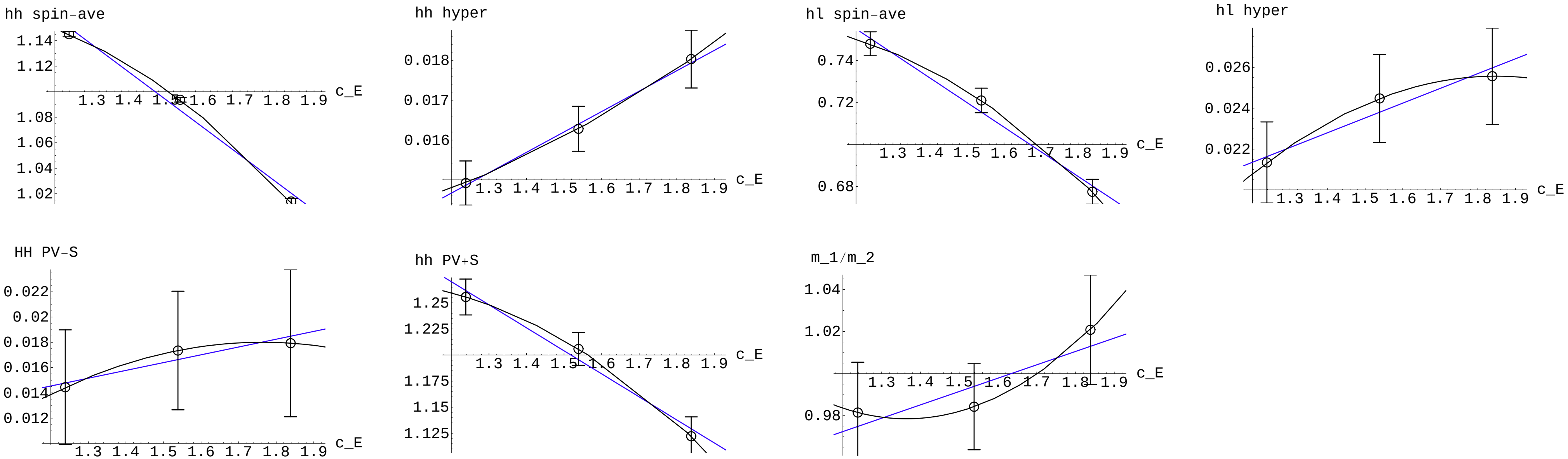
m_0	c_B	c_E	ζ
0.0330	1.609	1.538	1.044
0.01	1.700	1.574	1.022
0.00371	1.709	1.577	1.023
0.0138	1.715	1.579	1.025
0.02	1.719	1.580	1.026
0.03	1.725	1.582	1.027
0.08	1.707	1.576	1.023
0.09	1.713	1.578	1.025
0.1	1.719	1.580	1.026
Output			
0.08(3)	1.72(8)	1.58(3)	1.03(2)

- The output coefficients of c_B are consistent for two different regions of simulation.

Linearity?

- Center: $X_{RHQ} = \{0.0328, 1.511, 1.538, 1.036\}$
- "Linear" region for requiring the linear fit can only differ up to 5%: m_0: 0.1, c_B: 0.1, c_E: 0.3, ζ: 0.02.

- Plot for the change in c_E direction.



Eigenspace

- Take the above selection for example.
  - The eigenvalues of the $J^T \cdot J$ matrix are:
  { 9.58(15), 1.39(10), 0.000144(22), 0.000037(12)}.
  with corresponding eigenvectors:
  { 0.832(4), -0.1101(6), -0.1081(7), 0.533(6)}
  { -0.523(7), 0.062(6), 0.086(3), 0.846(4)}
  { 0.182(7), 0.81(7), 0.56(10), -0.004(6)}
  { 0.040(23), -0.57(10), 0.82(7), -0.0157(29)}

- The smallest eigenvalue is dominated by the eigenvector that has large component in c_E and that leads to large errorbar in the c_E coefficient.
- Improve the X^{out} $\Rightarrow$
  - Improve accuracy of smallest eigenvalue and eigenvector.
  - Improve accuracy of corresponding fine-lattice measurements.

Comparison with the perturbative coefficients

- on $\beta = 6.351$ lattice:
  - NP $c_{SW} = 1.544$ *M. Luscher et. al. (hep-lat/9609035)*
  - Wilson(G) + Clover(F): *H. Panagopoulos et. al. (hep-lat/0108021)*
  2-loop $\kappa_{crit} = 0.132$
  and $m_0 = 0.123$ gives m_q about 1.08 GeV for our simulation.

- Compared with the result from Tsukuba action calculation *S. Aoki et. al. (hep-lat/0309161)*
  after transformation (5 to 4 parameters):
  $X^{PT,out} = \{0.122, 2.370, 1.523, 1.216\}$
  while our previous values are
  $X^{out} = \{0.04(3), 1.70(22), 1.22(16), 1.047(27)\}$.

- No surprise. PT predicts $c_{SW} = 1.254$ vs NP one $= 1.544$.

Conclusion and Outlook

- It appears practical to determine m_0, c_B, c_E and ζ from finite-volume, non-perturbative matching.
- More statistics and better parameter coverage may reduce the matching errors to a few percent.
- The next step is to do this for full QCD.
  - Matching requires N_f=3 but not a physical value of m_{sea}.
  - Can make m_{sea} large and the calculation easy provided $m_{sea}a \ll 1$.
  - Must have m_{sea}/Λ_{QCD} equal for each pair of systems being matched.