Staggered chiral perturbation theory and heavy light form factors

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Introduction

1) Recent Fermilab results

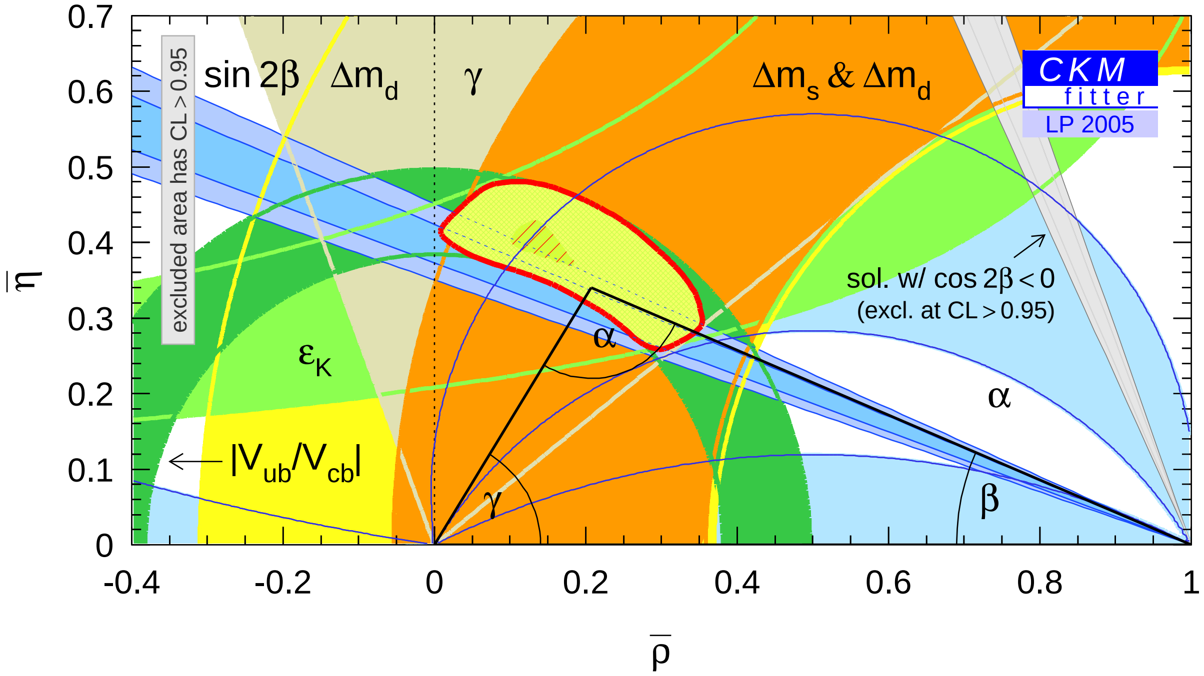
2) Intro to staggered chiral perturbation theory

3) Heavy light calculations in progress

CKM matrix from lattice QCD
hep-lat/0412044

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ & 0.225(2)(1) & 3.5(5)(5) \times 10^{-3} \\ & Kl3 & B \to \pi \\ \\ V_{cd} & V_{cs} & V_{cb} \\ 0.24(3)(2) & 0.97(10)(2) & 3.9(1)(3) \times 10^{-2} \\ D \to \pi & D \to K & B \to D \\ \\ V_{td} & V_{ts} & V_{tb} \\ \\ \end{pmatrix} \quad (1)$$

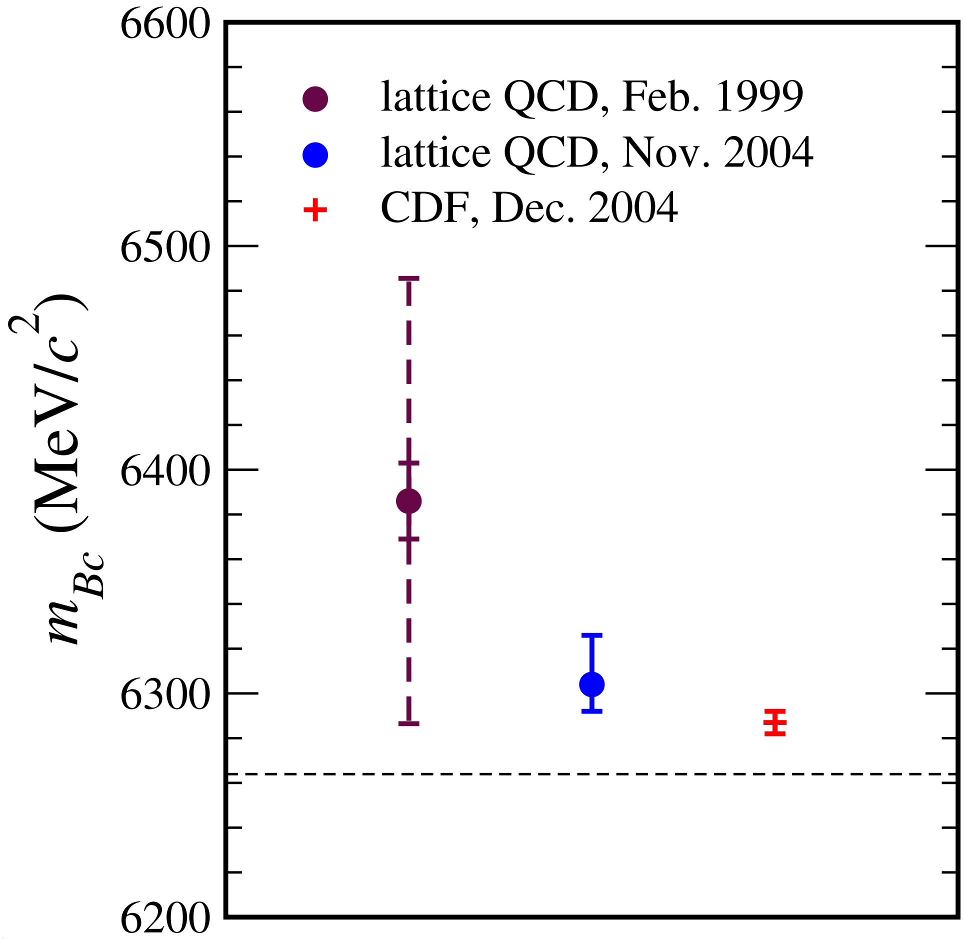
Unitarity Triangle



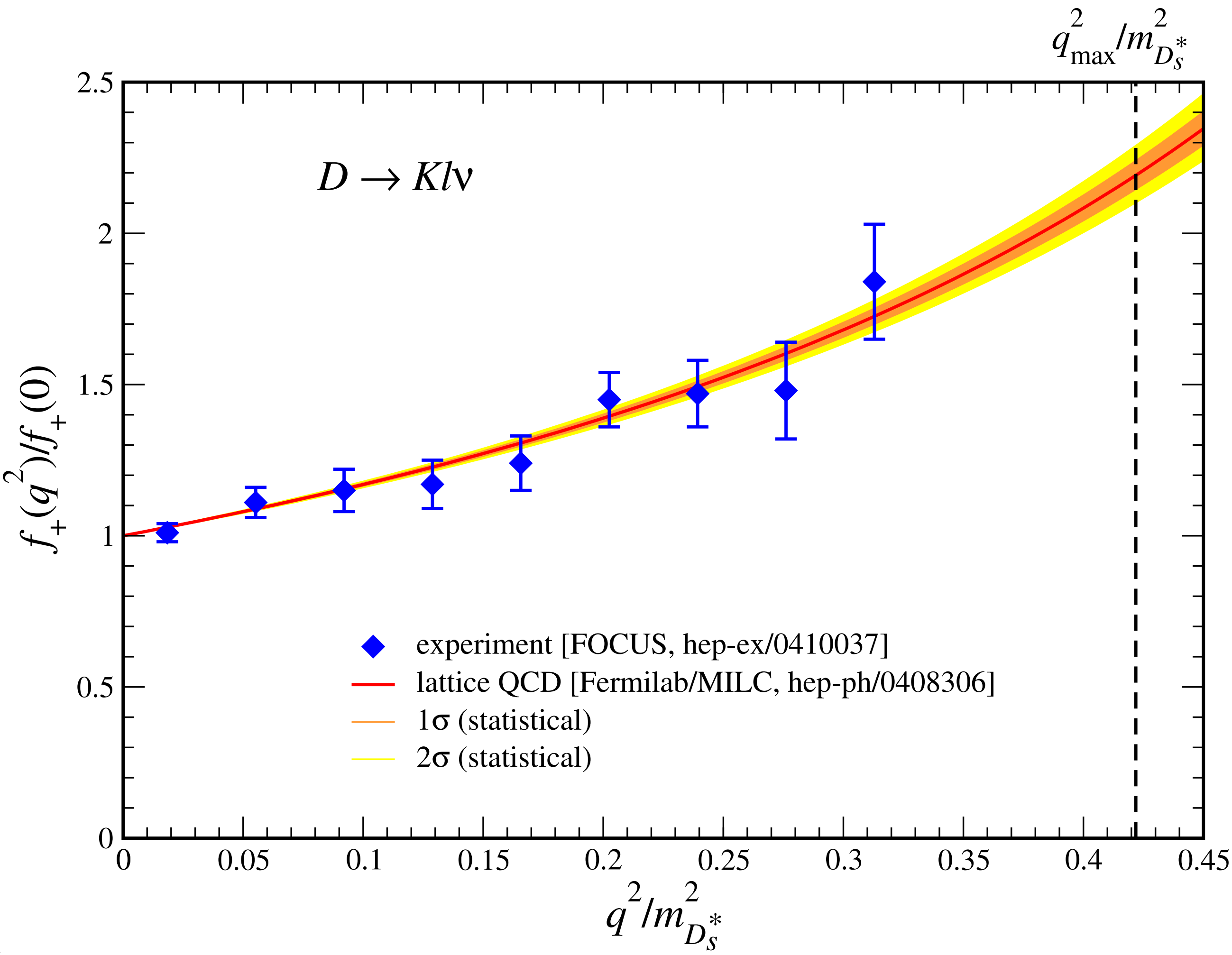
Some discretizations of fermions:

Staggered: A remnant of chiral symmetry is preserved, but additional flavors, called "tastes" are introduced. These vanish in the continuum limit, but must be accounted for in typical simulations. (Also concern over the "4th root trick.")

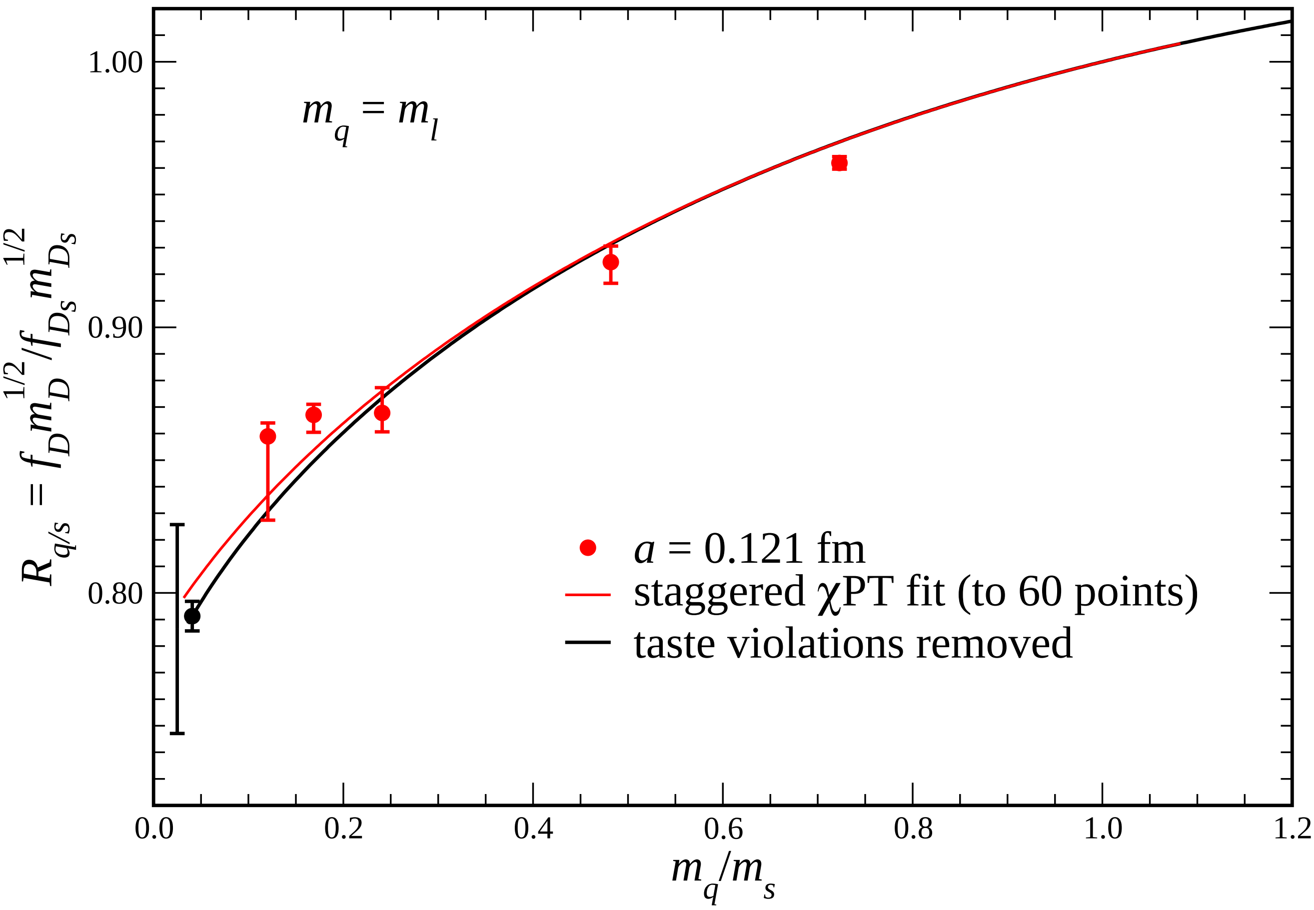
Domain wall: Good chiral properties due to an added dimension, but more expensive.

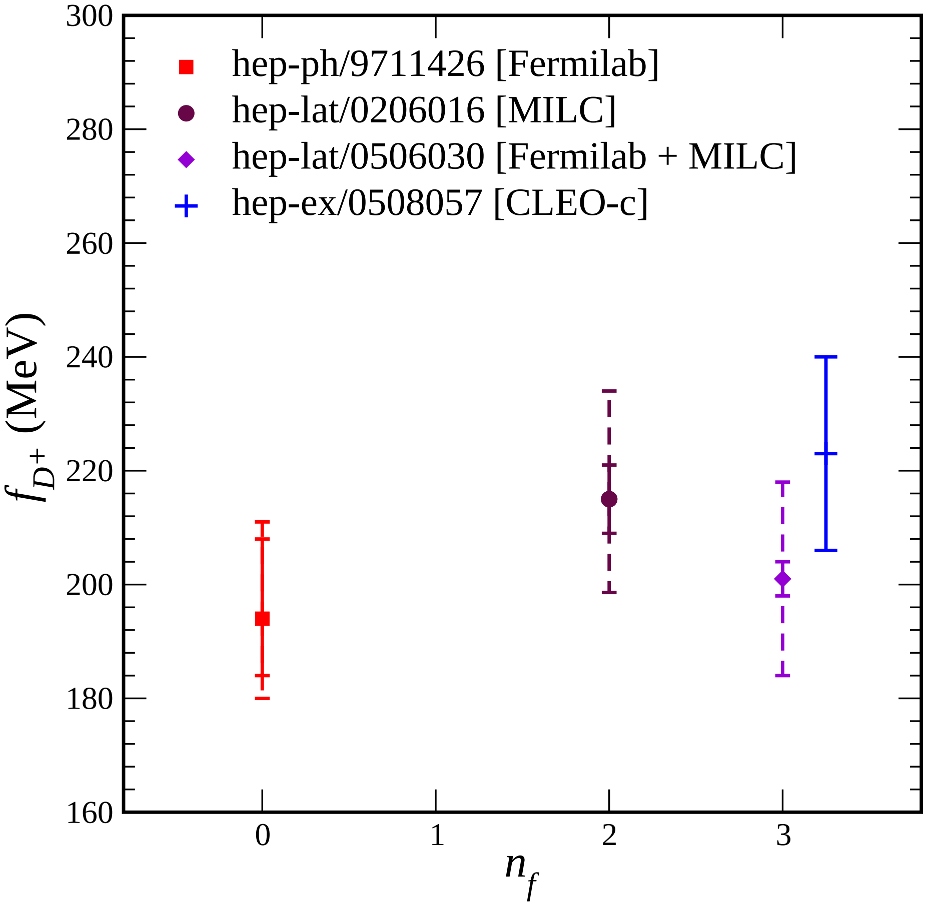


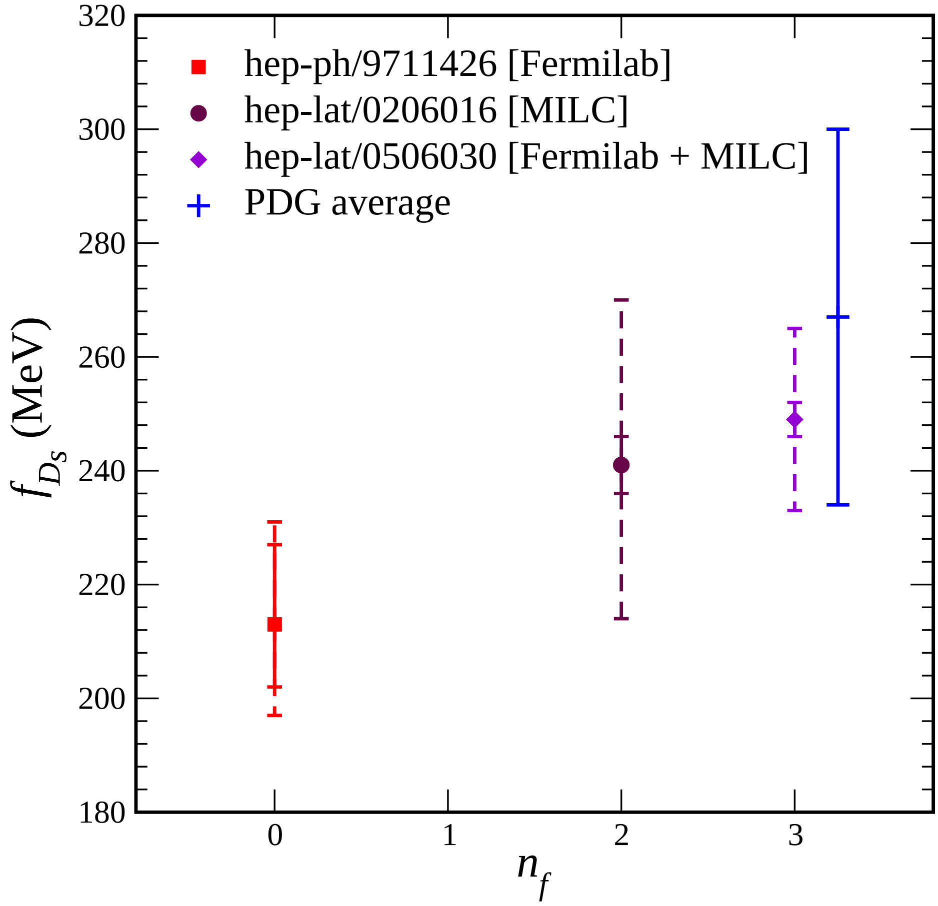
6600
6500
6400
6300
6200
m_{Bc} (MeV/c^2)
lattice QCD, Feb. 1999
lattice QCD, Nov. 2004
CDF, Dec. 2004



$q^2_{max}/m^2_{D_s^*}$
$D \to K l \nu$
$f_+(q^2)/f_+(0)$
$q^2/m^2_{D_s^*}$
experiment [FOCUS, hep-ex/0410037]
lattice QCD [Fermilab/MILC, hep-ph/0408306]
1σ (statistical)
2σ (statistical)


$m_q = m_l$
$R_{q/s} = f_D m_D^{1/2} / f_{D_s} m_{D_s}^{1/2}$
m_q/m_s
a = 0.121 fm
staggered χPT fit (to 60 points)
taste violations removed
1.00
0.90
0.80
0.0
0.2
0.4
0.6
0.8
1.0
1.2


hep-ph/9711426 [Fermilab]
hep-lat/0206016 [MILC]
hep-lat/0506030 [Fermilab + MILC]
hep-ex/0508057 [CLEO-c]
f_{D^+} (MeV)
n_f
300
280
260
240
220
200
180
160
0
1
2
3



hep-ph/9711426 [Fermilab]
hep-lat/0206016 [MILC]
hep-lat/0506030 [Fermilab + MILC]
PDG average
f_{D_s} (MeV)
n_f
320
300
280
260
240
220
200
180
0
1
2
3

Chiral Perturbation Theory

Operators are constructed out of the unitary chiral matrix field Σ,

$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{2}$$

where λ^a are proportional to the Gell-Mann matrices with $\mathrm{tr}(\lambda_a\lambda_b) = \delta_{ab}$, ϕ^a are the real pseudoscalar-meson fields, and f is the meson decay constant in the chiral limit.

Leading order Strong Lagrangian

At leading order $[O(p^2)]$ in ChPT, the strong Lagrangian is

$$\mathcal{L}_{st}^{(2)} = \frac{f^2}{8}\mathrm{tr}[\partial_\mu\Sigma\partial^\mu\Sigma] + \frac{f^2 B_0}{4}\mathrm{tr}[\chi^\dagger\Sigma + \Sigma^\dagger\chi], \tag{3}$$

where $\chi = (m_u, m_d, m_s)_{\mathrm{diag}}$ and

$B_0 = \frac{m^2_{\pi^+}}{m_u+m_d} = \frac{m^2_{K^+}}{m_u+m_s} = \frac{m^2_{K^0}}{m_d+m_s}$.

Strong Lagrangian at NLO

According to Gasser and Leutwyler, $\mathcal{L}_{st}^{(4)} = \sum L_i \mathcal{O}_i^{(st)}$. Here L_i is defined

$$L_i = L_i^r + \frac{1}{16\pi^2}\left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi)\right]\Gamma_i. \tag{4}$$

Partially Quenched ChPT of Bernard and Golterman

1) The valence quarks are quenched by introducing "ghost" quarks which have the same mass and quantum numbers as the valence quarks but opposite statistics.

2) Sea quarks are then introduced which can appear in the loops.

3) In the partially quenched case, the chiral field

$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{5}$$

has $\phi^a\lambda^a$ replaced by a larger matrix,

$$\Phi \equiv \begin{pmatrix} \phi & \chi^\dagger \\ \chi & \tilde{\phi} \end{pmatrix}. \tag{6}$$

Where does PQChPT get us?

1) The sea quarks generated in the configurations can have different masses from the valence quarks calculated in the propagator inversions.

2) N_{sea} can be arbitrary. When $N_{sea} = 3$ the LEC's take on the same values as in the full theory.

Extending to PQChPT

A general operator U is made up of products of Σ and its partial derivatives

$$U = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad (7)$$

The transition to the partially quenched theory is made by replacing $\phi^a \lambda^a$ by Φ, and replacing the traces in the operators with supertraces, defined as

$$\mathrm{str}(U) = \mathrm{tr}(A) - \mathrm{tr}(D). \qquad (8)$$

Staggered Chiral Perturbation Theory

$$\Sigma = \sigma^2 = \exp\left[\frac{2i\Phi}{f}\right], \tag{9}$$

$$\Phi = \begin{pmatrix} U & \pi^+ & K^+ \\ \pi^- & D & K^0 \\ K^- & \overline{K}^0 & S \end{pmatrix}, \tag{10}$$

$U = \sum_{a=1}^{16} U_a T_a$, *etc*, with

$$T_a = \{\xi_5, i\xi_{\mu 5}, i\xi_{\mu\nu}, \xi_\mu, \xi_I\} \tag{11}$$

Aubin and Bernard, hep-lat/0304014.

Leading order staggered Lagrangian

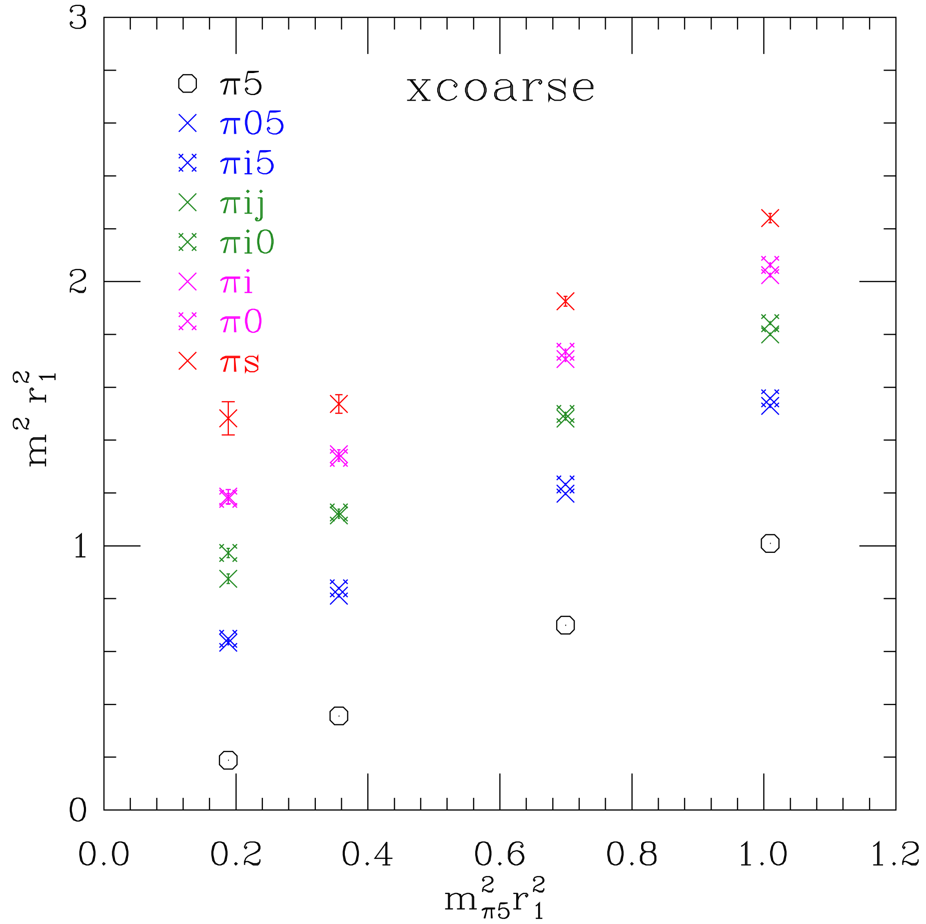
$$\mathcal{L}_S^{(2)} = \frac{f^2}{8}\text{tr}[\partial_\mu\Sigma\partial^\mu\Sigma] + \frac{f^2 B_0}{4}\text{tr}[\chi^\dagger\Sigma + \Sigma^\dagger\chi] + \frac{2m_0^2}{3}(U_I + D_I + S_I + ...)^2 + a^2\mathcal{V}, \quad (12)$$

where

$$\mathcal{V} = \sum_k C_k\mathcal{O}_k + \sum_{k'} C_{k'}\mathcal{O}_{k'}, \quad (13)$$

are taste breaking operators.

Staggered mass splittings



Heavy light ChPT

$$\mathcal{L}^{(2)} = i\mathrm{tr}_D[\overline{H}_a v^\mu(\delta_{ab}\partial_\mu + iV_\mu^{ba})H_b] + g_\pi \mathrm{tr}_D(\overline{H}_a H_b \gamma^\nu \gamma_5 A_\nu^{ba}) + \mathcal{L}_{S\chi PT}, \tag{14}$$

$$V_\mu \equiv \frac{i}{2}[\sigma^\dagger \partial\sigma + \sigma\partial_\mu\sigma^\dagger], \tag{15}$$

$$A_\mu \equiv \frac{i}{2}[\sigma^\dagger \partial\sigma - \sigma\partial_\mu\sigma^\dagger]. \tag{16}$$

Obtaining V_{cb} from $\overline{B} \to Dl\overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} \times |V_{cb}|^2 |\mathcal{F}_{B\to D}(w)|^2 \quad (17)$$

where $\mathcal{F}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)$.

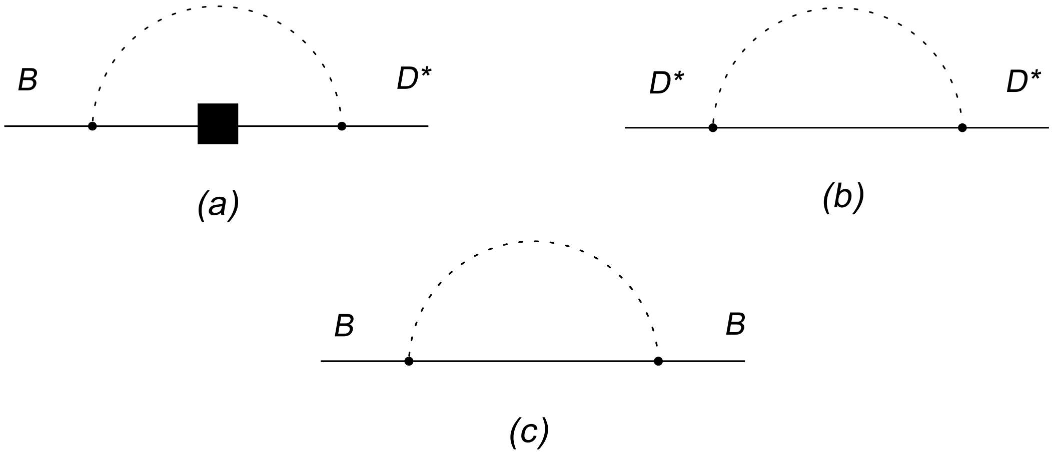
Double ratio method

$$\frac{\langle D|\bar{c}\gamma_0 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 c|D\rangle\langle\overline{B}|\bar{b}\gamma_0 b|\overline{B}\rangle} = \left|h_+(1)\right|^2. \qquad (18)$$

$h_+(1)$ is constrained by heavy quark symmetry:

$$h_+(1) \;=\; \eta_V \left[1 - l_P \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \qquad (19)$$

Diagrams contributing to $B \to D^{(*)}$

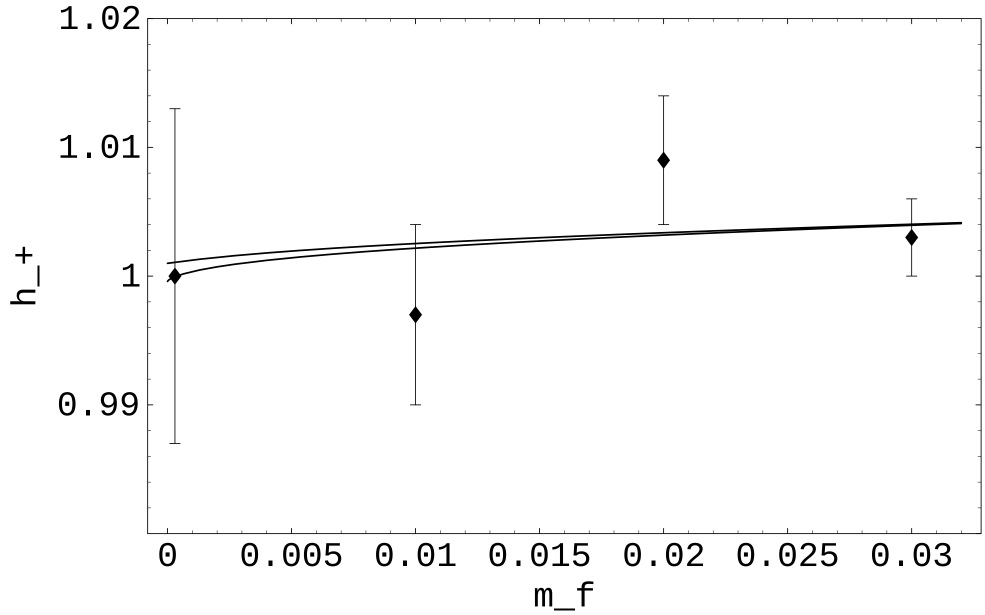


One-loop diagrams that contribute to $B \to D^{(*)}$. The solid line represents a meson containing a heavy quark, and the dashed line represents light mesons. The small solid circles are strong vertices and contribute a factor of g_π. The large solid square is a weak interaction vertex. Diagram (a) is a vertex correction, and (b) and (c) correspond to wavefunction renormalization.

Staggered ChPT formula

$$
\begin{aligned}
h_{+}^{2+1}(1) \quad = \quad & 1 + X_{+} + \frac{g_\pi^2}{16\pi^2 f^2}\Bigg[\frac{3}{2}F_{\pi_I} + F_{K_I} + \frac{1}{6}F_{\eta_I} \\
& + a^2\delta'_V\left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}F_{\pi_V}\right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}F_{\eta_V} \\
& \left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}F_{\eta'_V}\right) + (V \to A)\Bigg],
\end{aligned} \tag{20}
$$

Chiral extrapolation for $B \to D$



top curve: fit to staggered ChPT
bottom curve: staggered ChPT with order a^2 terms dropped

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1}\,\mathcal{G}(w) \times |V_{cb}|^2 |\mathcal{F}_{B\to D^*}(w)|^2 \qquad (21)$$

where $\mathcal{G}(w)$ is a kinematic factor and $\mathcal{F}_{B\to D^*}$ is a nonperturbative matrix element.

Calculating $B \to D^*$

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1), \tag{22}$$

$$\langle D^*(v)|\mathcal{A}^\mu|\overline{B}(v)\rangle \;\; = \;\; i\sqrt{2m_B 2m_{D^*}}\overline{\epsilon'}^\mu h_{A_1}(1). \tag{23}$$

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

$$h_{A_1}(1) \;\; = \;\; \eta_A\left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right] \tag{24}$$

Double ratio method

$$\frac{\langle D^*|\bar{c}\gamma_4 b|\overline{B}^*\rangle\langle\overline{B}^*|\bar{b}\gamma_4 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\overline{B}^*|\bar{b}\gamma_4 b|\overline{B}^*\rangle} = |h_1(1)|^2 . \tag{25}$$

$$\frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle\langle\overline{B}^*|\bar{b}\gamma_j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma_j\gamma_5 c|D\rangle\langle\overline{B}^*|\bar{b}\gamma_j\gamma_5 b|\overline{B}\rangle} = \left|\check{h}_{A_1}(1)\right|^2 . \tag{26}$$

$h_1(1)$ and $\check{h}_{A_1}(1)$ are constrained by heavy quark symmetry:

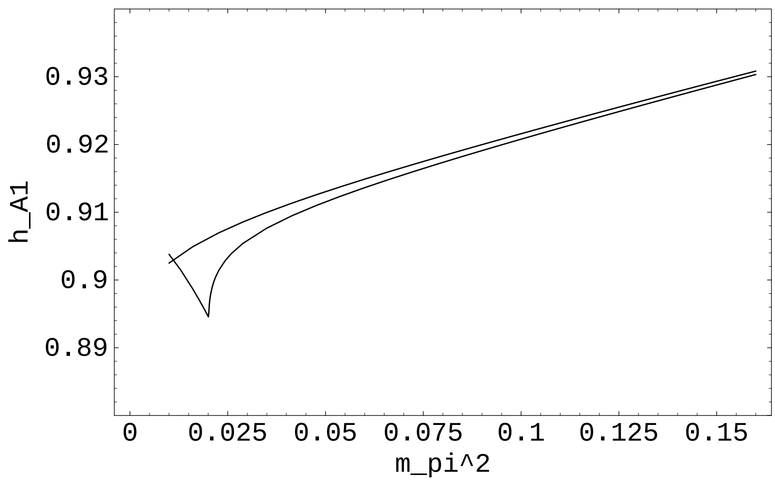
$$\begin{aligned} h_1(1) &= \eta_V \left[1 - l_V \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \\ \check{h}_{A_1}(1) &= \check{\eta}_A \left[1 - l_A \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]. \end{aligned} \tag{27}$$

Staggered ChPT formula

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \quad = \quad & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{3}{2}\overline{F}_{\pi_I} + \overline{F}_{K_I} + \frac{1}{6}\overline{F}_{\eta_I}\right. \\
& + a^2\delta'_V\left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}\overline{F}_{\pi_V}\right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left.\left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta'_V}\right) + (V \to A)\right],
\end{aligned}
\tag{28}
$$

where a is the lattice spacing, δ'_V, g_π and X_A are constants, and $\overline{F}$ is a complicated function involving logs.

Possible Chiral Extrapolation for $B \to D^*$



Lower (upper) curves add to linear behavior the contribution from chiral logs with $g_\pi = 0.60$ ($g_\pi = 0.27$). The curves with the large cusp are continuum extrapolated curves; the ones without the cusp include also staggered effects.

Conclusion

1) Staggered quarks trade living with the fourth root trick for speed. Good agreement with experiment is found for all quantities so far, including predictions (not "post-dictions").

2) Staggered lattice calculations require staggered ChPT because of taste-breaking. The ChPT necessary for $B \to D^{(*)}$ has been done. In $B \to D$, the extrapolation is nearly linear, and the error is small. In $B \to D^*$, the effect is larger due to the "cusp".

3) The lattice calculation of $B \to D^*$ is underway, and will lower the error on exclusive V_{cb}.