Quenched Scaling Study of Charm and Bottom Systems with a Relativistic Heavy Quark Action

Y.Kuramashi (U.Tsukuba)
for CP-PACS collaboration

October 3, 2005

§1. Introduction

cutoff effects for $m_Q a \sim O(1)$

$$f_0(m_Q a),\ \ f_k(m_Q a)(a\Lambda_{\rm QCD})^k \ \ \text{with}\ \ f_{0,k}(0) = \text{const.}$$

the relativistic heavy quark action for $O(f_1(m_Q a)a\Lambda_{\rm QCD})$ improvement

$$\begin{aligned}\mathcal{L}_q^{\rm RHQ} \ = \ & m_0\bar{q}q + \bar{q}\gamma_0 D_0 q + \nu\sum_i \bar{q}\gamma_i D_i q - \frac{r_t a}{2}\bar{q}D_0^2 q - \frac{r_s a}{2}\sum_i \bar{q}D_i^2 q \\ & -\frac{iga}{2}c_E\sum_i \bar{q}\sigma_{0i}F_{0i}q - \frac{iga}{4}c_B\sum_{i,j}\bar{q}\sigma_{ij}F_{ij}q,\end{aligned}$$

obtained by extending Symanzik's impr. program to massive case

r_t is redundant

ν, r_s, c_E, c_B should be adjusted in a $m_Q a$ dependent way

cf. r_t and r_s are redundant in Fermilab approach what we have achieved so far

- propose the relativistic heavy quark action
  Prog. Theor. Phys. 109(2003)383

- perturbative determination of ν, r_s, c_B, c_E in the action
  Nucl. Phys. B697(2004)271

- perturbative improvement of the vector and axial vector currents
  Nucl. Phys. B689(2004)127

formulation is completed at least perturbatively

next step is a systematic scaling study of charm and bottom systems in quenched QCD

Plan of this talk

§**2.** Perturbative determination of impr. coeffs.

§**3.** Simulation details

§**4.** Dispersion relation for heavy-heavy and heavy-light mesons

§**5.** $c\bar{c}$ and $b\bar{b}$ spectra

§**6.** f_{D_s} and f_{B_s}

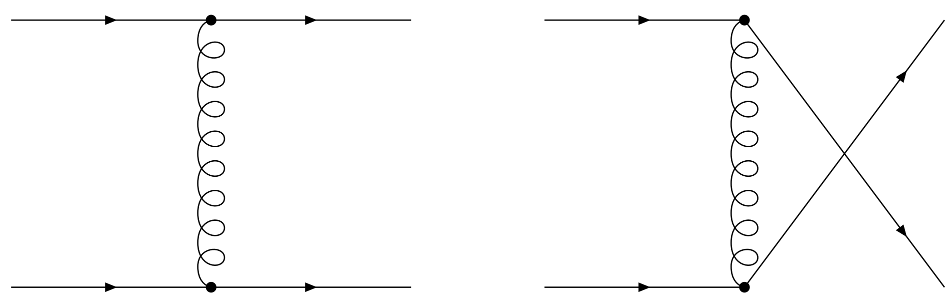
§**7.** $m_c^{\overline{\mathsf{MS}}}$, $m_b^{\overline{\mathsf{MS}}}$ from heavy-heavy and heavy-light AWI

§**8.** Summary

§2. Perturbative determination of impr. coeffs.
§2-1. Impr. coeffs. in the action at tree level

"$\cdots$ on-shell quantities(particle energies, scattering amplitudes, normalized matrix elements of local composite fields between particle states etc.) $\cdots$" Lüscher-Sint-Sommer-Weisz, NPB478(1996)365

ν, r_s, c_E, c_B can be determined by on-shell quark-quark scattering



continuum scattering amplitude should be reproduced

massless case done by Wohlert to determine c_{SW}

on-shell improvement condition yields

$$\nu^{(0)} = \frac{\sinh(m_p^{(0)})}{m_p^{(0)}} \longrightarrow 1$$

$$r_s^{(0)} = \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{{m_p^{(0)}}^2} \longrightarrow r_t$$

$$c_E^{(0)} = r_t \nu^{(0)} \longrightarrow r_t$$

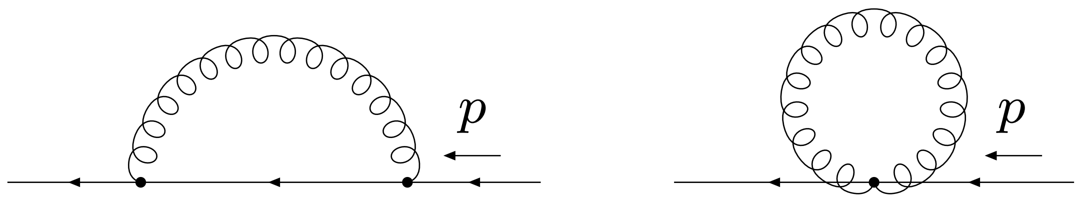
$$c_B^{(0)} = r_s^{(0)} \longrightarrow r_t$$

points

- 4 parameters are uniquely determined
- r_t is not fixed $\longrightarrow$ consistent with redundancy
- reproduce correct SW quark action in the chiral limit
- also possible to determine $\nu^{(0)}$, $r_s^{(0)}$ from quark propagator

§2-2. $\nu(m_Q a, g)$, $r_s(m_Q a, g)$ at one-loop level

quark self energy

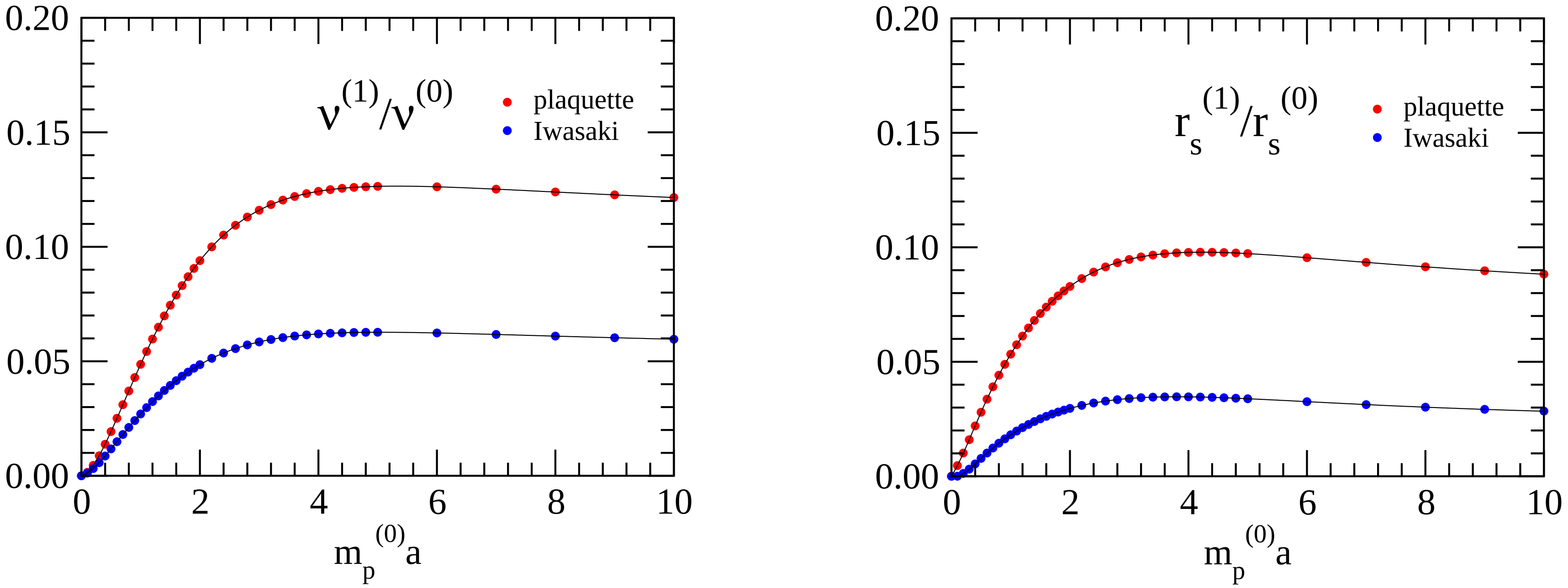


up to one-loop level

$$S_q^{-1}(p,m) = i\gamma_0 \sin p_0[1 - g^2 B_0(p,m)] + \nu i \sum_i \gamma_i \sin p_i [1 - g^2 B_i(p,m)]$$
$$+m + 2r_t \sin^2\left(\frac{p_0}{2}\right) + 2r_s \sum_i \sin^2\left(\frac{p_i}{2}\right) - g^2 \widehat{C}(p,m)$$

ν, r_s are determined by dispersion relation and spinor structure

$\nu^{(1)}$, $r_s^{(1)}$ from quark propagator

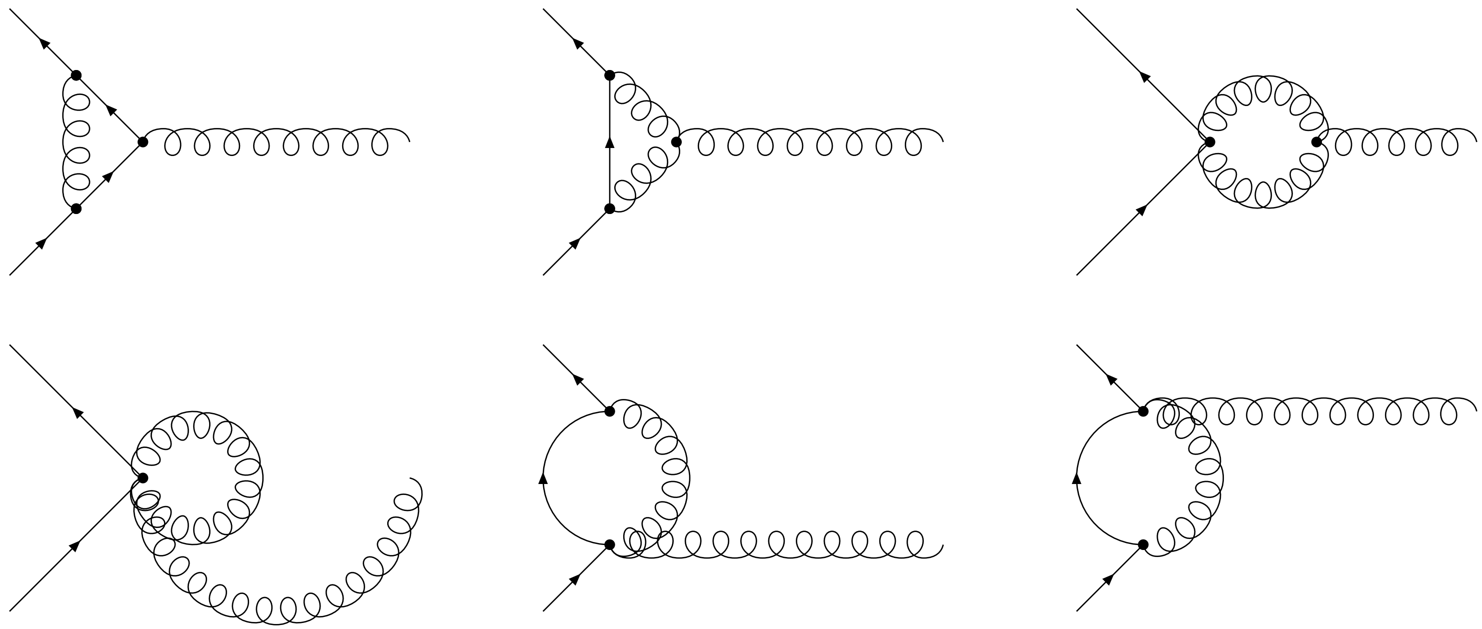


$\nu^{(1)}$, $r_s^{(1)}$ vanish for $m_p^{(0)} \to 0$

assures that $\nu \to 1$, $r_s \to r_t$ up to one-loop level

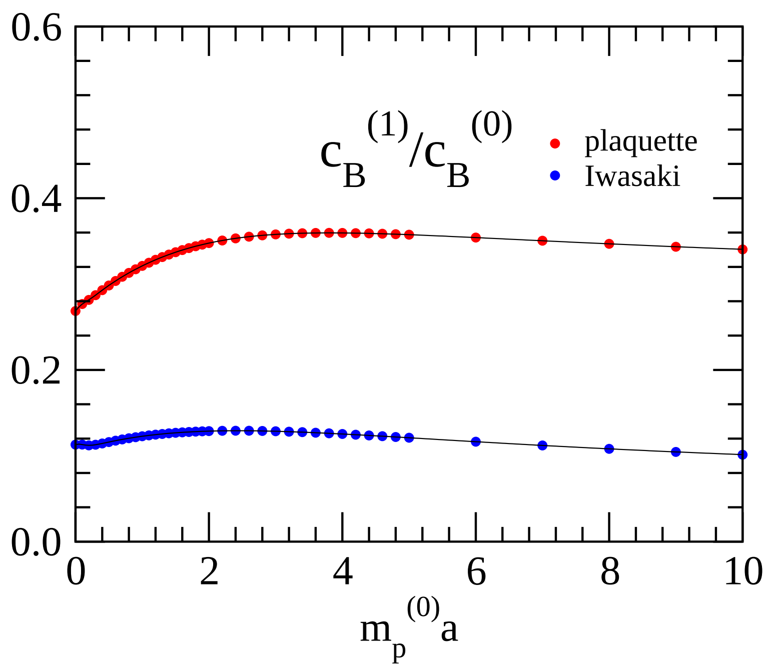
§2-3. c_B, c_E at one-loop level

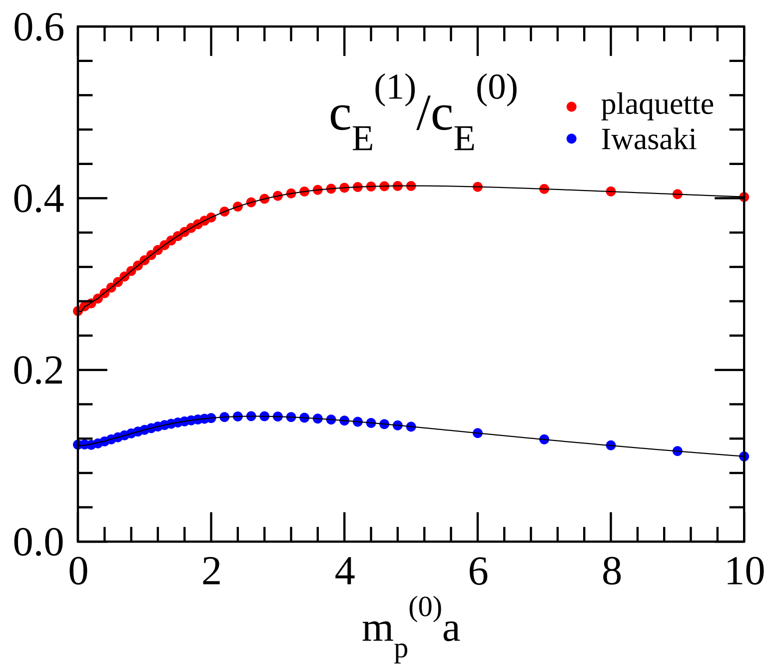
vertex corrections



infrared divergences are cancelled out after summation
if and only if $\nu^{(0)}$, $r_s^{(0)}$, $c_B^{(0)}$, $c_E^{(0)}$ are properly tuned

$c_B^{(1)}$, $c_E^{(1)}$ from quark-quark scattering





$c_B^{(1)}$ is increased $\longrightarrow$ hyperfine splitting will be enhanced

§2-4. Renorm. and impr. of the axial current

improvement of A_0 for heavy-light case

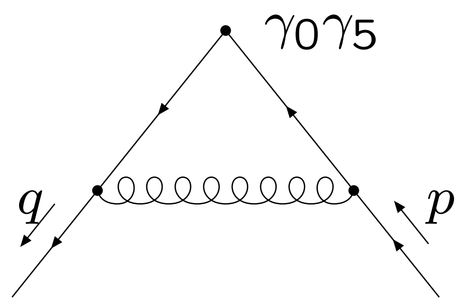
$$
\begin{aligned}
A_0^{\mathrm{latt},R}(x) &= Z_{A_0}^{\mathrm{latt}}\Big[\bar{q}(x)\gamma_0\gamma_5 Q(x) - g^2 c_{A_0}^{+}\partial_0^{+}\{\bar{q}(x)\gamma_5 Q(x)\} \\
&\quad - g^2 c_{A_0}^{-}\partial_0^{-}\{\bar{q}(x)\gamma_5 Q(x)\} + O(g^4)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\partial_0^{+}\{\bar{q}(x)\gamma_5 Q(x)\} &= \bar{q}(x)\gamma_5\{\partial_0 Q(x)\} + \{\partial_0\bar{q}(x)\}\gamma_5 Q(x) \\
\partial_0^{-}\{\bar{q}(x)\gamma_5 Q(x)\} &= \bar{q}(x)\gamma_5\{\partial_0 Q(x)\} - \{\partial_0\bar{q}(x)\}\gamma_5 Q(x)
\end{aligned}
$$

additional improvement coefficient $c_{A_0}^{-}$

general form of off-shell vertex function at one-loop level

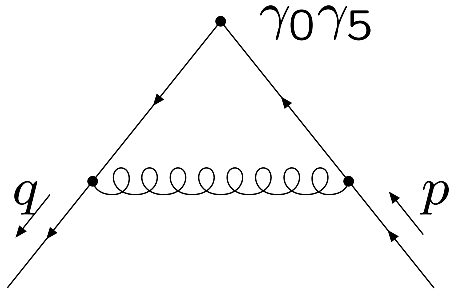
$$
\begin{aligned}
& \Lambda_{05}^{(1)}(p,q,m_H,m_L) \\
= \; & \gamma_0\gamma_5 F_1^{05} + \gamma_0\gamma_5 \not{p} F_2^{05} + \not{q}\gamma_0\gamma_5 F_3^{05} + \not{q}\gamma_0\gamma_5\not{p} F_4^{05} \\
& +(p_0 - q_0)\left[\gamma_5 G_1^{05} + \gamma_5\not{p} G_2^{05} + \not{q}\gamma_5 G_3^{05} + \not{q}\gamma_5\not{p} G_4^{05}\right] \\
& +(p_0 + q_0)\left[\gamma_5 H_1^{05} + \gamma_5\not{p} H_2^{05} + \not{q}\gamma_5 H_3^{05} + \not{q}\gamma_5\not{p} H_4^{05}\right] \\
& +O(a^2)
\end{aligned}
$$



general form of off-shell vertex function at one-loop level

$$
\begin{aligned}
&\Lambda^{(1)}_{05}(p,q,m_H,m_L) \\
=\ & \gamma_0\gamma_5 F_1^{05} + \gamma_0\gamma_5 \not{p} F_2^{05} + \not{q}\gamma_0\gamma_5 F_3^{05} + \not{q}\gamma_0\gamma_5\not{p} F_4^{05} \quad \Delta_{\gamma_0\gamma_5} \text{ in } Z_{A_0} \\
&+(p_0-q_0)\left[\gamma_5 G_1^{05} + \gamma_5\not{p} G_2^{05} + \not{q}\gamma_5 G_3^{05} + \not{q}\gamma_5\not{p} G_4^{05}\right] \quad c^+_{A_0} \\
&+(p_0+q_0)\left[\gamma_5 H_1^{05} + \gamma_5\not{p} H_2^{05} + \not{q}\gamma_5 H_3^{05} + \not{q}\gamma_5\not{p} H_4^{05}\right] \quad c^-_{A_0} \\
&+O(a^2)
\end{aligned}
$$

with $\not{p}u(p) = im_H u(p)$, $\bar{u}(q)\not{q} = im_L\bar{u}(q)$

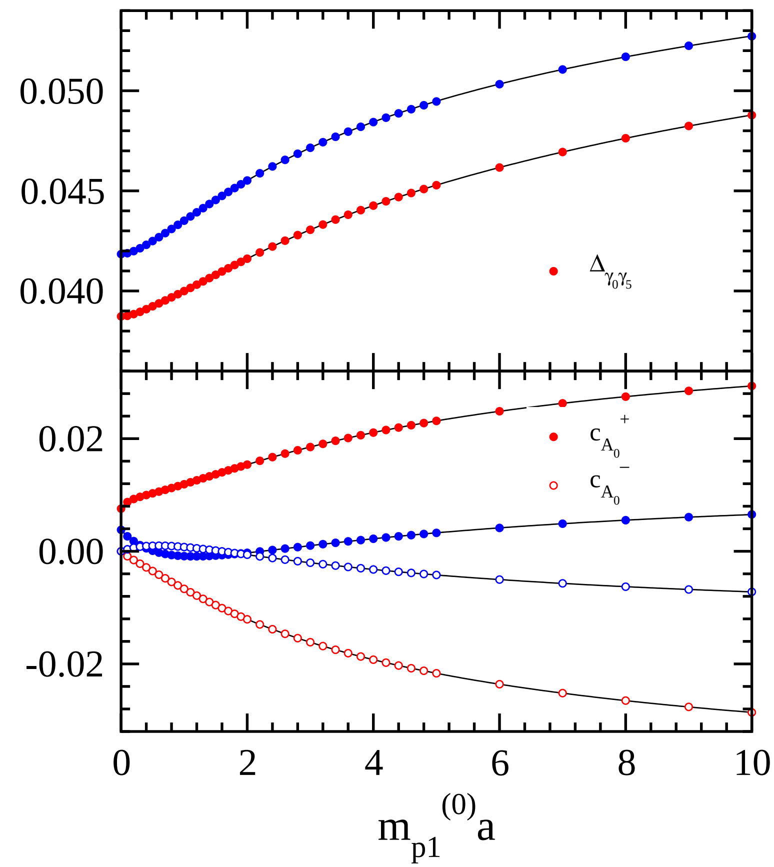


heavy-light case

red : plaquette

blue : Iwasaki

small values for $\Delta_{\gamma_0\gamma_5}$, $c^{\pm}_{A_0}$



$O(a)$ improvement of V_0, V_k, A_0, A_k is done for H-H and H-L cases

§3. Simulation details

light quarks

– clover with $c_{\mathrm{SW}}^{\mathrm{NP}}$

– 2 kappa to sandwich m_s determined by m_ϕ

heavy quarks

– RHQ(PT) with ν^{PT} vs. RHQ(NP) with ν^{NP}

– perturbative value for r_s

– $c_{B/E} = \{c_{B/E}^{\mathrm{PT}}(m_Q a) - c_{B/E}^{\mathrm{PT}}(0)\} + c_{\mathrm{SW}}^{\mathrm{NP}}$

$\rightarrow$ include nonperturbative contribution at $m_Q = 0$

– 6 kappa to cover m_c to m_b

axial vector currents

– perurbative renorm. factor and impr. coeffs.

remaing syst. errors in the heavy quark action except $O((a\Lambda_{\mathrm{QCD}})^2)$

<u>RHQ(PT)</u>

$O(\alpha_s^2) \sim 5\%$ from $\nu^{\mathrm{PT}} \sum_i \bar{q}\gamma_i D_i q$

$\longrightarrow$ responsible for M^{pole}-M^{kin} difference

<u>RHQ(NP)</u>

ν^{NP} is determined by adusting ν to satisfy $M_{hh}^{\mathrm{pole}} = M_{hh}^{\mathrm{kin}}$

$O(\alpha_s^2 a\Lambda_{\mathrm{QCD}}) \sim 1\%$ from Wilson and clover terms

$\longrightarrow$ negligible under current statistical errors

RHQ(NP) should show better scaling behavior simulation parameters

- Iwasaki gauge action
- the scale is set by $r_0 = 0.5$fm from CP-PACS, hep-lat/0408010
- $L = 1.8$fm with $a(r_0)$
- $|\vec{p}| = 2\pi/L$, $\sqrt{2} \times 2\pi/L$

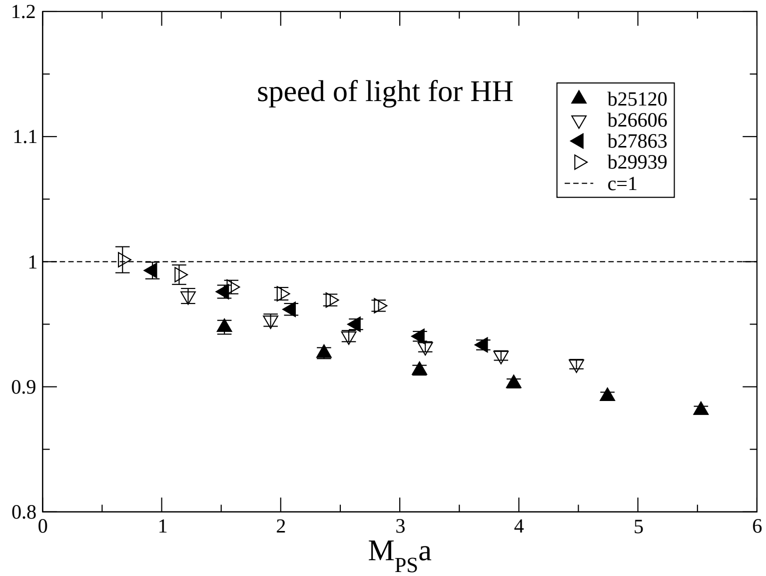
$L^3 \times T$	β	a[fm]	a^{-1}[GeV]	#conf(PT)	#conf(NP)
$16^3 \times 40$	2.5120	0.11250	1.7538	550	550
$20^3 \times 48$	2.6606	0.09000	2.1922	480	480
$24^3 \times 48$	2.7863	0.07500	2.6307	450	450
$32^3 \times 64$	2.9939	0.05625	3.5076	420	0

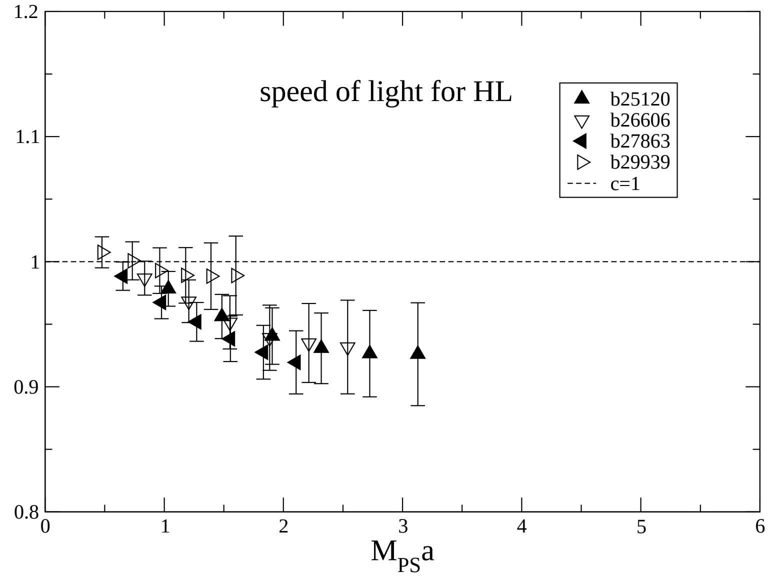
RHQ(PT) and RHQ(NP) are studied on the same gauge configs.

§4. Dispersion relation for HH and HL mesons

$E^2 = m_{\rm pole}^2 + {c_{\rm eff}}^2 |\vec{p}|^2$ for PS meson

<u>RHQ(PT)</u>

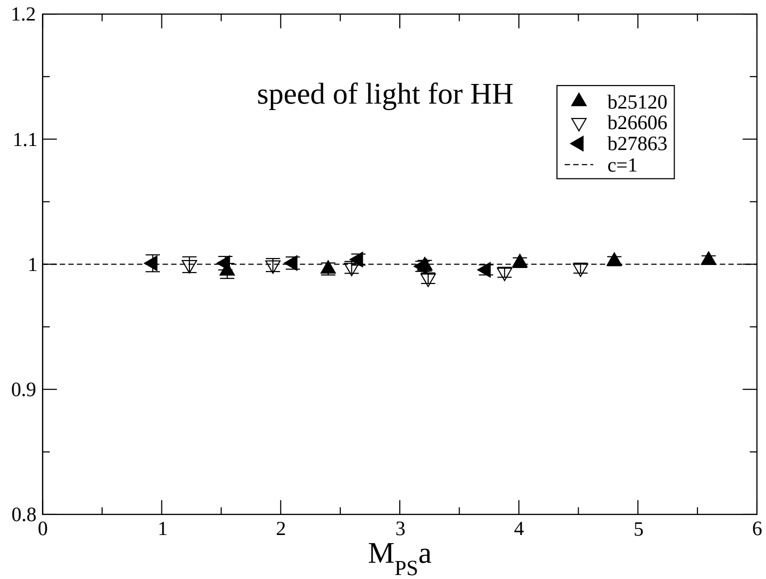


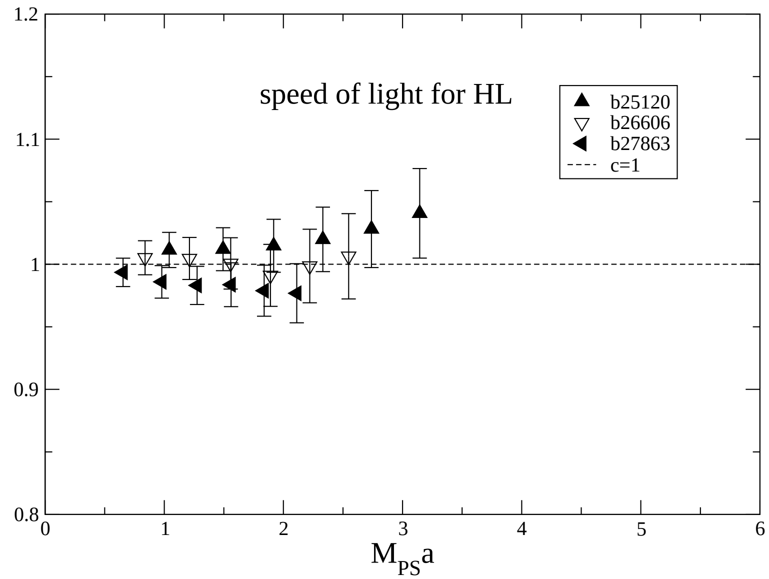


10%-15% deviation from 1 both for HH and HL cases

RHQ(NP)

ν is determined to satisfy $c_{\text{eff}} = 1$ for heavy-heavy PS meson





c_{eff} for heavy-light PS meson is automatically tuned to be 1

expected from our formulation

§5. $c\bar{c}$ and $b\bar{b}$ spectra

exploratory study of $b\bar{b}$ spectra with RHQ

charm and bottom quark masses are fixed by $M_{J/\Psi}$ and M_Υ

$^{2S+1}L_J$	J^{PC}	$c\bar{c}$	mass[GeV]	$b\bar{b}$	mass[GeV]	operator
$^1S_0$	0^{-+}	η_c	2.980	η_b	–	$\bar{q}\gamma_5 q$
$^3S_1$	1^{--}	J/ψ	3.097	Υ	9.460	$\bar{q}\gamma_i q$
$^1P_1$	1^{+-}	h_c	–	h_b	–	$\bar{q}\sigma_{ij} q$
$^3P_0$	0^{++}	χ_{c0}	3.415	χ_{b0}	9.860	$\bar{q}q$
$^3P_1$	1^{++}	χ_{c1}	3.511	χ_{b1}	9.893	$\bar{q}\gamma_i\gamma_5 q$

$\chi_{c2}(2^{++})$, $\chi_{b2}(2^{++})$ are omitted

Previous quenched scaling studies for $c\bar{c}$

- QCDTARO Collab., JHEP0308(2003)022
  - clover on isotropic lattice
  - nonperturbative c_{SW}
  - 4 β values with plaqutte gauge action

- CP-PACS Collab., PRD65(2002)094508
  - clover on anisotropic lattice
  - tadpole improved tree-level c_{SW}
  - anisotropic parameter $a_s/a_t = 3$
  - 4 β values with Iwasaki gauge action

compare scaling properties with clover results

Previous quenched scaling study for $b\bar{b}$

- Davies *et al.*, PRD58(1998)054505
  - NRQCD on isotropic lattice
  - spin-dependent v^4 terms are included
  - tadpole-improved perturbative values for coeffs.
  - 3 β values with plaqutte gauge action

compared with NRQCD results as a benchmark in quenched QCD

results are converted to physical units by $a(r_0)$ taken from Necco-Sommer, NPB622(2002)328 mass splittings

1. orbital excitation: $\Delta M(^3P_1 - {}^3S_1)$ not spin-avaraged

$\Delta M(\chi_{c1} - J/\Psi)$=414MeV

$\Delta M(\chi_{b1} - \Upsilon)$=433MeV

used for determination of α_s

2. spin-orbit splitting: $\Delta M(^3P_1 - {}^3P_0)$

$\Delta M(\chi_{c1} - \chi_{c0})$=96MeV

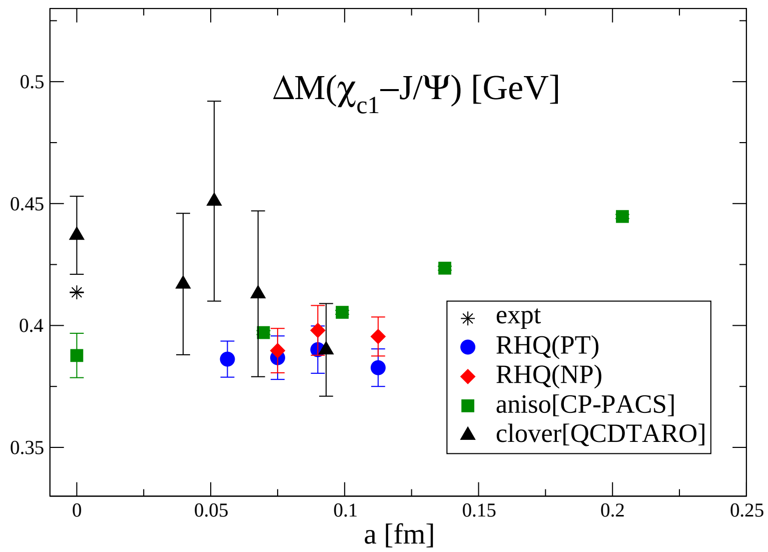
$\Delta M(\chi_{b1} - \chi_{b0})$=33MeV

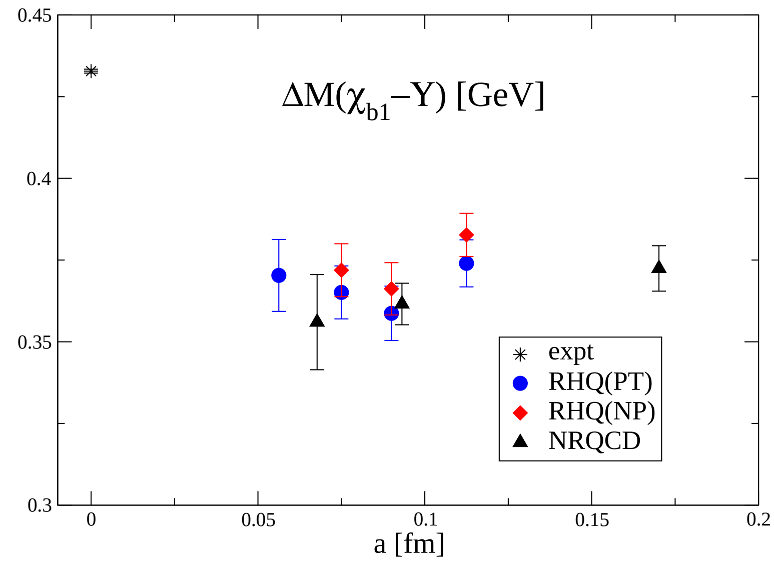
3. hyperfine splitting: $\Delta M(^3S_1 - {}^1S_0)$

$\Delta M(J/\Psi - \eta_c)$=117MeV

η_b requires experimental confirmation

orbital excitation: $\Delta M(^3P_1 - {}^3S_1)$

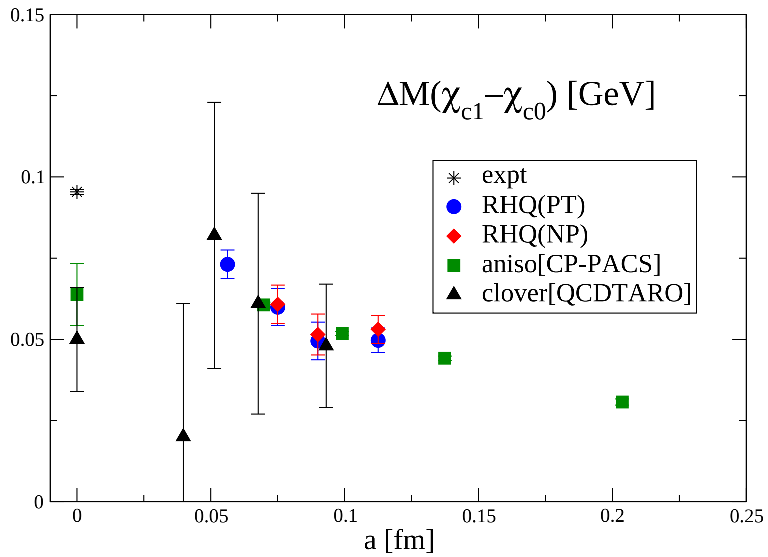


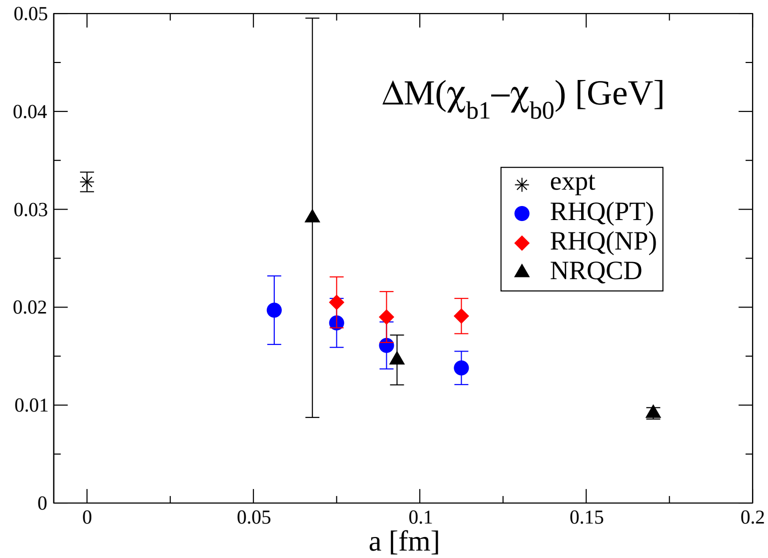


good scaling behavior

consistent with NRQCD results for $b\bar{b}$

spin-orbit splitting: $\Delta M(^3P_1 - {}^3P_0)$

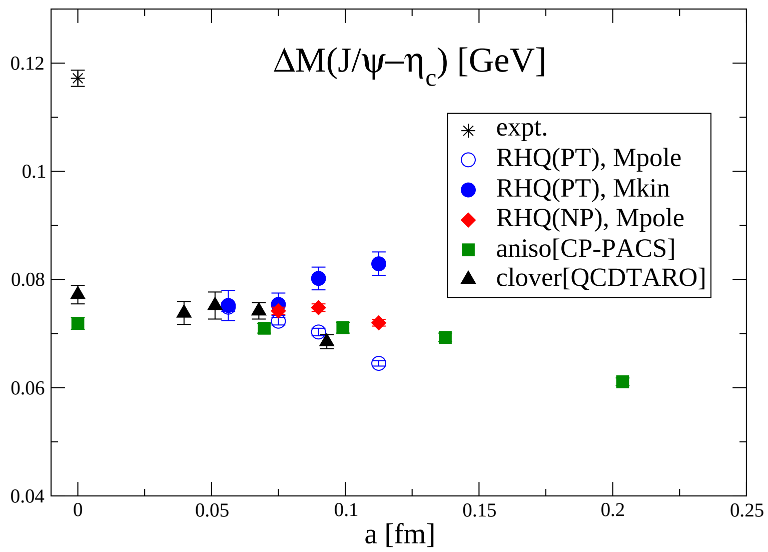


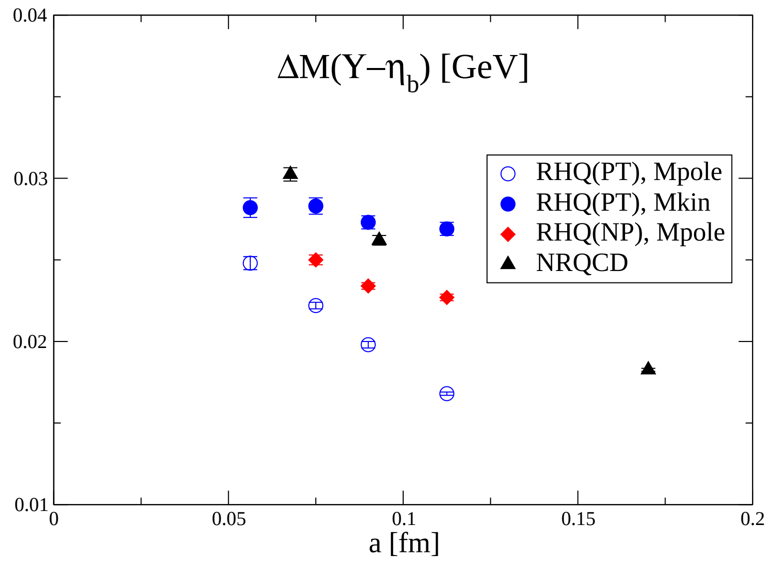


large error for NRQCD

scaling property is improved by RHQ(NP) for $b\bar{b}$

hyperfine splitting: $\Delta M(^3S_1 - {}^1S_0)$





better scaling behavior than NRQCD

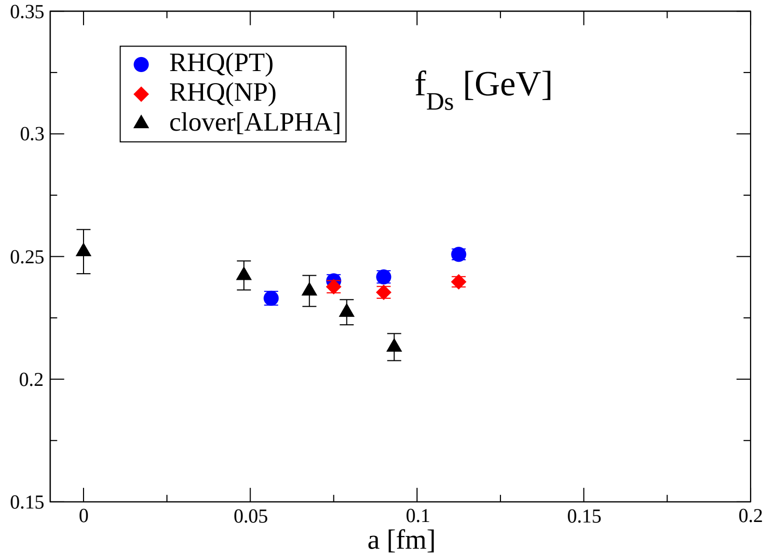
consistent with NRQCD results?

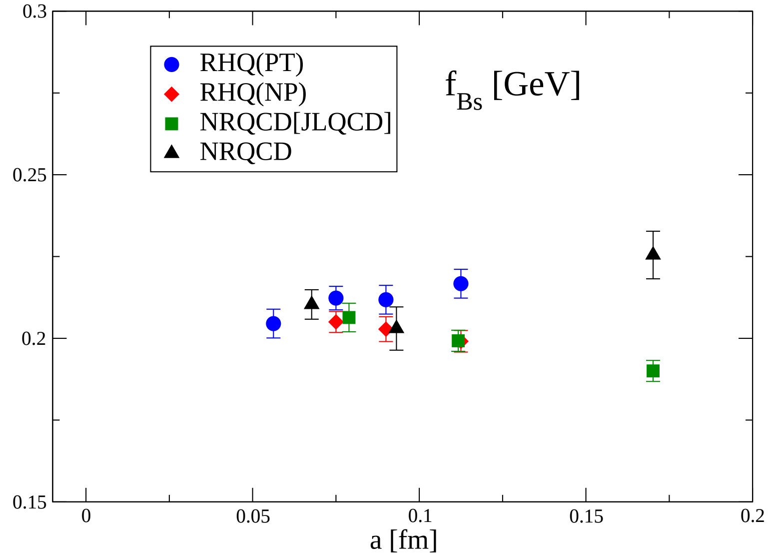
comments

- RHQ shows similar or better scaling behavior than NRQCD as far as the quantities we investigated
- scaling property is further improved by RHQ(NP)
- we also obtain good scaling behavior for $c\bar{c}$

§6. f_{D_s} and f_{B_s}

fitting function: $\Phi_P = \sqrt{M_P} f_P = c_0 + c_1/m_p^{(0)} + c_2/m_p^{(0)^2} + c_3/K_l$





good scaling behavior both for f_{D_s} and f_{B_s}

consistent with previous NRQCD results using $a(r_0)$

§7. $m_c^{\overline{\mathrm{MS}}}$, $m_b^{\overline{\mathrm{MS}}}$ from heavy-heavy and heavy-light AWI

method

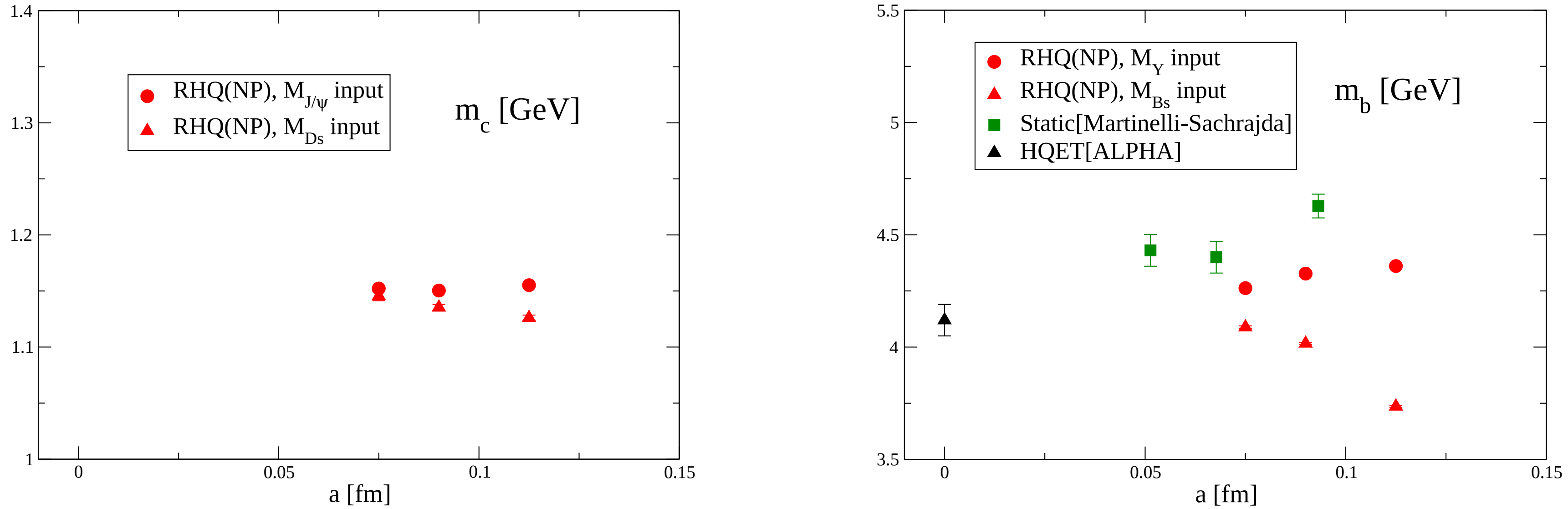
- axial Ward identities — input

$m_{D_s}\langle 0\vert A_0\vert D_s\rangle = (m_c + m_s)\langle 0\vert P\vert D_s\rangle$	m_{D_s}
$m_{B_s}\langle 0\vert A_0\vert B_s\rangle = (m_b + m_s)\langle 0\vert P\vert B_s\rangle$	m_{B_s}
$m_{\eta_c}\langle 0\vert A_0\vert \eta_c\rangle = 2m_c\langle 0\vert P\vert \eta_c\rangle$	$m_{J/\psi}$
$m_{\eta_b}\langle 0\vert A_0\vert \eta_b\rangle = 2m_b\langle 0\vert P\vert \eta_b\rangle$	m_Υ

- one-loop matching to $\overline{\mathrm{MS}}$ quark mass
  Nucl. Phys. B689(2004)127

- m_s determined by m_ϕ

$m_c^{\overline{\text{MS}}}(\mu = m_c^{\overline{\text{MS}}})$ and $m_b^{\overline{\text{MS}}}(\mu = m_b^{\overline{\text{MS}}})$



feasible to determine m_c, m_b directly from heavy-heavy and heavy-light meson pole masses

next task is nonperturbative determination of Z_A/Z_P

§8. Summary

- detailed scaling study in quenched QCD:

$$\mathcal{L}_q^{\text{RHQ}} = m_0\bar{q}q + \bar{q}\gamma_0 D_0 q + \nu\sum_i \bar{q}\gamma_i D_i q - \frac{r_t a}{2}\bar{q}D_0^2 q - \frac{r_s a}{2}\sum_i \bar{q}D_i^2 q$$
$$-\frac{iga}{2}c_E\sum_i \bar{q}\sigma_{0i}F_{0i}q - \frac{iga}{4}c_B\sum_{i,j}\bar{q}\sigma_{ij}F_{ij}q,$$

we have studied RHQ(PT) with ν^{PT} and RHQ(NP) with ν^{NP}
tadpole improved one-loop values for r_s, c_E, c_B

- good scaling behavior both for charm and bottom systems

- RHQ shows similar or better scaling behavior than NRQCD for $b\bar{b}$ spectra

- $m_c^{\overline{\mathrm{MS}}}$, $m_b^{\overline{\mathrm{MS}}}$ are determined directly from heavy-heavy and heavy-light spectra using AWI at one-loop level next step

- renormalization and improvement of 4-fermi ops. (under way)
- nonperturbative determination of Z_A, Z_P
- repeat the calculation on 2+1 flavor gauge configs. of CP-PACS
  the same formulation from sea to heavy valence quarks