Lattice QCD study of Radiative Transitions in Charmonium

(with a little help from the quark model)

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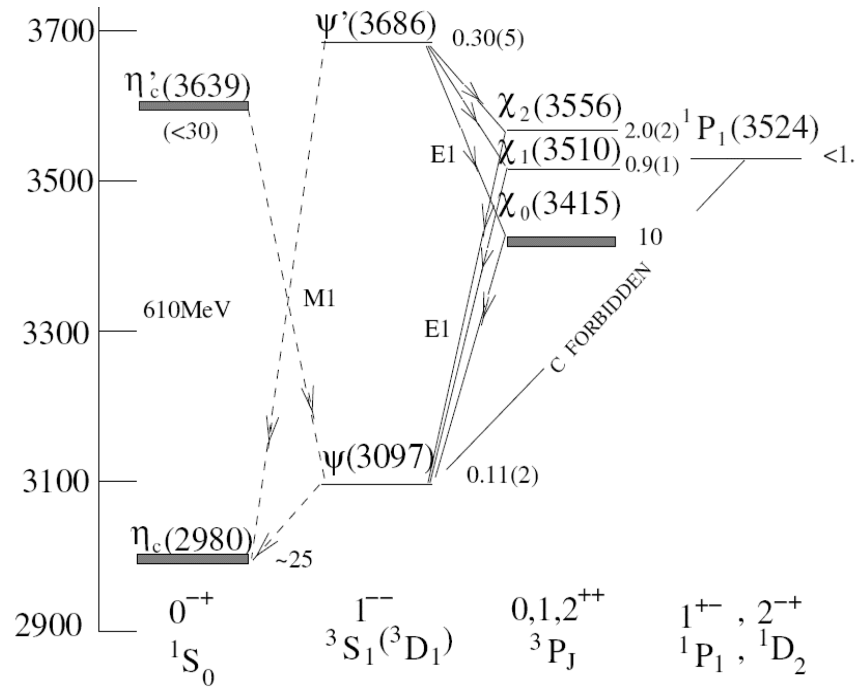
with Robert Edwards & David Richards

Charmonium spectrum & radiative transitions

- most charmonium states below $D\bar{D}$ threshold have measured radiative transitions to a lighter charmonium state

- transitions are typically ***E1*** or ***M1*** multipoles, although transitions involving χ_{c1}, χ_{c2} also admit ***M2*** & ***E3***, whose amplitude is measured through angular distributions



- these transitions are described reasonably well in quark potential models

Eichten, Lane & Quigg PRL.89:162002,2002

Transition	γ energy k (MeV)	Partial width (keV) Computed	Measured[a]
$\psi \xrightarrow{\text{M1}} \eta_c \gamma$	115	1.92	1.13 ± 0.41
$\chi_{c0} \xrightarrow{\text{E1}} J/\psi \gamma$	303	120 (105)[b]	98 ± 43
$\chi_{c1} \xrightarrow{\text{E1}} J/\psi \gamma$	390	242 (215)[b]	240 ± 51
$\chi_{c2} \xrightarrow{\text{E1}} J/\psi \gamma$	429	315 (289)[b]	270 ± 46
$h_c \xrightarrow{\text{E1}} \eta_c \gamma$	504	482	
$\eta'_c \xrightarrow{\text{E1}} h_c \gamma$	126	51	
$\psi' \xrightarrow{\text{E1}} \chi_{c2} \gamma$	128	29 (25)[b]	22 ± 5
$\psi' \xrightarrow{\text{E1}} \chi_{c1} \gamma$	171	41 (31)[b]	24 ± 5
$\psi' \xrightarrow{\text{E1}} \chi_{c0} \gamma$	261	46 (38)[b]	26 ± 5
$\psi' \xrightarrow{\text{M1}} \eta'_c \gamma$	32	0.04	
$\psi' \xrightarrow{\text{M1}} \eta_c \gamma$	638	0.91	0.75 ± 0.25

Simulation details

- anisotropic Wilson gauge action

$$\xi = \frac{a_s}{a_t} = 3.0 \qquad \beta = 6.1$$

$$a_s \sim 0.1\text{fm} \qquad 12^3 \times 48$$

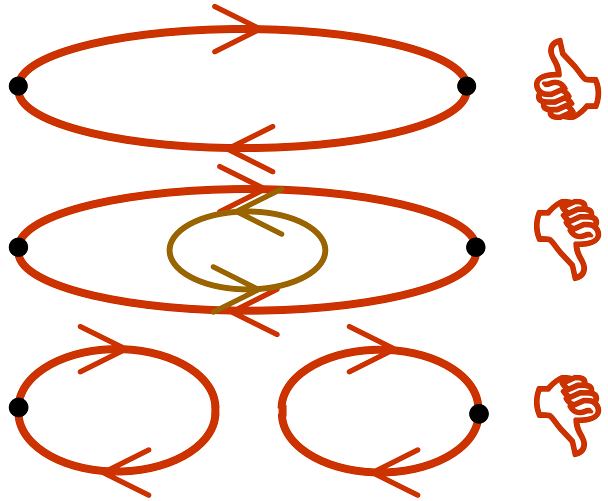
- domain wall fermions for charm quark propagators

$$M = 1.7 \quad L_5 = 16$$

- quenched approximation & no disconnected diagrams

e.g.

two-point

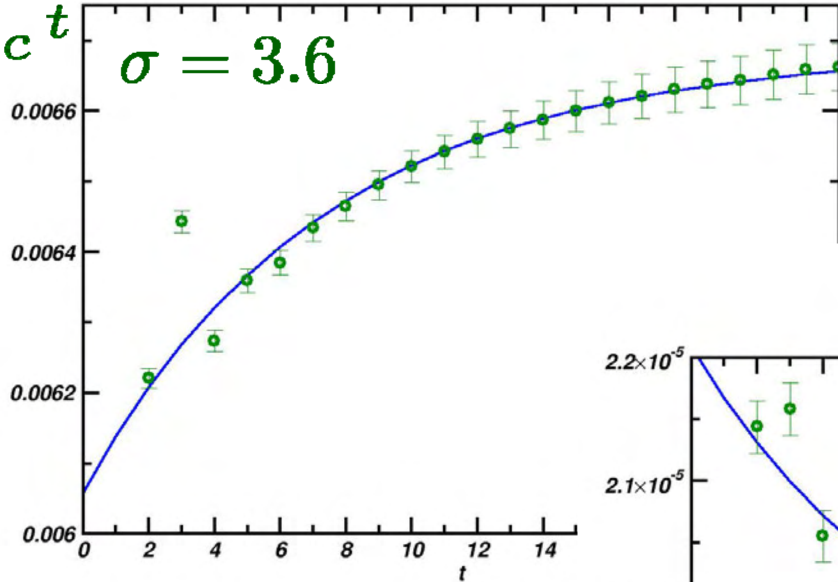


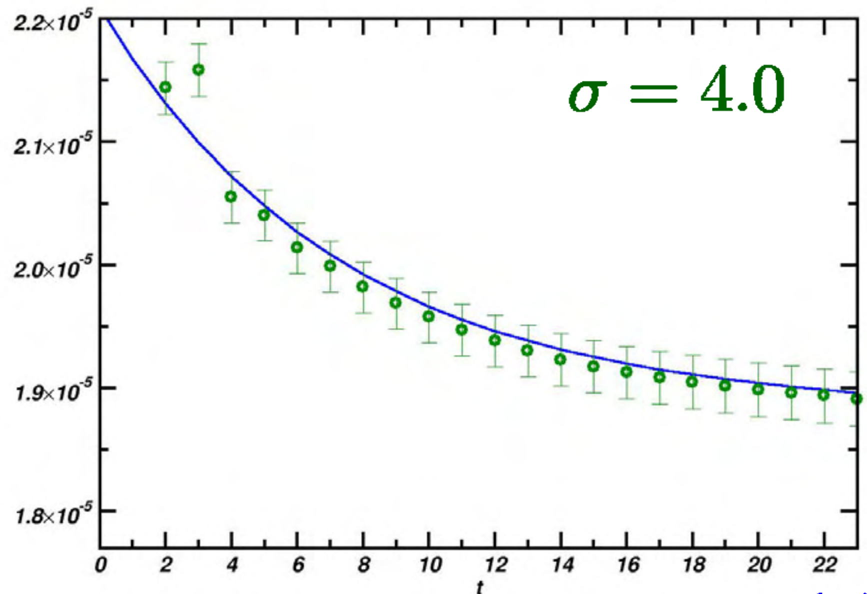
Smearing & two-point fits

$$\frac{C_{\eta_c}(t)}{e^{-m_{\eta_c} t}}$$

smeared – local

psuedoscalar two-point





simultaneous multi-exponential fit
to multiple correlators

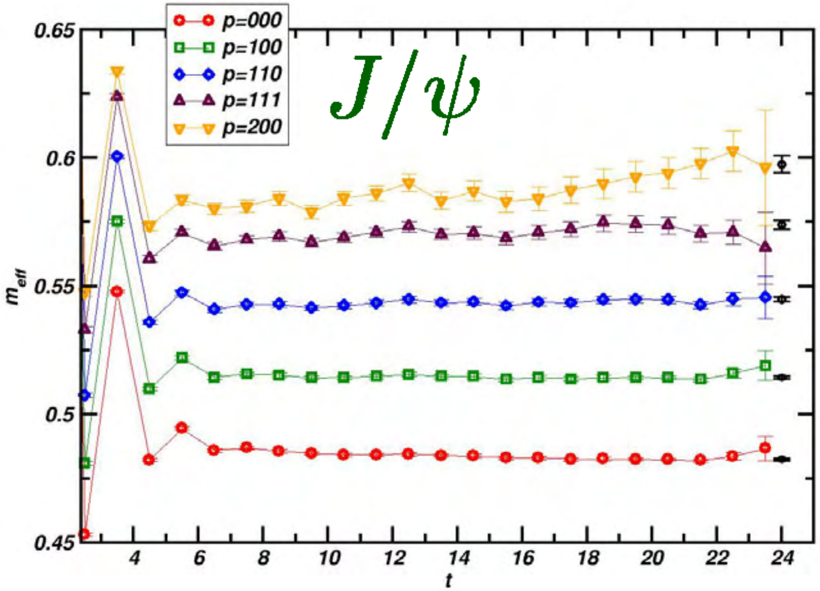
$$a_t m_{\eta_c} = 0.4659(6)$$

$$Z_{\eta_c}(\vec{p}=0) = \langle 0|\varphi_{\eta_c}(0)|\eta_c(\vec{p}=0)\rangle$$

$$Z^{(\sigma=3.6)}_{\eta_c}(\vec{p}=0) = 0.02474(2)$$

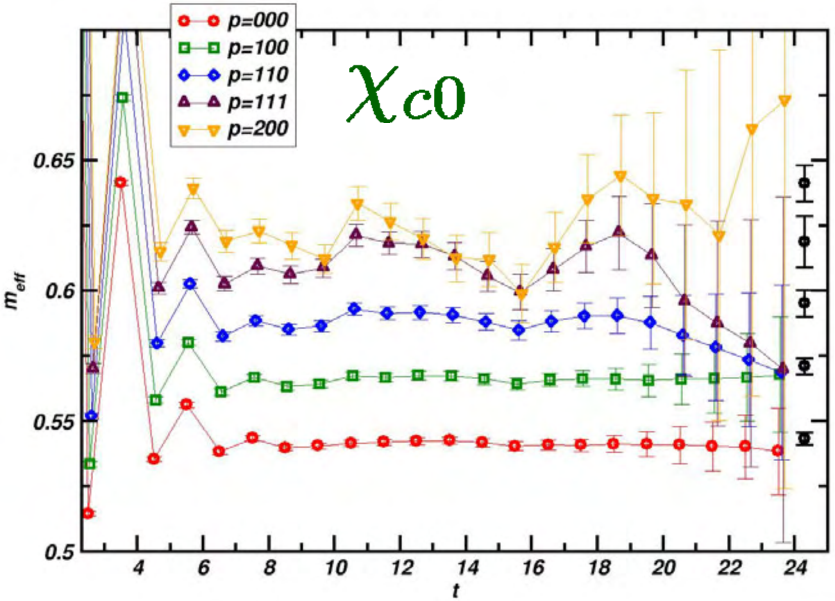
Smeared-local two-point functions

★ Require early-time plateau in two-points for good three-points



$$\sigma = 3.6$$

smeared – local



Scale setting & the spectrum

- Sommer scale and 1P-1S produce compatible lattice spacings

$$\Longrightarrow \; a_t^{-1} = 6.05(1)\mathrm{GeV}$$

$$m_{\eta_c} = 2819(7)\mathrm{MeV}$$

$$m_{\eta_c}(\mathrm{PDG}) = 2980(1)\mathrm{MeV}$$

- we didn't tune our charm quark 'mass'-parameter very accurately – our whole charmonium spectrum is too light

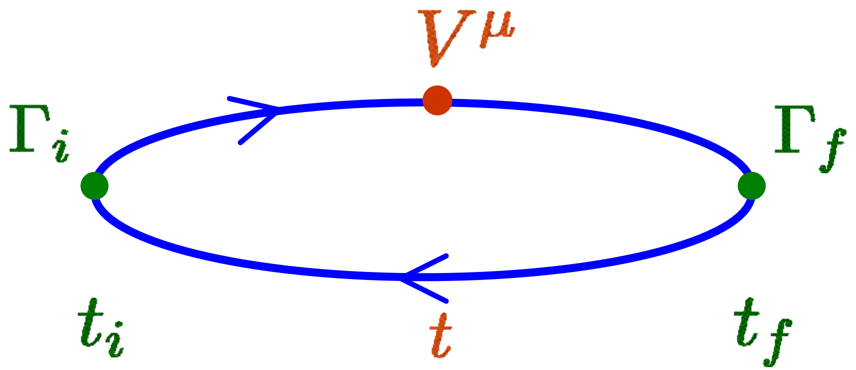
	lat.	PDG	diff.
η_c	2819(7)	2980(1)	161(7)
J/ψ	2917(7)	3097	180(7)
χ_{c0}	3288(15)	3415	127(15)
χ_{c1}	3401(29)	3511	110(29)
h_c	3351(19)	3526	175(19)

$$\overline{\delta m} = 151$$

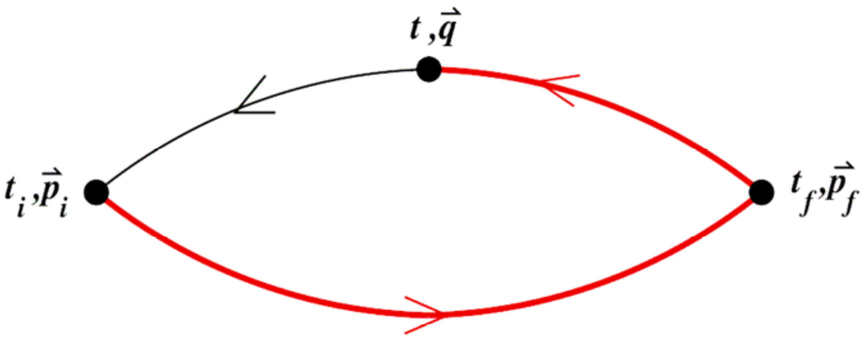
small residual differences from discretisation, quenching & lack of disconnected

Three-point functions

- radiative transitions involve the insertion of a vector current to a quark-line



- computation of this correlator is by the sequential-source method:



red line is one propagator – new calc<u>n</u> for each new $\Gamma_f, \vec{p}_f$

- used local vector current (hence not-conserved) $\bar{\psi}_x \gamma^\mu \psi_x$

  gets renormalised multiplicatively $\langle f|V^\mu|i\rangle_{\text{cont.}} = Z_V(a)\langle f|V^\mu|i\rangle_{\text{lat.}(a)}$

'Fit' method for three-point functions

$$\langle \varphi^\dagger_{\eta_c}(t_f)\, V^\mu(t)\, \varphi_{\eta_c}(t_i)\rangle$$
$$= \sum_{N,M} \frac{1}{4E_N E_M} \langle 0|\varphi^\dagger_{\eta_c}(t_f)|N\rangle \langle N|V^\mu(t)|M\rangle \langle M|\varphi_{\eta_c}(t_i)|0\rangle$$
$$\to \frac{e^{-(t_f-t)E_f}\, e^{-(t-t_i)E_i}}{4E_i E_f} Z^*_{\eta_c}(p_f) Z_{\eta_c}(p_i) \langle \eta_c(p_f)|V^\mu(0)|\eta_c(p_i)\rangle$$

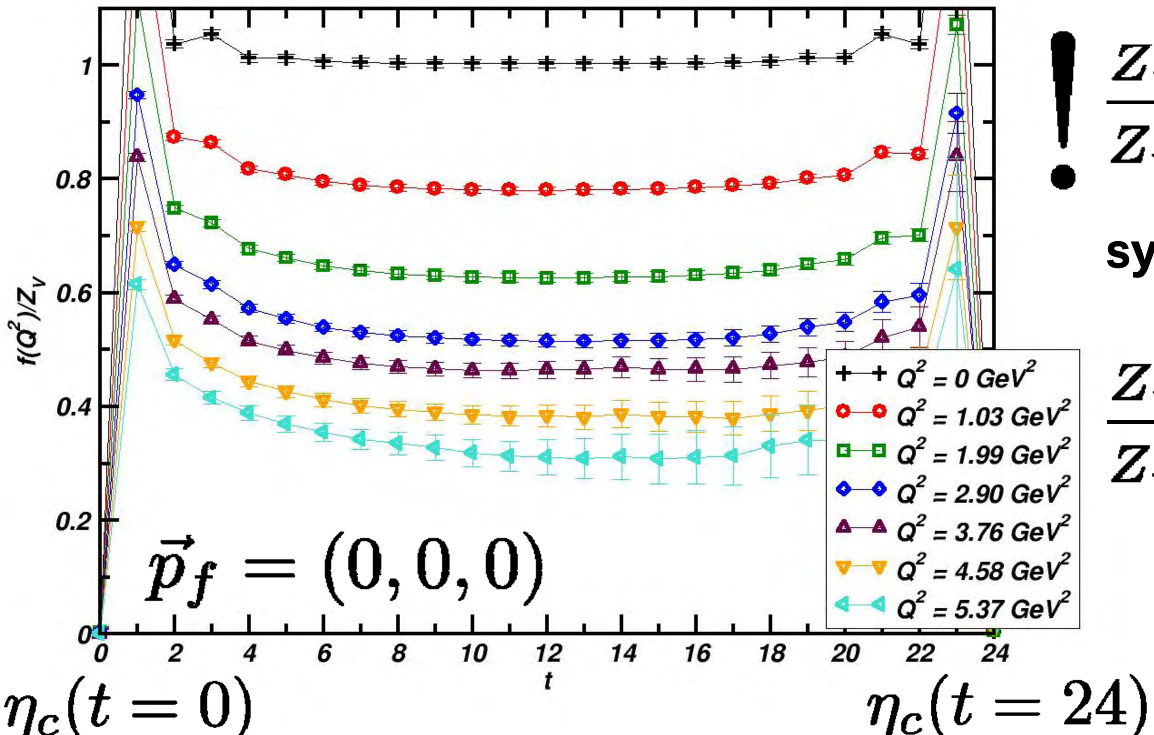
- $Z(p), E(p)$ are extracted from the two-point function fits
- $\langle \eta_c(p_f)|V^\mu(0)|\eta_c(p_i)\rangle$ is what we intend to extract
- alternative 'ratio' method is less simple to implement when we have multiple form-factors (multipoles)

η_c form-factor & setting Z_V

★ true η_c has no electromagnetic form-factor by charge conjugation invariance

- at quark level by coupling to $c \,\&\, \bar{c}$ - non-zero if just couple to c

$$\langle \eta_c(\vec{p}_f) \mid V^\mu(0) \mid \eta_c(\vec{p}_i)\rangle = f(Q^2)(p_f^\mu + p_i^\mu)$$



! $$\frac{Z_V^{(\pi)}(\vec{p}_f=(0,0,0))}{Z_V^{(\pi)}(\vec{p}_f=(1,0,0))} = 1.11(2)$$

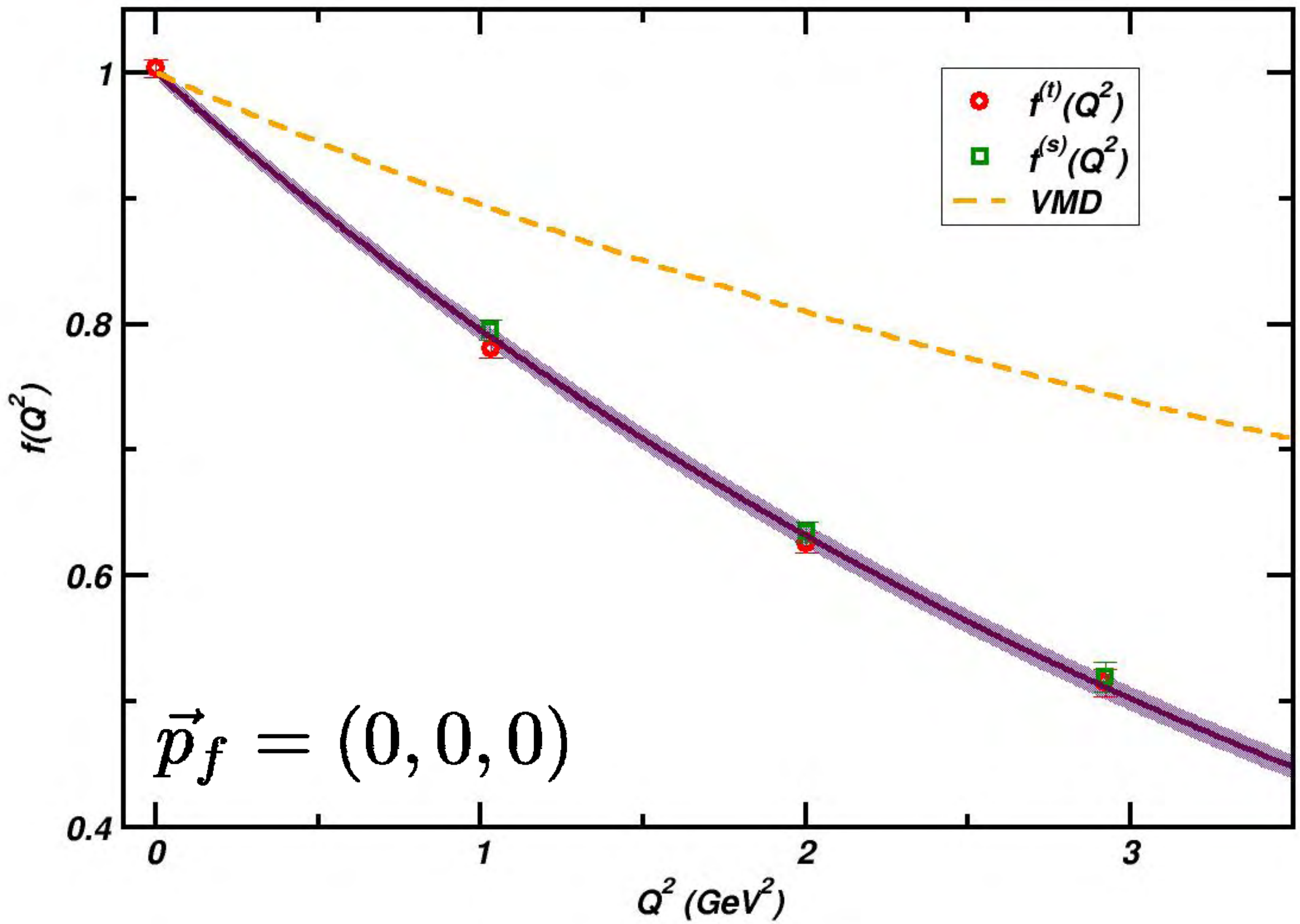
systematic problem

$$\frac{Z_V^{(\rho)}(\vec{p}_f=(0,0,0))}{Z_V^{(\rho)}(\vec{p}_f=(1,0,0))} = 1.03(2)$$

★ extract $f(Q^2)$ from the plateaux

η_c form-factor

- plot as a function of Q^2



$\vec{p}_f = (0,0,0)$

$$\left(1+\frac{Q^2}{m_\psi^2}\right)^{-1}$$

$$e^{-Q^2/16\beta^2}$$

$$\beta = 0.522(5)\mathrm{GeV}$$

$J/\psi \to \gamma\eta_c$ transition

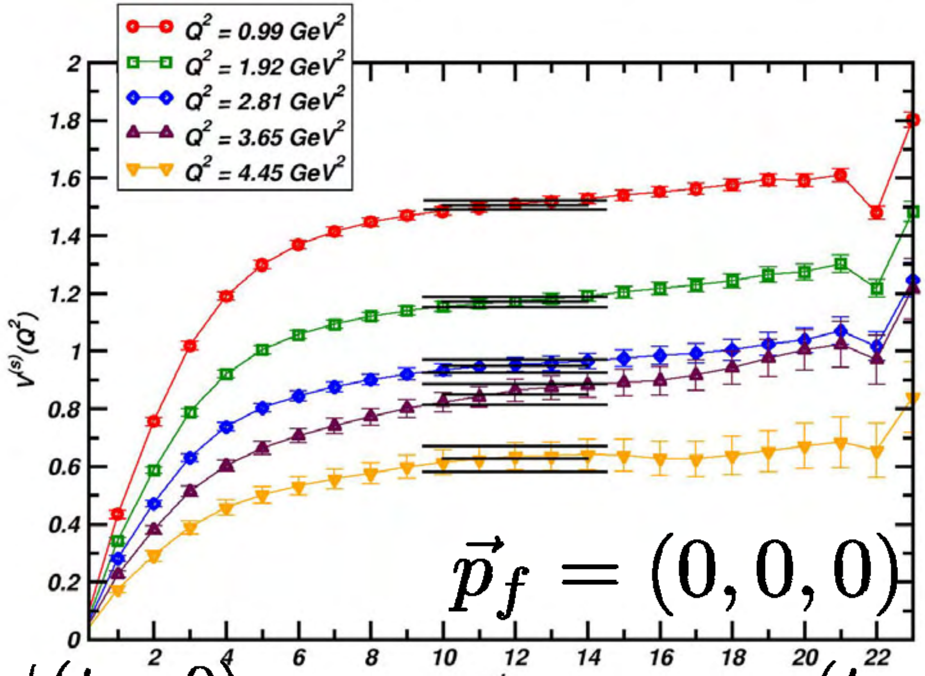
- transitions between different states are no more difficult

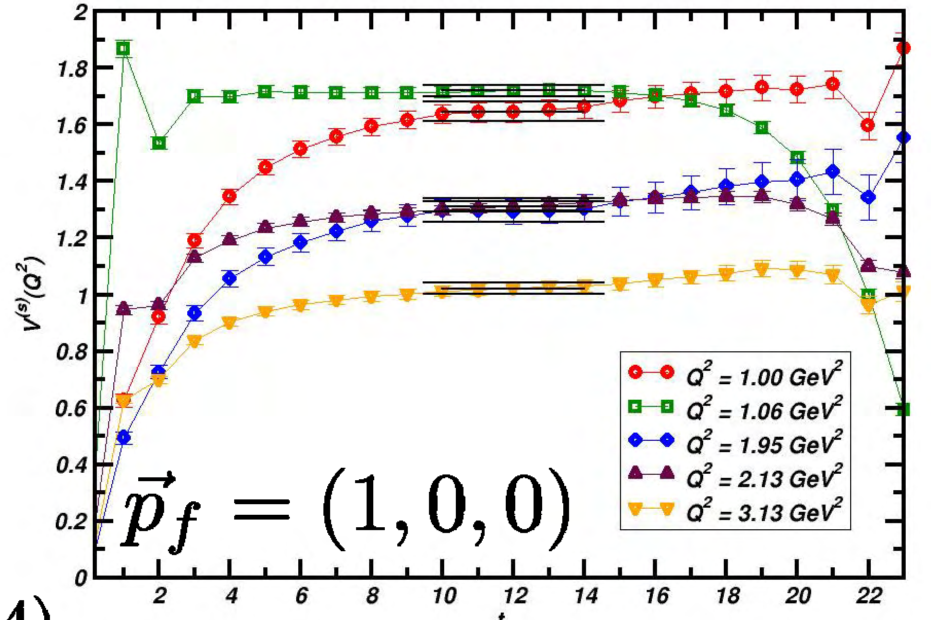
$$\langle \varphi^\dagger_{\eta_c}(t_f)\, V^\mu(t)\, \Omega^\nu_\psi(t_i)\rangle$$

J/ψ has a polarisation

- decomposition in terms of one form-factor

$$\langle \eta_c(\vec{p}_f) \mid V^\mu(0) \mid \psi(\vec{p}_i, r)\rangle = \frac{2V(Q^2)}{m_{\eta_c} + m_\psi}\epsilon^{\mu\alpha\beta\gamma} p_f^\alpha p_i^\beta \epsilon^\gamma(\vec{p}_i, r)$$

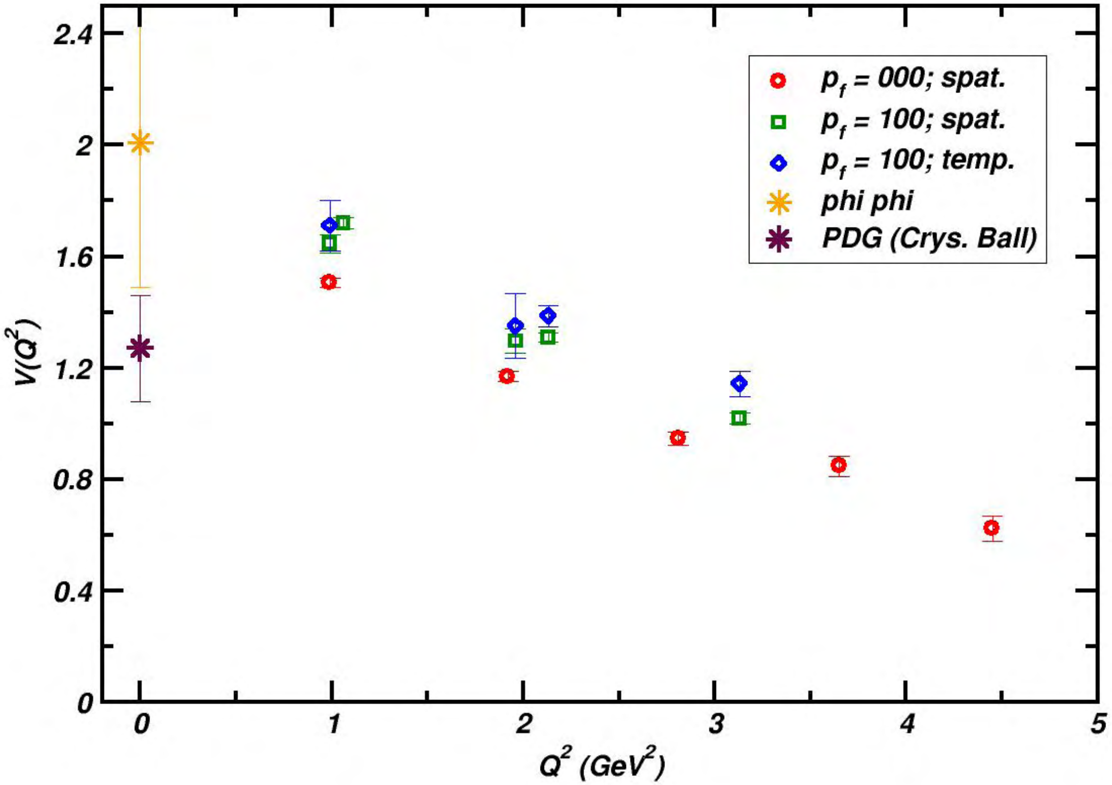




$\psi(t=0)$ $\eta_c(t=24)$

smeared-smeared $\sigma = 3.6$

$J/\psi \to \gamma\eta_c$ transition



★ radiative width: $\Gamma(J/\psi \to \eta_c\gamma) = \alpha \frac{|\vec{q}|^3}{(m_\psi + m_{\eta_c})^2} \frac{4}{3} |V(Q^2=0)|^2$

$|V(Q^2=0)|_{\mathrm{PDG}} = 1.27(19)$ Note that this is just one Crystal Ball measurement

I cooked up an alternative from $\mathcal{B}(J/\psi \to \eta_c\gamma) = \frac{\mathcal{B}(J/\psi \to \eta_c\gamma \to \phi\phi\gamma)}{\mathcal{B}(\eta_c \to \phi\phi)}$

$\Gamma(J/\psi \to \eta_c\gamma) = 2.9(1.5)\mathrm{keV}$ $|V(Q^2=0)|_{\phi\phi} = 2.01(52)$

$J/\psi \to \gamma\eta_c$ transition

- how do we extrapolate back to $Q^2 = 0$?

  - take advice from the non-rel quark-model:

  - this is an *M1* transition which proceeds by quark spin-flip

$$\vec{V} \sim \frac{|\vec{q}|}{m_c}\vec{\sigma}\, j_0(|\vec{q}|r)$$

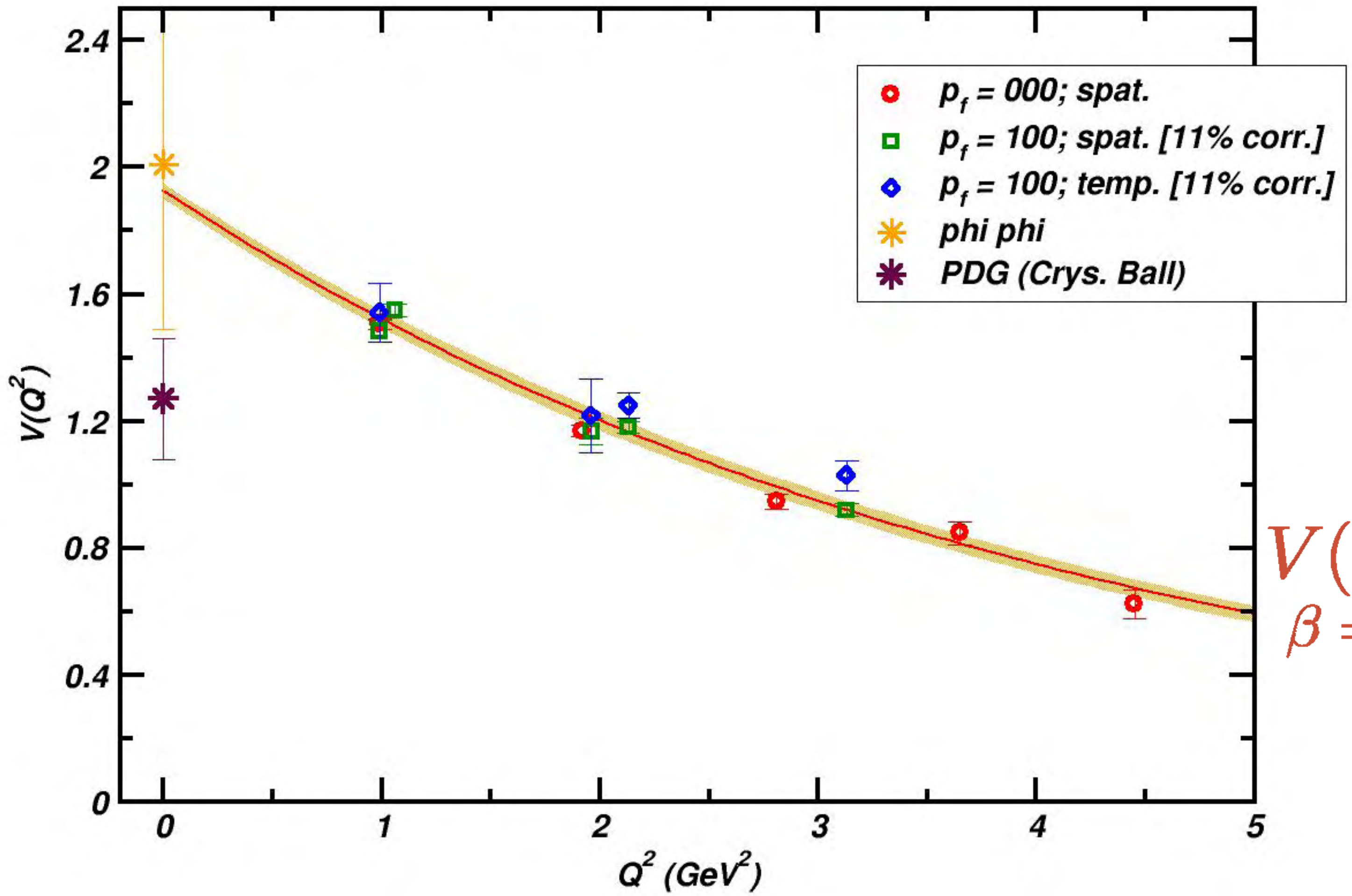
convoluting with Gaussian wavefunctions one obtains

$$V(Q^2) = V(0)e^{-Q^2/16\beta^2}$$

  - we can fit our lattice points with this form to obtain $V(0)$ and β

$J/\psi \rightarrow \gamma\eta_c$ transition

we have a clue about the systematic difference: scaling of $\vec{p}_f = (1,0,0)$ by 1.11



$V(0) = 1.93(2)$

$\beta = 0.515(7)\text{GeV}$

$\chi_{c0} \to \gamma J/\psi$ transition

- at $Q^2 = 0$, there are only transverse photons and this transition has only one multipole – *E1*

  with non-zero Q^2 , longitudinal photons provide access to a second: *C1*

- multipole decomposition:

$$\langle \chi_{c0}(\vec{p}_S) | V^\mu(0) | \psi(\vec{p}_V, \lambda) \rangle =$$

$$E_1(Q^2) \left[\epsilon^\mu(\vec{p}_V, \lambda) - \frac{\epsilon . p_S}{(p_V . p_S)^2 - m_V^2 m_S^2} m_V^2 p_{S\alpha} \mathcal{P}^{\mu\alpha}(p_V) \right] +$$

$$\frac{C_1(Q^2)}{\sqrt{Q^2}} \frac{m_V \epsilon . p_S}{(p_V . p_S)^2 - m_V^2 m_S^2} \left[m_V^2 p_{S\alpha} \mathcal{P}^{\mu\alpha}(p_V) + m_S^2 p_{V\alpha} \mathcal{P}^{\mu\alpha}(p_S) \right]$$

$\chi_{c0} \to \gamma J/\psi$ transition

- three-point function

$$\Gamma^{\mu\nu}(t_f, t, t_i; \vec{p}_f, \vec{p}_i) = \langle \phi^\dagger_{\chi_{c0}}(t_f, \vec{p}_f)\, V^\mu(t)\, \Omega^\nu_\psi(t_i, \vec{p}_i)\rangle$$

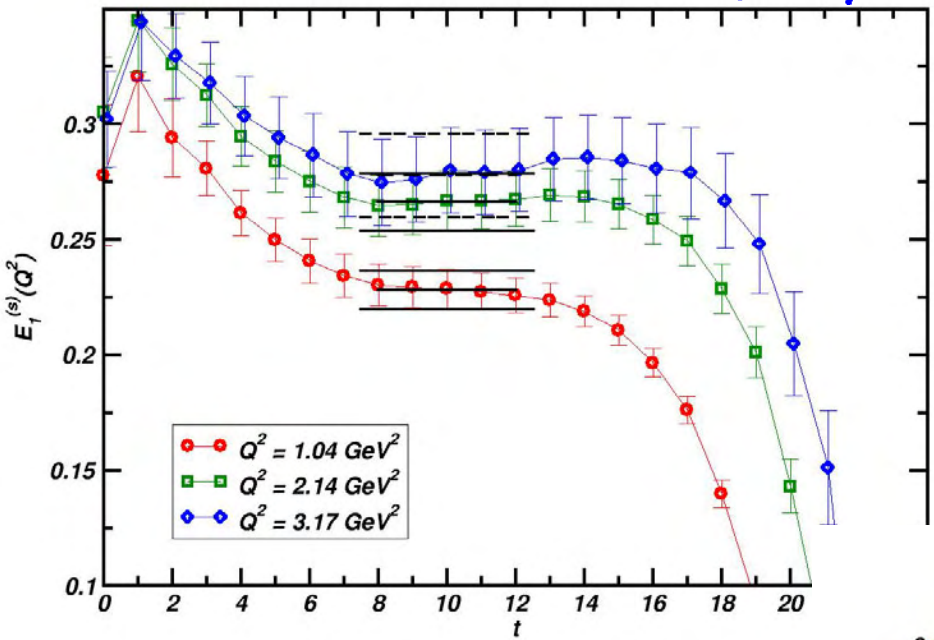
- several different $\vec{p}_f, \vec{p}_i, \mu, \nu$ combinations have the same Q^2 value

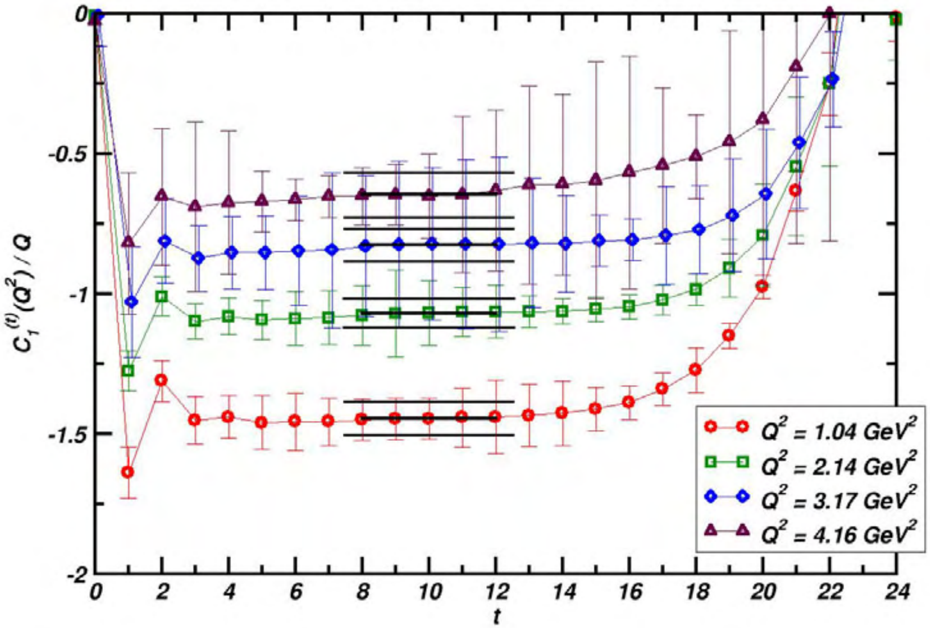
$$\begin{pmatrix} \Gamma^{11} \\ \Gamma^{12} \\ \Gamma^{13} \\ \vdots \end{pmatrix} = \begin{pmatrix} K_E^{11} & K_C^{11} \\ K_E^{12} & K_C^{12} \\ K_E^{13} & K_C^{13} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} E_1(Q^2) \\ C_1(Q^2) \end{pmatrix}$$

$K^{\mu\nu}_{E,C}$ are known functions

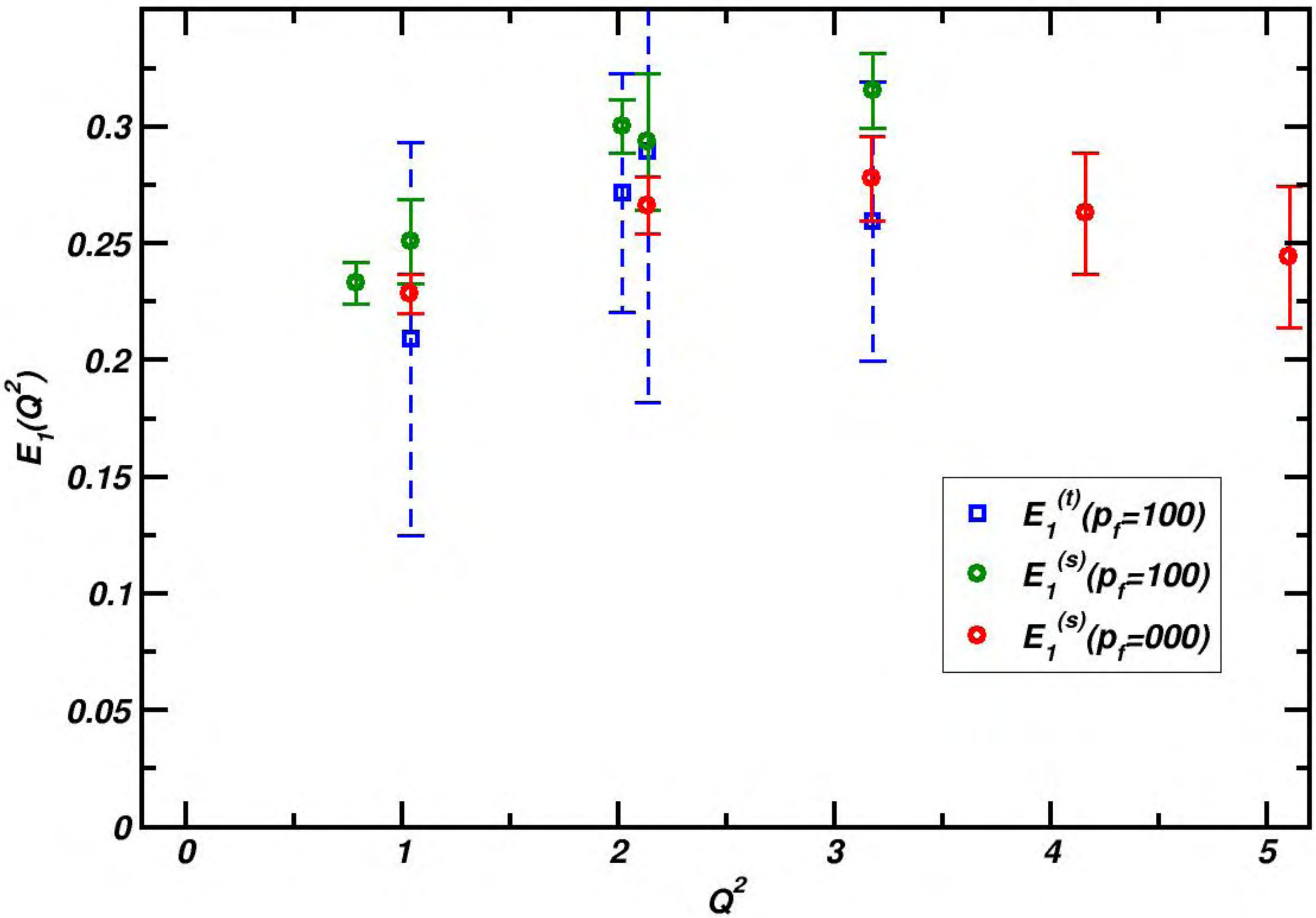
- do the inversion to obtain the multipole form-factors

$\chi_{c0} \to \gamma J/\psi$ transition





$\chi_{c0} \to \gamma J/\psi$ transition



$\chi_{c0} \to \gamma J/\psi$ transition

- we'll again constrain our $Q^2 \to 0$ extrapolation using a quark model form

- this is an *E1* transition which proceeds by the electric-dipole moment

$$\vec{V} \sim \vec{r}\, j_1(|\vec{q}|r)$$

convoluting with Gaussian wavefunctions one obtains

$$E_1(Q^2) = c|\vec{q}(Q^2)|\, e^{-Q^2/16\beta^2}$$

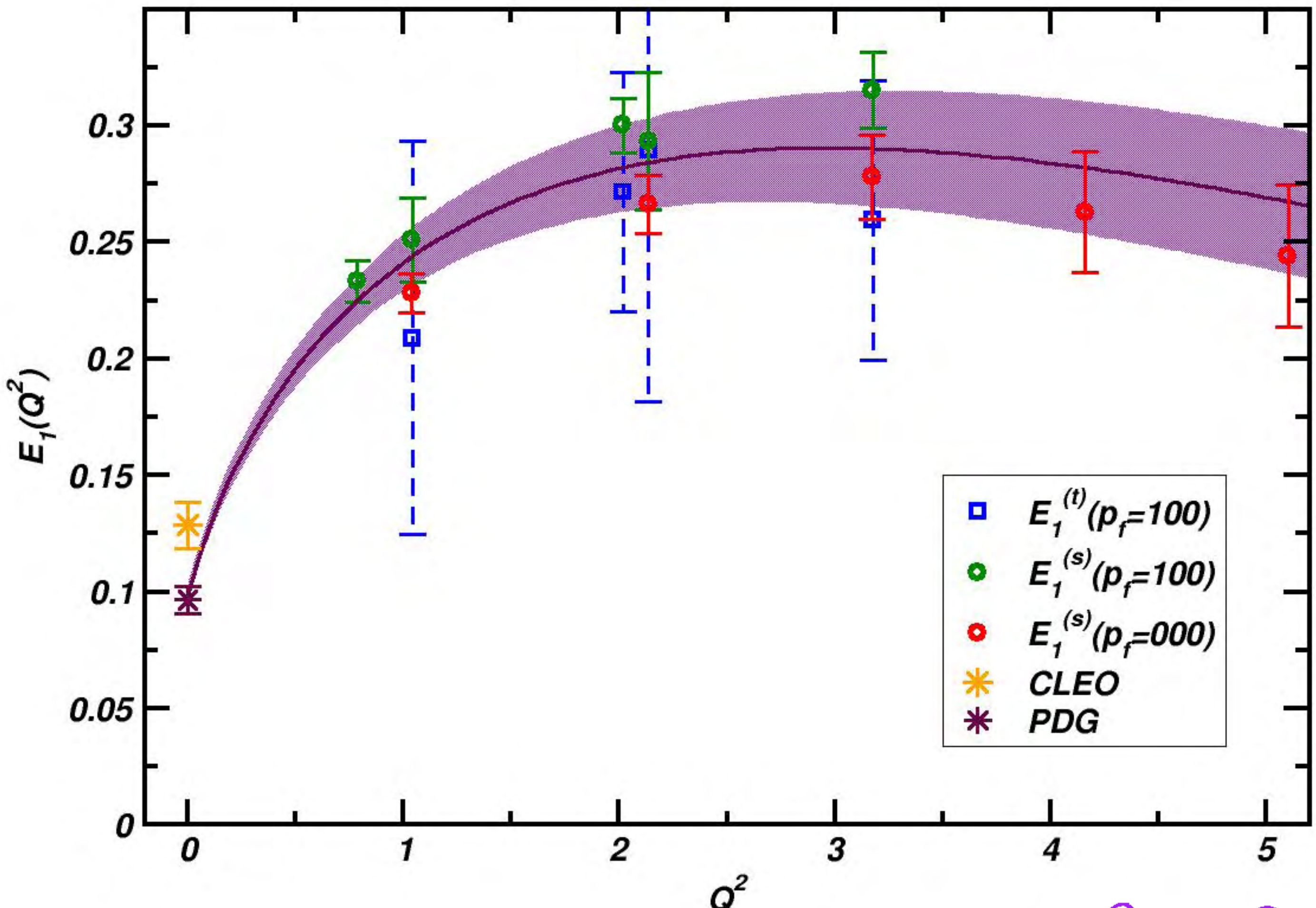
where the photon 3-momentum at virtuality Q^2 is given by

$$|\,\vec{q}(Q^2)\,|^2 = \frac{(m_\psi^2 - m_\chi^2)^2 + 2Q^2(m_\psi^2 + m_\chi^2) + Q^4}{4m_\chi^2}$$

in the rest frame of a decaying χ

$\chi_{c0} \to \gamma J/\psi$ transition

$$E_1(Q^2) = c|\vec{q}(Q^2)|\, e^{-Q^2/16\beta^2}$$

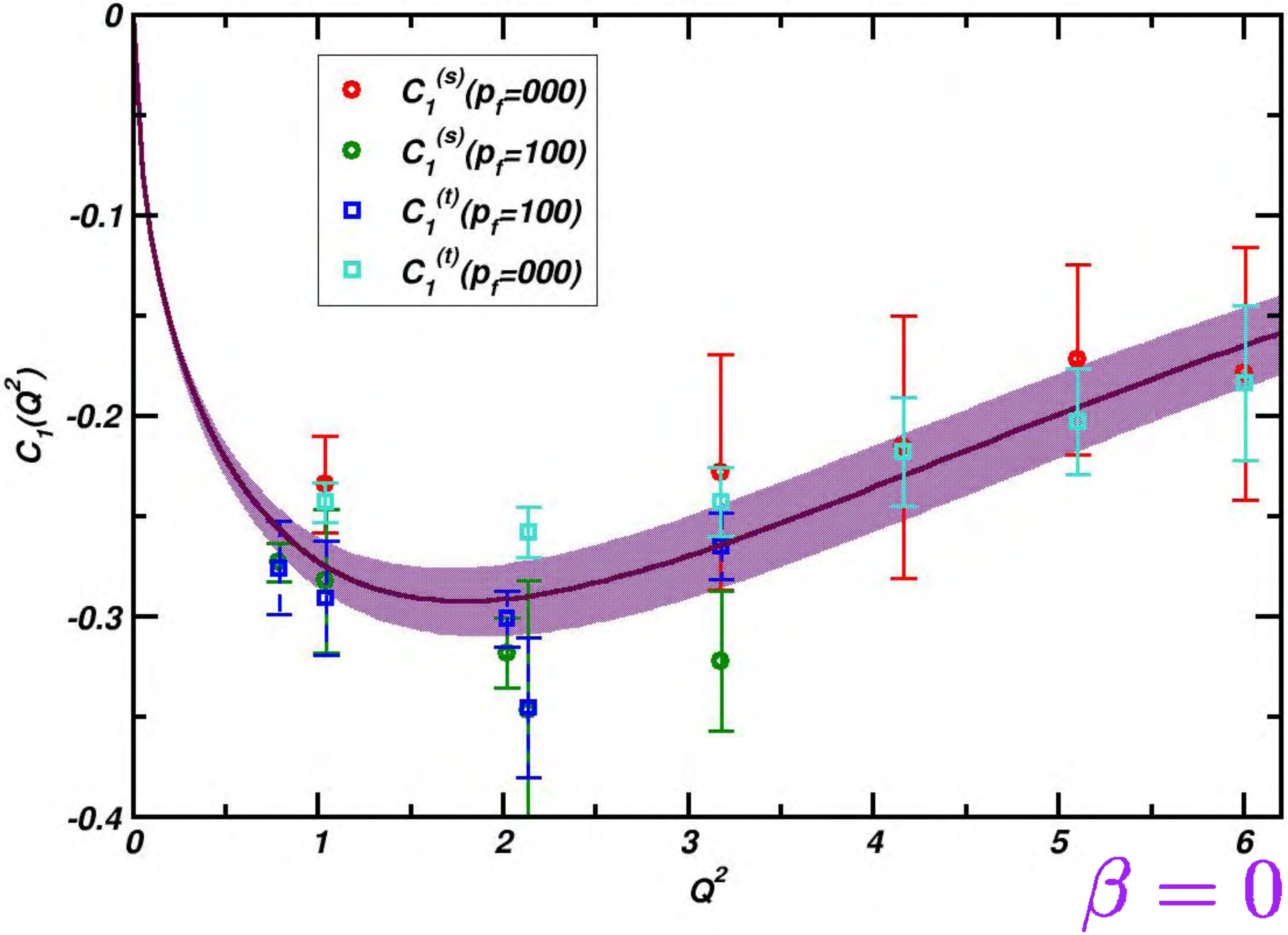


$$\beta = 0.601(28)\mathrm{GeV}$$

$\chi_{c0} \to \gamma J/\psi$ transition

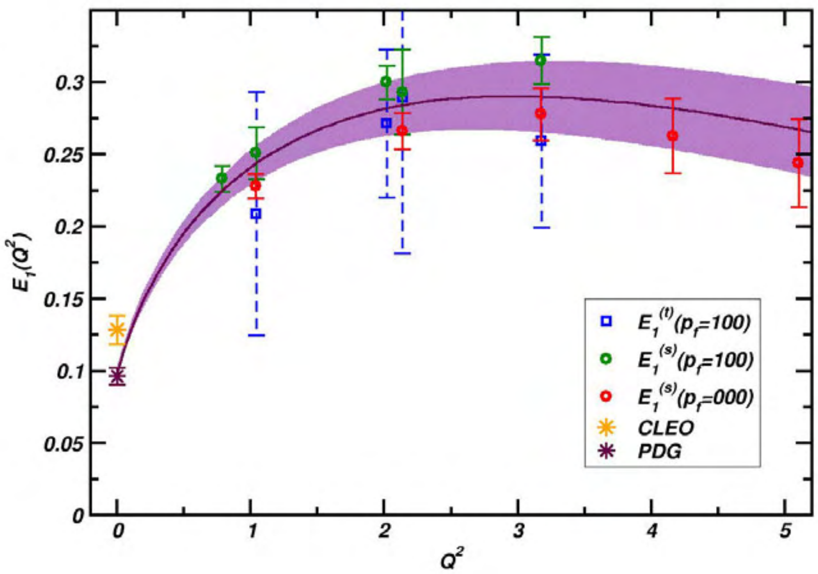
- also obtain the physically unobtainable *C1* multipole

$$C_1(Q^2) = \tilde{c}\sqrt{Q^2}\, e^{-Q^2/16\beta^2}$$



$$\beta = 0.473(13)\mathrm{GeV}$$

Quark model extrapolation



- error shrinks as $Q^2 \longrightarrow 0$!?

  property of the extrapolation form: $c\left|\vec{q}(Q^2)\right| e^{-Q^2/16\beta^2}$

  at $Q^2 = 0$ the error on β is irrelevant; only error on c matters

- great benefit of this form – but only if it's right!

$\chi_{c1} \to \gamma J/\psi$ transition

- two physical multipoles contribute: *E1, M2*

  also one longitudinal multipole: *C1*

- multipole decomposition:

$$\langle \chi_{c1}(\vec{p}_A, r_A)|V^\mu(0)|\psi(\vec{p}_V, r_V)\rangle = \frac{i}{4\sqrt{2}\Omega(q^2)}\epsilon^{\mu\nu\rho\sigma}(p_A - p_V)_\sigma \times$$

$$\times \Big[E_1(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p}_A, r_A).p_V]\epsilon_\nu(\vec{p}_V, r_V) + 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_V, r_V)\Big)$$

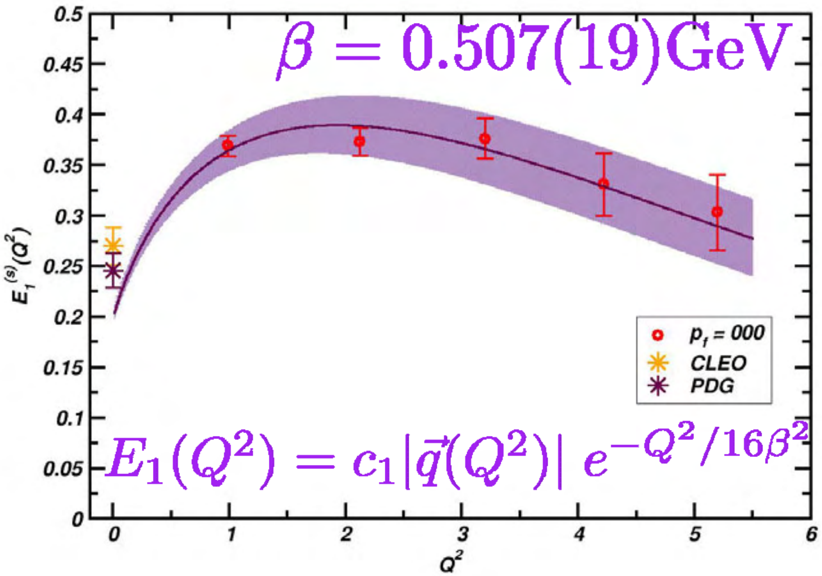
$$+M_2(Q^2)(p_A + p_V)_\rho \Big(2m_A[\epsilon^*(\vec{p}_A, r_A).p_V]\epsilon_\nu(\vec{p}_V, r_V) - 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_V, r_V)\Big)$$

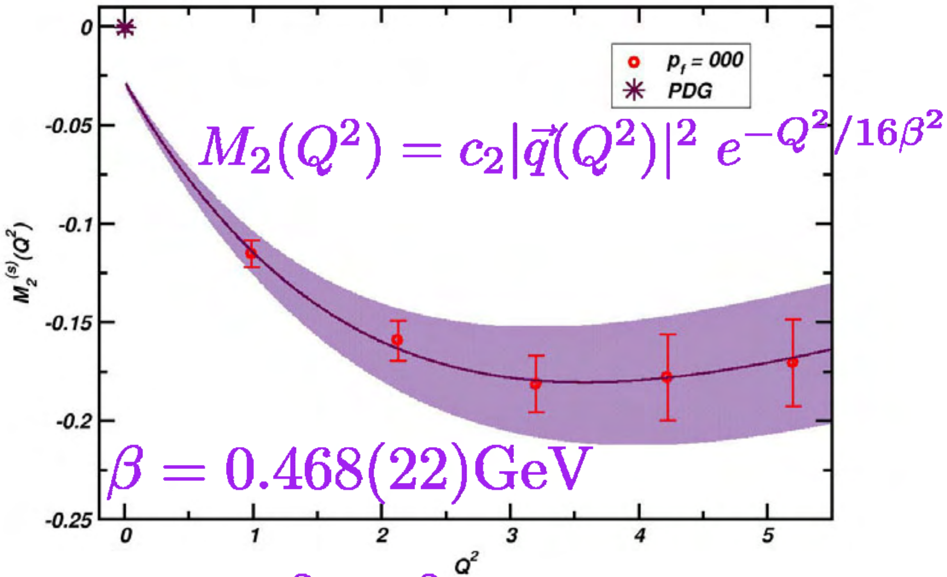
$$+\frac{C_1(Q^2)}{\sqrt{q^2}}\Big(- 4\Omega(q^2)\epsilon^*_\nu(\vec{p}_A, r_A)\epsilon_\rho(\vec{p}_V, r_V)$$

$$+ (p_A + p_V)_\rho \Big[(m_A^2 - m_V^2 + q^2)[\epsilon^*(\vec{p}_A, r_A).p_V]\, \epsilon_\nu(\vec{p}_V, r_V)$$

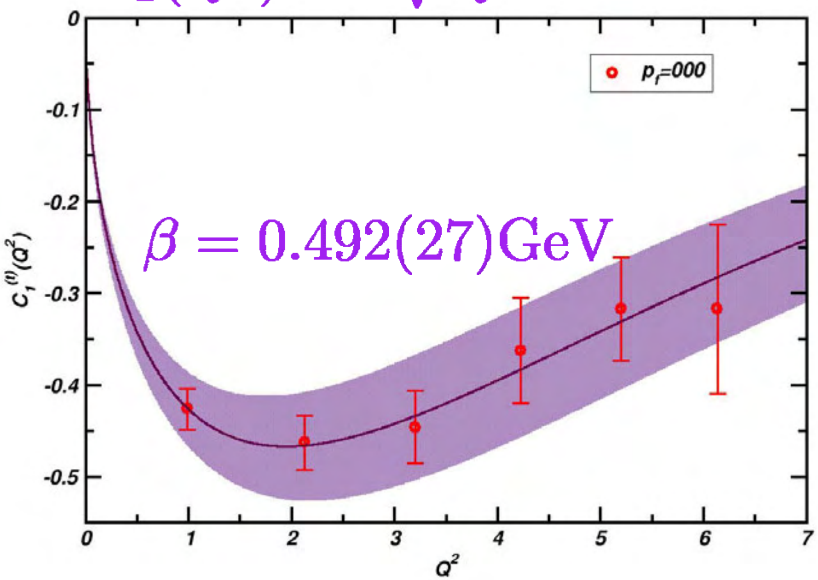
$$+ (m_A^2 - m_V^2 - q^2)[\epsilon(\vec{p}_V, r_V).p_A]\, \epsilon^*_\nu(\vec{p}_A, r_A)\Big]\Big)\Big]$$

$\chi_{c1} \rightarrow \gamma J/\psi$ transition





$$C_1(Q^2) = \tilde{c}\sqrt{Q^2}\ e^{-Q^2/16\beta^2}$$

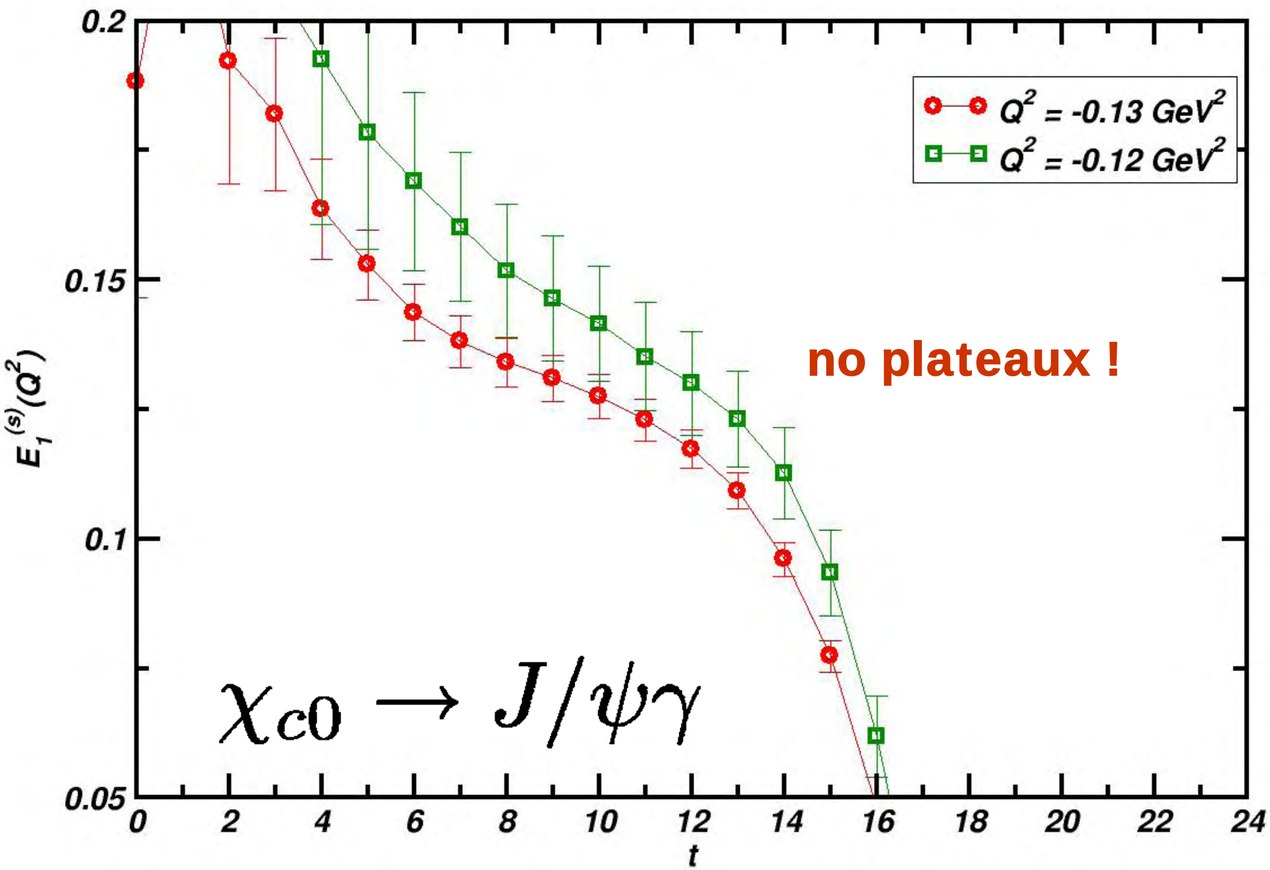


nearer to $Q^2 = 0$

- our simulations do have points very near to $Q^2 = 0$:

$$\vec{p}_f = \vec{p}_i \qquad Q^2 = -(E_f - E_i)^2$$

- very small, negative Q^2



how well do we do?

★ wavefunction extent:

	$"\eta_c" \to "\eta_c"\gamma$	$J/\psi \to \eta_c\gamma$	$\chi_{c0} \to J/\psi\gamma$		$\chi_{c1} \to J/\psi\gamma$		
	C_0	M_1	E_1	C_1	E_1	M_2	C_1
β/MeV	$522(5)$	$510(20)$	$601(28)$	$473(13)$	$507(19)$	$468(22)$	$492(27)$

quark model charmonium wavefunctions (from Coulomb + Linear potential) have typically $\beta/\mathrm{MeV} \sim 500 - 600$

★ transition widths & multipole ratios:

	$\Gamma(J/\psi \to \eta_c\gamma)$	$\Gamma(\chi_{c0} \to J/\psi\gamma)$	$\Gamma(\chi_{c1} \to J/\psi\gamma)$
lat*	$1.11(16)\mathrm{keV}$	$135(13)\mathrm{keV}$	$252(20)\mathrm{keV}$
PDG	$1.14(33)\mathrm{keV}$	$115(14)\mathrm{keV}$	$303(44)\mathrm{keV}$
CLEO		$204(31)\mathrm{keV}$	$364(51)\mathrm{keV}$

	$\frac{M_2}{E_1}(\chi_{c1} \to J/\psi\gamma)$
lat	$-0.14(1)$
PDG	$-0.002^{+0.008}_{-0.017}$
Grotch et al	-0.07

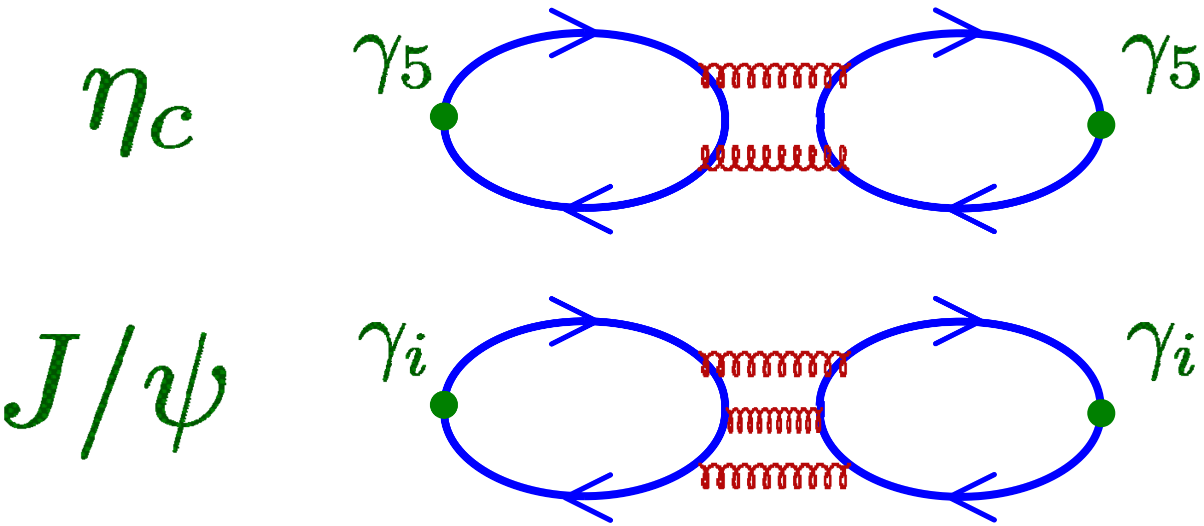
*using lattice simul. masses

extras

Approximations in the spectrum?

- what about the disconnected diagrams

  - in the perturbative picture



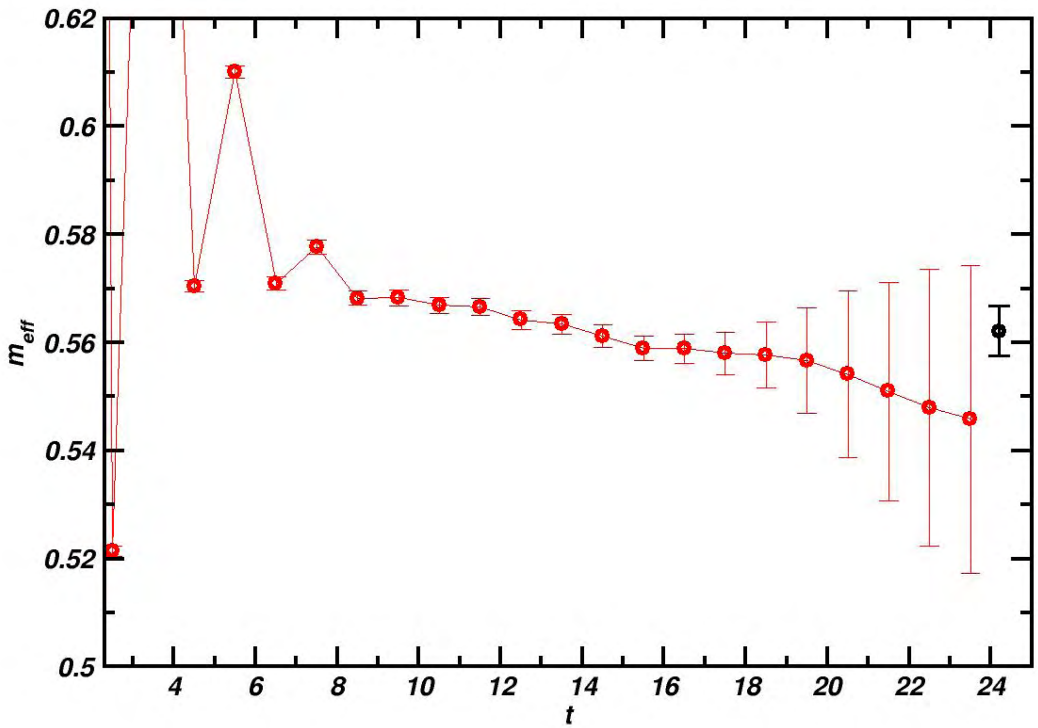
- disconnected diagrams might contribute to the hyperfine splitting

  - studies (QCD-TARO, Michael & McNeile) suggest an effect of order 10 MeV in the right direction

$\chi_{c1} \to \gamma J/\psi$ transition

- we might have some trouble with the three-point functions

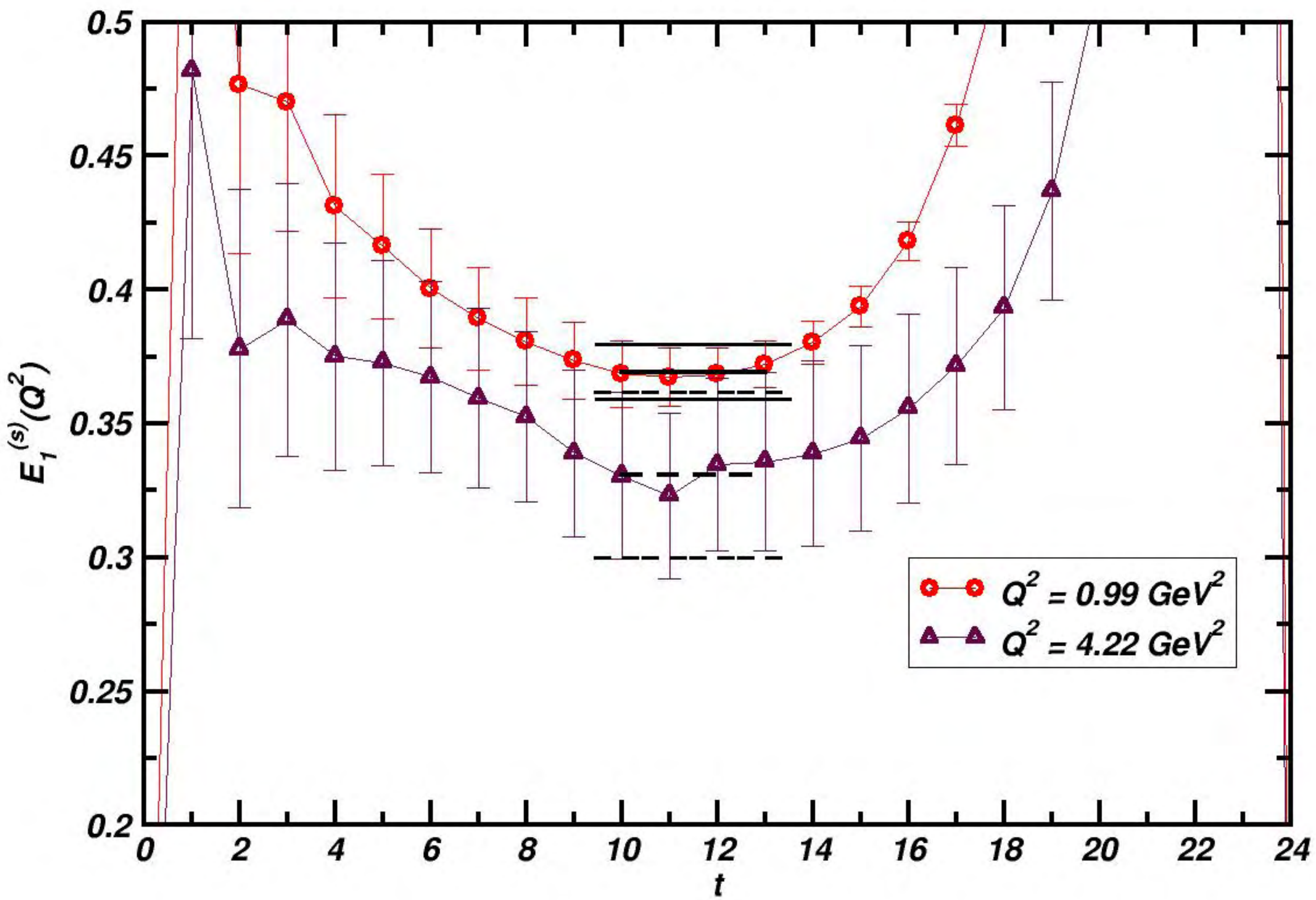
  - go back to the χ_{c1} at rest two-point function



- plateau is borderline – our smearing isn't ideal for this state
- non-zero momentum states are even worse

$\chi_{c1} \to \gamma J/\psi$ transition

- we anticipate borderline plateaux:



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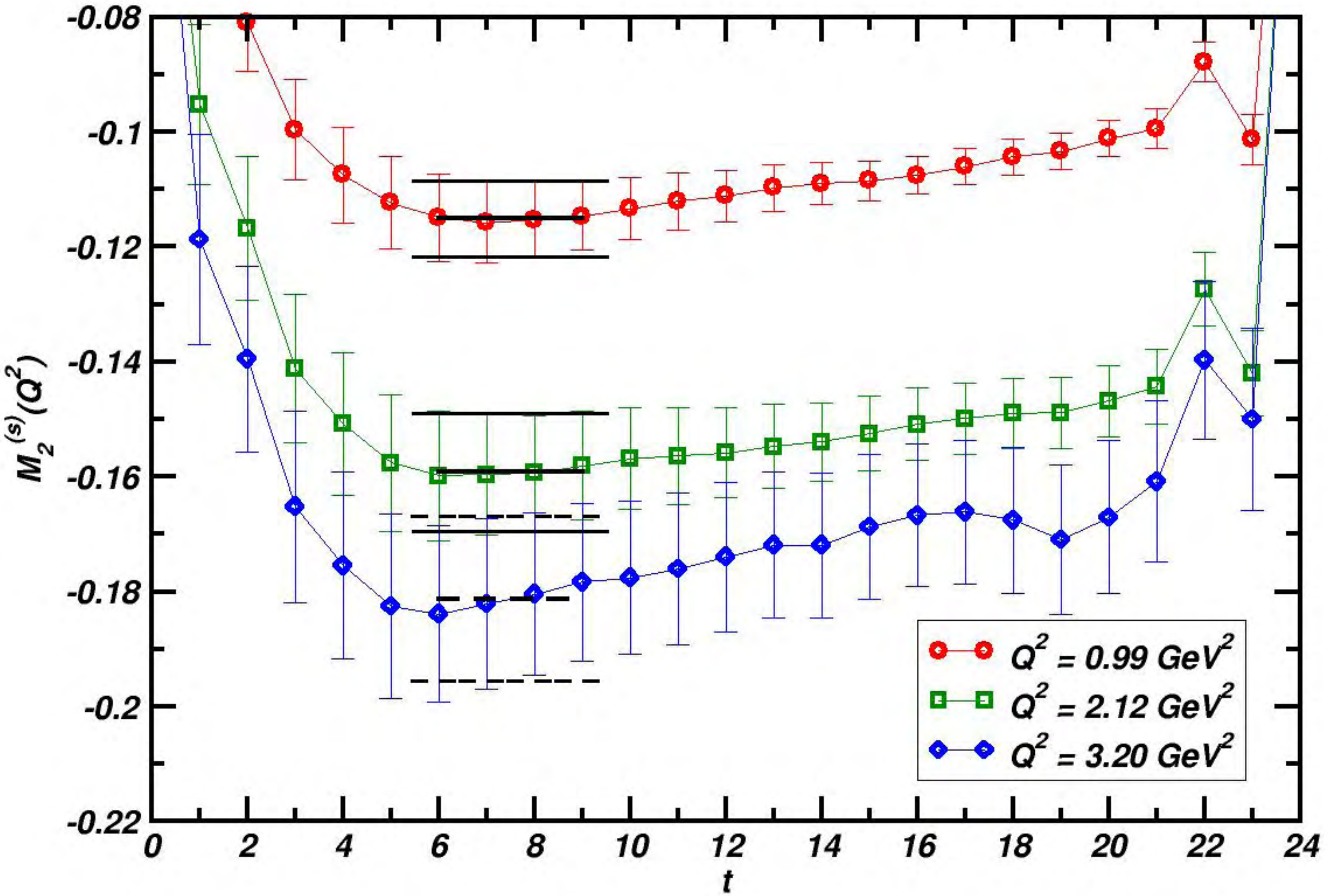

$\chi_{c1} \rightarrow \gamma J/\psi$ transition

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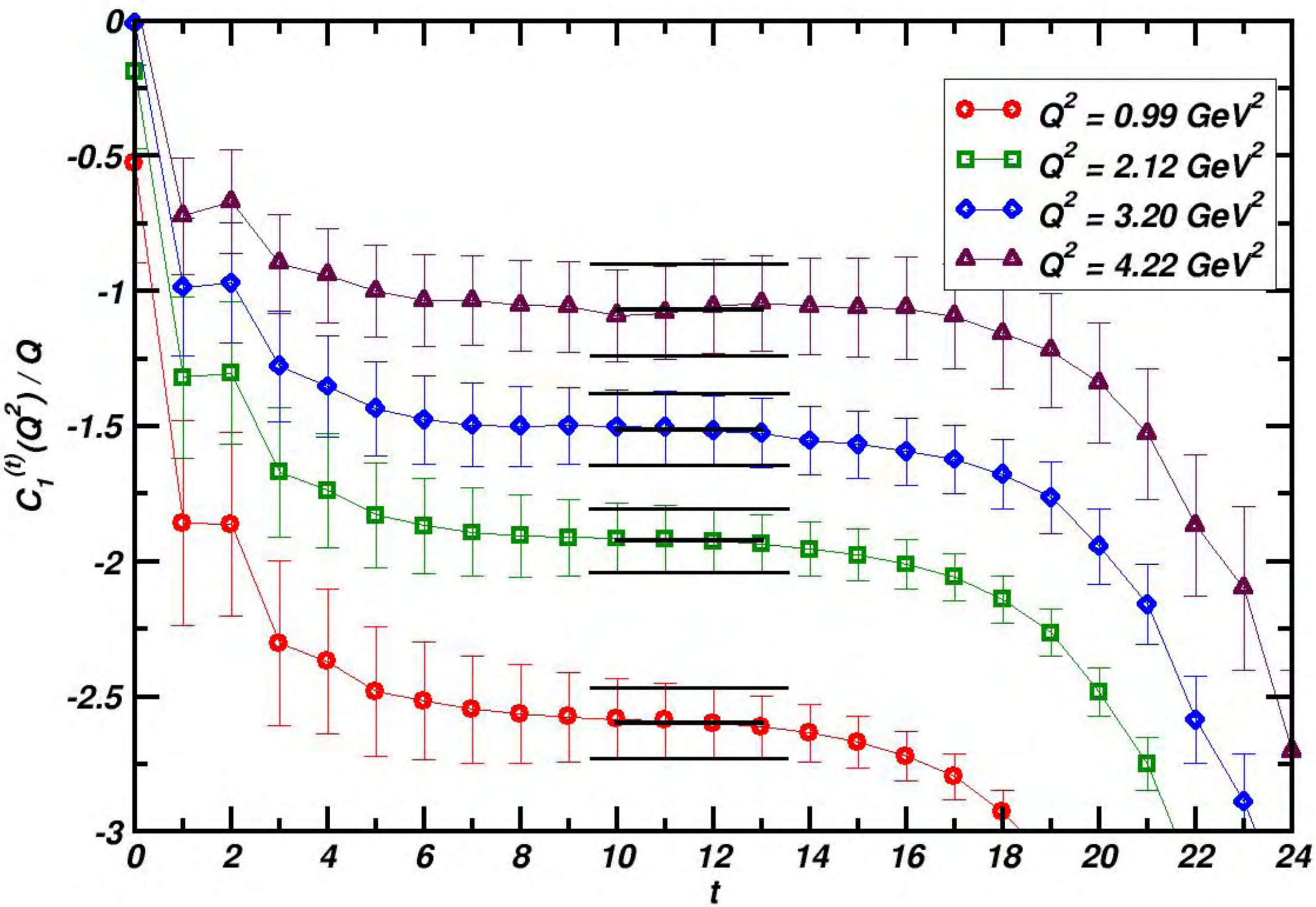


Anisotropic Lattices

- the gluon (Yang-Mills) piece of the action gains a parameter, ξ_0

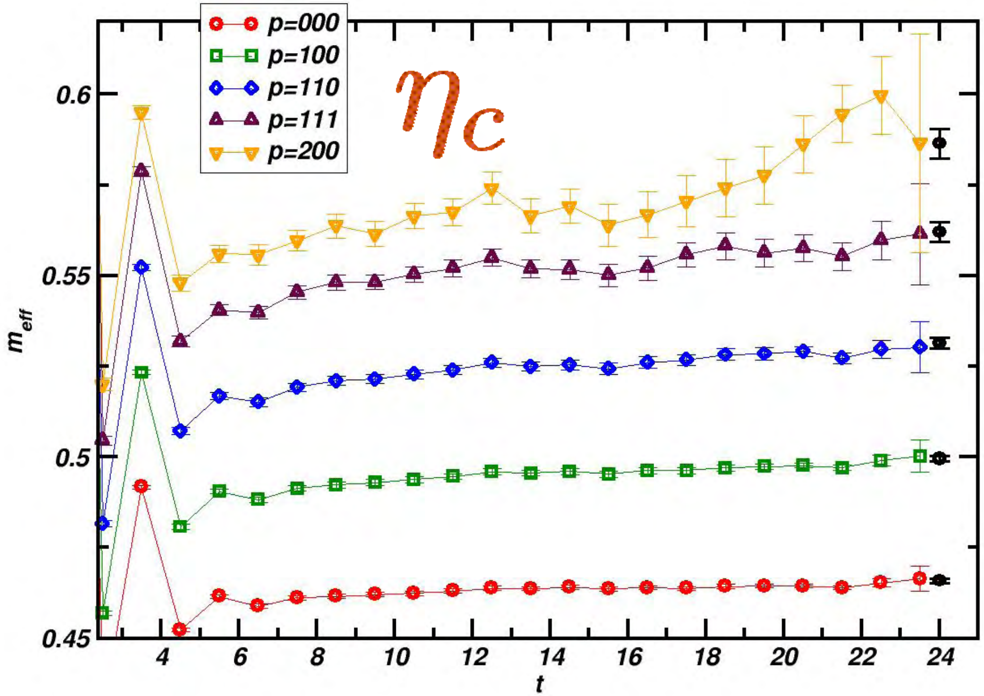
$$S_G^{\xi} = \frac{\beta}{N_c}\left[\frac{1}{\xi_o}\sum_{x,s>s'} \mathrm{Re}\,\mathrm{Tr}\,\left[1 - P_{ss'}(x)\right] + \xi_o \sum_{x,s} \mathrm{Re}\,\mathrm{Tr}\,\left[1 - P_{st}(x)\right]\right]$$

  this is chosen to get the desired anisotropy $\xi = \frac{a_s}{a_t}$

- the quark-gluon piece of the action features both ξ_0 and a second parameter, ν, sometimes called the “bare speed-of-light” (ratio of spat. to temp. derivatives)

  ν is tuned to ensure physical particles have the correct dispersion relation

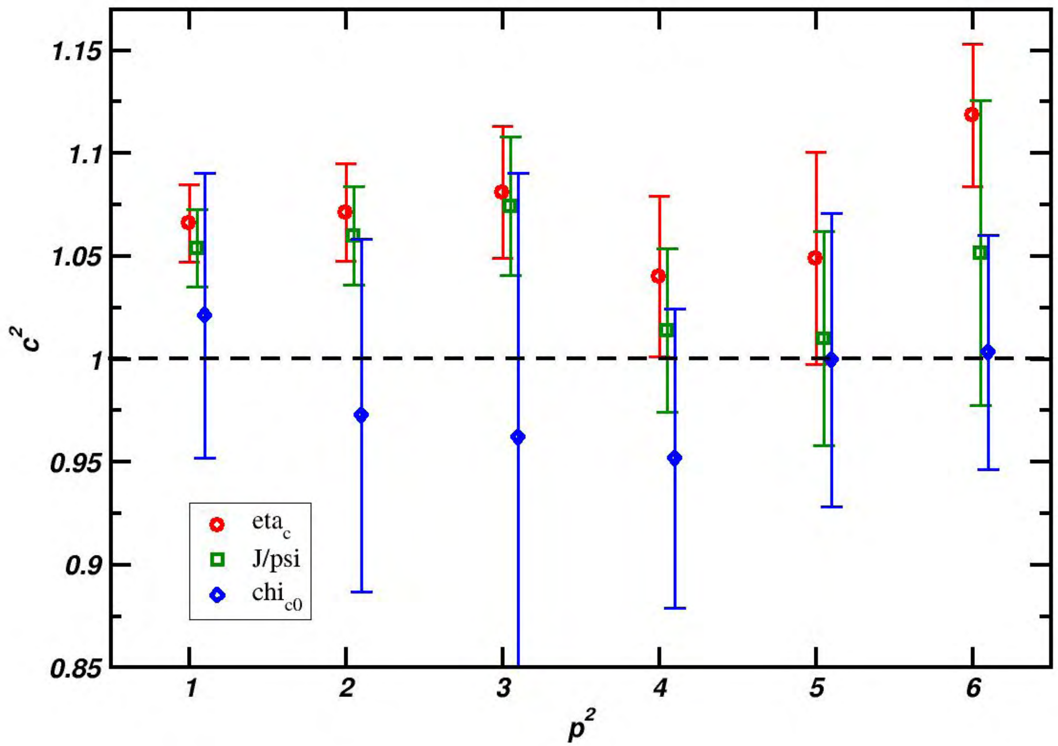
  i.e. $E^2 = p^2 + m^2$ *(up to lattice artifacts)*


p=000
p=100
p=110
p=111
p=200
η_c
0.6
0.55
0.5
0.45
m_{eff}
4
6
8
10
12
14
16
18
20
22
24
t

Dispersion relation tests

★ display the dispersion relation via the quantity $c^2(p^2) \equiv \frac{E^2 - m^2}{p^2}$

perfect tuning would be $c^2(p^2) = 1$



★ we've not tuned perfectly – a hazard of using anisotropy!