Pseudoscalar Flavor-Singlet Physics with Staggered Fermions

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UK QCD collaboration

Overview

- Quick review of singlet physics
- Singlets as a probe of validity of the $\det^{1/4}$ trick in staggered formulation
- Very preliminary results
- Future plans

The η' propagator

$$G_{\eta'}(x', x) = \langle \sum_i \overline{q}_i(x')(\gamma_5 \otimes \mathbf{1})q_i(x') \sum_j \overline{q}_j(x)(\gamma_5 \otimes \mathbf{1})q_j(x)\rangle$$

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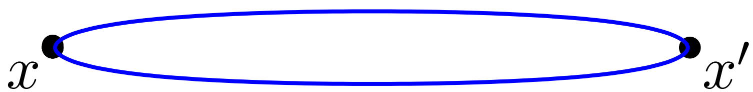
N_f terms from

$$\langle \textstyle\sum_i \overbrace{\overline{q}_i(x')(\gamma_5 \otimes \mathbf{1})\, \underbrace{q_i(x') \displaystyle\sum_j \overline{q}_j(x)}(\gamma_5 \otimes \mathbf{1})q_j(x)}\rangle$$



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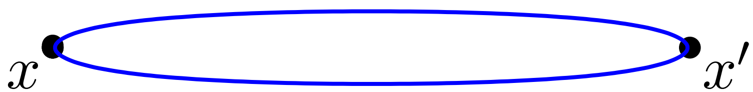
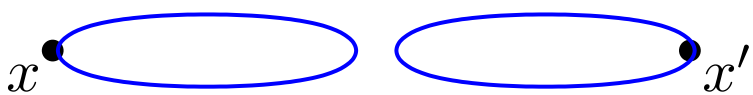


N_f^2 terms from

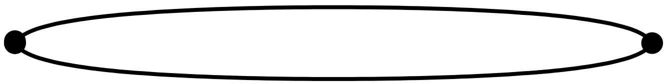
$$\langle \textstyle\sum_i \overbrace{\overline{q}_i(x')(\gamma_5 \otimes \mathbf{1})q_i}(x') \sum_j \overbrace{\overline{q}_j(x)(\gamma_5 \otimes \mathbf{1})q_j}(x)\rangle$$



Singlet propagator

μ^2 represents the coupling between 2 octet PS mesons.

$\tilde{G}_{\eta'} =$



$\frac{1}{p^2+m_\pi^2}$

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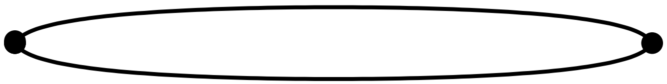

$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$

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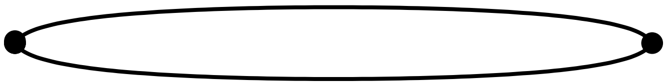
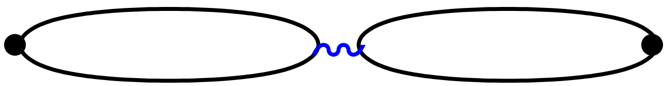


$$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$$



$$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

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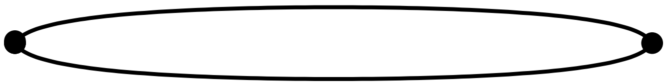
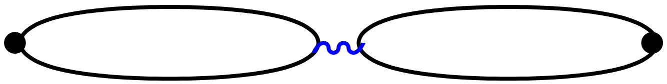


$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$



$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$ $\longleftarrow$ Quenched prop stops

Singlet propagator

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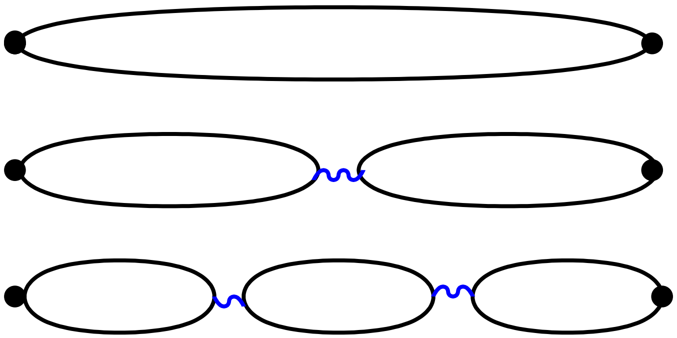

$$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$$

$$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2} \longleftarrow \text{Quenched prop stops}$$

$$+\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

Singlet propagator

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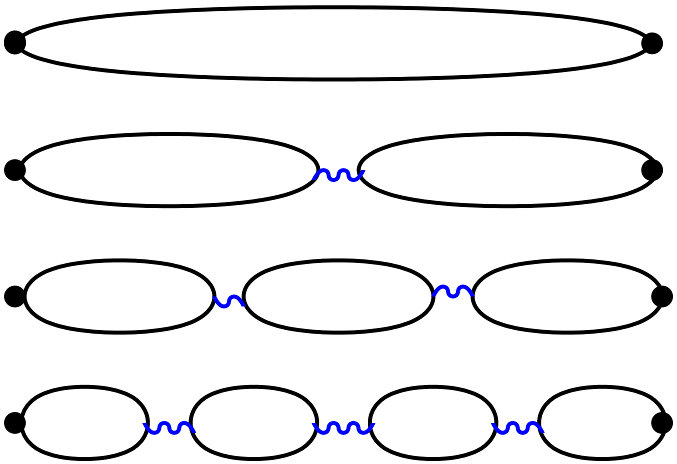

$$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$$

$$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2} \longleftarrow \text{Quenched prop stops}$$

$$+\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

$$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

$$+\ldots$$

Singlet propagator

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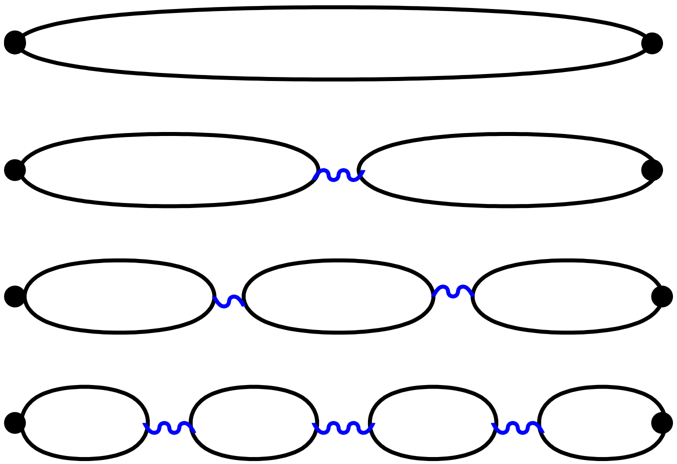

$$\frac{1}{p^2+m_\pi^2} \longleftarrow G_\pi$$

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$$+\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

$$-\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}$$

$$+\ldots$$

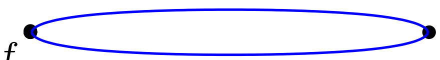
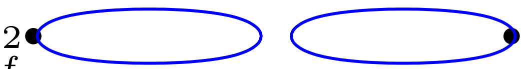
$$\tilde{G}_{\eta'} = \frac{1}{p^2+m_\pi^2+\mu^2}$$

$D(t)/C(t)$ **ratio**

For N_f flavors

$G_\pi(t) = N_f$ 

$= N_f C(t)$

$G_{\eta'}(t) = N_f$  $- N_f^2$ 

$= N_f C(t) - N_f^2 D(t)$

In full QCD we expect

$G_{\eta'}(t) \sim e^{-m_{\eta'} t}$ and $G_\pi(t) \sim e^{-m_\pi t}$.

So

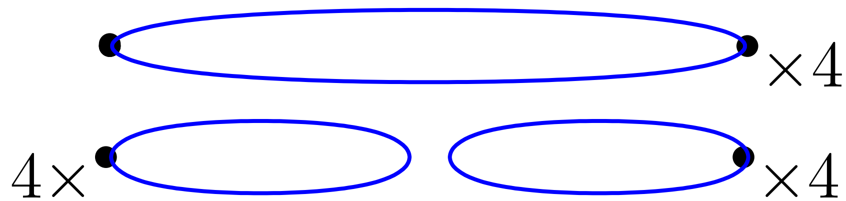
$$R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = \frac{N_f C(t) - G_{\eta'}(t)}{N_f C(t)} = 1 - A \exp\left[-(m_\eta - m_\pi)t\right]$$

Whereas in quenched QCD

$$R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = A + Bt$$

Or something else if $(\det M)^{1/4}$ introduces some strange pathology like $m_{\text{val}} \neq m_{\text{sea}}$ or $N_{\text{val}} \neq N_{\text{sea}}$.

Caveat:



In fact, since the staggered formulation is a theory of four valence tastes, D is too large by a factor of 4.

Rescaling of D by a factor of $\frac{1}{4}$ is implicit in the rest of the talk.

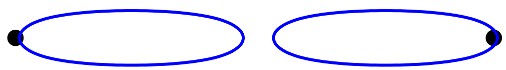
2+1 flavors, $m_u = m_d \neq m_s$

Two connected correlators represent three diagrams:

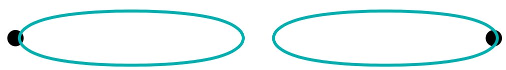
$C_{uu} = C_{dd} \equiv C_{qq}$

C_{ss}

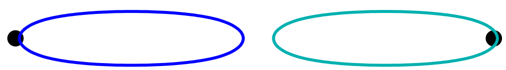
Three disconnected correlators represent nine diagrams:



$$D_{uu} = D_{dd} = D_{ud} = D_{du} \equiv D_{qq}$$



$$D_{ss}$$

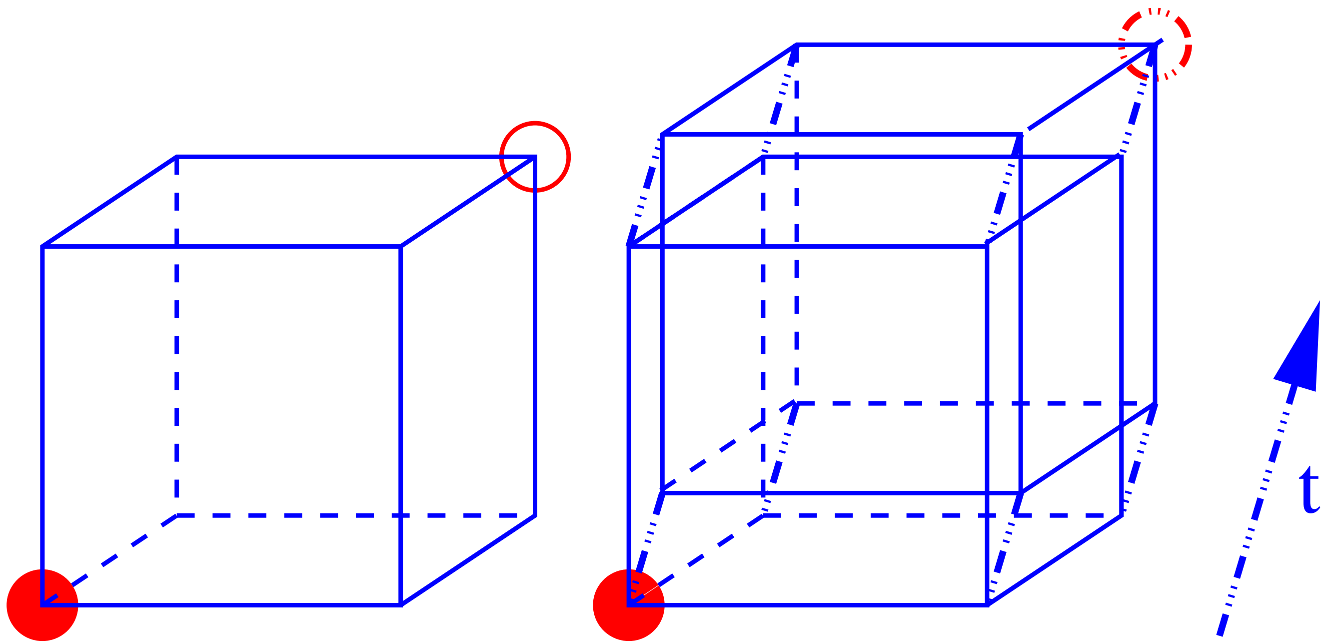


$$D_{us} = D_{ds} = D_{su} = D_{sd} \equiv D_{qs}$$

So construct:

$$R(t) = \frac{N_f^2 D(t)}{N_f C(t)} \longrightarrow \frac{4D_{qq} + 4D_{qs} + D_{ss}}{2C_{qq} + C_{ss}}$$

Staggered PS Singlets



$\gamma_4\gamma_5 \otimes \mathbf{1}$ $\gamma_5 \otimes \mathbf{1}$

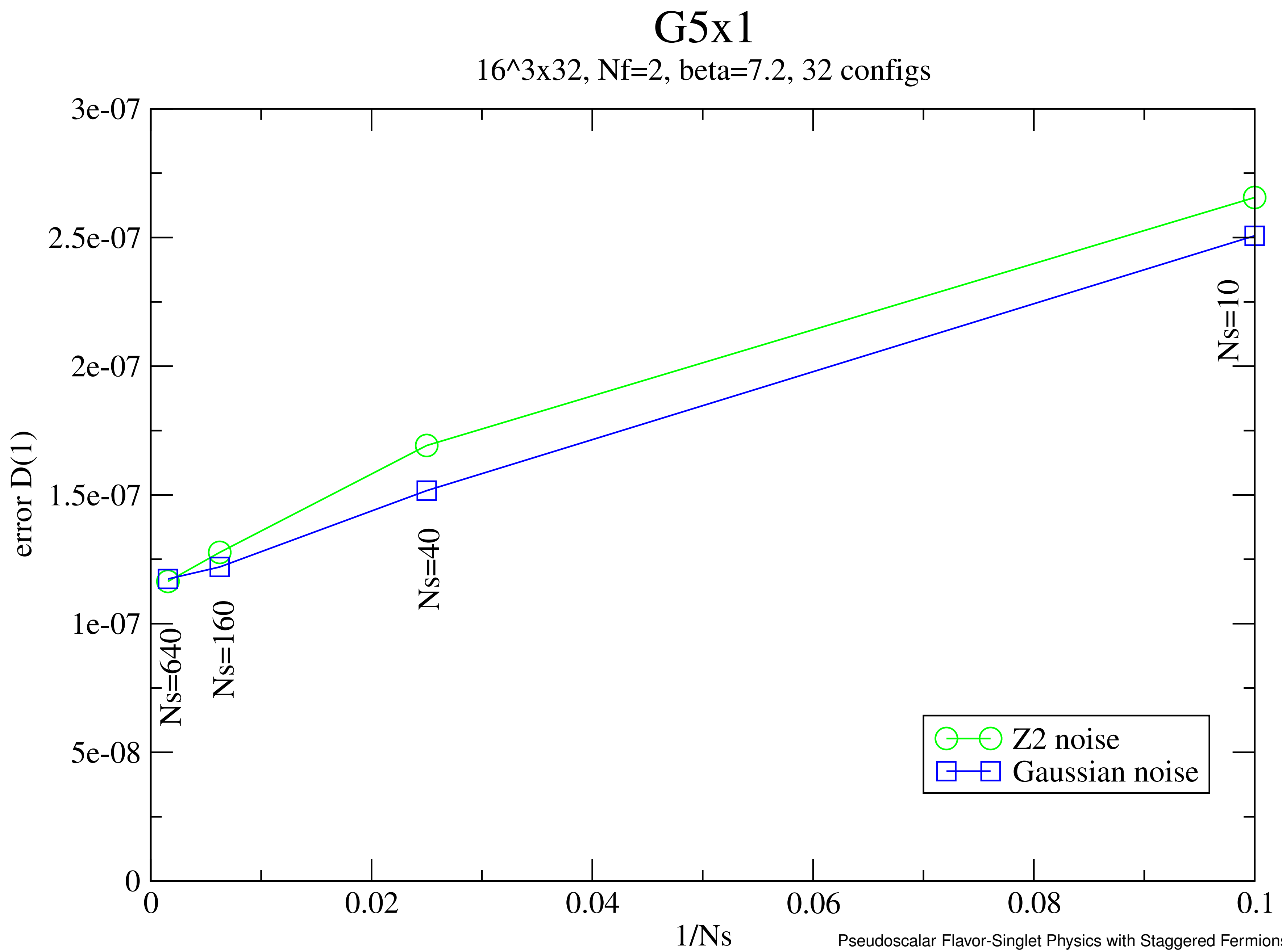
Measuring Correlators

- Measure connected correlators with standard point sources.
- Measure disconnected correlators with gaussian stochastic volume sources η.
- Invert and solve for

$$\mathcal{O}(t) = \mathrm{Tr}\eta^\dagger \Delta_{\gamma 5 \otimes \mathbf{1}} M^{-1} \eta,$$

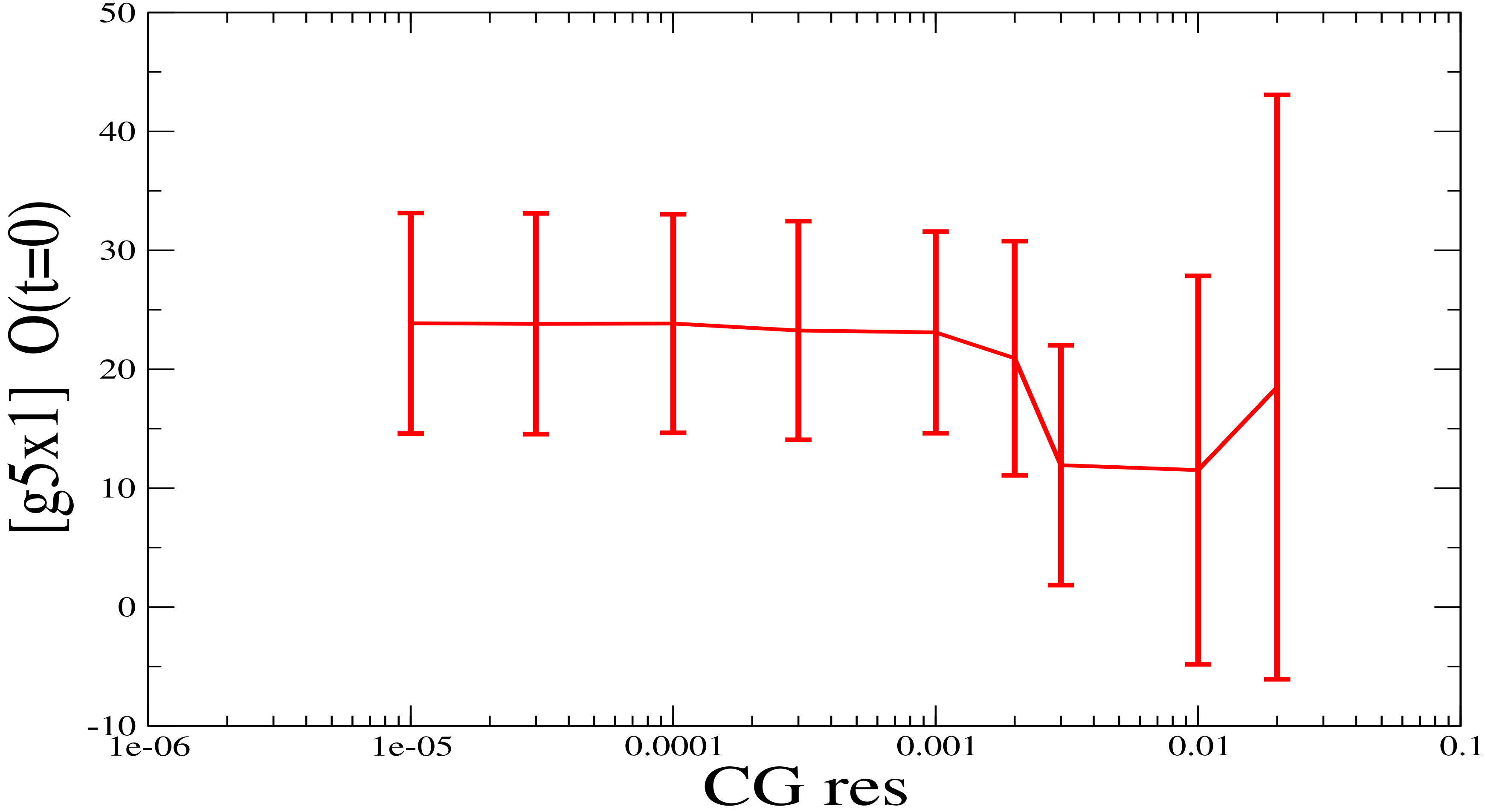
  (summed over timeslice t) where $\Delta_{\gamma 5 \otimes \mathbf{1}}$ effects the shifts and phasing appropriate to the staggered $\gamma 5 \otimes \mathbf{1}$ operator.
- We are currently using $N_s = 40$ stochastic sources per gauge configuration.

Noise dependence of errors



CG residual dependence of PS operators

$\beta = 6.26 \; am = 0.010, 0.050 \; (\gamma_5 \times \mathbf{1})$



Very preliminary results

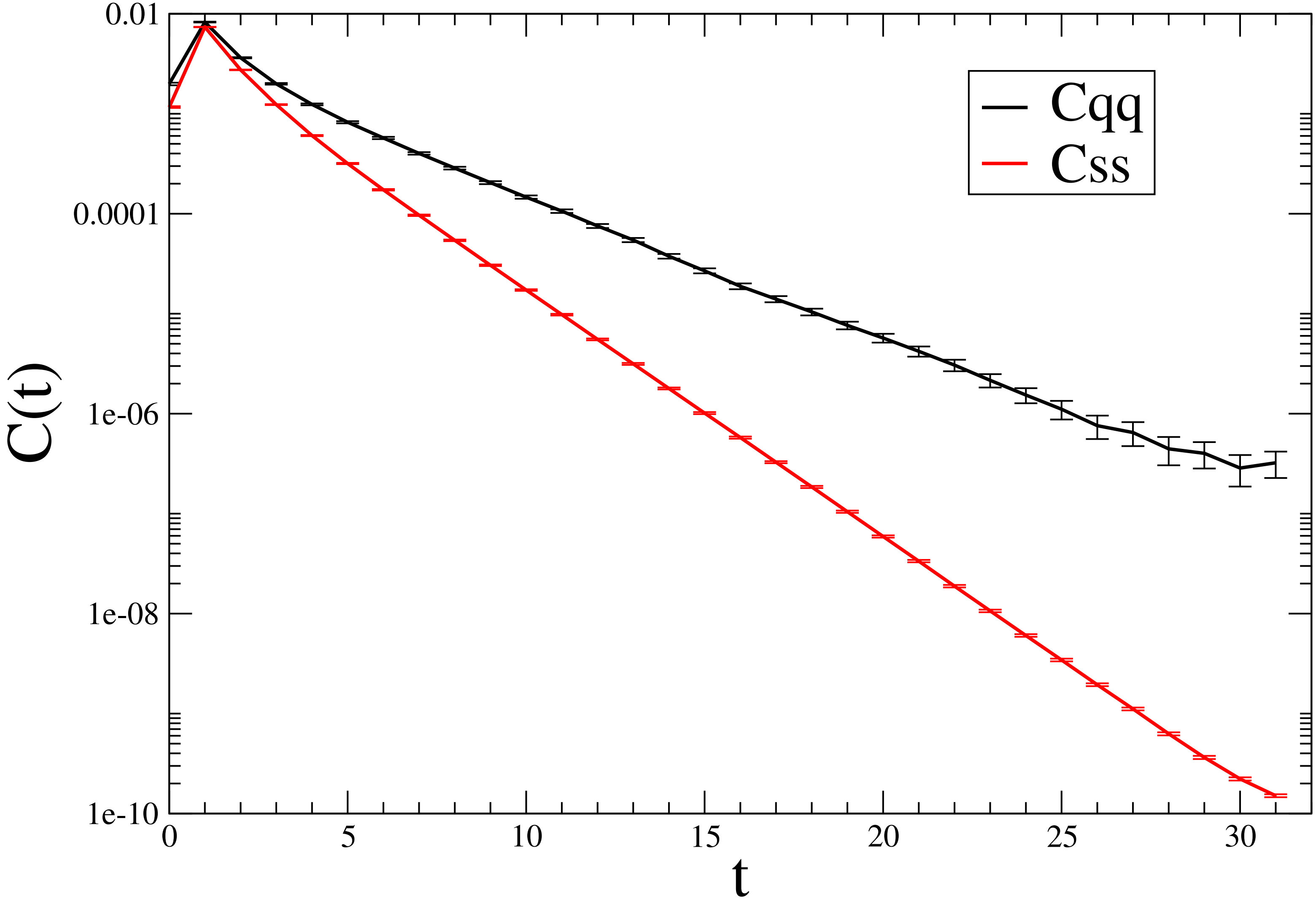
To date have run small test ensembles:

- $N_f = 2$, $\beta = 7.2$ $16^3 \times 32$, $am = 0.02$
- $N_f = 0$, $\beta = 8.0$ $16^3 \times 32$, $am_{\mathrm{val}} = 0.02$

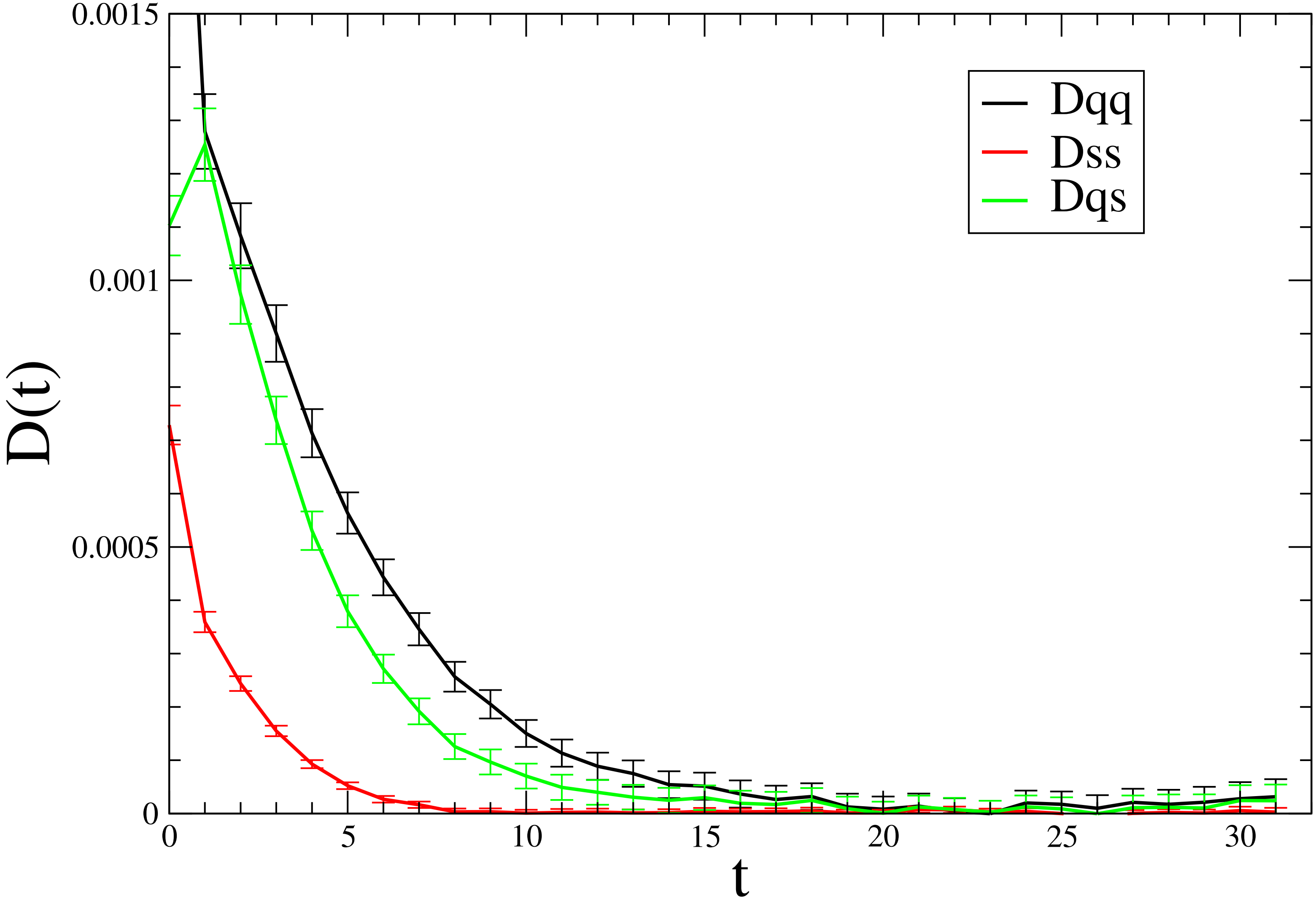
and two larger $N_f = 2 + 1$ MILC ensembles:

- $N_f = 2 + 1$, $\beta = 6.76$ $20^3 \times 64$, $am = 0.007, 0.05$ 422 cfgs
- $N_f = 2 + 1$, $\beta = 6.76$ $20^3 \times 64$, $am = 0.010, 0.05$ 644 cfgs

$\beta = 6.76\ am = 0.007, 0.050$



$\beta = 6.76 \; am = 0.007, 0.050$



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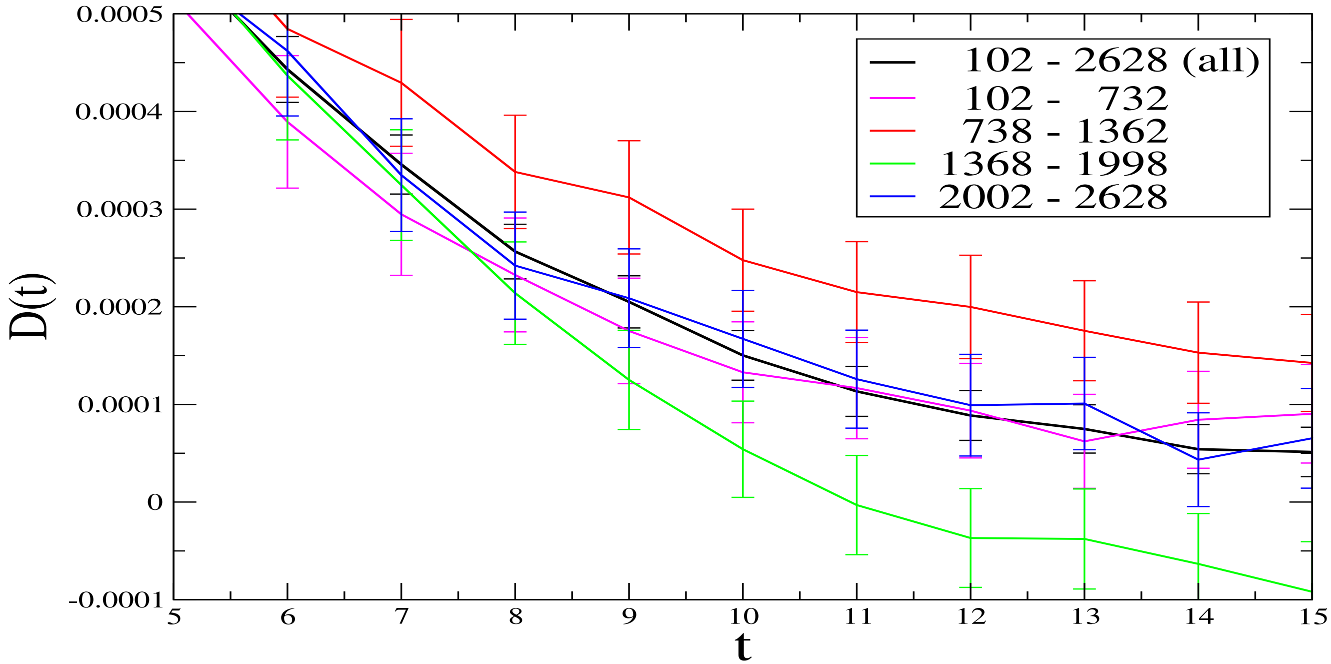
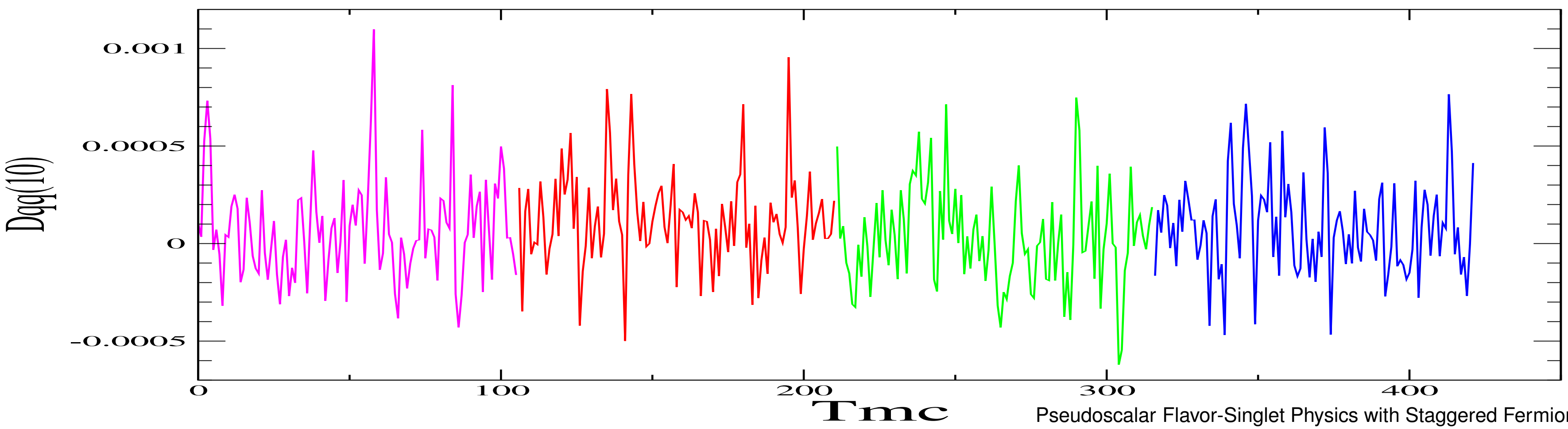
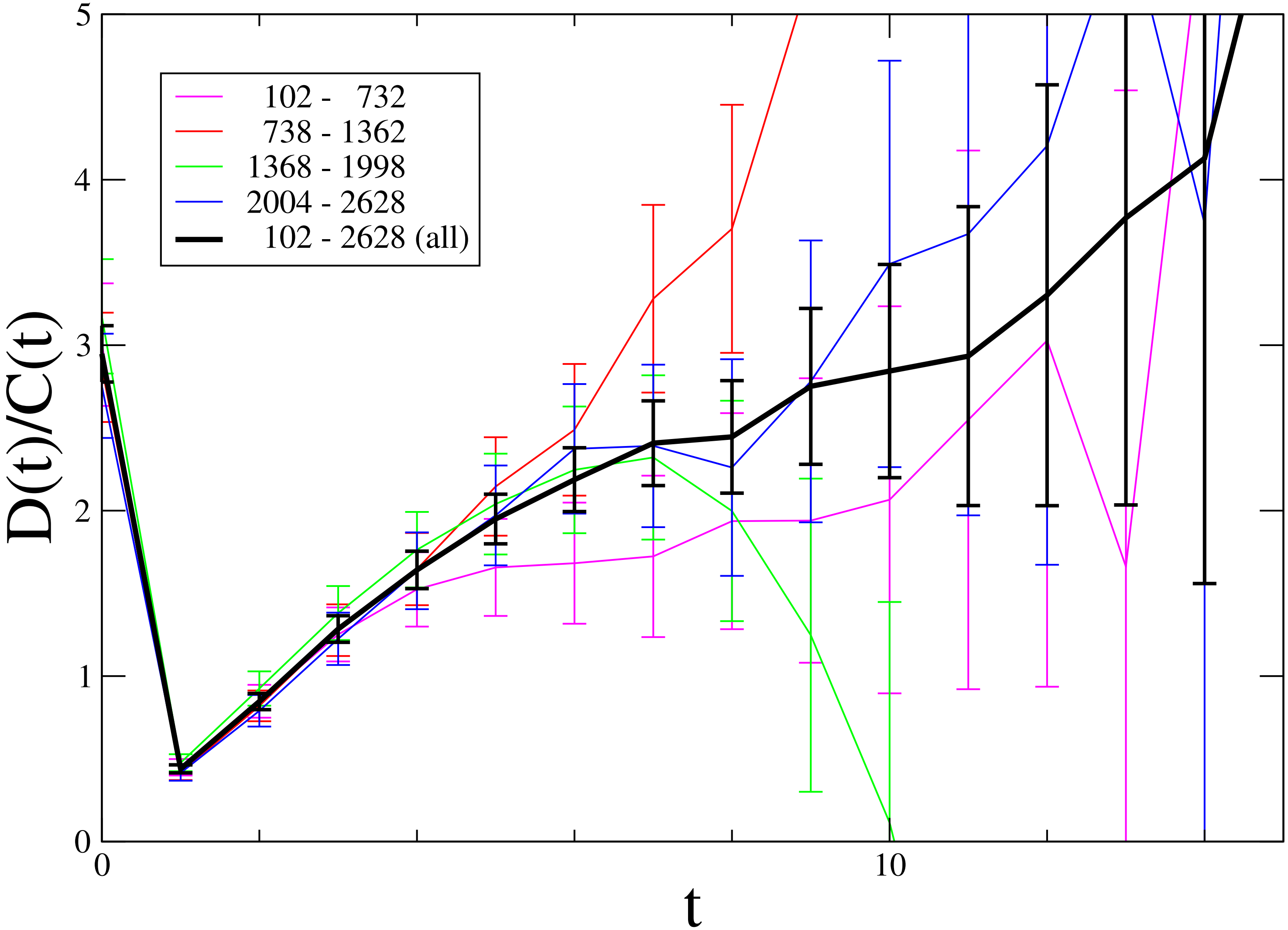




$\beta = 6.76\ am = 0.007, 0.050\ (\gamma_5 \times \mathbf{1})$



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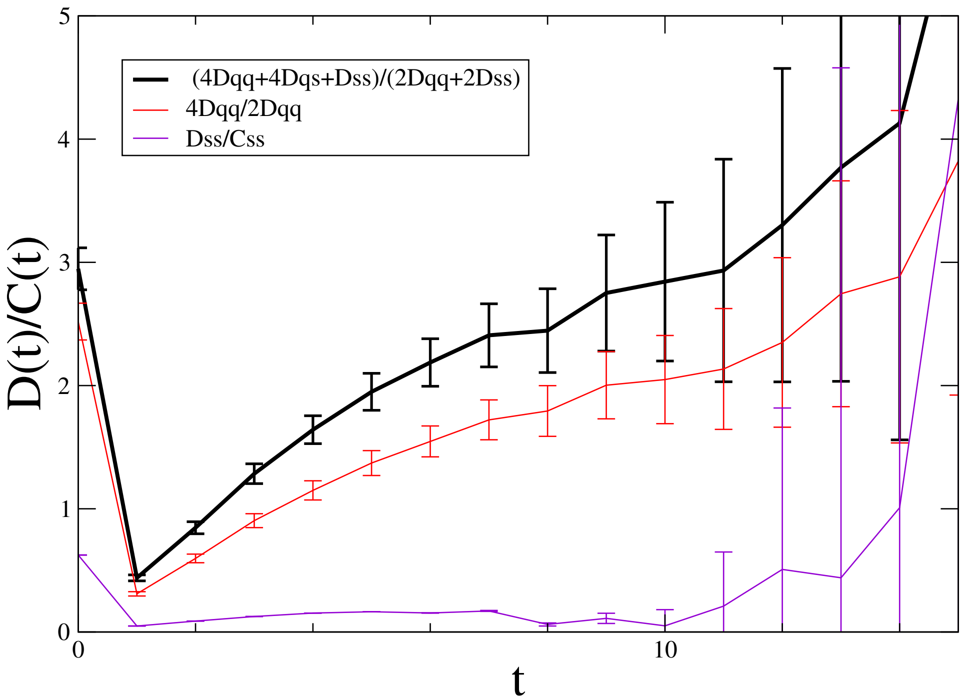
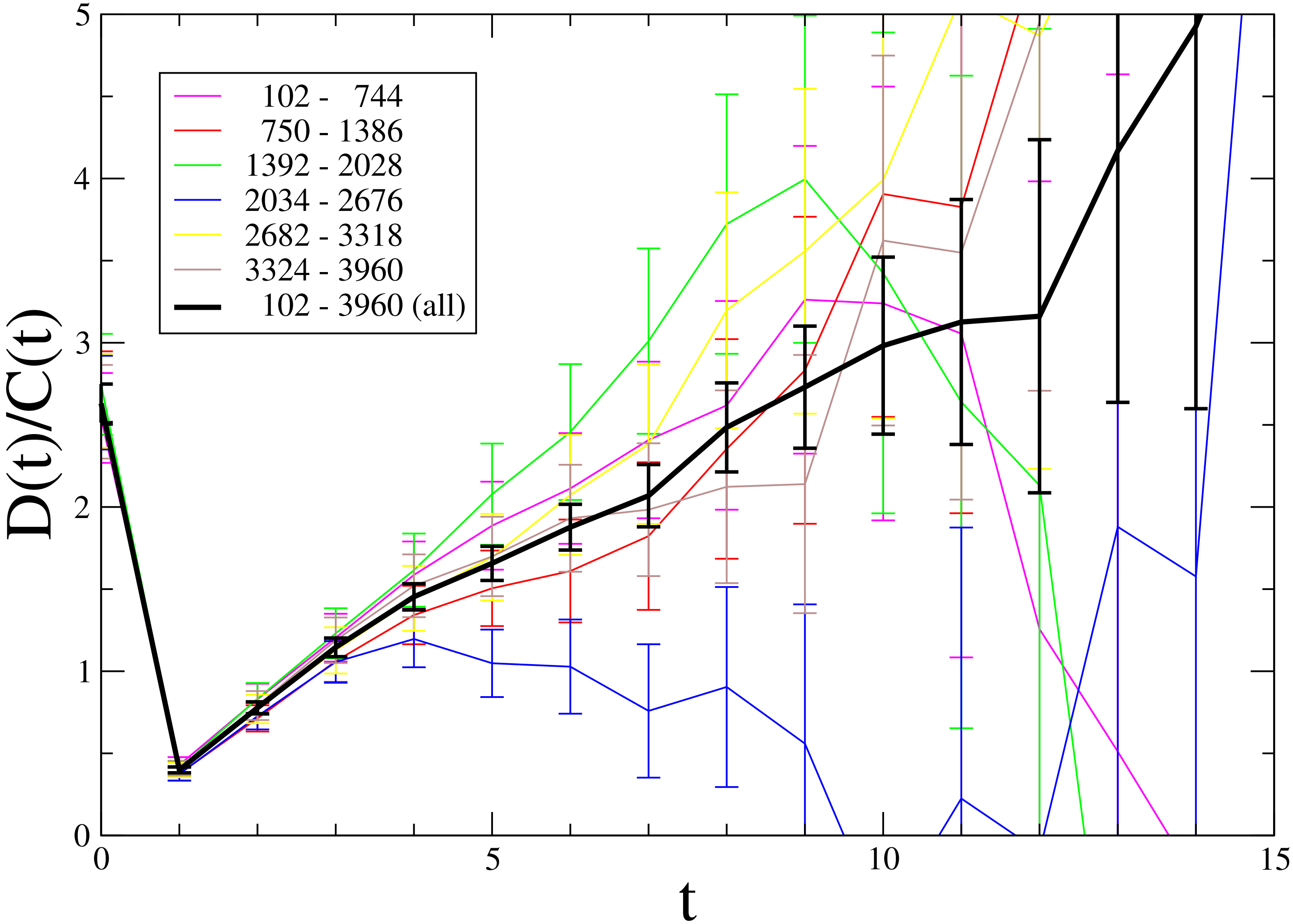


Obviously some normalization issues!

$\beta = 6.76\ am = 0.010, 0.050\ (\gamma_5 \times \mathbf{1})$



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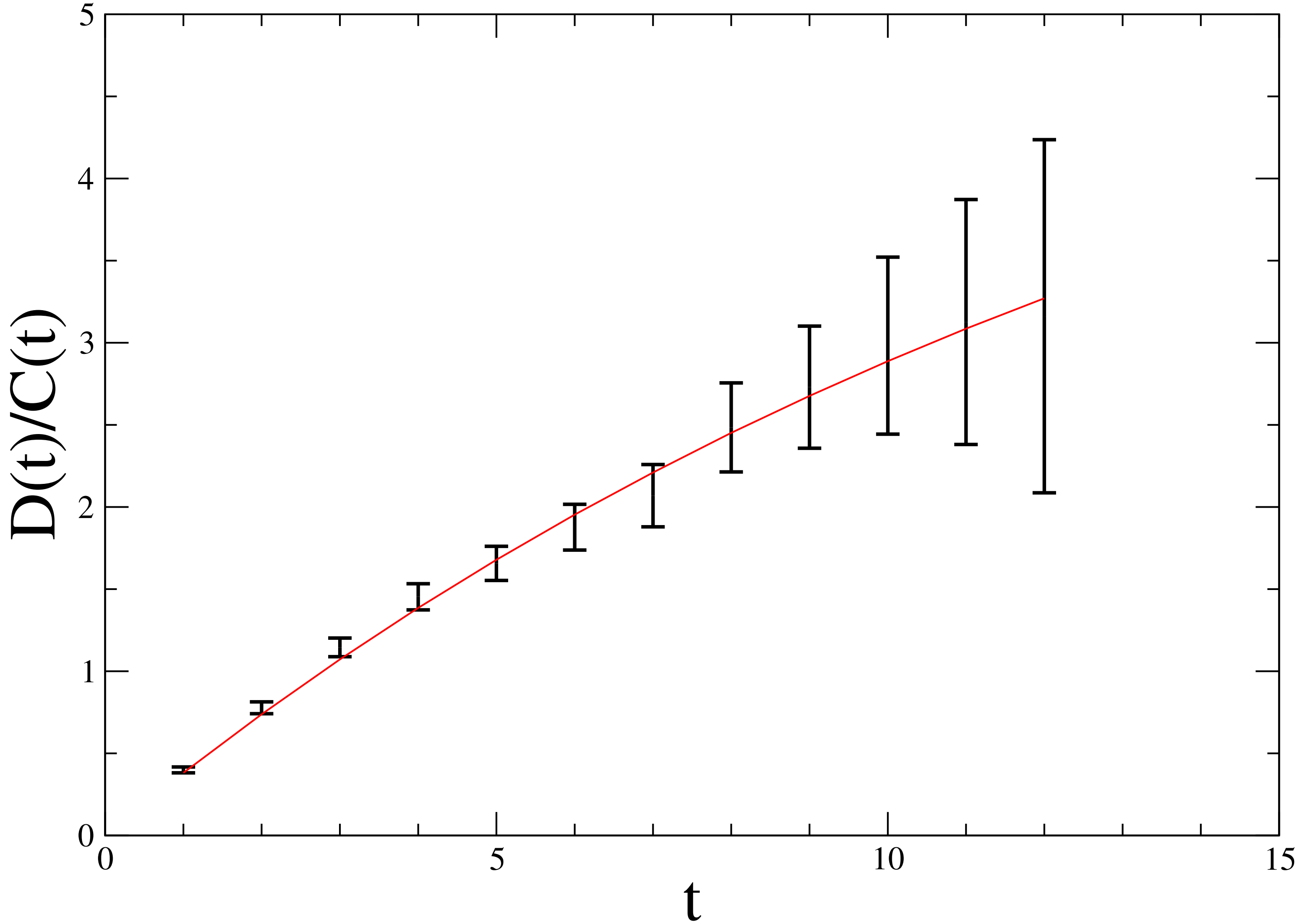
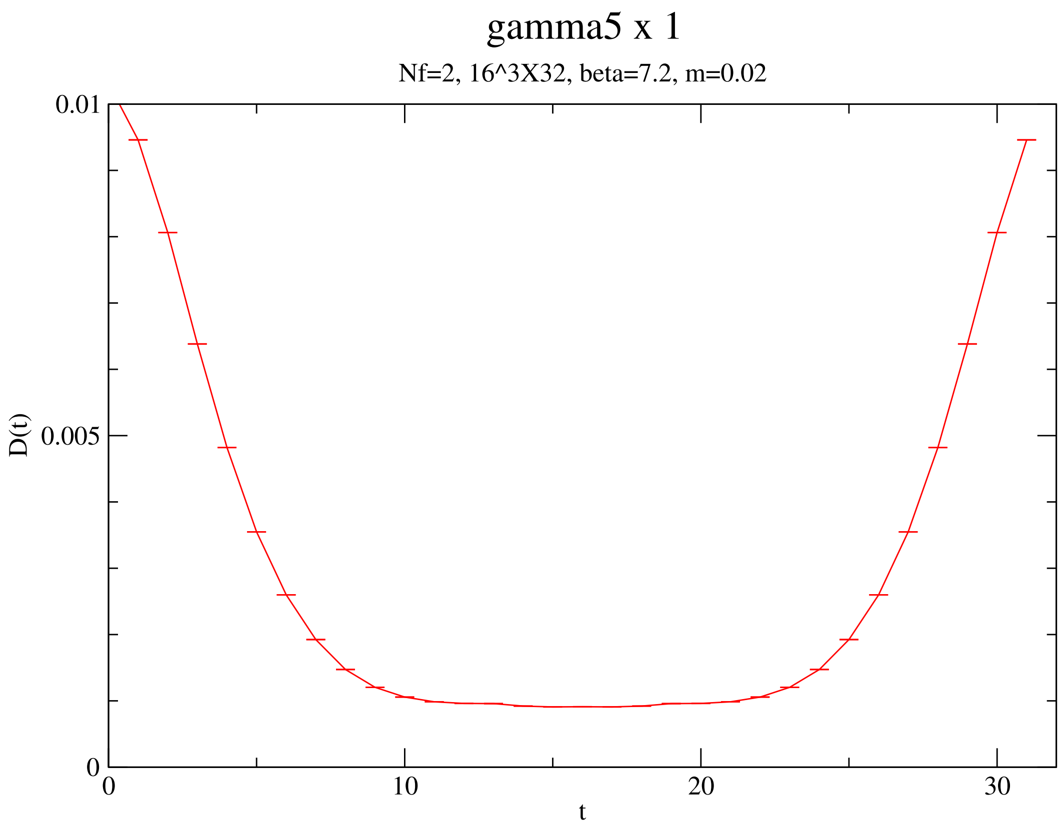


curve: $R(t) = 6\,[1 - \exp(-(0.066)t)]$

$N_f = 2$ test run

Interesting quirk:



$\langle \overline{q}(\gamma_5 \otimes \mathbf{1})q \rangle \approx 0$, but long range correlations.

Stay tuned for results on larger $(20^3 \times 64)$ lattices.

$\eta\ /\eta'$ spectrum

Would expect mixing, so to fit masses diagonalize

$$C(t) = \begin{pmatrix} C_{qq}(t) - 2Dqq(t) & \sqrt{2}D_{qs}(t) \\ \sqrt{2}D_{sq}(t) & C_{ss}(t) - Dss(t) \end{pmatrix}$$

(properly normalized!)

Bag of tricks

- Dilution: Define noise source only on some subset of the lattice (e.g., one timeslice, one color, one hyper-cube corner ....)
- "Kilkup & Venkataraman method"
  Instead of $\mathcal{O} = \langle \phi_x^\dagger \eta_y \rangle$, measure

$$\mathcal{O} = m\langle \phi_x^\dagger \phi_y \rangle = (\underset{eo}{\not{D}} + m)\langle \phi_x^\dagger \phi_y \rangle = \langle \phi_x^\dagger \eta_y \rangle,$$

  where $(\not{D}_{eo} + m)\phi = \eta$. (Good only for 4-link $(\gamma_5 \otimes \mathbf{1})$.
- eigen modes?

Future plans

- Continue measurements on $N_f = 0, 2, 2+1$ lattices
- Understand normalizations
- Optimizations
- Fit for η' mass
- Need long time series ensembles to be produced this fall on QCDOC

a (fm)	m_q/m_s	vol	trajectories
0.125	0.2	$32^3 \times 64$	30000
0.125	0.2	$24^3 \times 96$	20000
0.125	0.2	$48^3 \times 144$	2000