Lattice QCD for Nuclear Physics

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Outline

- What is Nuclear Physics?
- Tutorial:
  - NN
  - $\mathrm{EFT}(\not\pi)$
  - Finite-Volume
  - $\mathrm{EFT}(\pi)$
  - NNN
- Computational Resources
- Scattering Examples:
  - $\pi\pi$
  - $\Lambda\Lambda$
- Conclusion

What is Nuclear Physics?

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(C) A kind of Hadronic physics, only dirtier and less interesting.

(D) A system of at least two interacting baryons.

$$A(p) \;=\; \frac{4\pi}{Mp}\sin\delta(p)e^{i\delta(p)} \;=\; \frac{4\pi}{M}\,\frac{1}{p\cot\delta(p)-ip}$$

Low-Energy S-wave Nucleon-Nucleon Scattering

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contact interactions : $$p\cot\delta(p) \;=\; -\frac{1}{a_s} + \frac{1}{2}r_s\; p^2\;\sum_{i=0}^{\infty}\,(r_i^2\;p^2)^i$$

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neutron-proton (np) S-wave:

$$a_s^{^1S_0} = -23.714 \text{ fm} \qquad r_s^{^1S_0} = 2.73 \text{ fm}$$
$$a_s^{^3S_1} = 5.425 \text{ fm} \qquad r_s^{^3S_1} = 1.749 \text{ fm}$$

$$a_s \gg \Lambda_{QCD}^{-1} \text{ !!}$$

EFT $\not\pi$

$p \ll m_\pi \qquad \Longrightarrow \qquad$ Integrate out the pion

Expansion in $\frac{p}{m_\pi}$, $\frac{p}{M}$

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EFT of contact operators:

$$\mathcal{L} = -\ C_0\ (N^\dagger N)^2 -\ C_2\ (N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \dots$$

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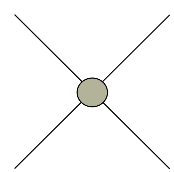
Expansion in $\frac{p}{m_\pi}$, $\frac{p}{M}$

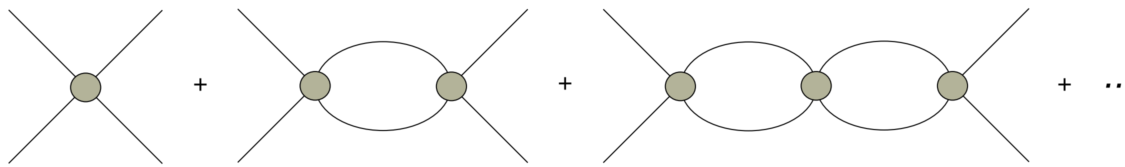
EFT of contact operators:

$$\mathcal{L} = -\, C_0\,(N^\dagger N)^2 - \, C_2\,(N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \dots$$

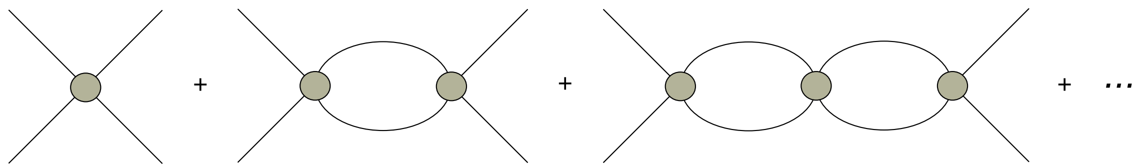
$$V(p) = C_0 \; + \; C_2\, p^2 \; + \; \dots \equiv$$



$A(p) =$



$$= \frac{\sum C_{2n}(\mu)\ p^{2n}}{1 - I_0(\infty) \sum C_{2n}(\mu)\ p^{2n}}$$

$$A(p) =$$ 

$$= \frac{\sum C_{2n}(\mu)\ p^{2n}}{1 - I_0(\infty) \sum C_{2n}(\mu)\ p^{2n}}$$

$$I_0(\infty) \quad = \quad \left(\frac{\mu}{2}\right)^{4-D} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}} \frac{1}{E - \frac{|\mathbf{q}|^2}{M} + i\epsilon}$$

$$\underset{PDS}{\longrightarrow} \quad -\frac{M}{4\pi}\ (\mu + ip)$$

Non-Trivial Fixed Point

$$C_0(\mu) \;=\; -\frac{4\pi}{M}\;\frac{1}{\mu\;-\;1/a_s}$$

$$\mu\frac{d}{d\mu}\hat{C}_0(\mu) \;=\; \hat{C}_0(\mu)\left(1+\hat{C}_0(\mu)\right)$$

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$$\mu \frac{d}{d\mu} \hat{C}_0(\mu) \; = \; \hat{C}_0(\mu) \left(1 + \hat{C}_0(\mu)\right)$$

$$a_s \to \pm\infty \leftrightarrow \hat{C}_0(\mu) = -1$$

$$\Downarrow$$

$\mathrm{EFT}(\not\pi)$ defines conformal field theory!!

$$I_0(L) = \frac{1}{L^3} \sum_{\mathbf{k}} \frac{1}{E - \frac{|\mathbf{k}|^2}{M}}$$

$$I_0^{(PDS)}(L) = -\frac{M}{4\pi}\mu + \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{E - \frac{|\mathbf{k}|^2}{M}} + M \int^{\Lambda} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{|\mathbf{k}|^2}$$

EFT $\not\pi$ in a Box

$$I_0(L) \quad = \quad \frac{1}{L^3} \sum_{\mathbf{k}} \frac{1}{E - \frac{|\mathbf{k}|^2}{M}}$$

$$I_0^{(PDS)}(L) \quad = \quad -\frac{M}{4\pi}\mu \; + \; \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{E - \frac{|\mathbf{k}|^2}{M}} \; + \; M \int^{\Lambda} \frac{d^3\mathbf{k}}{(2\pi)^3} \, \frac{1}{|\mathbf{k}|^2}$$

$$Re(\, A(p)^{-1}\,) \; = \; 0$$

$p^2 \; = \; M\,E$ gives all eigenstates in the box

Exact Eigenvalue Equation

Lüscher (1986,1991)

$L \gg 1/m_\pi$:

$$p\cot\delta(p) \; - \; \frac{1}{\pi L}\sum_{\mathbf{j}}^{\Lambda_j}\frac{1}{|\mathbf{j}|^2 - (\frac{Lp}{2\pi})^2} \; + \; \frac{4\Lambda_j}{L} \; = \; 0$$

$$\Lambda_j \to \infty$$

$$p\cot\delta(p) = -\frac{1}{a_s} \; + \; \frac{1}{2}\, r_s \; p^2 \; + \; \ldots$$

Lüscher's formula

Lüscher (1986)

$L \gg a_s, r_s$:

$$\Delta E_0 = +\frac{4\pi a_s}{ML^3}\left[1 - c_1\left(\frac{a_s}{L}\right) + c_2\left(\frac{a_s}{L}\right)^2 + \ldots\right]$$

$$\Delta E_1 = \ldots$$

$$c_1 = -2.837297 \qquad c_2 = 6.375183$$

Useful for natural size scattering lengths (e.g. $\pi\pi$)

The Infrared Fixed Point

Bedaque,Parreño,Savage,SB (2003)

$a_s \gg L \gg r_s$:

$$\Delta E_0 = +\frac{4\pi^2}{ML^2}\left[d_1 - d_2 \left(\frac{L}{a_s}\right) + \ldots \right]$$
$$\Delta E_1 = \ldots$$

$$d_1 = -0.095896 \qquad d_2 = 0.025434$$

Appropriate limit for nuclear physics?

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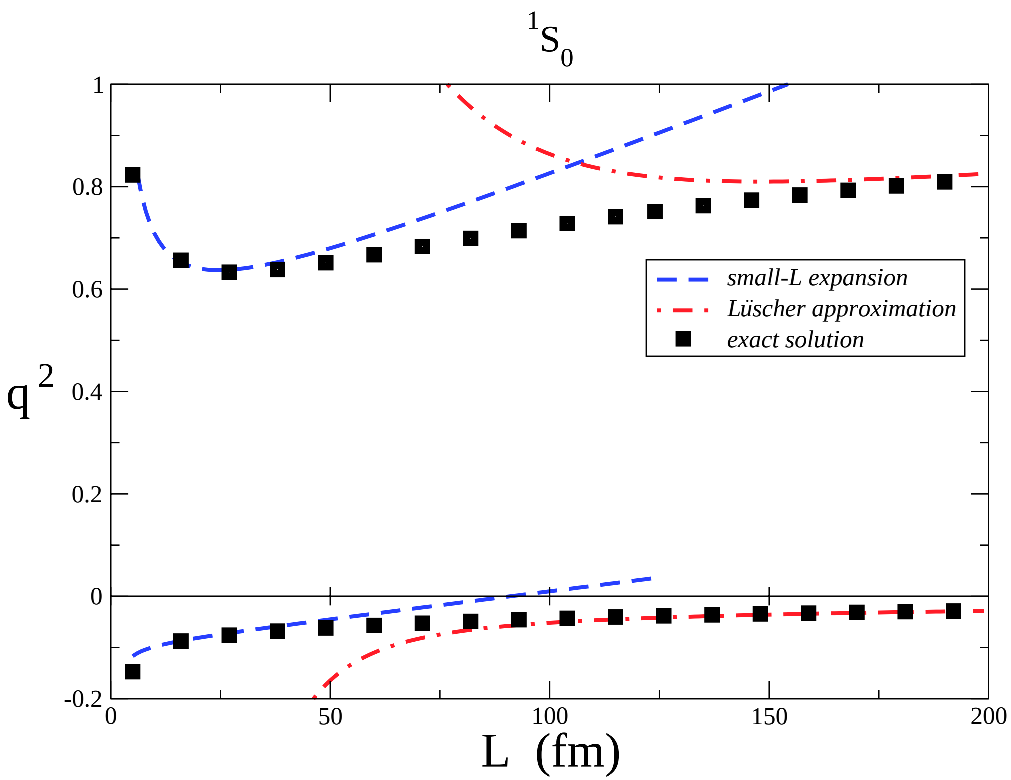
Appropriate limit for nuclear physics?

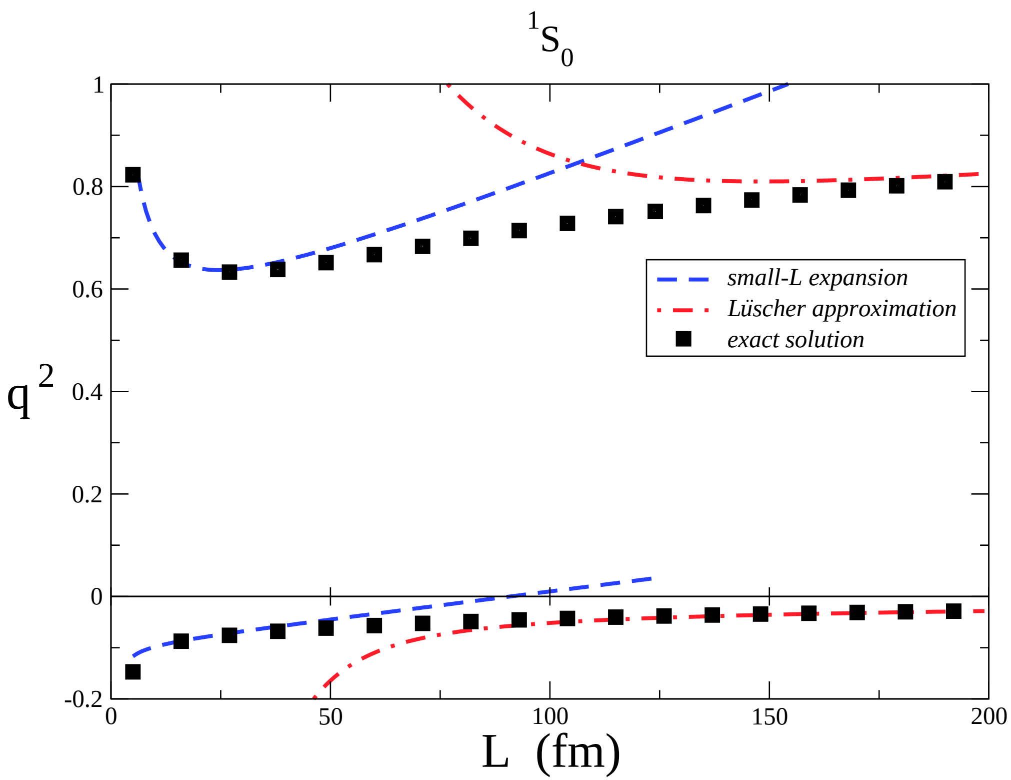
Consider finite-volume analysis with physical np parameters.

Spin-Singlet Eigenvalues



	$^1S_0$ $\|\mathbf{p}\|$ (MeV)	
L (fm)	1st	2nd
1000	0.1 i	1.21
100	2.6 i	10.52
25	13.8 i	39.3
15	24.6 i	67.0
10	39.0 i	104.3 (*)
5	94.4 i (*)	224.7 (*)

Spin-Singlet Eigenvalues



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$L \lesssim 5$ fm : outside of EFT $\not\pi$

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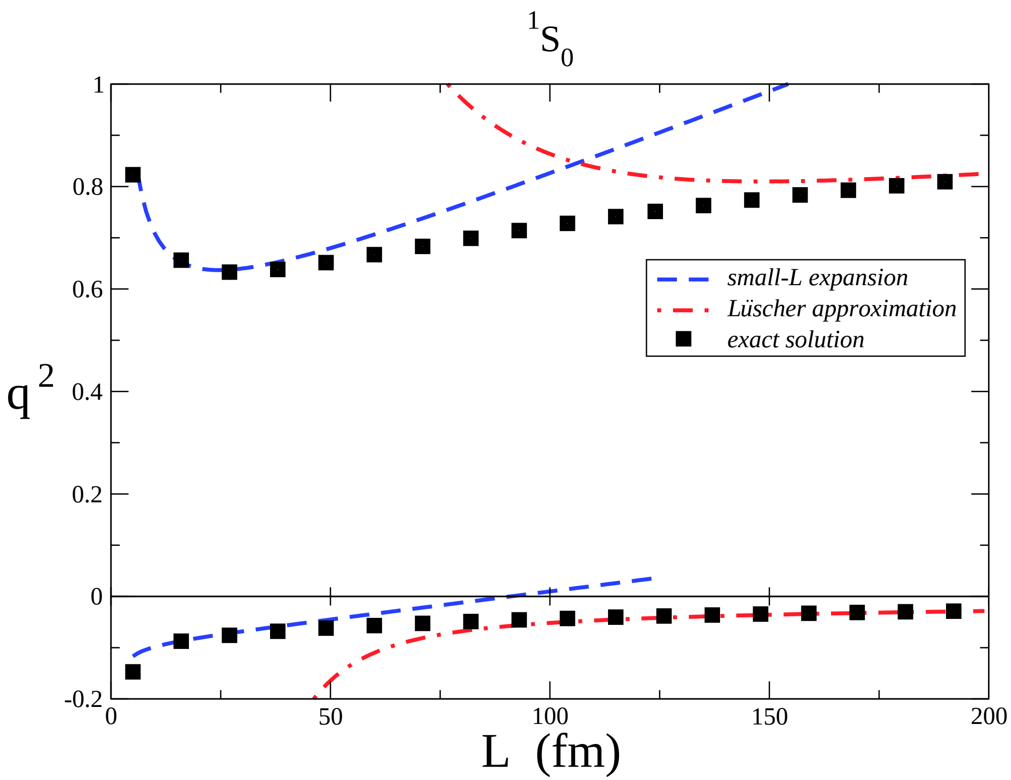

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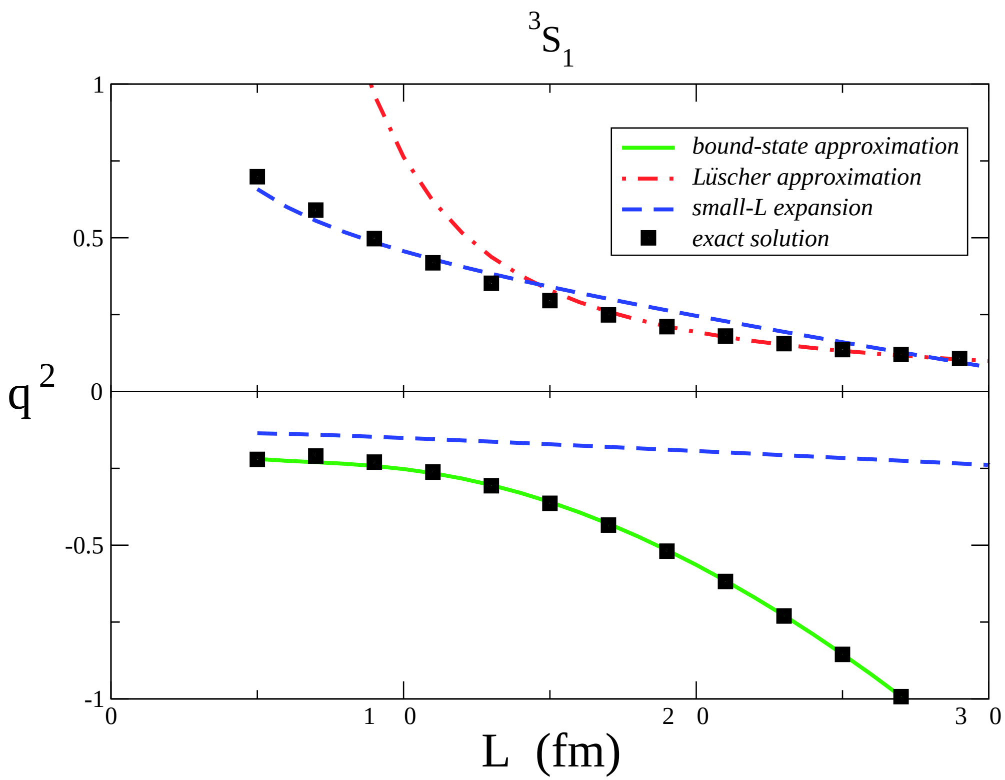
$L \lesssim 5$ fm : outside of EFT $\not\pi$

Maximal m_π $\longrightarrow$ Minimal L !!

Spin-Triplet Eigenvalues



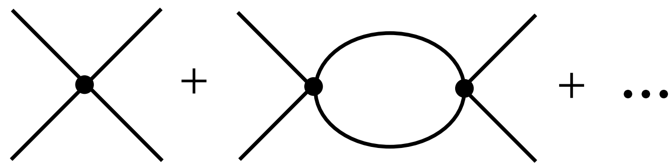
	$^3S_1$ $\lvert\mathbf{p}\rvert$ (MeV)	
L (fm)	Deuteron	1st
1000	45.5 i	0.052
100	45.5 i	1.76
25	45.8 i	18.25
15	49.9 i	44.61
10	61.3 i	83.1 (*)
5	116.5 i (*)	206.5 (*)

$$bound - state: \quad E_{-1} = -\frac{\gamma^2}{M}\left[1 + \frac{12}{\gamma L}\,\frac{1}{1-2\gamma(p\cot\delta)'}\,e^{-\gamma L} + \ldots\right]$$

Need control of the quark-mass dependence of the np parameters.

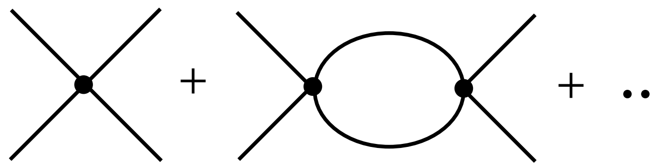
$\mathrm{EFT}(\pi)$: $^1S_0$

LO :

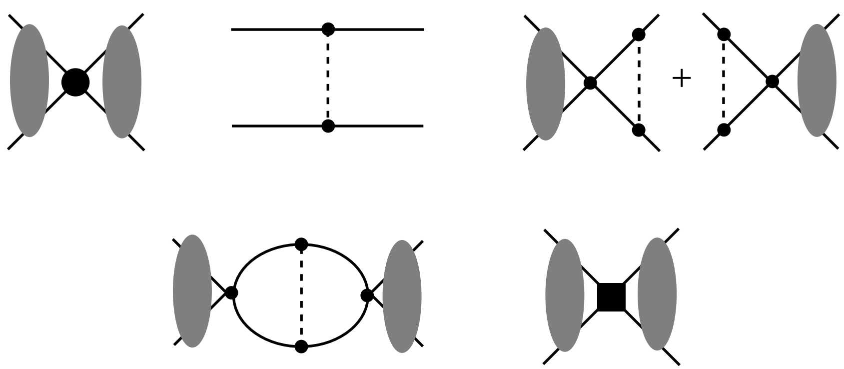


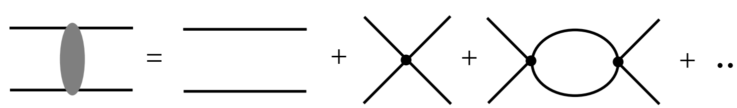
EFT(π): $^1S_0$

LO :



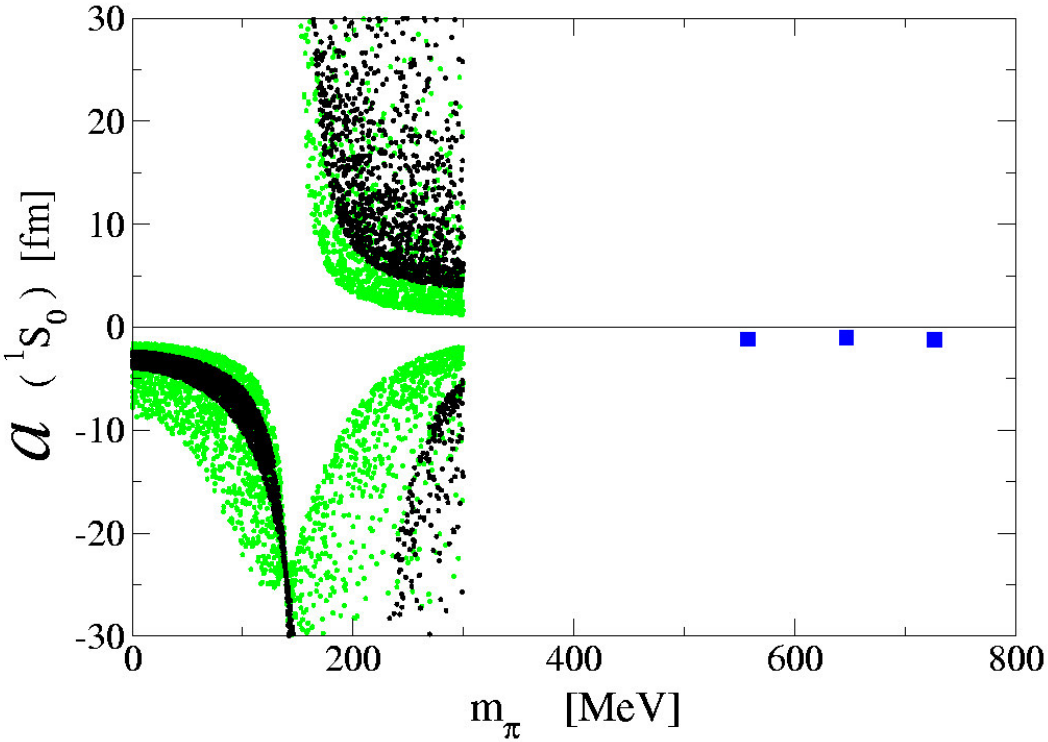
NLO :





$^1S_0$ of NN

Savage, SB (2002)   Epelbaum *et al* (2002)



$D_2 = 1/5, 1/15$

Lattice data : QQCD from Fukugita *et al* (1995)

$^3S_1$ of NN

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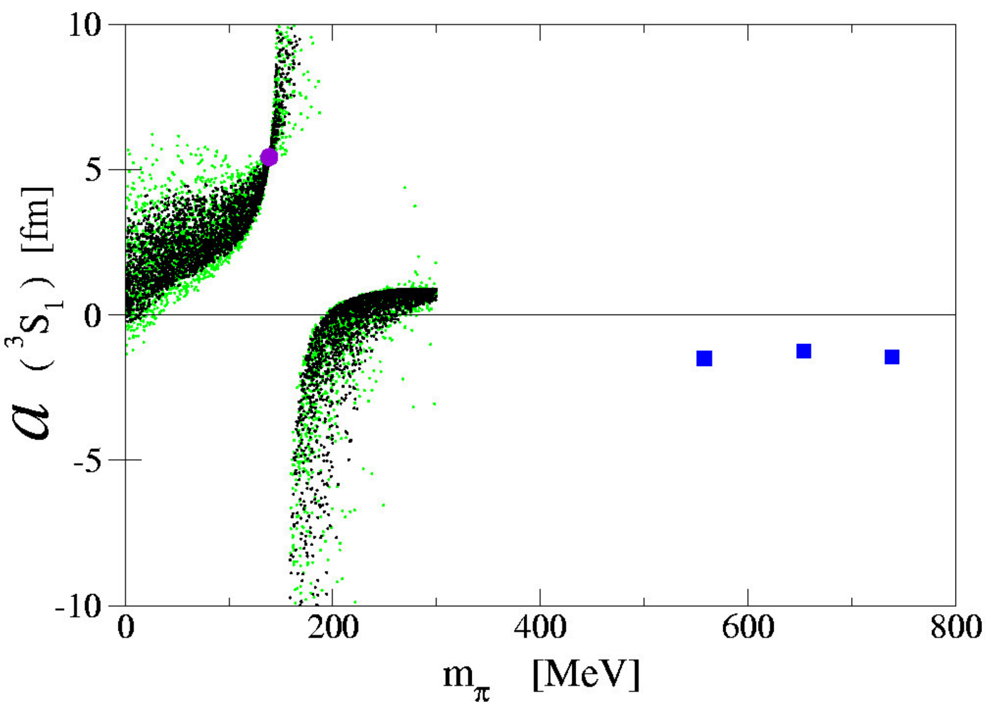
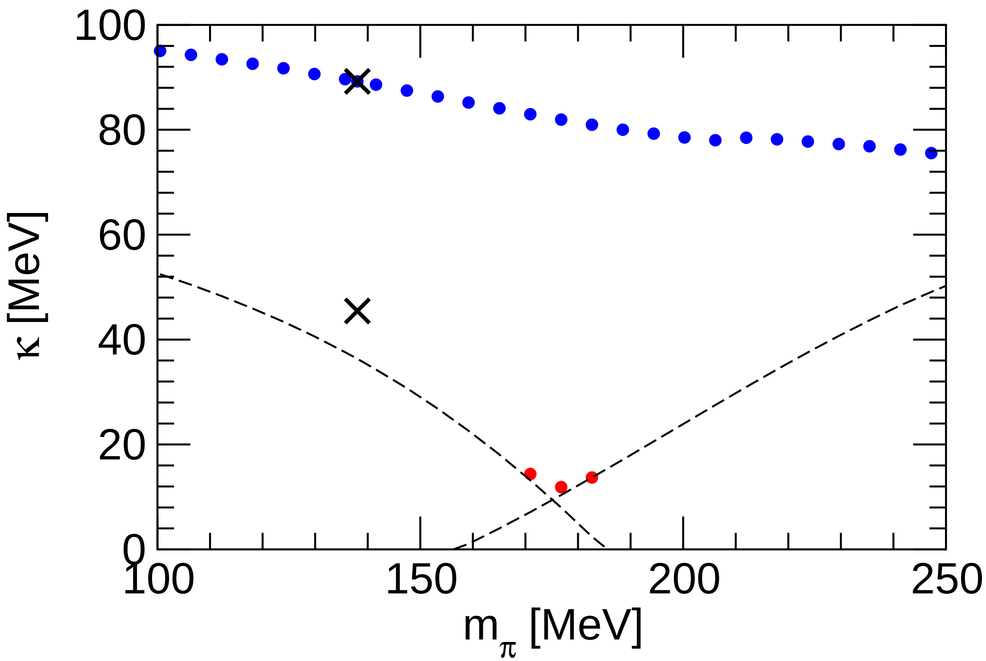


$D_2 = 1/5, 1/15 \qquad -2.61\ \mathrm{GeV}^{-2} < \bar{d}_{16} < -0.17\ \mathrm{GeV}^{-2} \qquad \bar{d}_{18} = -1.54\ \mathrm{GeV}^{-2}$

Lattice data : QQCD from Fukugita *et al* (1995)

NNN

BRAATEN,HAMMER(2003)



$$a_s^{^3S_1} \longrightarrow \infty \ , \ a_s^{^1S_0} \longrightarrow \infty$$

Efimov effect: infrared limit cycle in QCD ?

Resources Available

SciDAC:

- JLab Clusters – 8% Resources – 40 Gflop-yrs
- Chroma / QDP++
- MILC Staggered Lattices
- DWF Propagators of LHPC

Approved Exploratory:

- $\Lambda\Lambda$, NN, $N\Lambda$, $\Sigma\Sigma$, $N\Sigma$
- M_N – σ-term – strong-isospin-breaking (PQ)

$\pi\pi$ Scattering

Bedaque,Orginos,Savage,SB (2005)

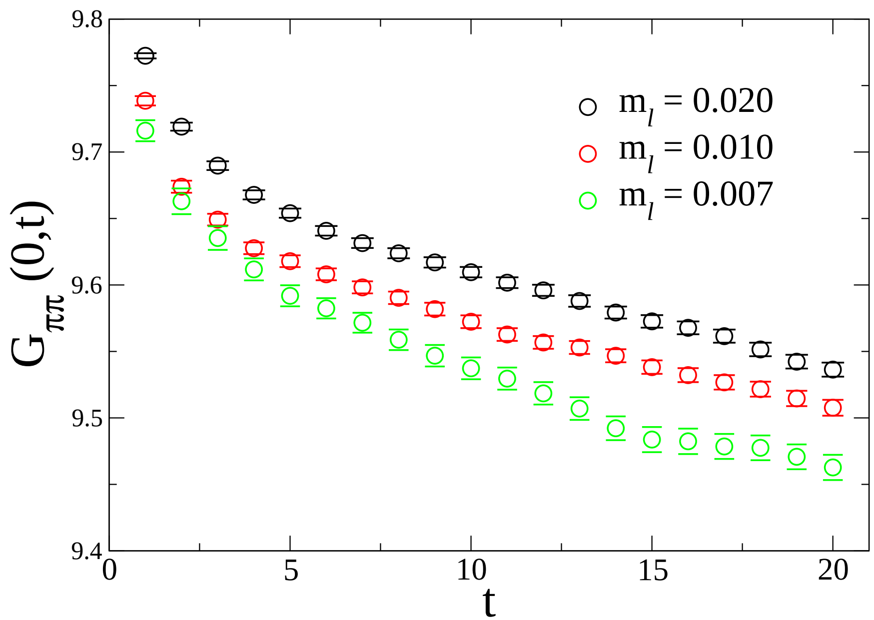
I=2 $\pi - \pi$

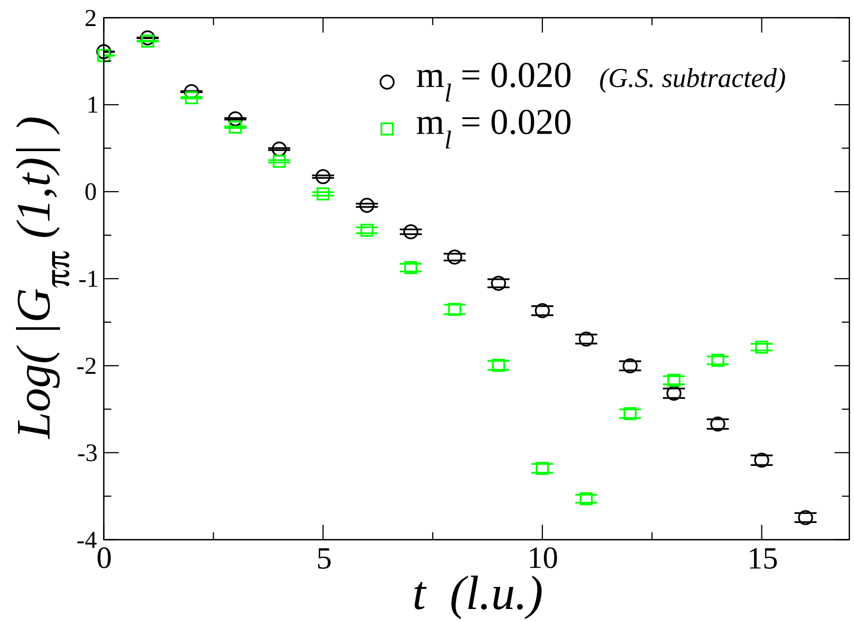
Config Set (MILC_)	Dimensions	bm_l	bm_s	bm_{dwf}	m_π	# configs
2064f21b679m007m050	$20^3 \times 64$	0.007	0.05	0.0081	294 MeV	319
2064f21b679m010m050	$20^3 \times 64$	0.010	0.05	0.0138	348 MeV	649
2064f21b679m020m050	$20^3 \times 64$	0.020	0.05	0.0313	484 MeV	453

Hybrid of staggered sea quarks (MILC) and domain-wall valence quarks (LHPC)

(2+1) dynamical flavors

Data





$$G_{\pi\pi}(p,t) \equiv \frac{\sum_{|\vec{p}|=p} \sum_{\vec{x},\vec{y}} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \langle \pi^\dagger(t,\vec{x})\pi^\dagger(t,\vec{y})\pi(0,\vec{0})\pi(0,\vec{0})\rangle}{(\sum_{\vec{x}} \langle \pi^\dagger(t,\vec{x})\pi(0,\vec{0})\rangle)^2} = \sum_{n=0}^{\infty} \mathcal{A}_n \ e^{-\Delta E_n \ t}$$

Analysis

$$\Delta E_n \equiv E_n - 2m_\pi = 2\sqrt{\vec{p}_n^{\,2} + m_\pi^2} - 2m_\pi$$

$$\Delta E_0 = -\frac{4\pi a_2}{m_\pi L^3}\left[1 + c_1 \frac{a_2}{L} + c_2 \left(\frac{a_2}{L}\right)^2 + \mathcal{O}(\left(\frac{a_2}{L}\right)^3) \right]$$

Analysis

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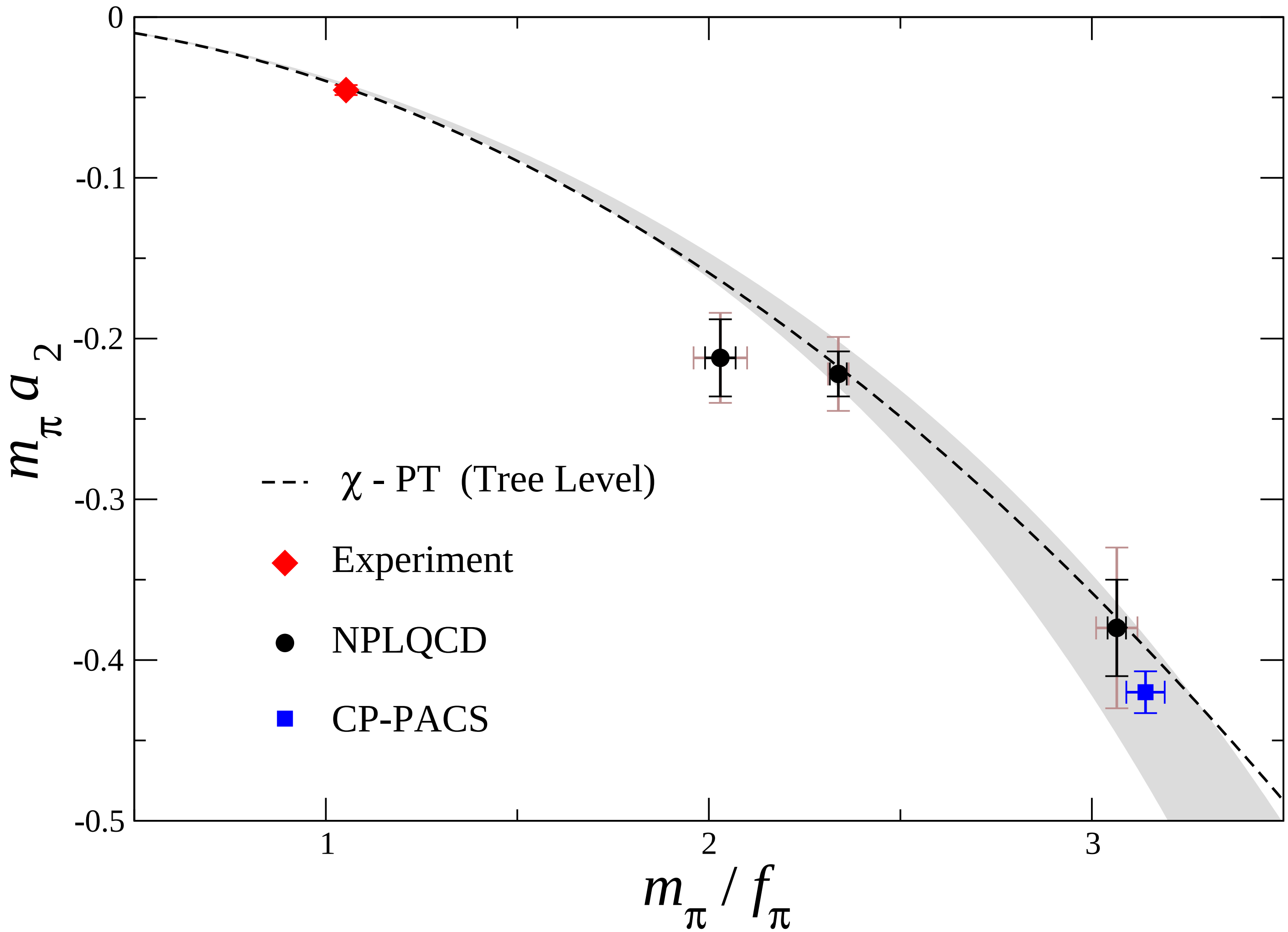
EXTRAPOLATION: χ-PT at one loop

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{16\pi^2 f_\pi^2} + l_{\pi\pi}\right)\right]$$

Results



0
-0.1
-0.2
-0.3
-0.4
-0.5
1
2
3
$m_\pi a_2$
m_π / f_π
χ - PT (Tree Level)
Experiment
NPLQCD
CP-PACS

Extrapolation Errors

EXTRAPOLATION: χ-PT at two loops

$$
\begin{aligned}
m_\pi a_2 \quad = \quad & -\frac{m_\pi^2}{8\pi f_\pi^2} \left\{ 1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2} \left[\log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi} \right] \right. \\
& \left. + \frac{m_\pi^4}{64\pi^4 f_\pi^4} \left[\frac{31}{6} \left(\log\frac{m_\pi^2}{\mu^2} \right)^2 + l_{\pi\pi}^{(2)} \log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi}^{(3)} \right] \right\}
\end{aligned}
$$

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ERROR ESTIMATES:

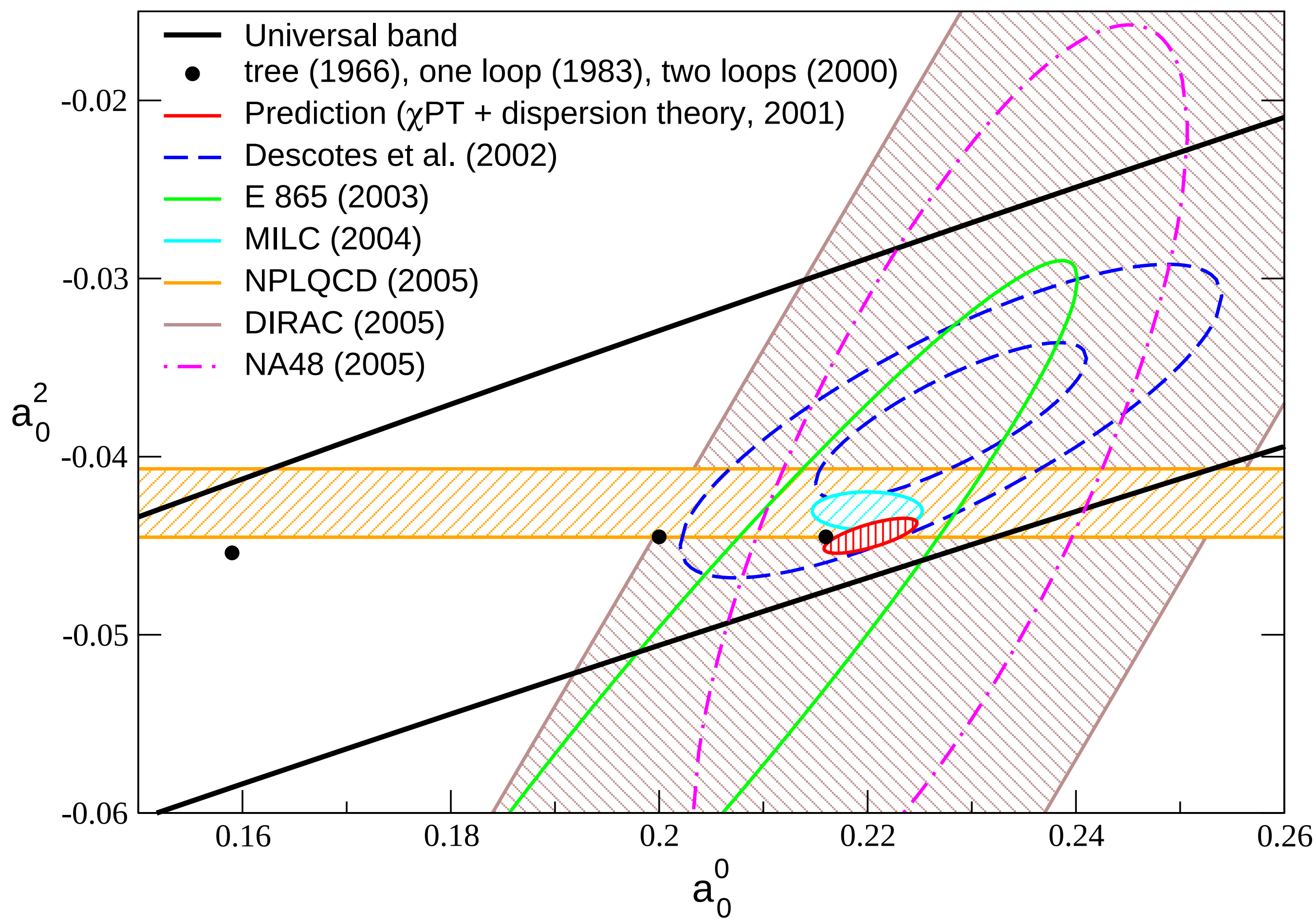
(A) FIT $l_{\pi\pi}$ $(l_{\pi\pi}^{(2)} = l_{\pi\pi}^{(3)} = 0)$

(B) FIT $l_{\pi\pi}$ AND $l_{\pi\pi}^{(2)}$ $(l_{\pi\pi}^{(3)} = 0)$

The Bottom Line

$$m_\pi a_2 = -0.0426 \pm 0.0006 \pm 0.0003 \pm 0.0018 \qquad NPLQCD$$

$$l_{\pi\pi} = 3.3 \pm 0.6 \pm 0.3$$



Caprini, Colangelo, Leutwyler(2005)

$\Lambda\Lambda$ Scattering

Bedaque,Orginos,Savage,SB (2005)

$\Lambda - \Lambda$

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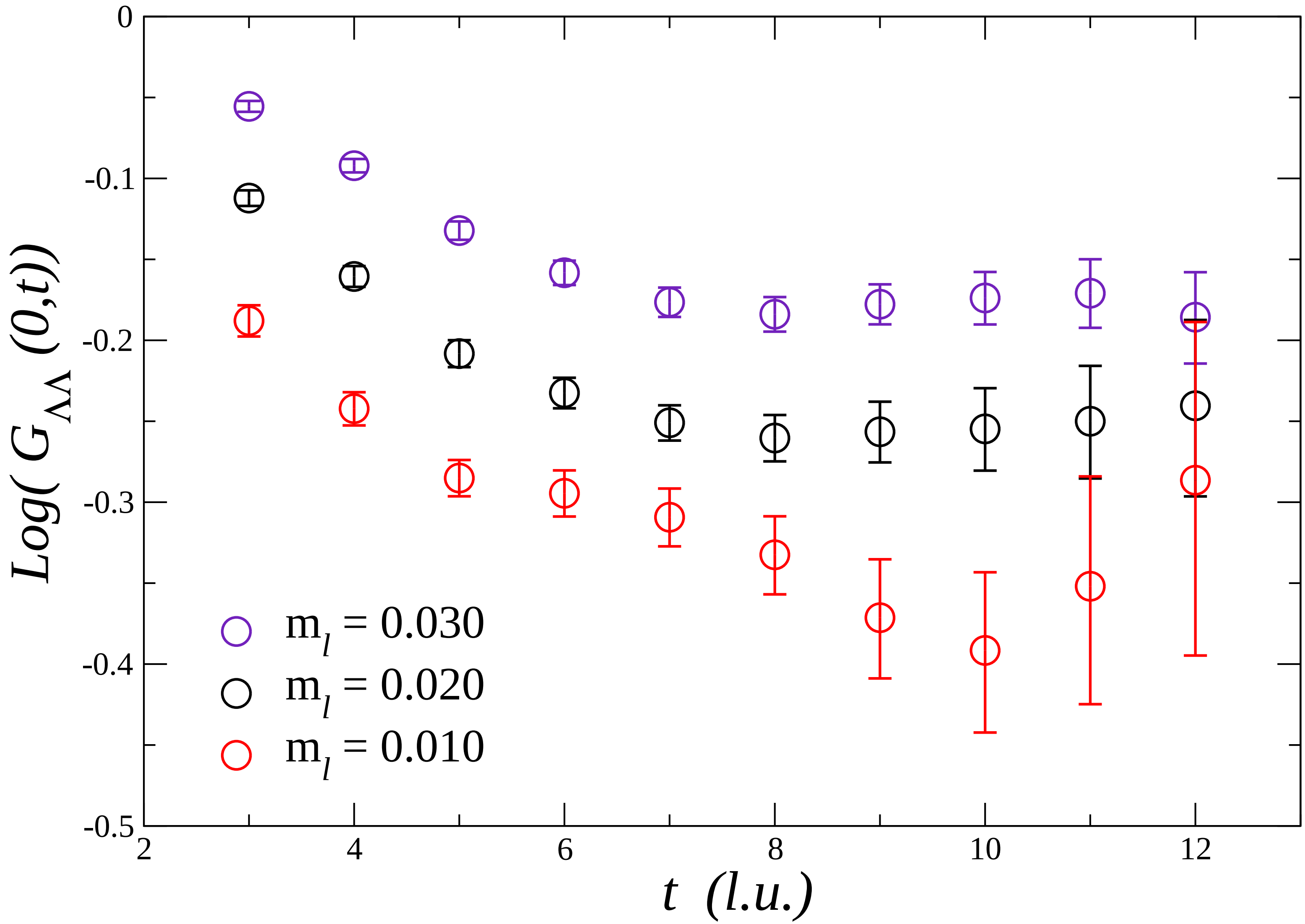
2064f21b679m020m050	$20^3 \times 64$	0.020	0.05	0.0313	484 MeV	486
2064f21b679m030m050	$20^3 \times 64$	0.030	0.05	0.0474	565 MeV	564

Hybrid of staggered sea quarks (MILC) and domain-wall valence quarks (LHPC)

(2+1) dynamical flavors


Log(G_ΛΛ (0,t))
t (l.u.)
m_l = 0.030
m_l = 0.020
m_l = 0.010
0
-0.1
-0.2
-0.3
-0.4
-0.5
2
4
6
8
10
12

Conclusion

- Many theoretical tools have been developed which will allow computation of properties of light nuclei from lattice QCD.

- As a warm-up we have obtained realistic results for the $I = 2$ $\pi\pi$ scattering length in fully-dynamical lattice QCD.

- We need more statistics in order to extract information about $\Lambda\Lambda$ scattering parameters. Other baryon-baryon systems are currently under investigation.