Muon $g-2$ from Staggered Chiral Perturbation Theory

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Motivation

- Muon $g-2$—Sign of new physics
  - Precision measurements (Muon $g-2$ Collaboration, BNL-E821)
  - Theoretical estimates:
    - For QCD part: use $e^+e^- \to \mathrm{hadrons}$ cross section or τ decay + dispersion relations)
    - Currently low
    - Errors dominated by hadronic contribution
  - Lattice: first principles method to extract hadronic contributions

Outline

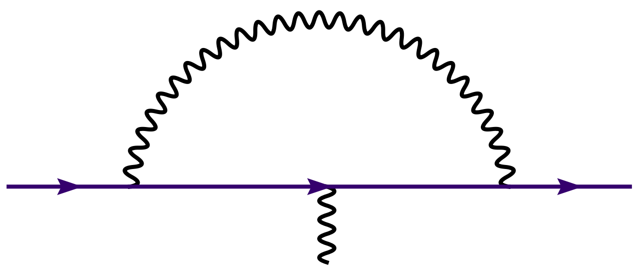
- Some details of the lattice calculation
- Staggered Chiral Perturbation Theory (SχPT) with photons
  - Pions and Kaons
  - Vector contributions
- Fits and preliminary results

Muon $g-2$

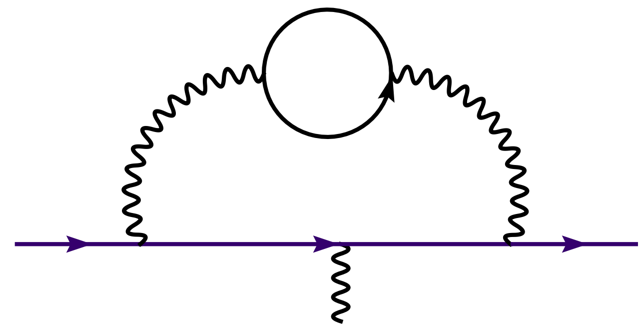
Full muon-photon vertex:

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\mu}{2m_\mu} F_2(q^2) \qquad a_\mu = \frac{g-2}{2} = F_2(0)$$

$O(\alpha)$:



$O(\alpha^2)$:



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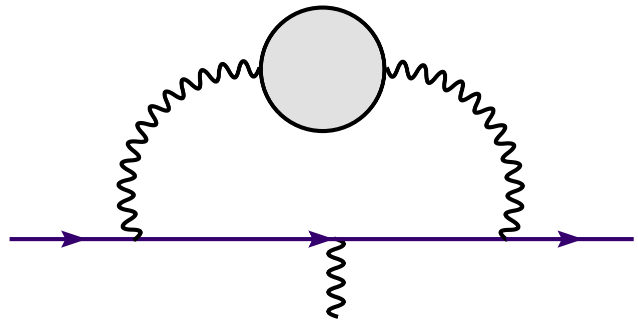
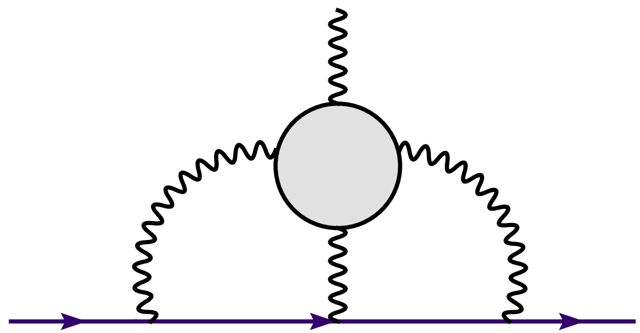
Leading Hadronic Contributions

$O(\alpha^2)$, Hadronic contribution to the photon vacuum polarization:

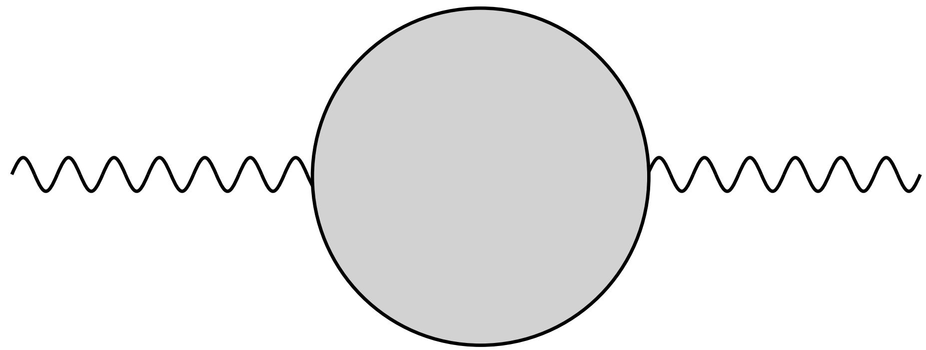


$O(\alpha^3)$, Light-by-light scattering [M. Hayakawa *et al.*, hep-lat/0509016]:



- Hadronic contributions are 7×10^{-5} times smaller than leading corrections

Vacuum Polarization



- We can extract the $O(\alpha^2)$ hadronic contribution to a_μ from the vacuum polarization using (Blum, 2003)

$$a_\mu^{(2)\text{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\Pi(K^2)$$

- $f(K^2)$ diverges as $K^2 \to 0 \Longrightarrow$ dominated by low momentum region—Need large lattices to simulate these low momenta accurately

Lattice Calculation

- Calculate the vacuum polarization using the conserved current

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iq\cdot(x-y)}\langle J^\mu(x)J^\nu(y)\rangle = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$$

$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

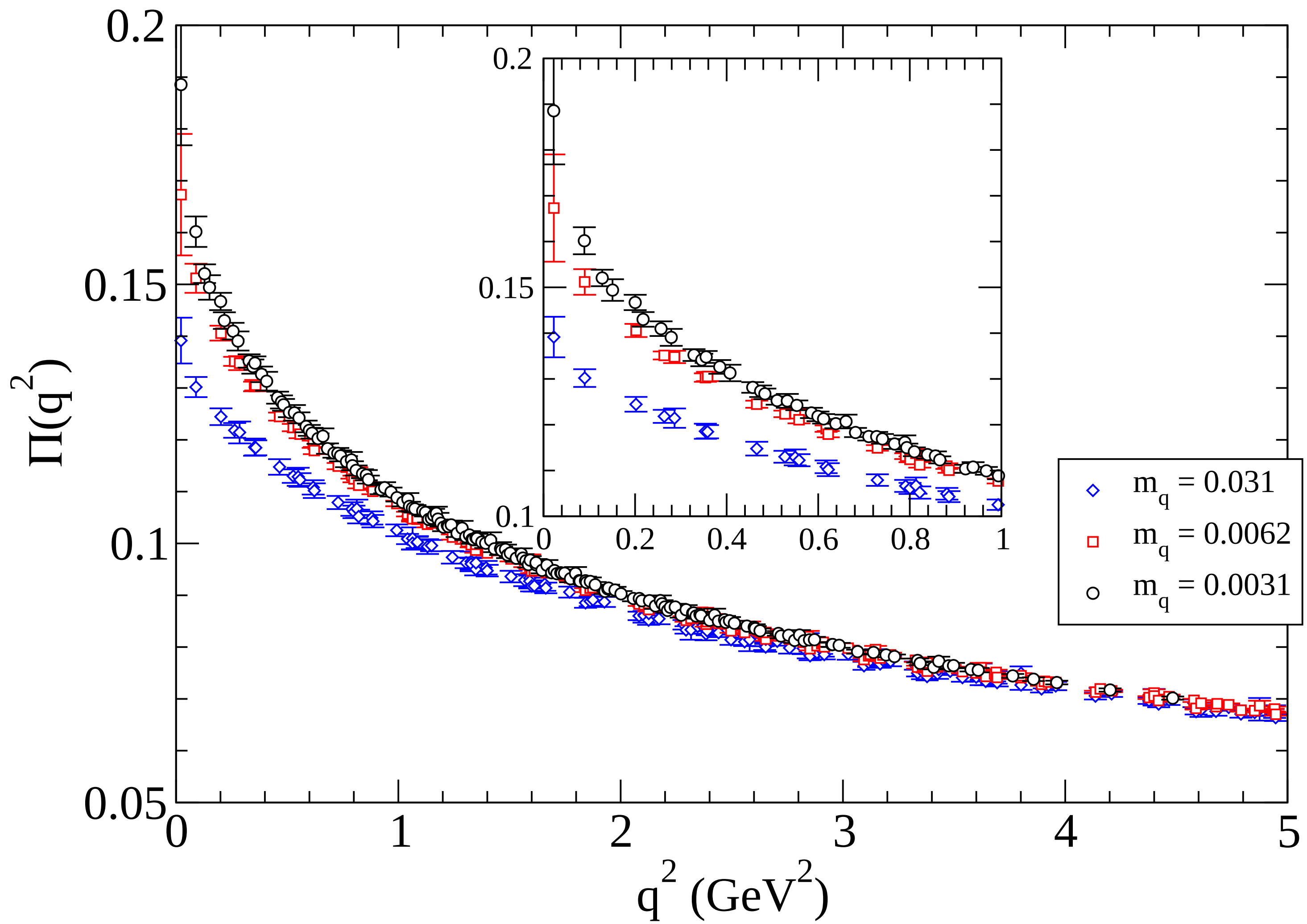
- On the lattice:
$J^\mu(x) = \frac{1}{2}\left[\bar{\psi}(x+\hat{\mu})U^\dagger(x)(1+\gamma^\mu)\psi(x) - \bar{\psi}(x)U(x)(1-\gamma^\mu)\psi(x+\hat{\mu})\right]$
- hard to fit low-q^2 region$\Rightarrow$ χPT
- For more details on the lattice calculation, see
  - T. Blum, PRL 91 052001, 2003—Quenched DWF
  - T. Blum, Confinement 2003 (hep-lat/0310064)—Dynamical Staggered

Simulation parameters

MILC $2+1$ Asqtad Configurations

a (fm)	Volume	am_l	am_s	$am_{\rm val}$
0.086(2)	$28^3 \times 96$	0.0062	0.031	0.031
0.086(2)	$28^3 \times 96$	0.0062	0.031	0.0062
0.086(2)	$40^3 \times 96$	0.0031	0.031	0.0031

Simulation Results ($2+1$ Staggered)



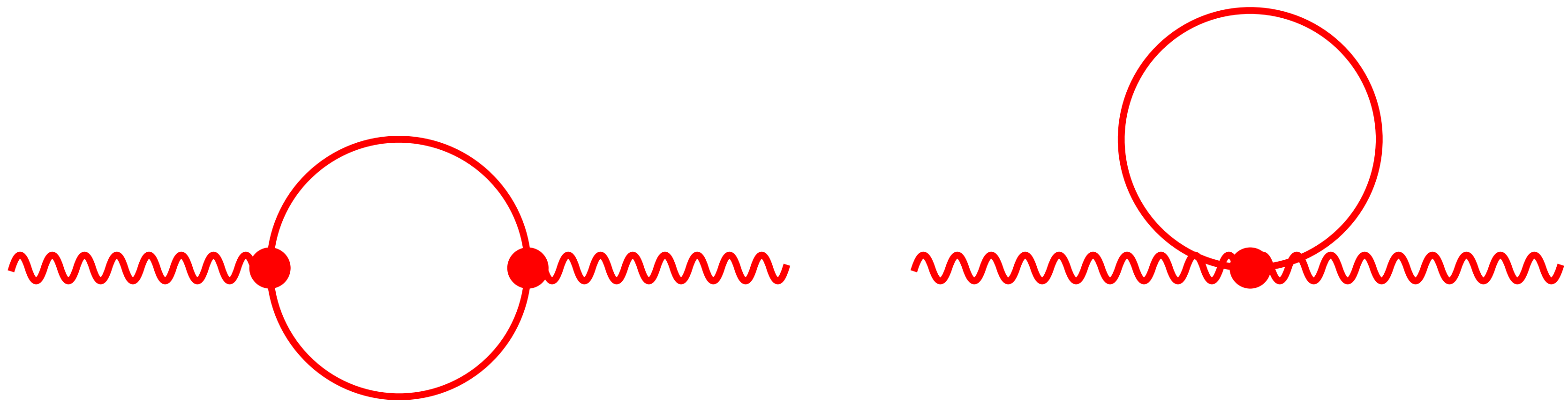
SχPT

- Calculate vacuum polarization with SχPT coupled to photons.
- Taste violations: easy to incorporate (no new TV terms in lagrangian with photons)

$$\mathcal{L} = \frac{f^2}{8}\,\mathrm{Tr}[D_\mu \Sigma D^\mu \Sigma^\dagger] + \frac{\mu f^2}{4}\,\mathrm{Tr}[\mathcal{M}\Sigma + \Sigma^\dagger \mathcal{M}] - a^2 \mathcal{V}$$

  - $D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu [Q, \Sigma]$
  - A_μ: photon field
  - Q: quark charge matrix
  - $\mathcal{M}$: light quark mass matrix
  - $\mathcal{V}$: taste symmetry breaking potential

SχPT Diagrams



SχPT Result

- One-loop pion/kaon contribution:

$$\Pi_{M_t}(p^2) = \frac{\alpha}{4\pi}\left\{\frac{1}{3}\left(1+x_{M_t}\right)^{3/2}\ln\left(\frac{\sqrt{1+x_{M_t}}+1}{\sqrt{1+x_{M_t}}-1}\right) - \frac{2x_{M_t}}{3} - \frac{8}{9} + \frac{1}{3}\ln\left(\frac{m_{M_t}^2}{\Lambda^2}\right)\right\}$$

$$\Pi(p^2) = \frac{1}{16}\sum_{t,M}\Pi_{M_t}(p^2) + \text{c. t.}$$

$$x_{M_t} = \frac{4m_{M_t}^2}{p^2} \quad , \quad m_{M_t}^2 = \mu(m_x + m_y) + a^2\Delta_t$$

$$M \in \{\pi^+, K^+\} \quad , \quad t \in \{P, A, T, V, S\}$$

- Nice:
  - No free parameters (besides counterterm—this is just a constant)
  - Taste violations enter trivially
- Bad: Two orders of magnitude too small!!

SχPT with vectors

- Quenched result is dominated by the vectors (QCDSF)—but the vector masses are heavy...
- Use resonance formalism of Ecker, Gasser, and Pich [NPB 321 311 (1989)]
- Incorporate vectors into field $V_{\mu\nu}$ so that under chiral $SU(12) \times SU(12)$:

$$V_{\mu\nu} \to U V_{\mu\nu} U^\dagger$$

where $U \in SU(12)$ is defined by

$$\sigma \to L \sigma U^\dagger = U \sigma R^\dagger$$

with $\sigma^2 = \Sigma$

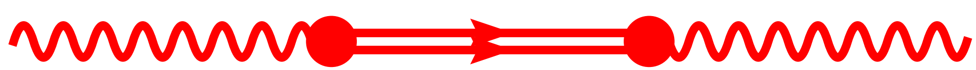
SχPTwith vectors

- So we have the interaction Lagrangian

$$\mathcal{L}_{\rm vec} = \frac{f_V}{2\sqrt{2}} \,\mathrm{Tr}\left[V_{\mu\nu}(\sigma F^{\mu\nu}\sigma^\dagger + \sigma^\dagger F^{\mu\nu}\sigma)\right] + \ldots$$

$$F^{\mu\nu} = eQ(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

- $V_{\mu\nu}$ is a 12×12 matrix with the 9 vector mesons (each with 16 tastes)
- Leading contribution to the photon vacuum polarization is at tree level:



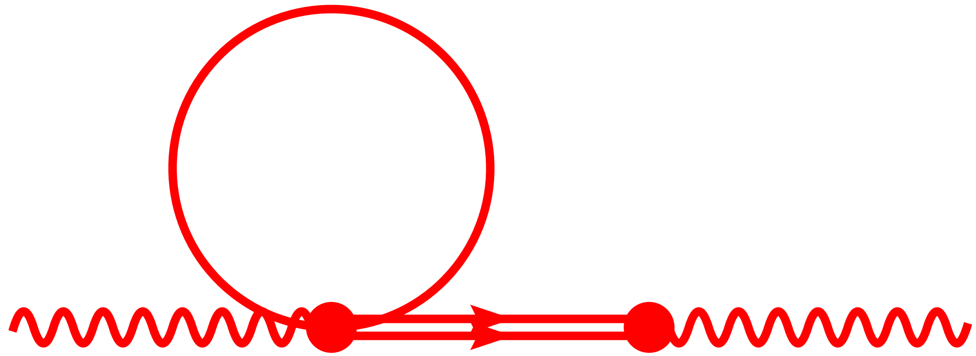
Tree-level result

- Tree-level result:

$$\Pi_V(p^2) \quad = \quad -\frac{\alpha}{4\pi}\frac{(4\pi)^2 f_V^2}{3}\left[\frac{3}{p^2+m_{\rho^0}^2}+\frac{1}{p^2+m_\omega^2}\right]$$

- Although the masses are heavy, the numerator has enhancement of $(4\pi f_V)^2$.
- There are no free parameters: The masses and f_V can be measured directly in the simulations (f_V not measured yet, in progress)
- What about taste violations?
  - Taste splitting is negligible for the vectors.
  - Since there are no pions, these don't arise here.

One-loop with vectors

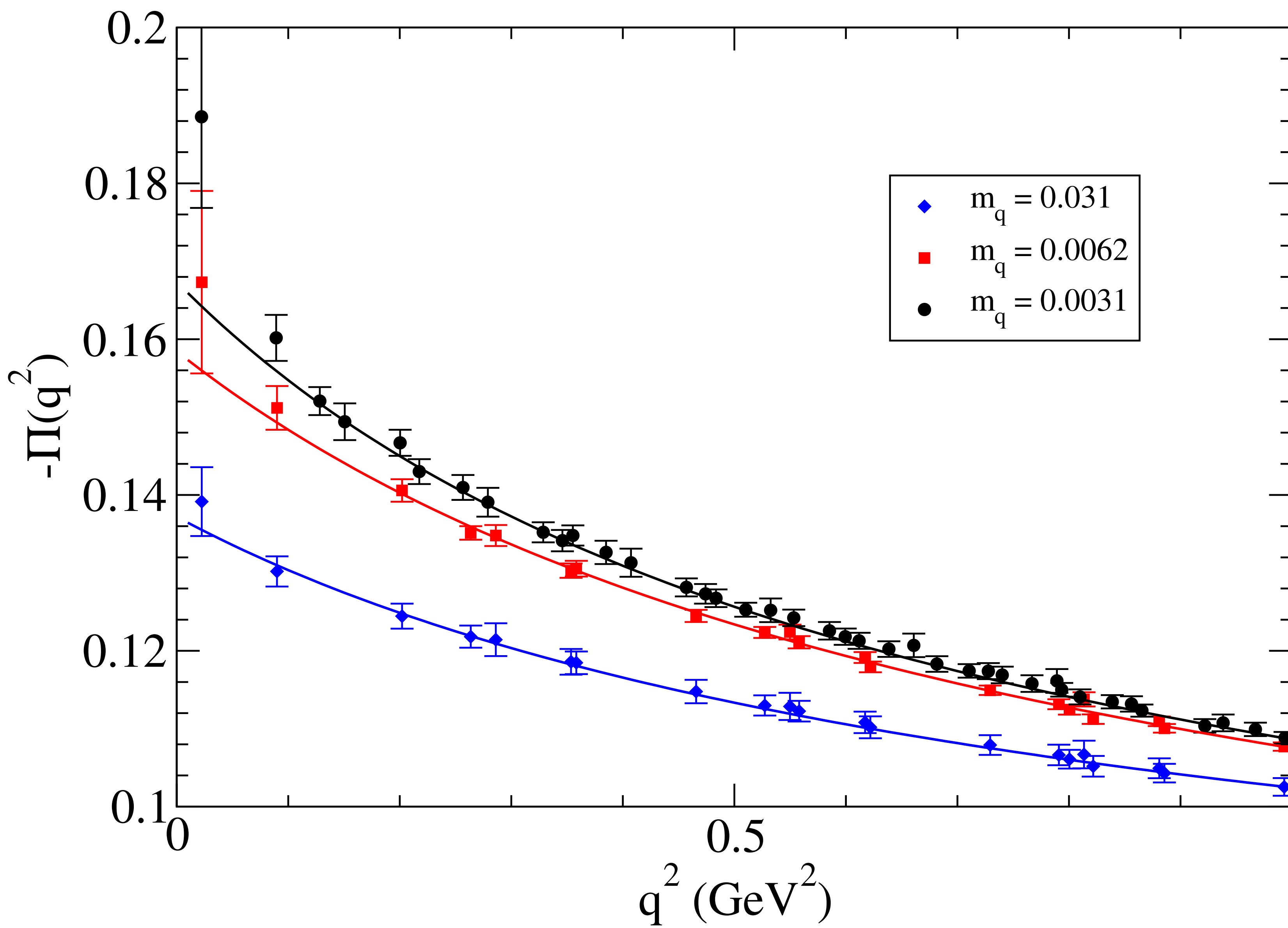


- One-loop calculation: only tadpole corrections to $\rho-$photon vertex

$$\Pi_V^{1-\mathrm{loop}}(p^2) = \frac{\alpha}{4\pi}\left(\frac{4f_V^2}{f^2}\right)\frac{1}{16}\sum_t\left[2\frac{m_{\pi_t}^2\ln m_{\pi_t}^2}{p^2+m_\rho^2}+\frac{m_{K_t}^2\ln m_{K_t}^2}{p^2+m_\rho^2}+\frac{m_{K_t}^2\ln m_{K_t}^2}{p^2+m_\omega^2}\right]$$

- Again, no hairpins: no neutral pions in the loop
- Magnitude of this term is same as one-loop pion contribution

Fit to χPT result



Fits & Preliminary Results

- Keep f_V as a free parameter—fits well and gives $\approx 170 - 180$ MeV

$$
\begin{aligned}
a_\mu^{\mathrm{had,VP}}(0.0062) &= 539(11) \times 10^{-10} \\
a_\mu^{\mathrm{had,VP}}(0.0031) &= 618(15) \times 10^{-10} \\
a_\mu^{\mathrm{had,VP}}(\mathrm{phys}) &= 657(20) \times 10^{-10} \\
a_\mu^{\mathrm{had,VP,pert}}(\mathrm{phys}) &\lesssim 10 \times 10^{-10} \\
a_\mu^{\mathrm{had,disp}}(\mathrm{phys}) &= 693.4(5.3)(3.5) \times 10^{-10}
\end{aligned}
$$

- Statistical errors only
- This is increased from previous result of $545(65) \times 10^{-10}$ [Blum, hep-lat/0310064]
- Last line is from e^+e^- data and dispersion relation [A. Hocker, review talk, ICHEP 2004 (Beijing)]

Discussion

- Possible issues:
  - Haven't included disconnected diagrams in lattice calculation (very noisy)
  - No Naik term in valence quark propagators
  - Finite Volume Effects?
- Pion Contribution is negligible
- Rho Contribution:
  - Dominates (VMD)
  - Model dependence?

Summary & Outlook

- Functional form from SχPT fits well to lattice data with few unknown parameters
- Need to understand why fit undershoots data (finite volume or other artifact?)
- How model dependent is the ρ contribution?