N_f=2 DWF ΔI=1/2 Rule

Shu Li
Columbia University
For RBC Collaboration

3rd International
Lattice Field Theory
Network Workshop

Jefferson Lab
Newport News, VA
October 3-6, 2005

RBRC-BNL-CU (RBC) Collaboration

RBRC	BNL	Columbia
Blum, Tom (U Conn)	Creutz, Mike	Aubin, Christopher
Dawson, Chris	Jung, Chulwoo	Cheng, Michael
Doi, Takumi	Karsch, Frithjof	Christ, Norman
Hashimoto, Koichi	Petreczky, Peter	Cohen, Saul
Izubuchi, Taku (Kanazawa)	Petrov, Konstantin	Li, Shu "Sam"
Kaneko, Takashi	Schmidt, Christian	Lin, Meifeng
Ohta, Shigemi (KEK)	Soni, Amarjit	Lin, HueyWen
Sasaki, Shoichi (KEK)		Loktik, Oleg
Yamazaki, Takeshi		Mawhinney, Robert

1. $K \to \pi\pi$ in the Standard Model

- At energies below electro-weak scale, can use local four-fermion operators. (OPE)

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i A_i(\mu, m_t, m_W, m_Z, \alpha_S, \alpha, V_{lm}) (\bar{q}_i \Gamma_i q_i^{'}) (\bar{q}_i^{''} \Gamma_i^{'} q_i^{'''})$$

- For the $K \to \pi\pi$ decay process, the operators with *ΔS*=1, *ΔD*=-1 are relevant.

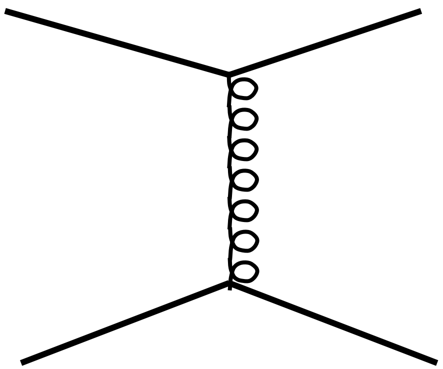
E>m$_C$: $$H_C^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{2} C_i(\mu) \left[P_i + (\tau - 1) P_i^C \right] + \tau \sum_{i=3}^{10} C_i(\mu) P_i \right\}$$

E<m$_C$: $$H^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

Vector boson exchange op.'s

Vector boson exchange operators are the most important, because of their large Wilson coefficients.



$$Q_1 \equiv \left(\bar{s}_\alpha d_\alpha\right)_{V-A}\left(\bar{u}_\beta u_\beta\right)_{V-A}$$ Color diagonal

$$Q_2 \equiv \left(\bar{s}_\alpha d_\beta\right)_{V-A}\left(\bar{u}_\beta u_\alpha\right)_{V-A}$$ Color mixed

$(V\text{-}A) = \bar{q}\gamma_\mu(1-\gamma_5)q$

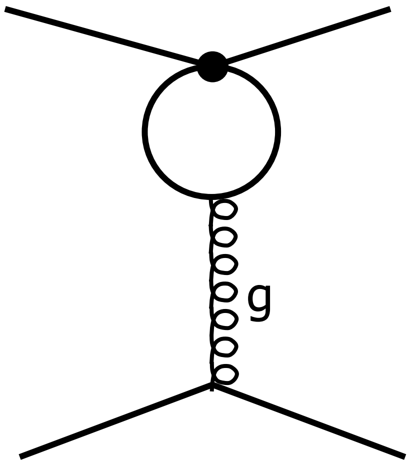
QCD penguin operators

$$Q_3 = \left(\bar{s}_\alpha d_\alpha\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\beta\right)_{V-A}$$

$$Q_4 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\alpha\right)_{V-A}$$

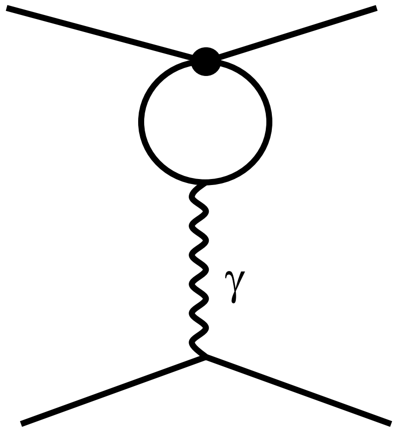
$$Q_5 = \left(\bar{s}_\alpha d_\alpha\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\beta\right)_{V+A}$$

$$Q_6 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$



Q_6 is especially important for ε'/ε because it makes the largest contribution to Im(A_0)

Electroweak penguin operators



$$Q_7 = \frac{3}{2}\left(\bar{s}_\alpha d_\alpha\right)_{V-A} \sum_{q=u,d,s} e_q \left(\bar{q}_\beta q_\beta\right)_{V+A}$$

$$Q_8 = \frac{3}{2}\left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} e_q \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

Q_8 is important to ε'/ε too, because it dominates Im(A_2)

$$Q_9 = \frac{3}{2}\left(\bar{s}_\alpha d_\alpha\right)_{V-A} \sum_{q=u,d,s} e_q \left(\bar{q}_\beta q_\beta\right)_{V-A}$$

$$Q_{10} = \frac{3}{2}\left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_{q=u,d,s} e_q \left(\bar{q}_\beta q_\alpha\right)_{V-A}$$

2. ΔI=1/2 Rule in $K \to \pi\pi$

An example:

$$\frac{\Gamma\left(K_s^0 \to \pi^+\pi^-\right)}{\Gamma\left(K^+ \to \pi^+\pi^0\right)} \cong 450$$

Final isospin (0,0) or (2,0)

ΔI=1/2 or 3/2

Final isospin (2,1)

ΔI=3/2

Initial I=1/2

The Wilson coefficients calculation helped to explain part of this effect. However, there is still a factor that must be ascribed to non-perturbative effect.

SU(3)/Isospin of operators

LL Op.	$(8_L,1_L)$, 1/2, S	$(8_L,1_L)$, 1/2, A	$(27_L,1_L)$, 1/2, S	$(27_L,1_L)$, 3/2, S
Q_1	1/10	1/2	1/15	1/3
Q_2	1/10	-1/2	1/15	1/3
Q_3	1/2	1/2		
Q_4	1/2	-1/2		
Q_9	-1/10	1/2	1/10	1/2
Q_{10}	-1/10	-1/2	1/10	1/2
LR Op.	**$(8_L,1_R)$, 1/2, S**	**$(8_L,1_R)$, 1/2, A**	**$(8_L,8_R)$, 1/2**	**$(8_L,8_R)$, 3/2**
Q_5	1/2	1/2		
Q_6	1/2	-1/2		
Q_7			1/2	1/2
Q_8			1/2	1/2

Isospin definite operators

ΔI=1/2 :

$$Q_1^{(1/2)} = \frac{2}{3}Q_1 - \frac{1}{3}Q_2 + \frac{1}{3}(\bar{s}d)_{V-A}(\bar{d}d)_{V-A}$$

$$Q_2^{(1/2)} = \frac{2}{3}Q_2 - \frac{1}{3}Q_1 + \frac{1}{3}(\bar{s}d)_{V-A}(\bar{d}d)_{V-A}$$

$Q_{3-6}^{(1/2)}$ are pure Isospin(1/2) operators

$$Q_{7,9}^{(1/2)} = \frac{1}{2}\left[(\bar{s}d)_{V-A}(\bar{u}u)_{V\pm A} - (\bar{s}u)_{V-A}(\bar{u}d)_{V\pm A} - (\bar{s}d)_{V-A}(\bar{s}s)_{V\pm A}\right]$$

$$Q_{8,10}^{(1/2)} = \frac{1}{2}\left[(\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V\pm A} - (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V\pm A} - (\bar{s}_\alpha d_\beta)_{V-A}(\bar{s}_\beta s_\alpha)_{V\pm A}\right]$$

ΔI=3/2 :

$$Q_i^{(3/2)} = Q_i - Q_i^{(1/2)} \quad (i = 1 \sim 10)$$

(Maiani-Testa theorem)

3. χPT for $K \to \pi\pi$

We can relate $K \to \pi\pi$ matrix elements to $K \to 0$ and $K \to \pi$ matrix elements by Chiral Perturbation Theory.

ΔI=3/2:

$$\left\langle \pi^+ \middle| \Theta^{(27,1)} \middle| K^+ \right\rangle = -\frac{4m_M^2}{f^2}\alpha^{(27,1)}$$

$$\left\langle \pi^+\pi^- \middle| \Theta^{(27,1)} \middle| K^0 \right\rangle = -\frac{4i}{f^3}\left(m_{K^0}^2 - m_{\pi^+}^2\right)\alpha^{(27,1)}$$

$$\left\langle \pi^+ \middle| \Theta^{(8,8)} \middle| K^+ \right\rangle = \frac{12}{f^2}\alpha^{(8,8),(3/2)}$$

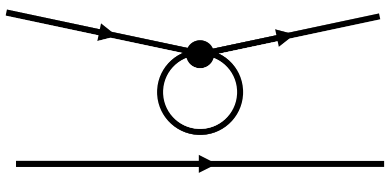
$$\left\langle \pi^+\pi^- \middle| \Theta^{(8,8)} \middle| K^0 \right\rangle = \frac{-12i}{f^3}\alpha^{(8,8),(3/2)}$$

χPT for ΔI=1/2 operators

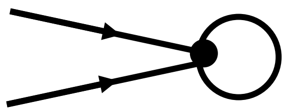
For ΔI=1/2 operators, as their amplitudes in $K \to \pi$ are quadratically divergent, also need $K \to 0$ matrix elements as counterterm.

ΔI=1/2:

$$\langle \pi^+ | \Theta^{(8,1)} | K^+ \rangle = \frac{4m_M^2}{f^2} \left(\alpha_1^{(8,1)} - \alpha_2^{(8,1)} \right)$$



$$\langle 0 | \Theta^{(8,1)} | K^0 \rangle = \frac{16 i \nu}{f^3} \left(m_s' - m_d' \right) \quad \alpha_2^{(8,1)}$$



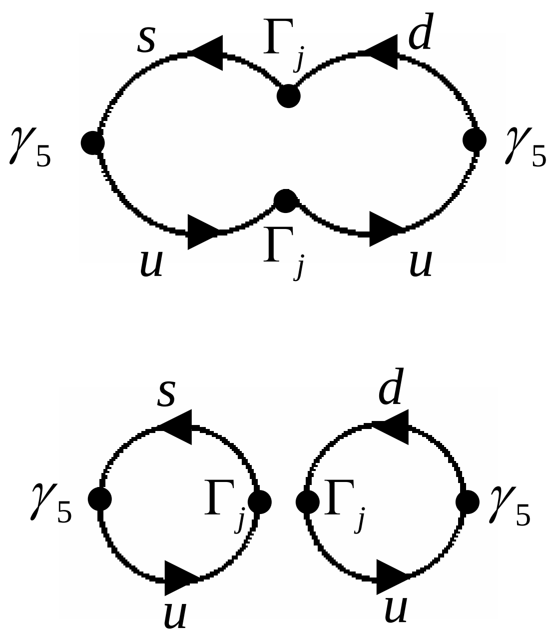
$$\Downarrow$$

$$\langle \pi^+ \pi^- | \Theta^{(8,1)} | K^0 \rangle = \frac{4i}{f^3} \left(m_{K^0}^2 - m_{\pi^+}^2 \right) \quad \alpha_1^{(8,1)}$$

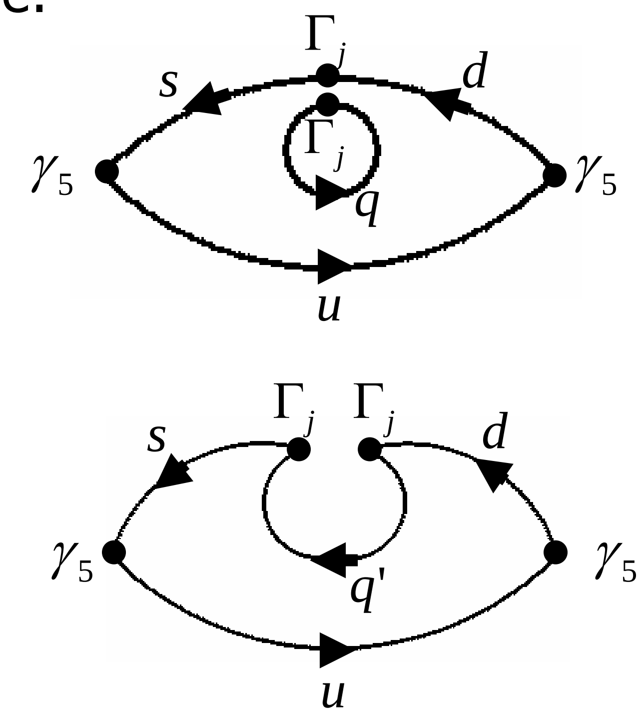
4. Contractions for $K \to \pi$

To get the values of $\langle \pi^+ | Q_i | K^+ \rangle$, we used Wick theorem and the following types of contractions are calculated:

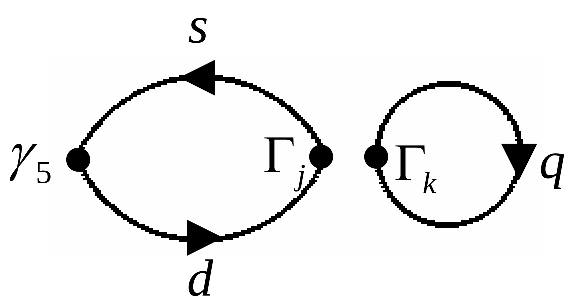
Figure-8:

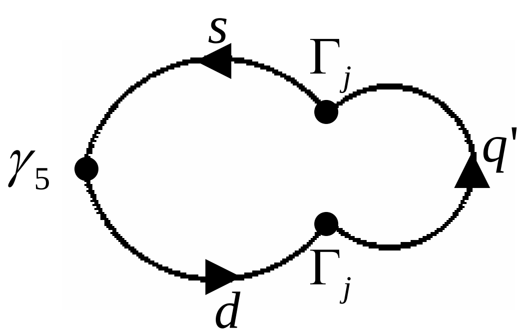


Eye:



Contractions for $K \to 0$





To construct the “pupils” in the eye diagram, we made use of random sources technique.

5. Ensemble: DWF 2f 1.7GeV

Generation details:

Volume	L_s	M_5	Gauge	β	Fermion	N_f	m_{sea}
16^3x32	12	1.8	DBW2	0.80	DWF	2	0.02, 0.03, 0.04

Measurements:

m_{val}	Trajectories	Meas. Separated by	Measure-ments #	Wall sources
0.01 ~ 0.05,	0.02: 5361 0.03: 6195 0.04: 5605	50 steps	94	4, 28

Physical results:

a^{-1}	$a\ m_{res}$	m_{res}	Z_A
1.691(53) GeV	0.001372(49)	2.32(15) MeV	0.75734(55)

m_π on the ensemble

- The pseudo-scalar mass m_π was measured on this ensemble with periodic+antiperiodic $\left\langle \pi^a(x) A_0^a(0) \right\rangle$ correlators.

Ansatz: $\left\langle \pi^a(x) A_0^a(0) \right\rangle = a \left\{ \exp\left[-m_\pi (x - x_0)\right] - \exp\left[-m_\pi (64 + x_0 - x)\right] \right\}$

m_{val}	m_{sea}=0.02	m_{sea}=0.03	m_{sea}=0.04
0.01	0.2215(21)	0.2263(21)	0.2222(19)
0.02	0.2941(16)	0.2989(18)	0.2972(16)
0.03	0.3536(15)	0.3576(18)	0.3573(15)
0.04	0.4055(16)	0.4090(17)	0.4097(14)
0.05	0.4525(16)	0.4556(17)	0.4571(14)

Fitting m_π vs. m_f

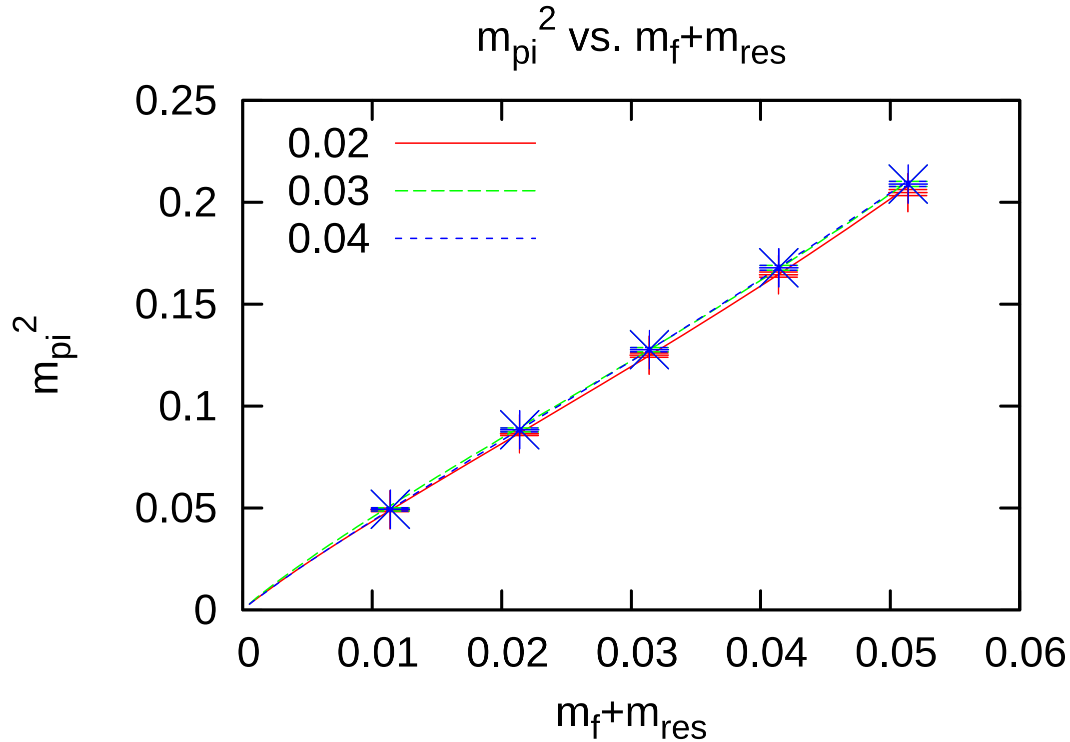
hep-lat/0411006

Using Partially-Quenched Chiral Perturbation Theory (PQχPT):

$$M^2_{PS(1-loop)} = M^2\left\{1+\frac{1}{8N\pi^2 f^2}\left[M^2 - M^2_{SS} + \left(2M^2 - M^2_{SS}\right)\ln\left(\frac{M^2}{\Lambda^2_{\chi PT}}\right)\right] - \frac{16}{f^2}\left[\left(L_5 - 2L_8\right)M^2 + \left(L_4 - 2L_6\right)NM^2_{SS}\right]\right\}$$

$$M^2 = 2B_0\left(m_{val} + m_{res}\right)$$

$$M^2_{SS} = 2B_0\left(m_{sea} + m_{res}\right)$$

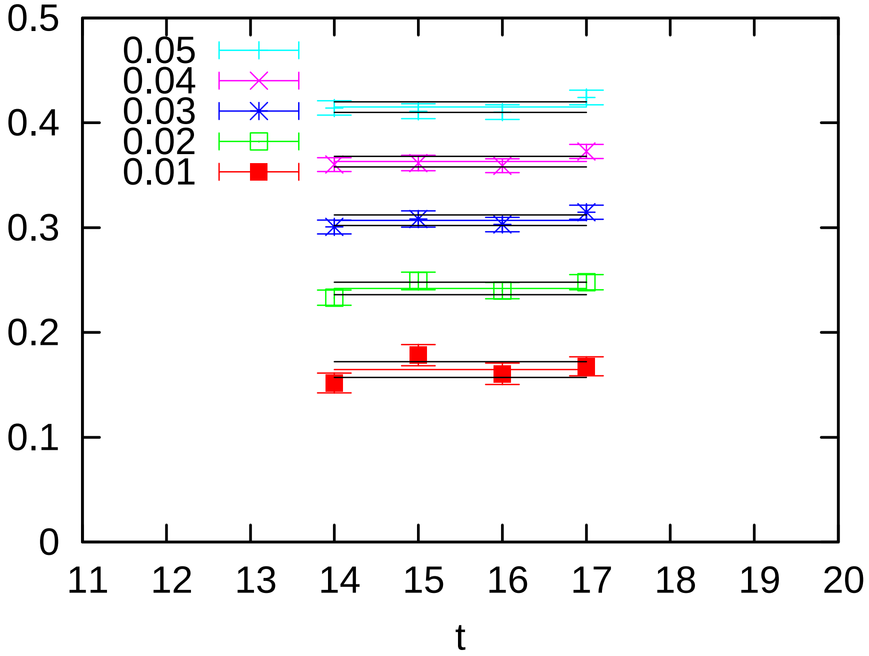


6. ΔI=3/2 operators

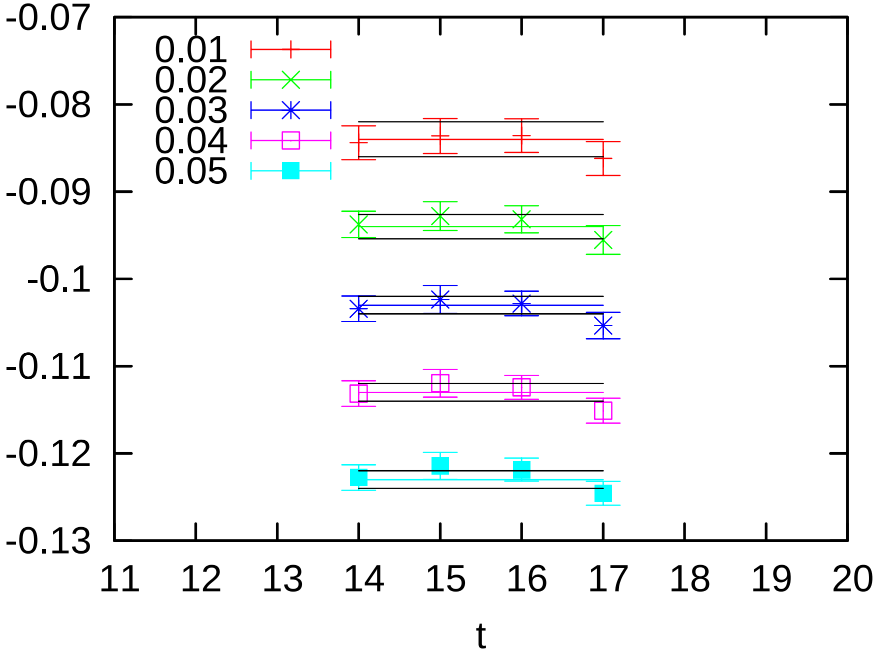
Extract matrix elements from the ratios of Green functions:

$$\lim_{t_K >> t >> t_\pi} \frac{G_{\pi O K}(t)}{G_{WW}(t_\pi, t_K)} = \frac{\langle \pi^+ | O | K^+ \rangle}{2m_\pi}$$

Q_6 vs. time-slice



Q_7 vs. time-slice



$\Delta I=3/2$: (1) $O^{(27,1),(3/2)}$

$$\langle \pi^+ | O^{(27,1)(3/2)}_{\text{lat}} | K^+ \rangle = 3 \langle \pi^+ | O^{(3/2)}_{1\text{ or }2,\text{ lat}} | K^+ \rangle = 2 \langle \pi^+ | O^{(3/2)}_{9\text{ or }10,\text{ lat}} | K^+ \rangle$$

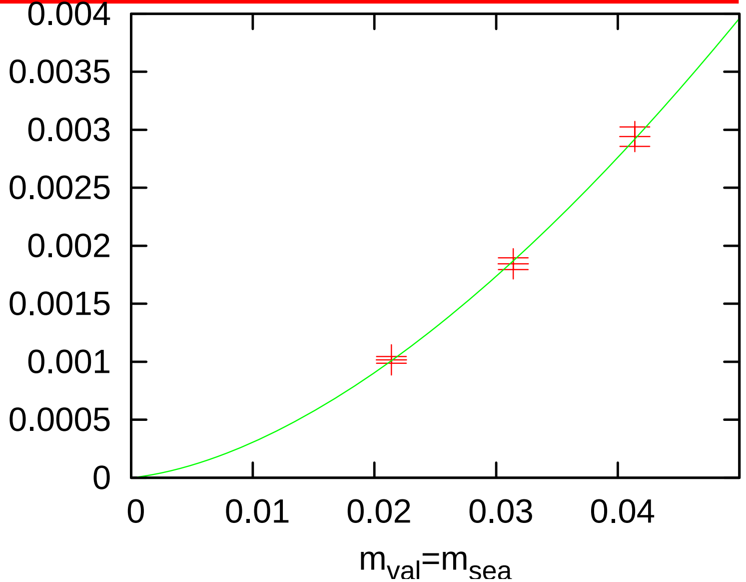
$$\langle \pi^+ | O^{(27,1),(3/2)}_{\text{lat}} | K^+ \rangle_{PQ\chi PT}$$

Golterman & Pallante 2000

$$= -\frac{4\alpha^{(27,1)} M^2}{f^2}\left(1 - \frac{1}{16\pi^2 f^2}\left[2NM^2_{vS}\left(\ln\frac{M^2_{vS}}{\Lambda^2_{\chi PT}} - 1\right) + 2M^2\left(3\ln\frac{M^2}{\Lambda^2_{\chi PT}} - 2\right) + \frac{2}{N}\left(\left(M^2_{SS} - 2M^2\right)\ln\frac{M^2}{\Lambda^2_{\chi PT}} + M^2\right)\right]\right)$$
$$-\frac{M^2}{2\pi^2 f^4}\left[2\xi^{(27,1)}M^2 + \beta_7^{27} NM^2_{SS} - 256\pi^2\alpha^{(27,1)}\left(\left(L_4 - L_6\right)NM^2_{SS} + \left(L_5 - L_8\right)M^2\right)\right]$$

$$\xi^{(27,1)} = \beta_2^{27} + \beta_4^{27}$$

Fitting for $\alpha^{(27,1)}$, $\xi^{(27,1)}$ and β_7^{27}

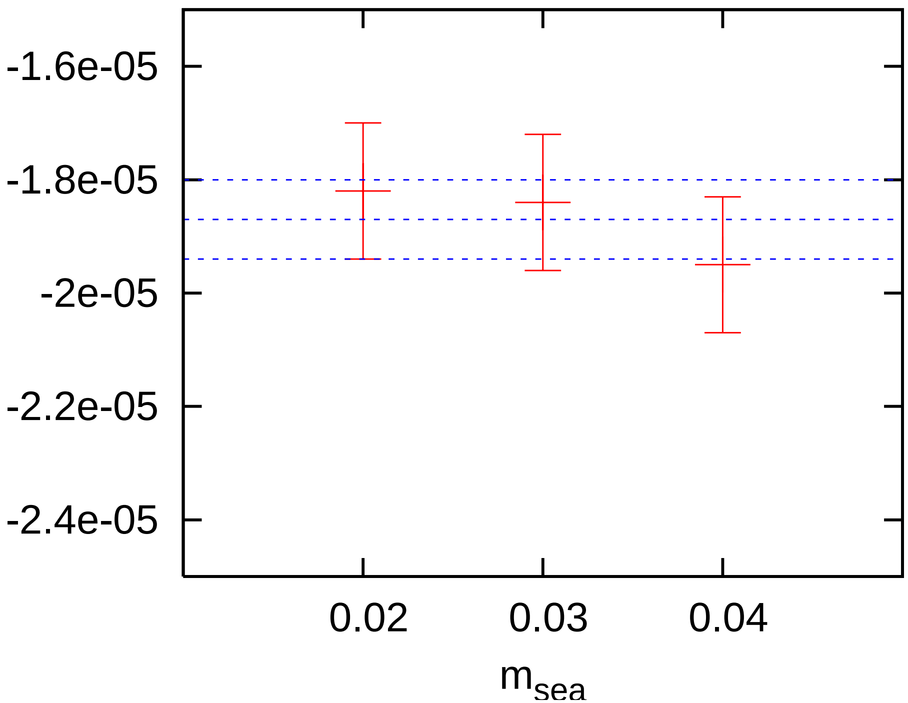


Compare with RBC Quenched Lattices:

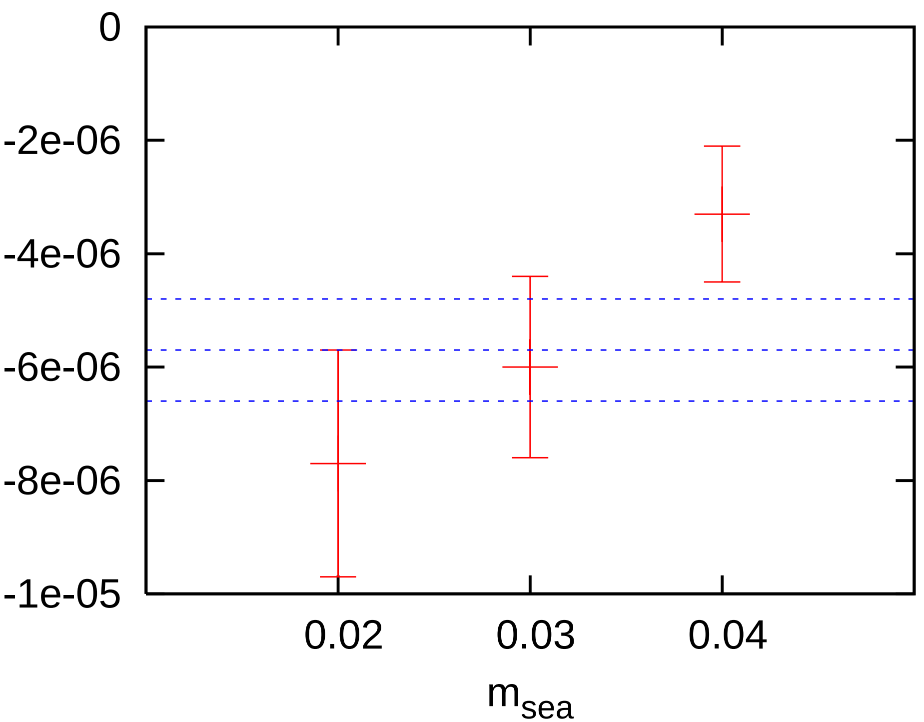
	This work	RBC 2002
$\alpha^{(27,1)}$(GeV^4)	-3.08(51)x10^{-5}	-5.64(25)x10^{-5}

$O^{(27,1),(3/2)}$ Fitting

$\xi^{(27,1)}$ vs. m_{sea}



β_7^{27} vs. m_{sea}



(2) Electroweak penguins

Also: Jun Noaki, Edinburgh meeting, March 2005

For the (8,8) operators with isospin = 3/2 (i.e, $O_7^{(3/2)}$ & $O_8^{(3/2)}$), for degenerate quark masses:

$$\left\langle \pi^+ \left| O^{(8,8),(3/2)} \right| K^+ \right\rangle = \frac{4\alpha_{88}}{f^2}\left\{1 - \frac{2}{16\pi^2 f^2}\left[M^2 \ln\frac{M^2}{\Lambda^2_{\chi PT}} + N\left(M^2 + M^2_{SS}\right)\ln\left(\frac{M^2 + M^2_{SS}}{2\Lambda^2_{\chi PT}}\right) + M^2\right] - \frac{2\Delta f}{f}\right\}$$

$$+\xi^{(3/2)}\frac{4}{f^2}M^2 + c_6^r \frac{8N}{f^2}M^2_{SS}$$

Laiho & Soni 2003

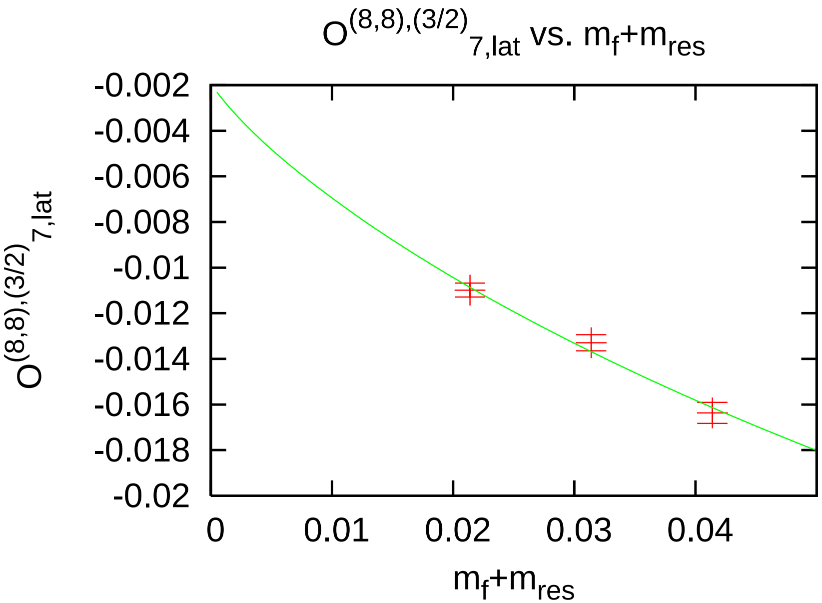
$$\frac{\Delta f}{f} = -\frac{N}{16\pi^2 f^2}M^2_{vS}\ln\frac{M^2_{vS}}{\Lambda^2_{\chi PT}} + \frac{8}{f^2}\left(L_5 M^2 + L_4 N M^2_{SS}\right)$$

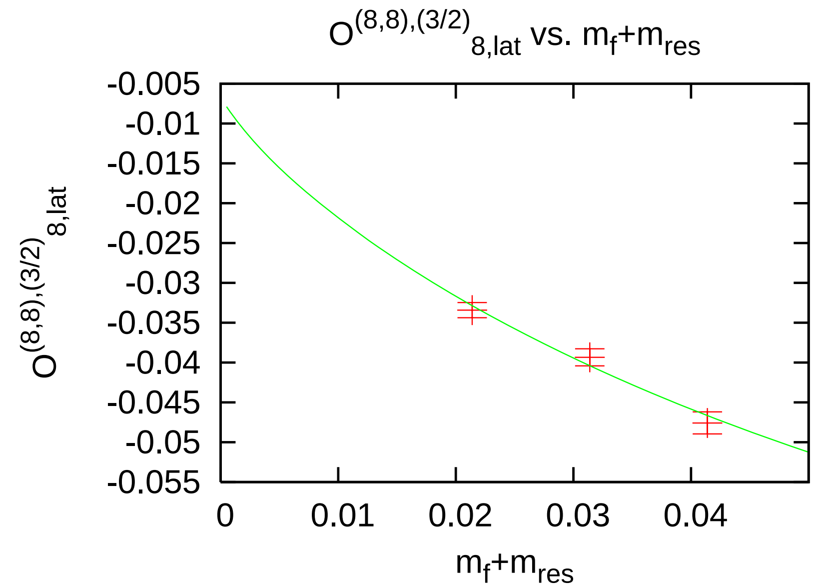
$$\xi^{(3/2)} = -c_1^r - c_2^r + 4c_4^r + 4c_5^r$$

Fitting for α_{88}, $\xi^{(3/2)}$ and c_6^r

Fitting $O^{(8,8),(3/2)}$: α^{88}

Dynamical extrapolation

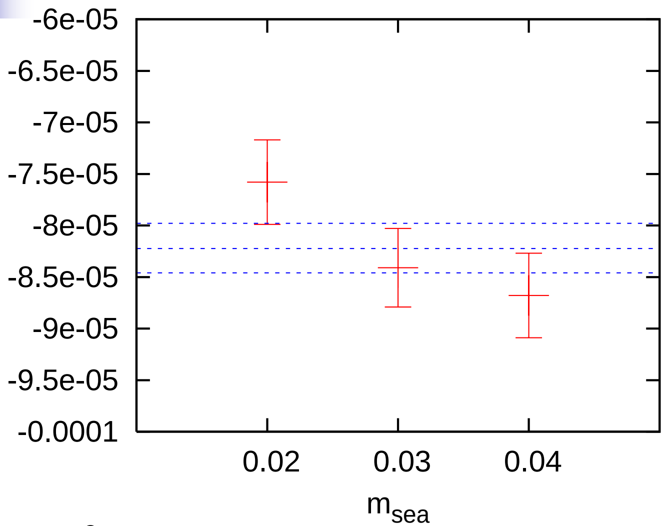




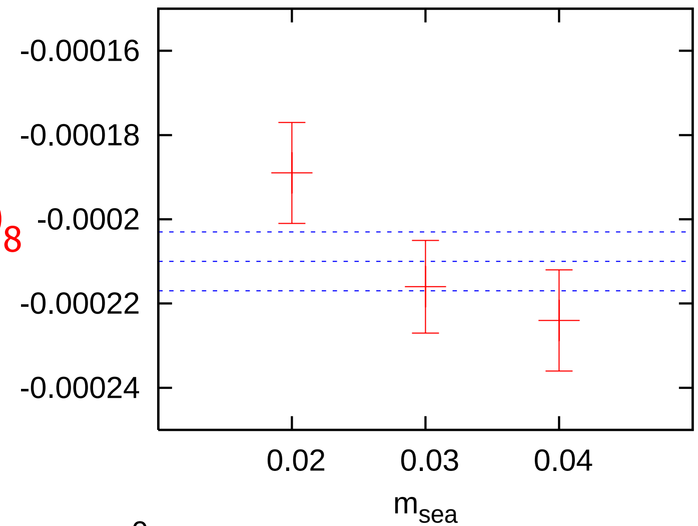
α_{88}(GeV^6)	This work	RBC 2002 Quenched
Q_7	$-6.7(1.0)\times10^{-5}$	$-8.11(40)\times10^{-5}$
Q_8	$-2.31(30)\times10^{-4}$	$-2.50(14)\times10^{-4}$

Fitting $\mathcal{O}^{(8,8),(3/2)}$: $\xi^{(3,2)}$ and c_6^r

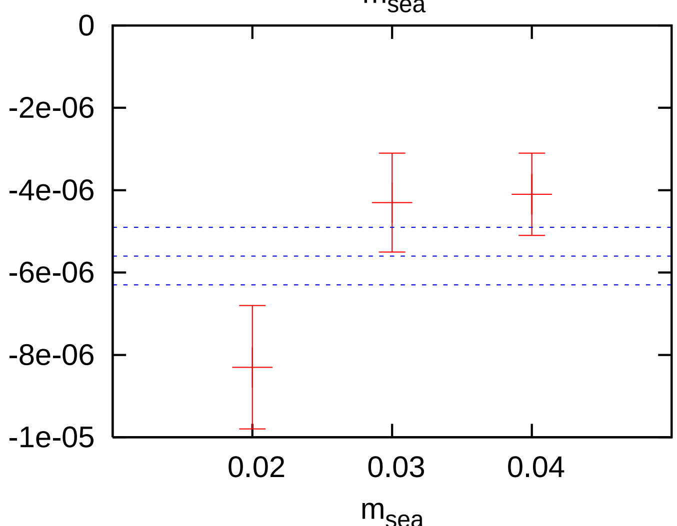
$\xi^{(3,2)}$ for O_7



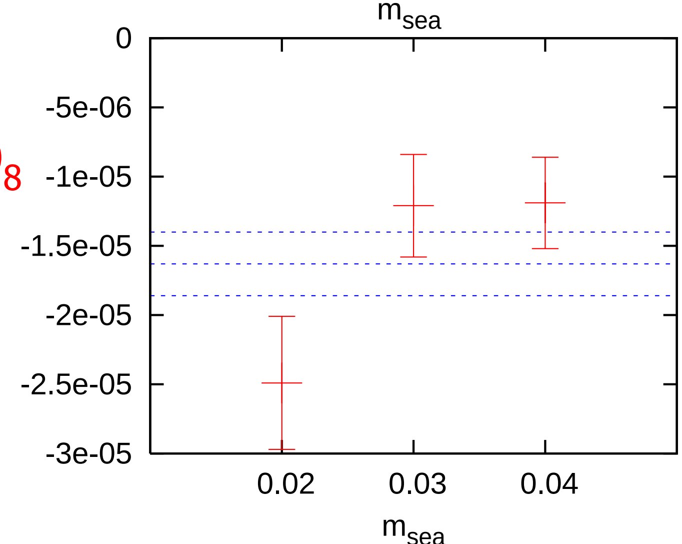
$\xi^{(3,2)}$ for O_8



c_6^r for O_7



c_6^r for O_8



7. ΔI=1/2 operators

For the ΔI=1/2 operators, they mix with the operator $B^{(3,\bar{3})}$

$$B^{(3,\bar{3})} = (m_s + m_d)\bar{s}d - (m_s - m_d)\bar{s}\gamma_5 d$$

$$\Rightarrow Q_{i,\mathrm{lat}} = Q_{i,\mathrm{phys}} + \eta_i (m_s + m_d)\bar{s}d - \eta'_i (m_s - m_d)\bar{s}\gamma_5 d$$

To get η_i, fit $$\frac{\langle 0|Q_{i,\mathrm{lat}}|K^0\rangle}{\langle 0|(\bar{s}\gamma_5 d)_{\mathrm{lat}}|K^0\rangle} = \eta_{0,i} - \eta_{1,i}(m'_s - m'_d)$$

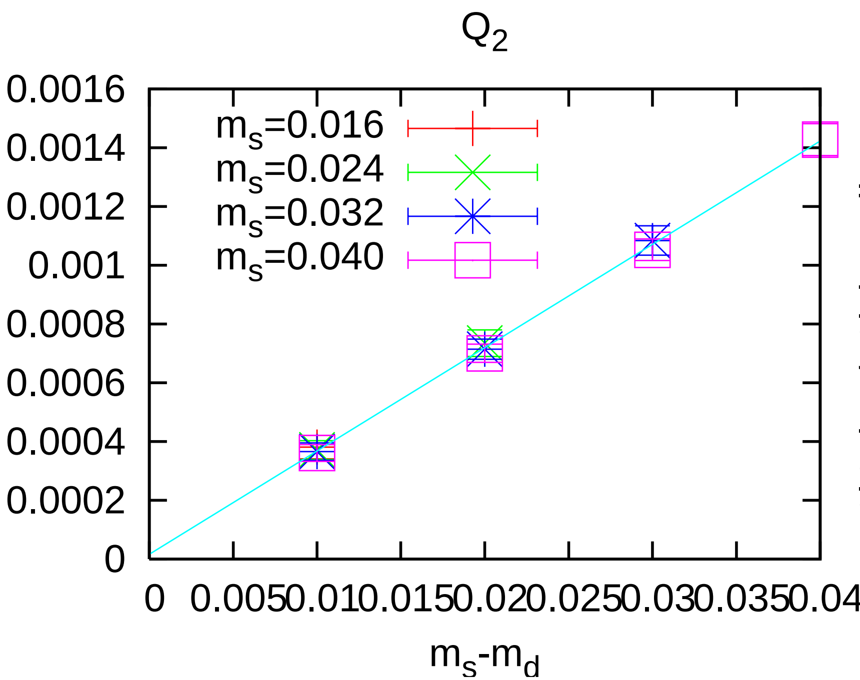
And then comes the subtraction:

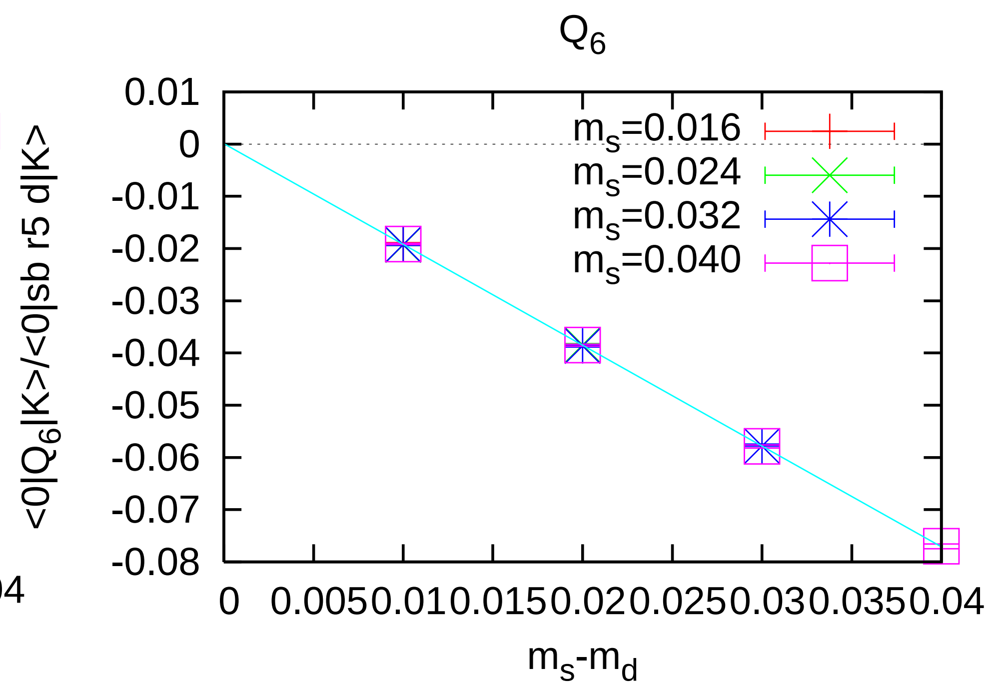
$$\langle \pi^+|Q^{(1/2)}_{i,\mathrm{phys}}|K^+\rangle = \langle \pi^+|Q^{(1/2)}_{i,\mathrm{lat}}|K^+\rangle - \eta_{1,i}(m_s + m_d)\langle \pi^+|\bar{s}d|K^+\rangle$$

- With preserved chiral symmetry, $\eta_i = \eta'_i$. And this subtraction is exact. However, with Domain-wall Fermions, they are slightly different.

- And power-divergent operators has non-exact chiral limit in DWF, it could bring $O(m_{\mathrm{res}})$ error to the subtracted results.

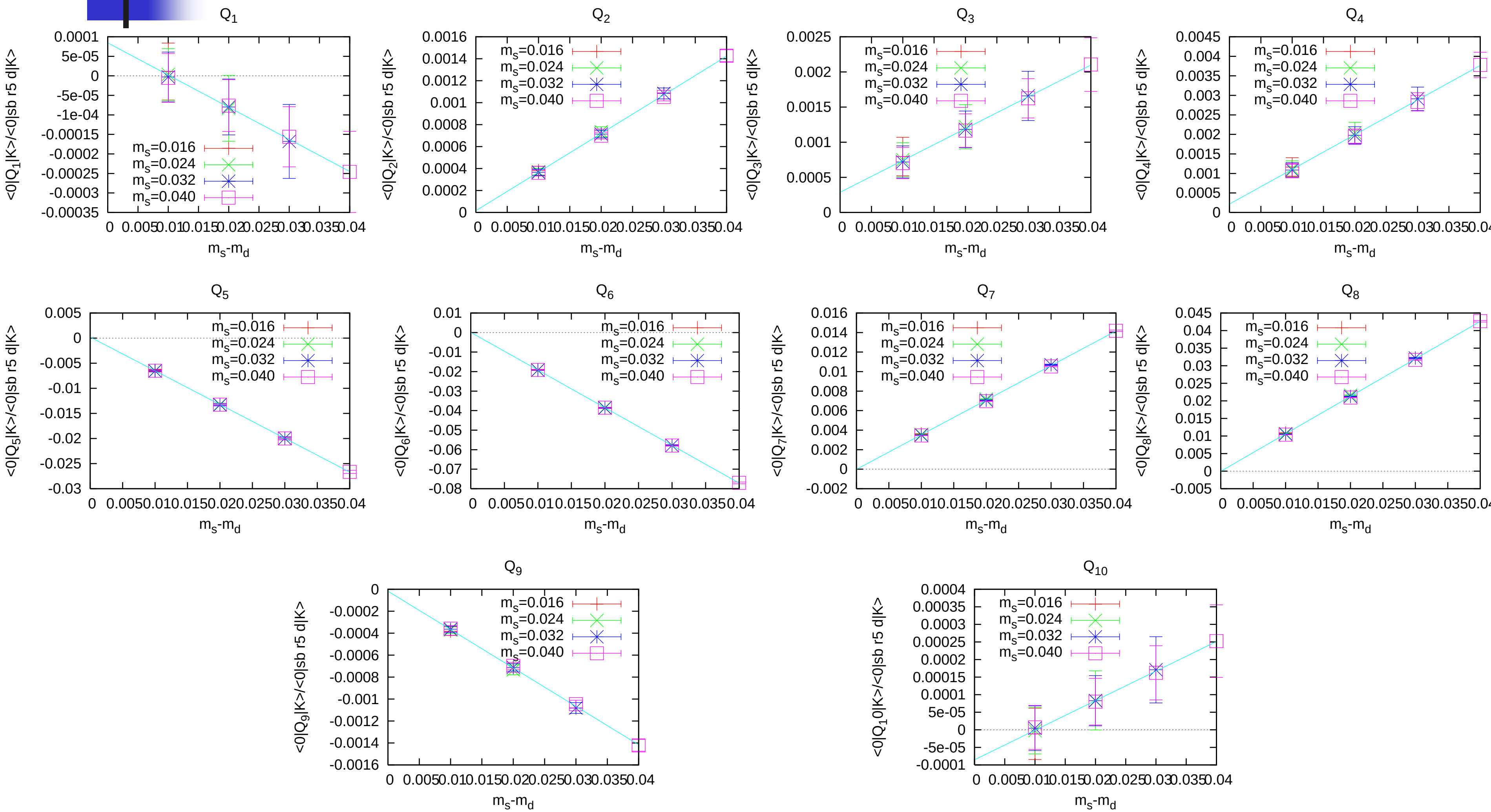
$K \to 0$ fitting





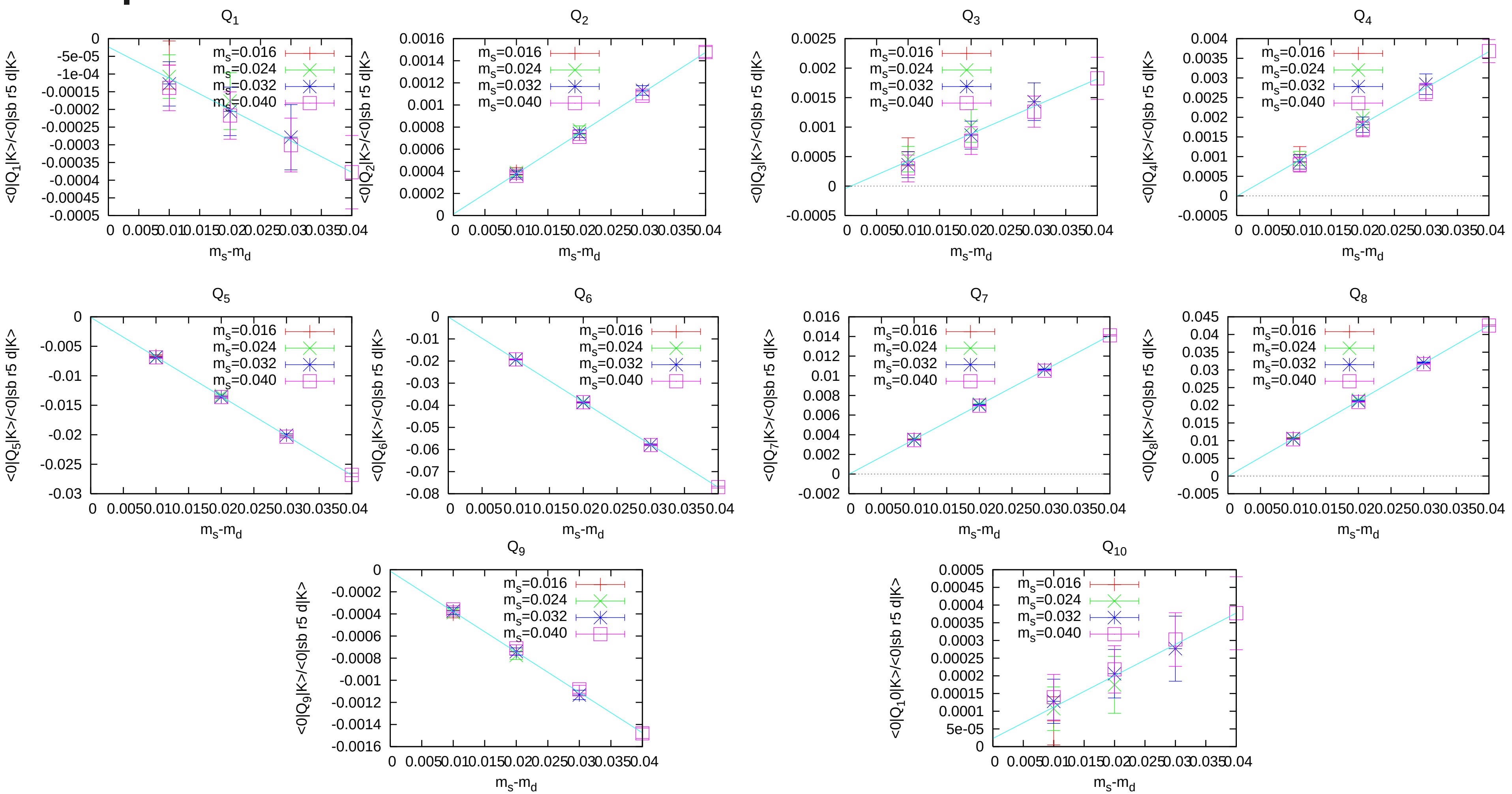
$K \to 0$ fitting

$m_{sea} = 0.02$



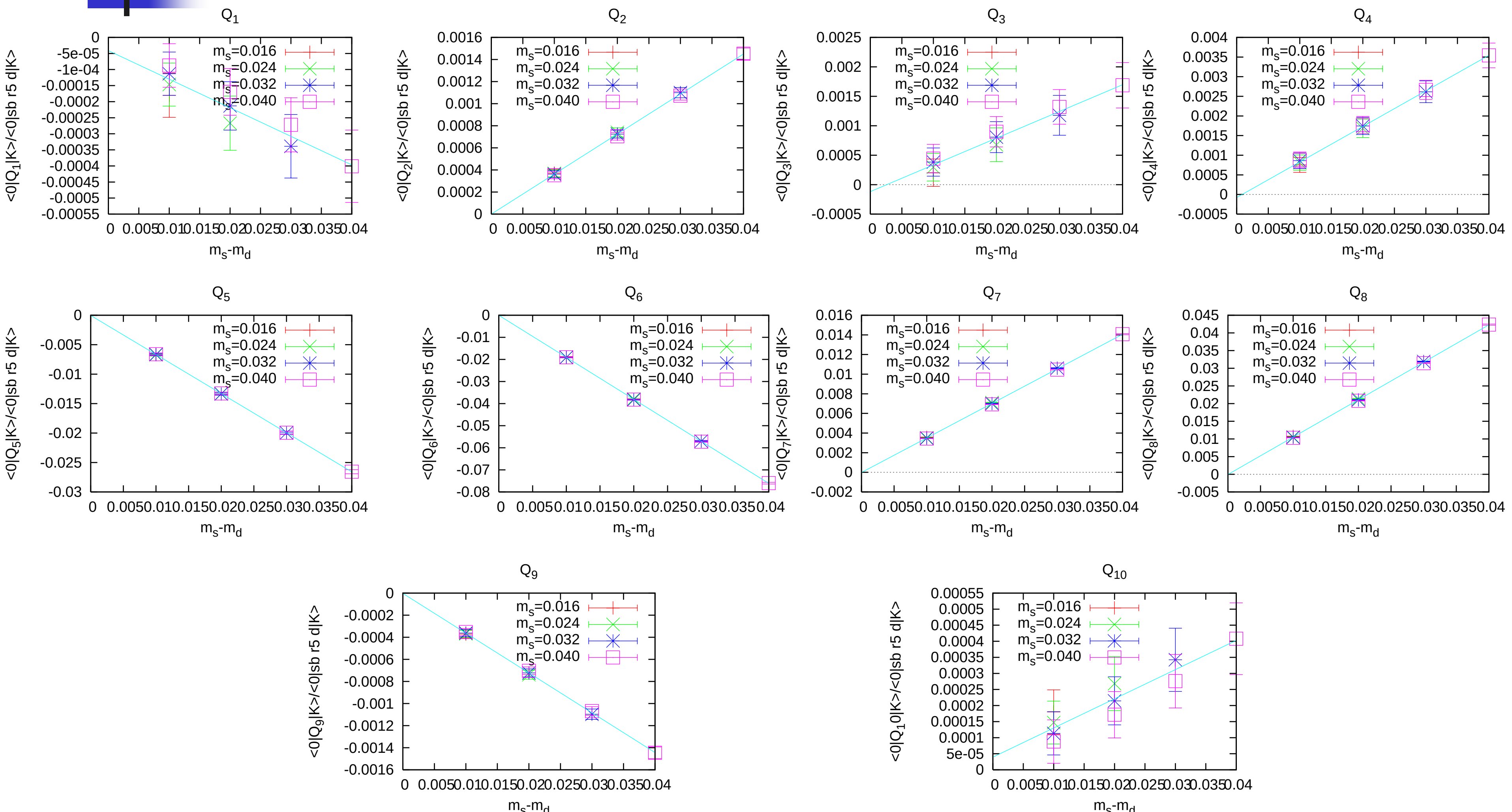
$K \to 0$ fitting

m_{sea}=0.03

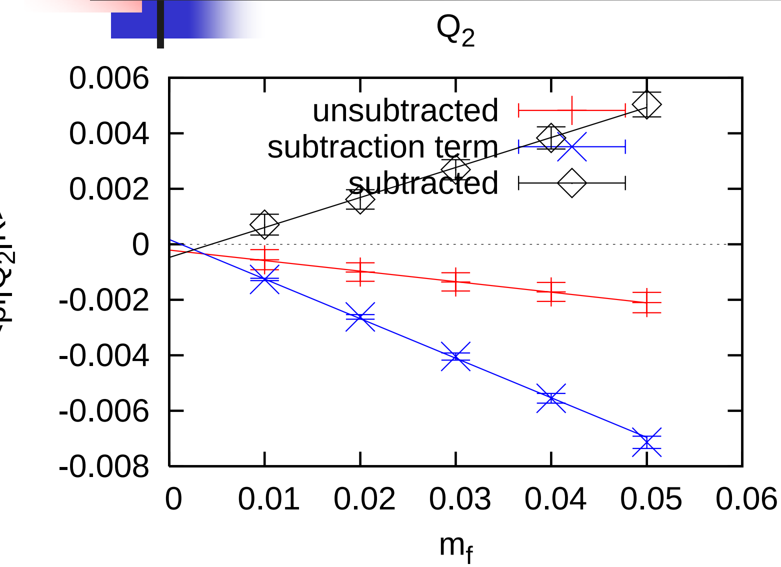


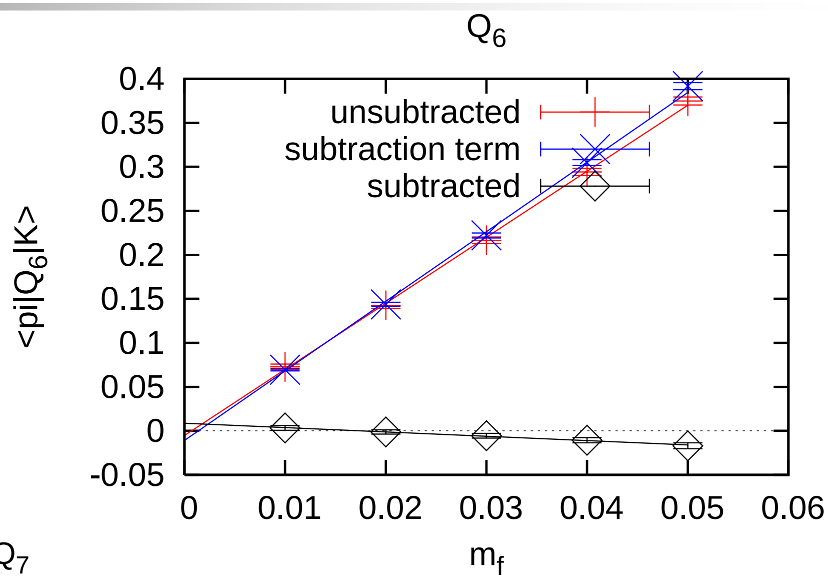
$K \to 0$ fitting

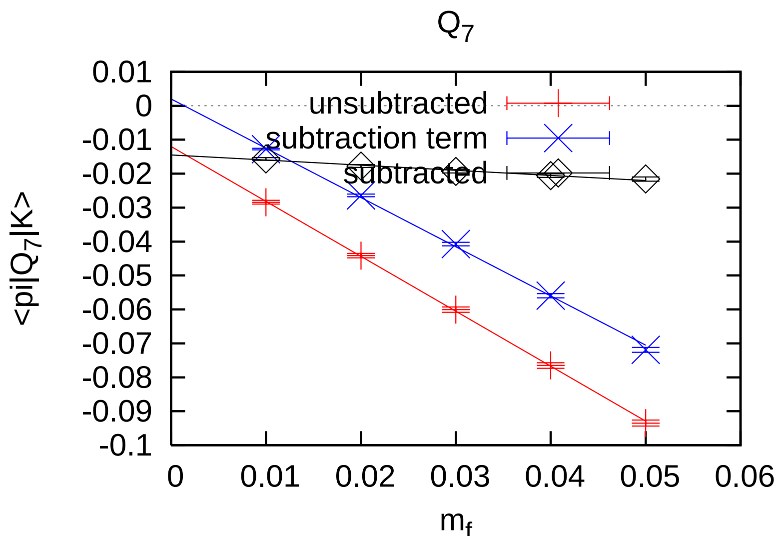
$m_{sea}=0.04$



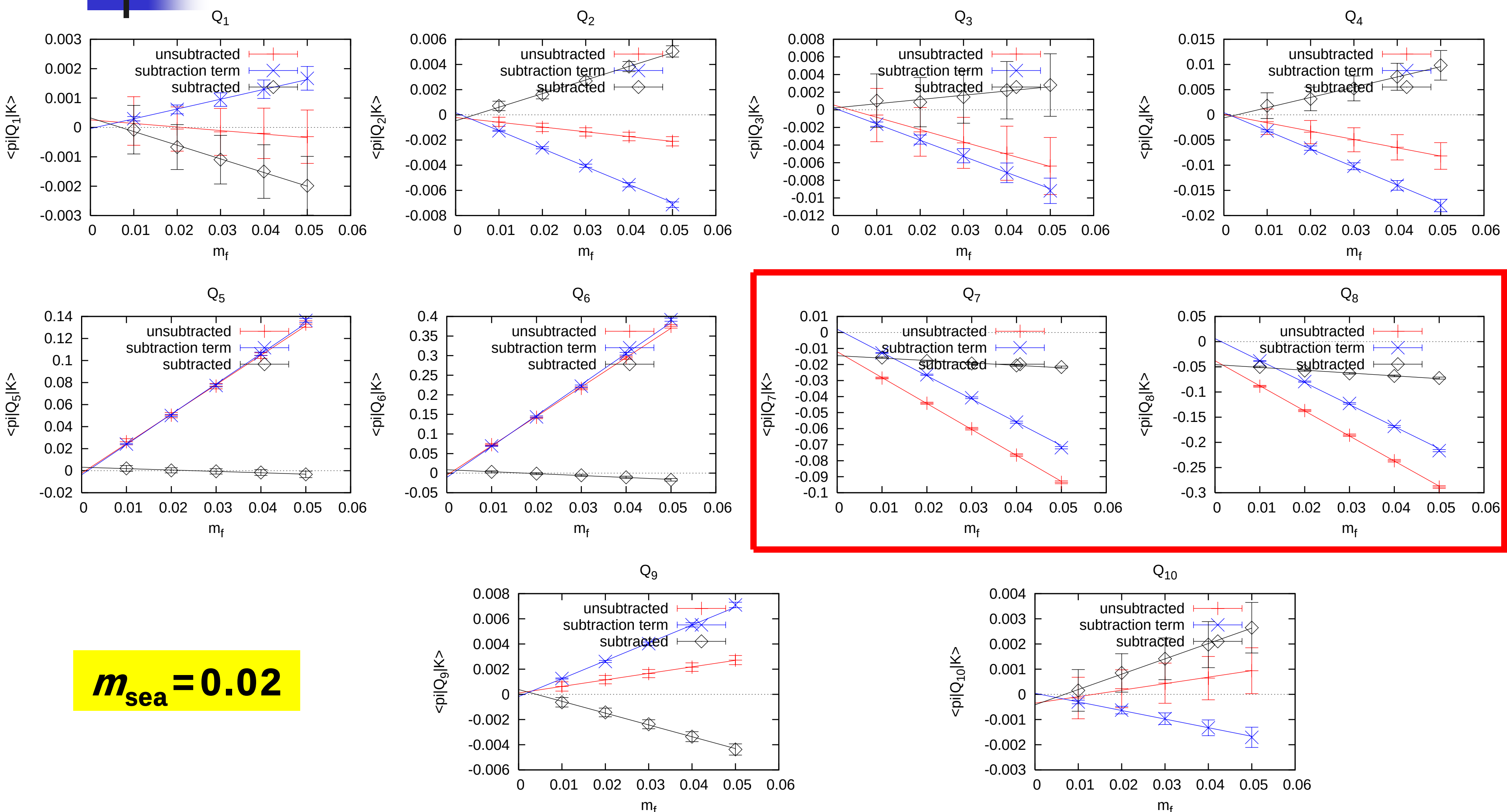
Subtractions





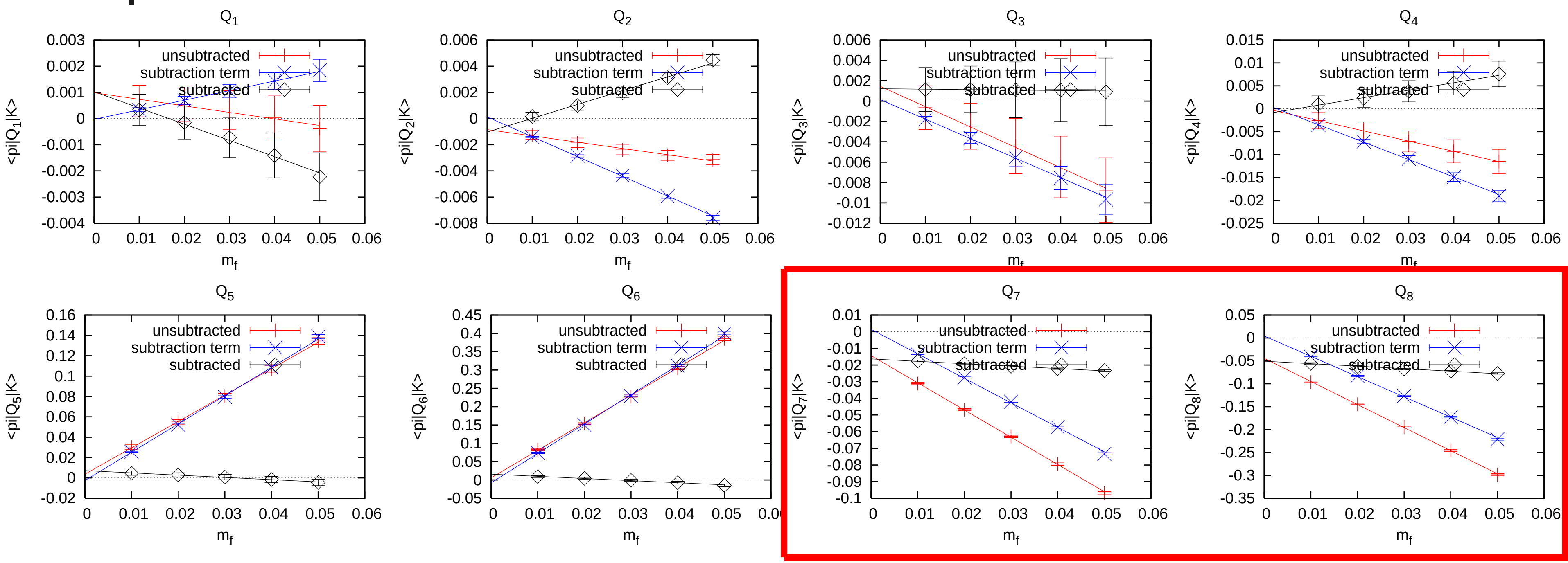


Subtractions

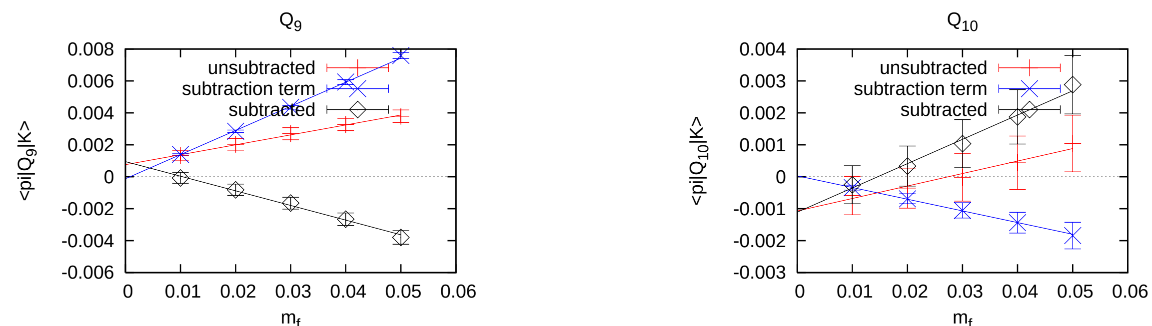


***m_{sea}*=0.02**

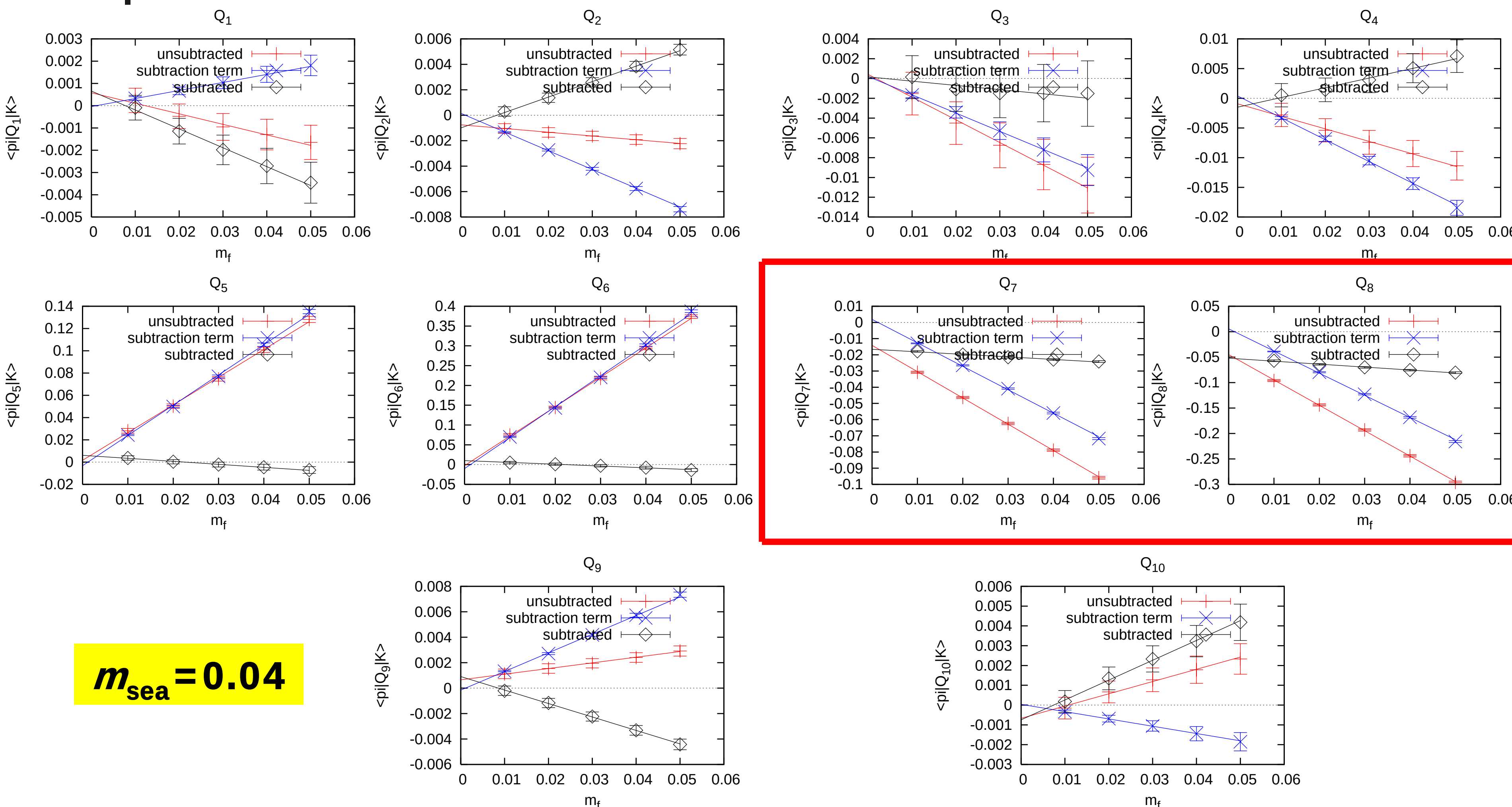
Subtractions



$m_{sea}=0.03$



Subtractions



$m_{sea} = 0.04$

Why don't they hit 0 exactly?

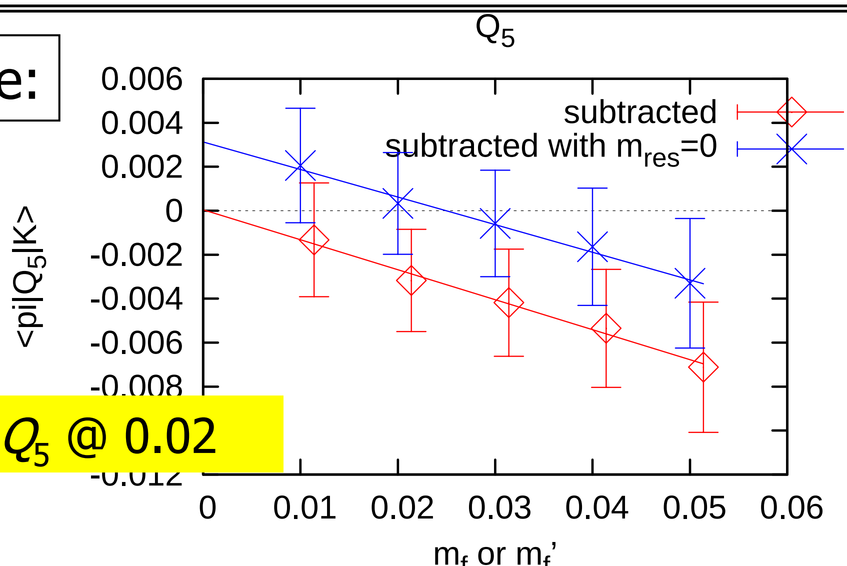
What is the "m_{res}" here?

The subtraction:

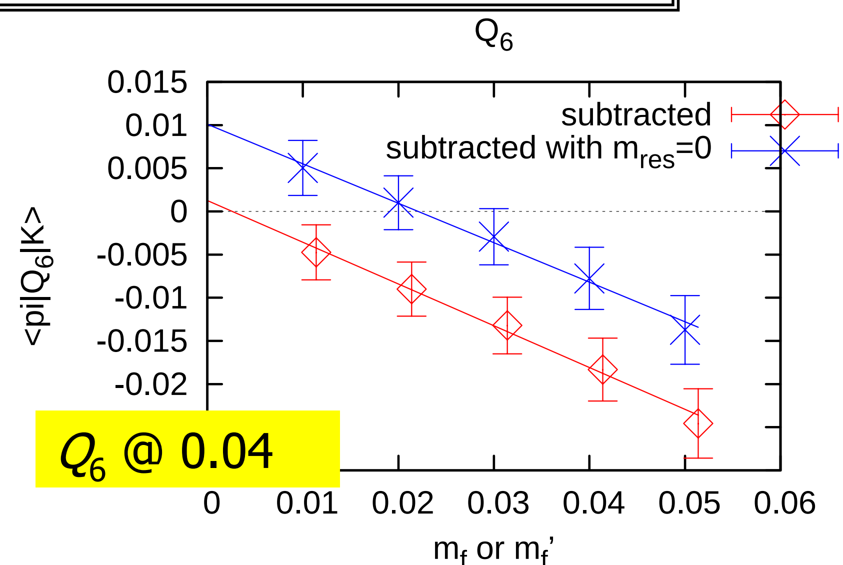
$$\langle \pi^+ | Q^{(1/2)}_{i,\mathrm{phys}} | K^+ \rangle = \langle \pi^+ | Q^{(1/2)}_{i,\mathrm{lat}} | K^+ \rangle - \eta_{1,i} (m_s + m_d) \langle \pi^+ | \bar{s} d | K^+ \rangle$$

- For power-divergent operators, their additive mass renormalization constant is different from the m_{res}.
- We only know they should add something of $O(m_{res})$.

Example:



Q_5 @ 0.02

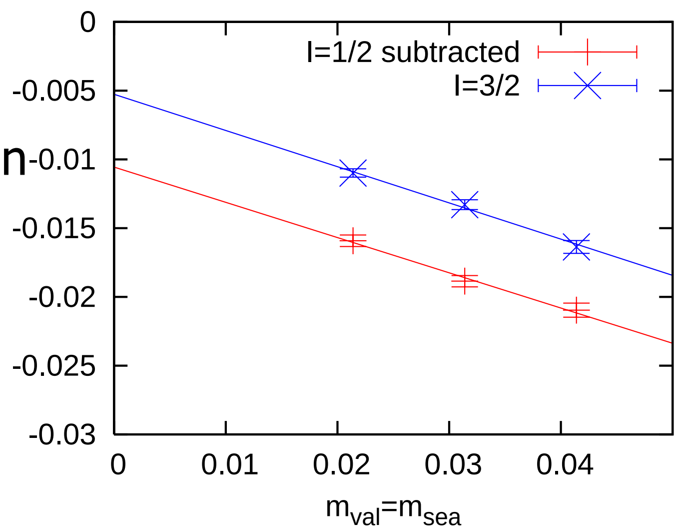


Q_6 @ 0.04

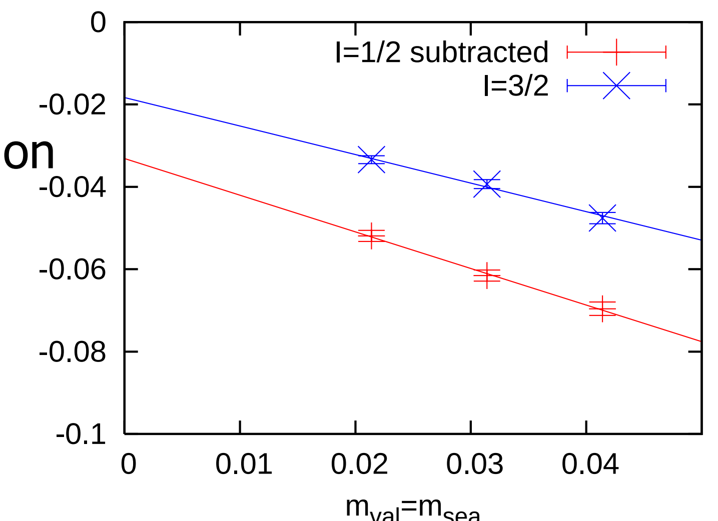
$Q^{(8,8),(1/2)}$ and $Q^{(8,8),(3/2)}$

- Each of the (8,8) operators are in a single representation of $SU(3)_L x SU(3)_R$, thus, their I=1/2 and 3/2 matrix elements are related by Wigner-Eckert theorem.
- This helps to avoid the problem induced by non-exact chiral limit on Q_7 and Q_8. Also, by comparing the I=1/2 and 3/2 contributions, we got confidence in our subtraction of power divergent operators.

Q7:
Dynamical extrapolation



Q8:
Dynamical extrapolation



LECs and feeling $\Delta I=1/2$ rule

Q_i	$\alpha^{(1/2)}_{lat}$	$\alpha^{(3/2)}_{lat}$
1	-3.7(1.9) x 10^{-5}	-1.26(20) x 10^{-6}
2	3.98(92) x 10^{-5}	-1.26(20) x 10^{-6}
3	-4.2(6.9) x 10^{-5}	0.0
4	3.5(5.8) x 10^{-5}	0.0
5	-8.8(6.0) x 10^{-5}	0.0
6	-1.14(76) x 10^{-4}	0.0
7	-5.72(86) x 10^{-6}	-2.86(43) x 10^{-6}
8	-1.98(26) x 10^{-5}	-9.9(1.3) x 10^{-6}
9	-3.38(91) x 10^{-5}	-2.52(40) x 10^{-6}
10	4.3(1.9) x 10^{-5}	-2.52(40) x 10^{-6}

8. Future Work

- After getting the lattice matrix elements, we can combine them into $K \to \pi\pi$ decay amplitudes (on lattice).
- Matching the *NPR* and $\overline{MS}$ renormalization. Combined with Wilson Coefficients, get the physical decay amplitude.
- Verify the ΔI=1/2 rule.
- Calculate ε'/ε and CP violation properties.
- Apply this calculation to the (upcoming) (2+1)f lattices, to get the physical results on "Full QCD".

Summary

- $K \to \pi\pi$ decay can be described by dim-6 local operators, classified as "Vector boson exchange", "QCD penguin" and "E-W penguin".
- Use χPT to relate $K \to \pi\pi$ to $K \to \pi$ and $K \to 0$.
- ΔI=1/2 operators involve power divergent pieces and needs vacuum subtraction to remove. (but still noisy)
- For O(8,8) operators, their ΔI=1/2 part and ΔI=3/2 part are related by Wigner-Eckert theorem.
- NPR, Wilson coefficients and (2+1) flavor….

Thank you!