Finite density simulations via canonical approach

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Outline

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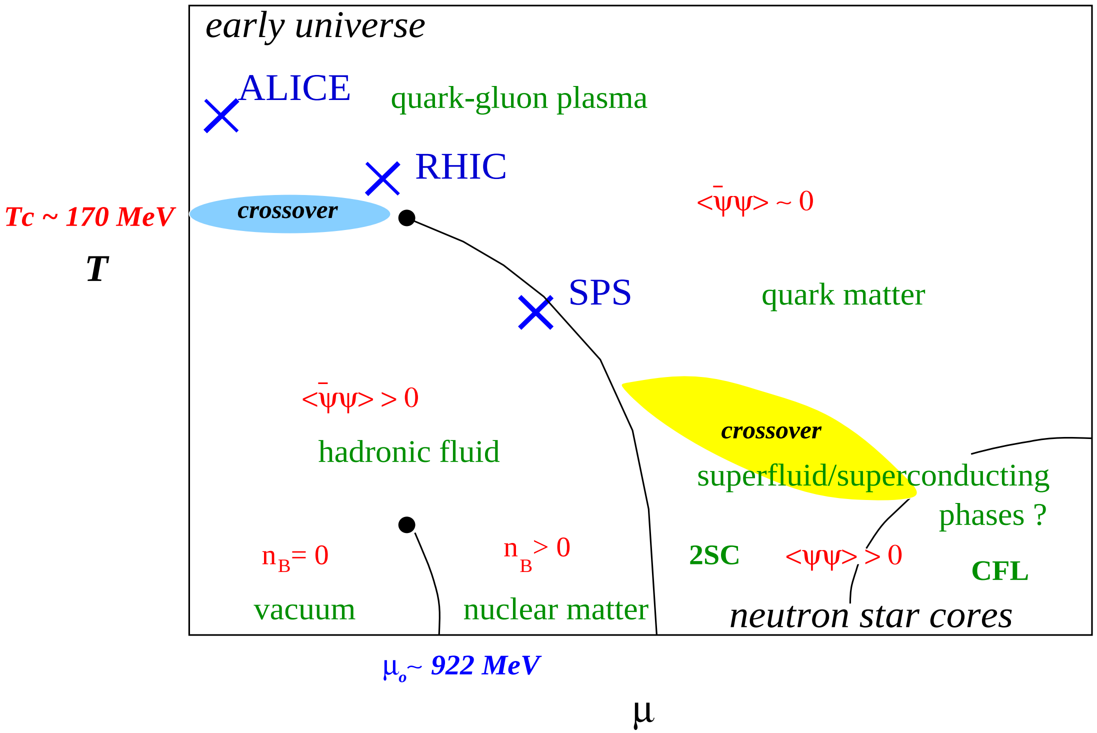
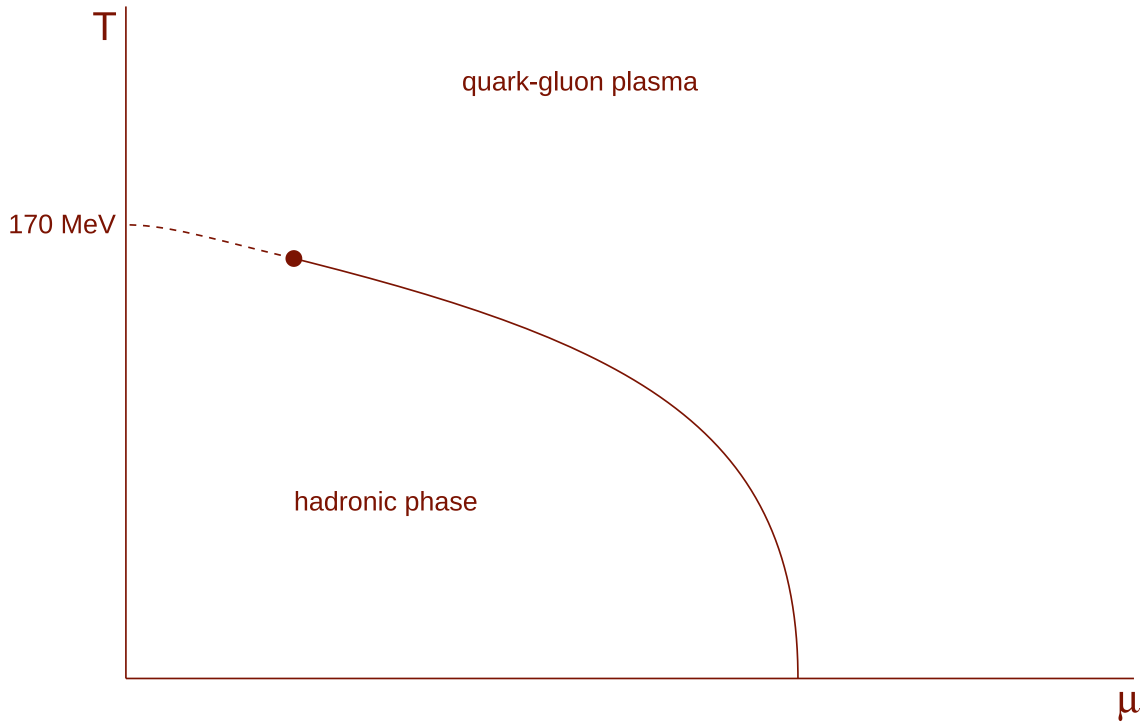
- Canonical partition function
- Algorithm
- Results
- Outlook

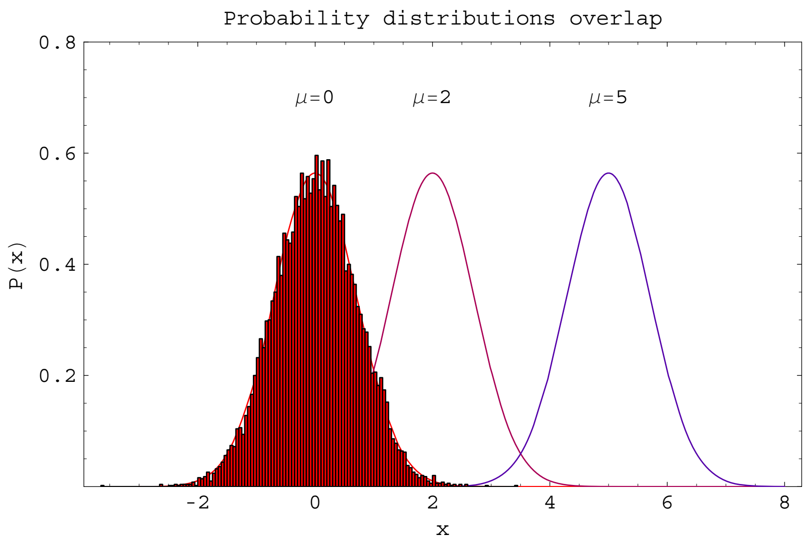
Motivation – Phase diagram

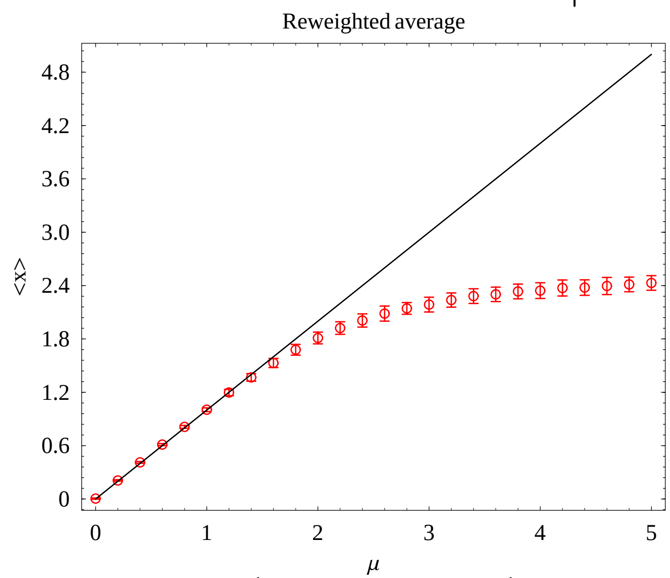


Motivation



Overlap problem



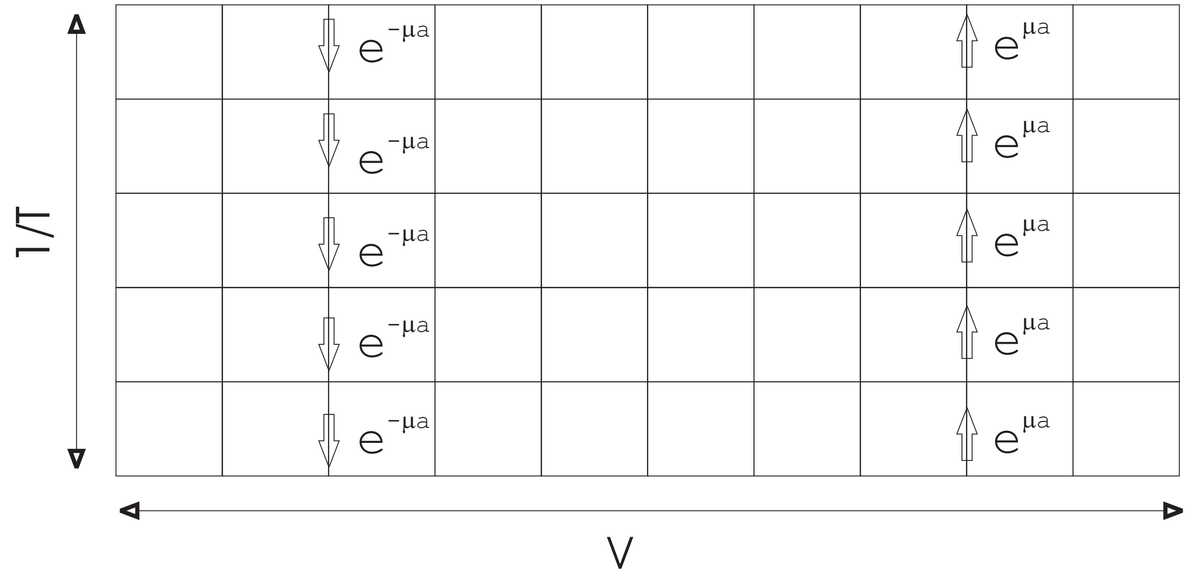


$$P(\mu; x) = \frac{1}{\sqrt{\pi}} e^{-(x-\mu)^2}$$

$$\langle x \rangle_\mu = \frac{\left\langle x \dfrac{P(\mu; x)}{P(0; x)} \right\rangle_0}{\left\langle \dfrac{P(\mu; x)}{P(0; x)} \right\rangle_0}$$

Grand canonical partition function

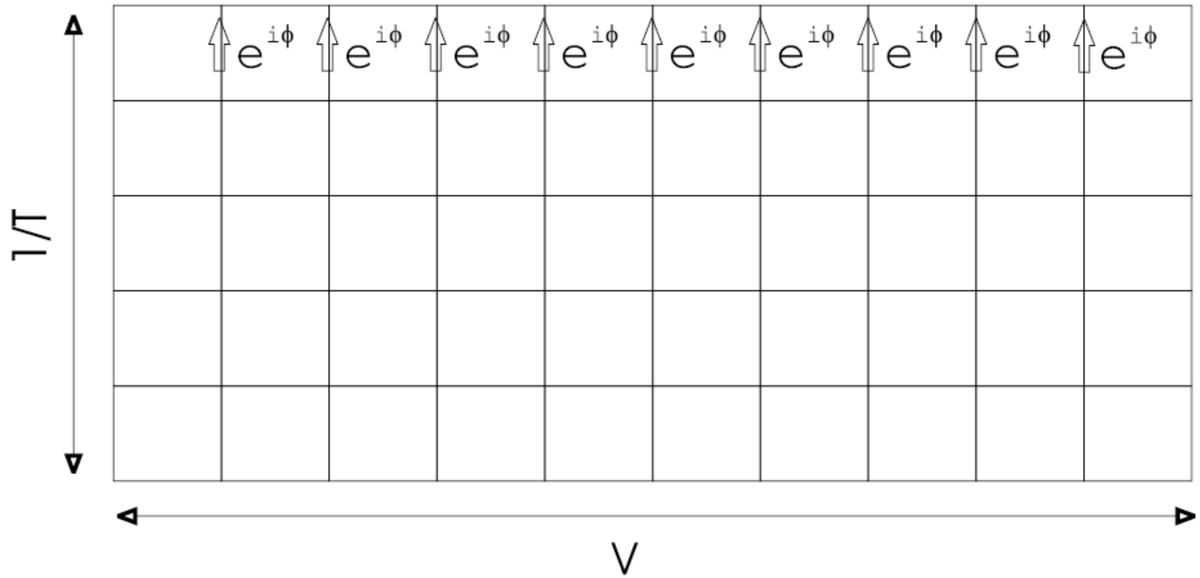
$$Z_{GC}(V,\mu,T)=\int DUD\overline{\psi}\,D\psi \quad e^{-S_G[U]-S_F[\mu;U,\overline{\psi},\psi]}$$



$$S_F[\mu;U,\overline{\psi},\psi]=\overline{\psi}\;M[\mu;U]\psi \qquad U_4\rightarrow U_4e^{-\mu a}$$

$$\overline{\psi}\;M[\mu;U]\psi=\sum_n(\overline{\psi}_n\psi_n+\kappa\overline{\psi}_{n+\hat{t}}(1+\gamma_4)U_4^{+}(n)e^{\mu a}\psi_n+\kappa\overline{\psi}_n(1-\gamma_4)U_4(n)e^{-\mu a}\psi_{n+\hat{t}}+\ldots)$$

Canonical partition function



Using the fugacity expansion $Z_{GC}(V,\mu,T) = \sum_{n=-4V}^{n=4V} Z_C(V,n,T) e^{\frac{\mu}{T}n}$ we get

$$Z_C(V,n,T) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-in\varphi} Z_{GC}(V,\mu = i\varphi T, T)$$

$$Z_C(V,n,T) = \int DU\, e^{-S_G[U]} \det{}_n M^2(U)$$

Projected determinant

$$\det{}_n M^2(U) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-in\varphi} \det M^2(U, \mu = i\varphi T)$$

$$\downarrow$$

$$\det{}'_n M^2(U) = \frac{1}{N}\sum_{j=0}^{N-1} e^{-in\varphi_j} \det M^2(U, \mu = i\varphi_j T), \quad \varphi_j = \frac{2\pi}{N} j$$

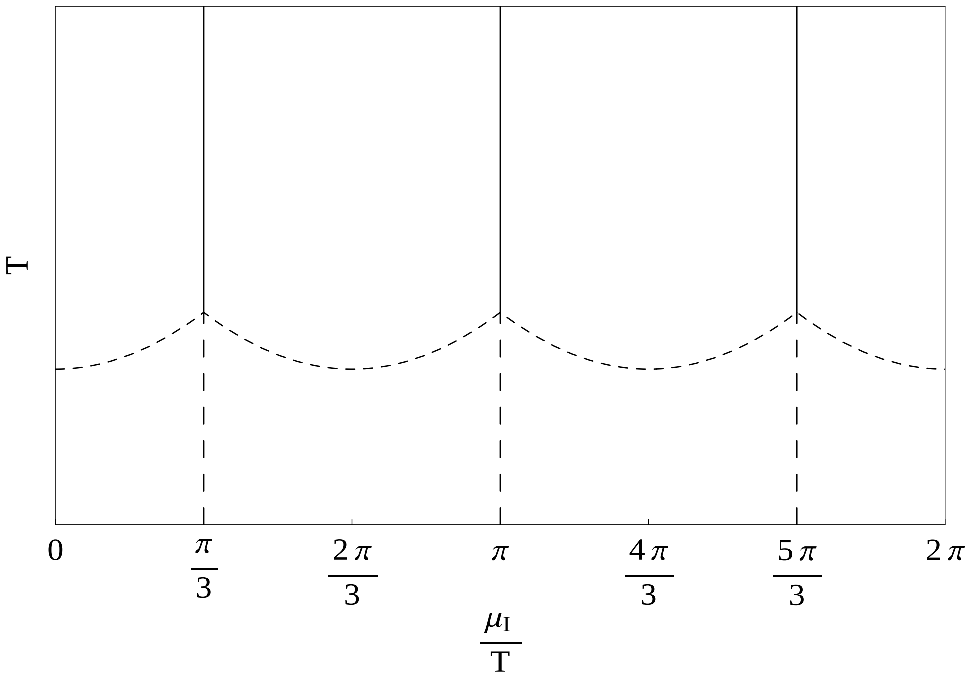
From the fugacity expansion we see that

$$Z'_C(n) = \sum_{k=-\infty}^{+\infty} Z_C(n+kN) = \ldots + Z_C(n-N) + Z_C(n) + Z_C(n+N) + \ldots$$

The most important mixing comes from the n-N sector. This should be suppressed by a factor of exp(-[F(N-n)-F(n)]/T).

Triality

$$Z_{GC}(\mu = \mu_R + i(\mu_I + \frac{2\pi T}{3})) = Z_{GC}(\mu = \mu_R + i\mu_I)$$



$$Z_C(V, n, T) = 0 \; if \;\; n \neq 3B$$

Algorithm

$$Z_C(V,n,T) = \int DU\; e^{-S_G(U)} \det{}_n M^2(U) =$$

$$\underbrace{\int DU\; e^{-S_G(U)} \det M^2(U,0)}_{\text{Standard HMC}} \underbrace{\frac{\left|\mathrm{Re}\det{}_n M^2(U)\right|}{\det M^2(U,0)}}_{\text{Accept/Reject}} \underbrace{\frac{\det{}_n M^2(U)}{\left|\mathrm{Re}\det{}_n M^2(U)\right|}}_{\text{Phase}}$$

Standard HMC — Accept/Reject — Phase

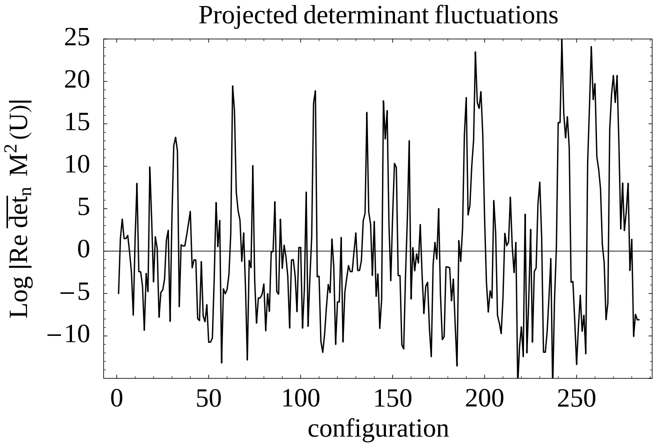
For HMC we use the phi algorithm for 2 degenerate flavors.

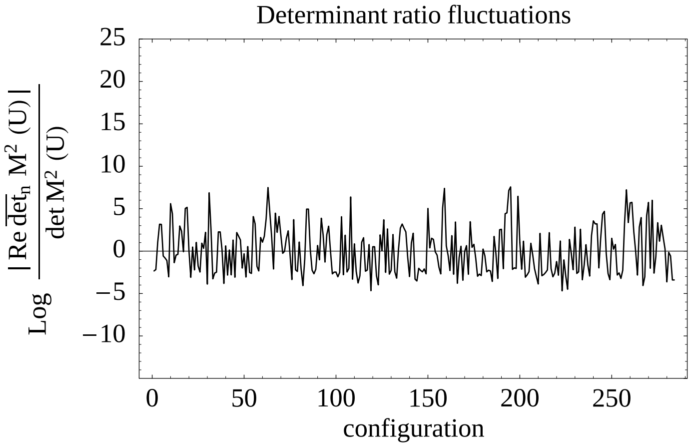
The HMC proposal is accepted/rejected based on the determinant ratio.

Z(3) hopping is performed at the end of each HMC trajectory.

Fluctuations

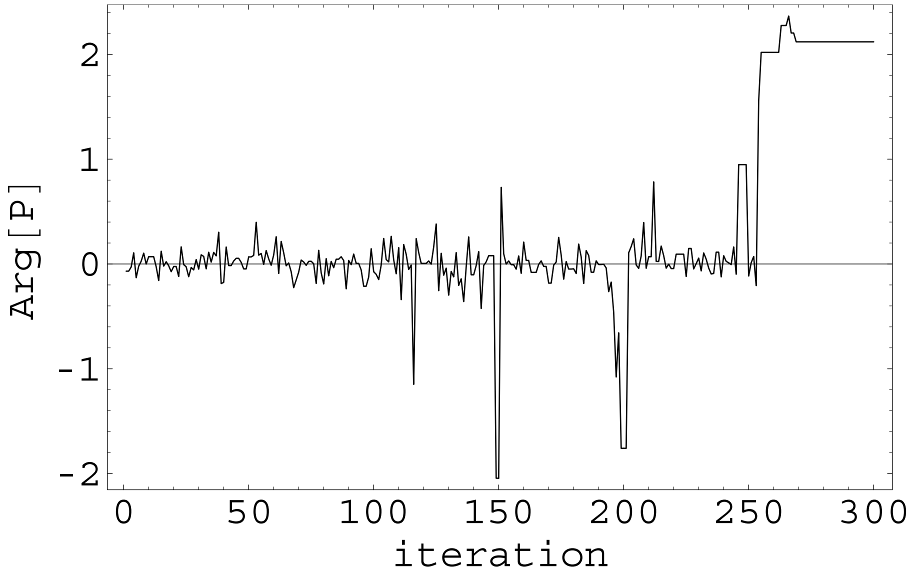
Using the HMC as a proposal rather than pure gauge update decreases fluctuations and improves the acceptance rate.





Z(3) hopping

- The canonical partition function is Z(3) symmetric
- To preserve this in the discretized version N has to be a multiple of 3
- The proposal mechanism (HMC) breaks this symmetry and can freeze the simulation



$$U \to U(\pm 2\pi / 3)$$

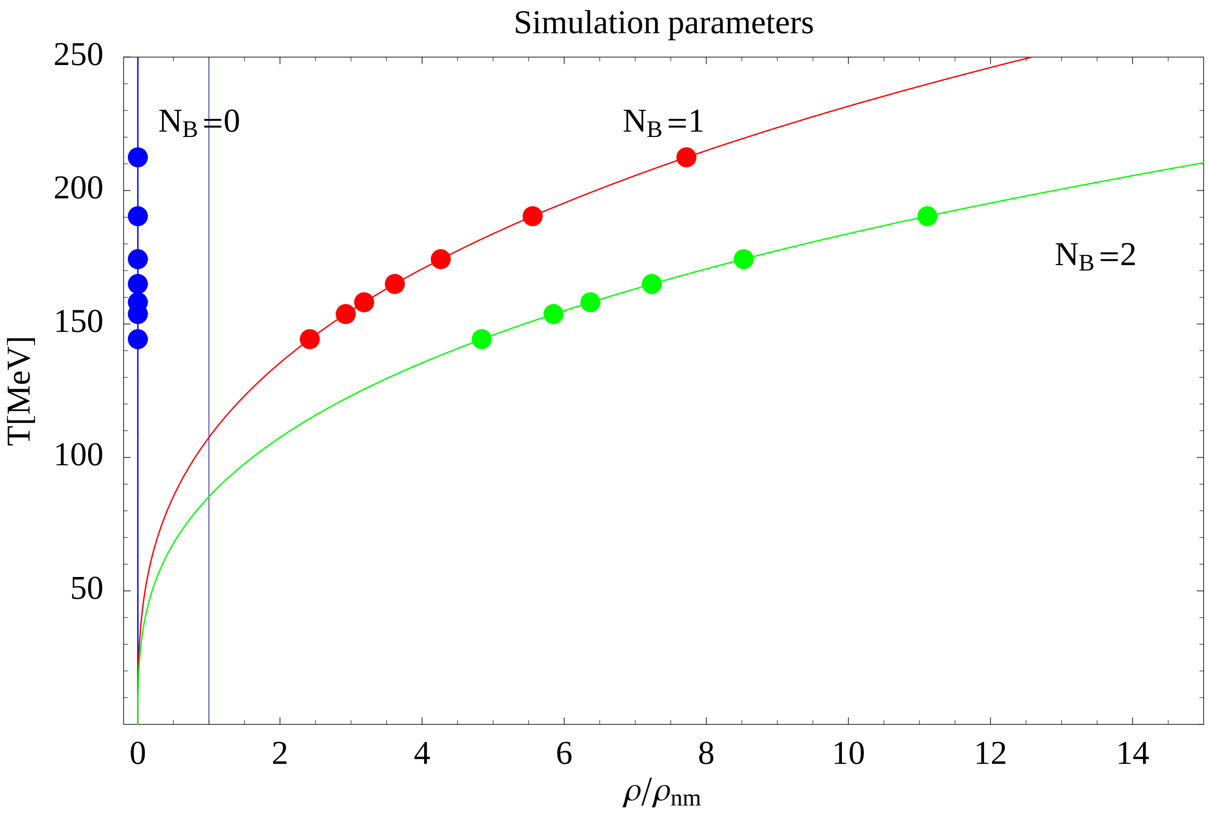
Run parameters

All runs are on a 4^4 lattice with Wilson fermions at $\kappa = 0.158$.

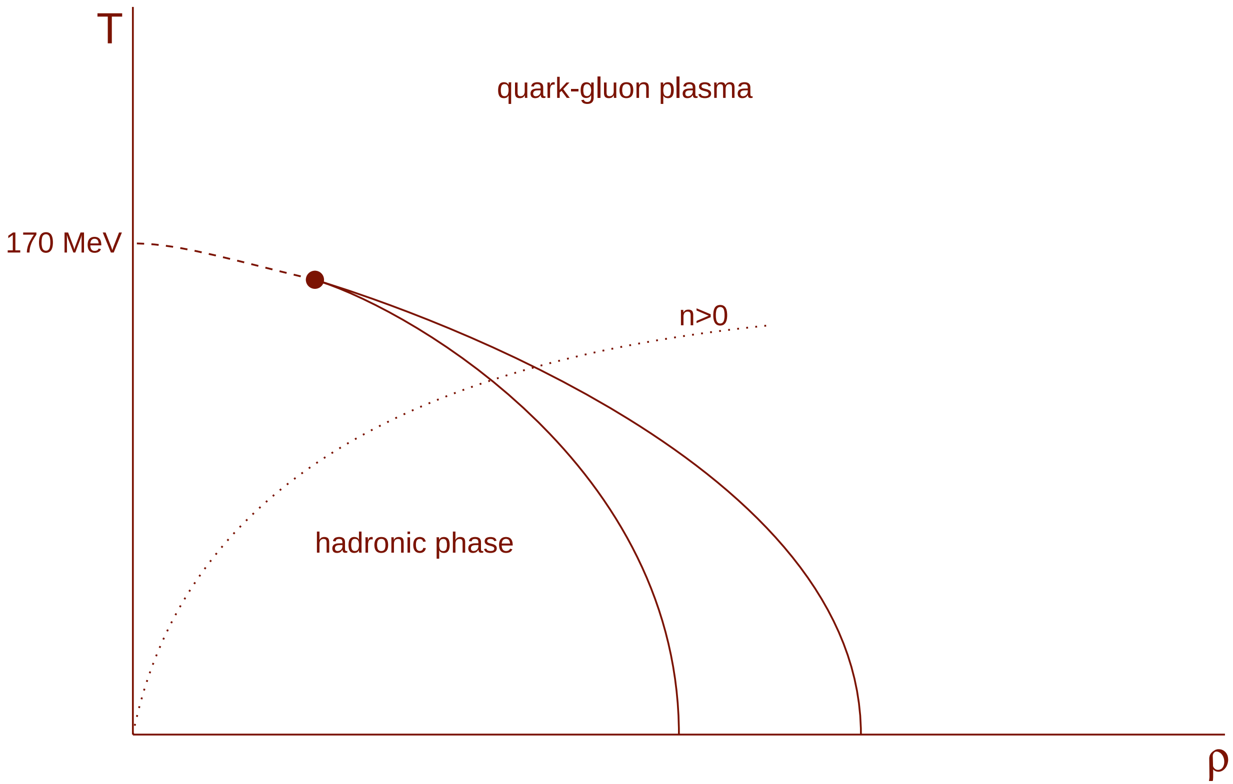
β	a(fm)	m_π(MeV)	$V^{-1}(fm^{-3})$	T(MeV)
5.00	0.343(2)	926(7)	0.387(7)	144(1)
5.10	0.322(4)	945(13)	0.468(17)	153(2)
5.15	0.313(3)	942(11)	0.510(15)	157(2)
5.20	0.300(1)	945(5)	0.579(6)	164(1)
5.25	0.284(5)	945(20)	0.682(36)	173(3)
5.30	0.260(1)	973(9)	0.889(10)	189(1)
5.35	0.233(2)	959(14)	1.235(32)	211(2)

We adjusted the length of the HMC trajectories to keep the acceptance rate at about 15-30%.

Run parameters

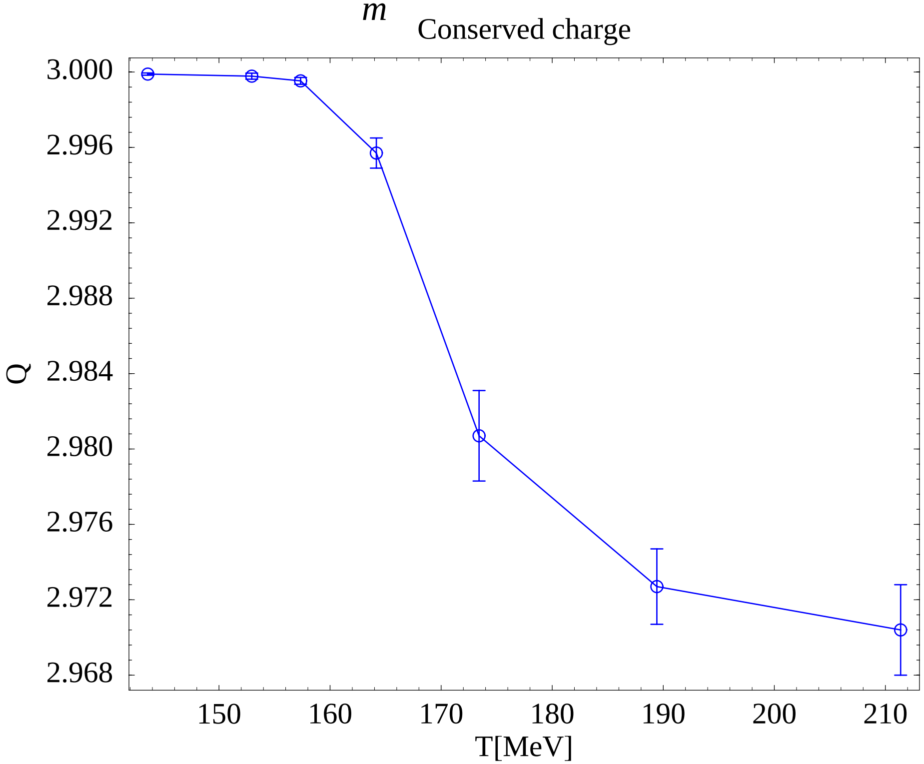


Phase diagram



Sector mixing

$$\langle Q \rangle_{Z_C'(n)} = \frac{\sum_m (n+mN) Z_C(n+mN)}{\sum_m Z_C(n+mN)}$$

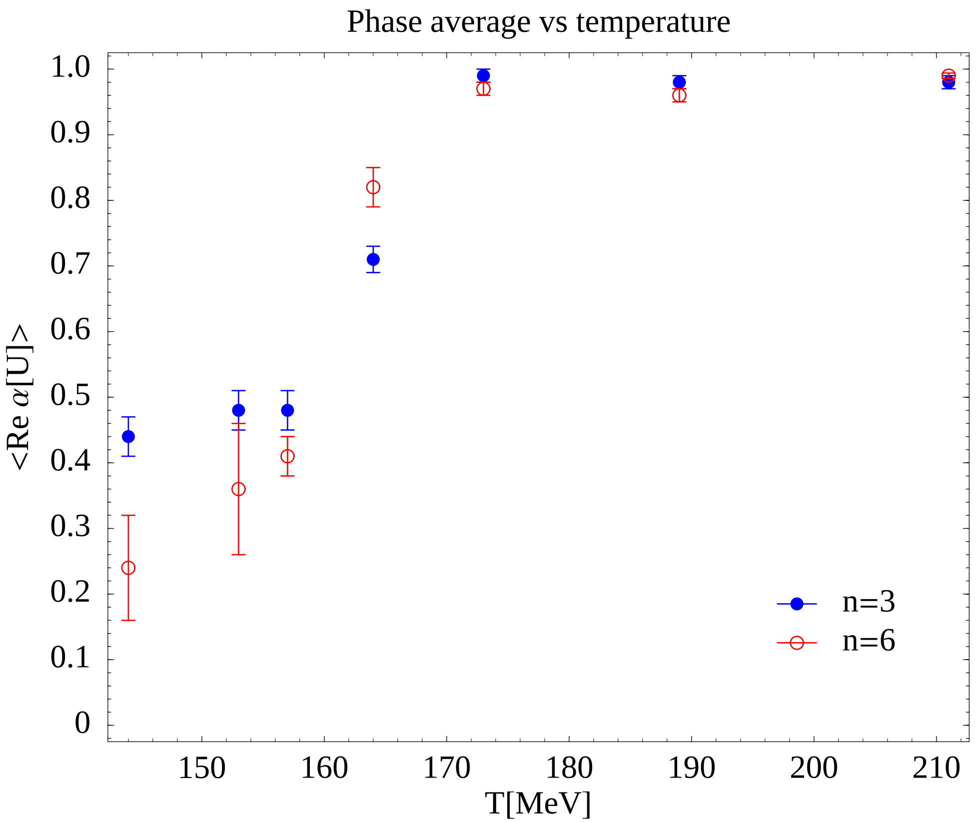


For N=12 we have Q=0 for n=0 and n=6. The only non-trivial case is n=3.

Below T~170MeV there is almost no mixing.

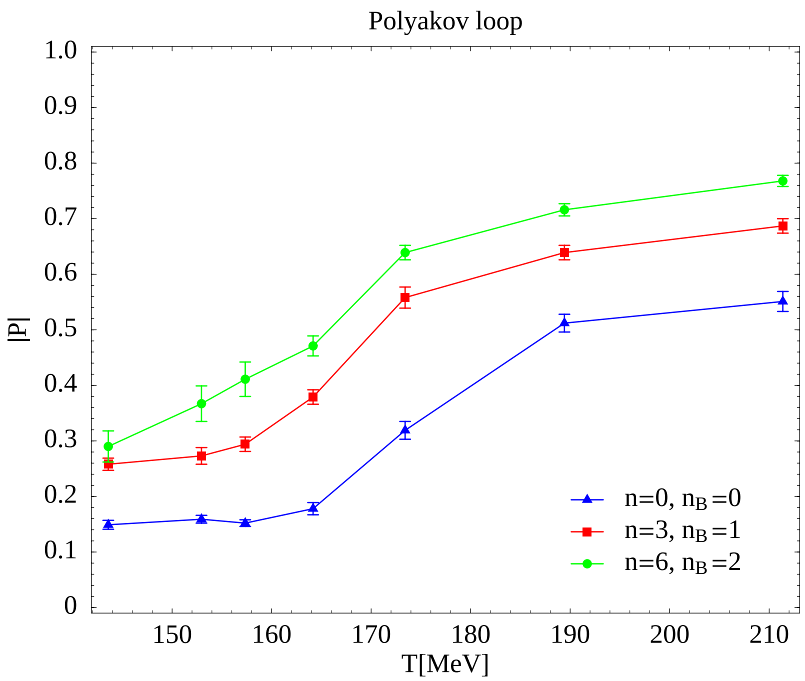
Sign problem

$$\alpha(U) = \frac{\det_n M(U)}{\left|\mathrm{Re}\det_n M(U)\right|} \qquad \left\langle O \right\rangle_n = \frac{\left\langle O(U)\alpha(U) \right\rangle}{\left\langle \alpha(U) \right\rangle}$$



Polyakov loop

We have to reintroduce the phase factor $\langle |P| \rangle_{\det_n M} = \dfrac{\langle |P| \alpha \rangle_{|\mathrm{Re}\det_n M|}}{\langle \alpha \rangle_{|\mathrm{Re}\det_n M|}}, \quad \alpha = \dfrac{\det_n M}{|\mathrm{Re}\det_n M|}$



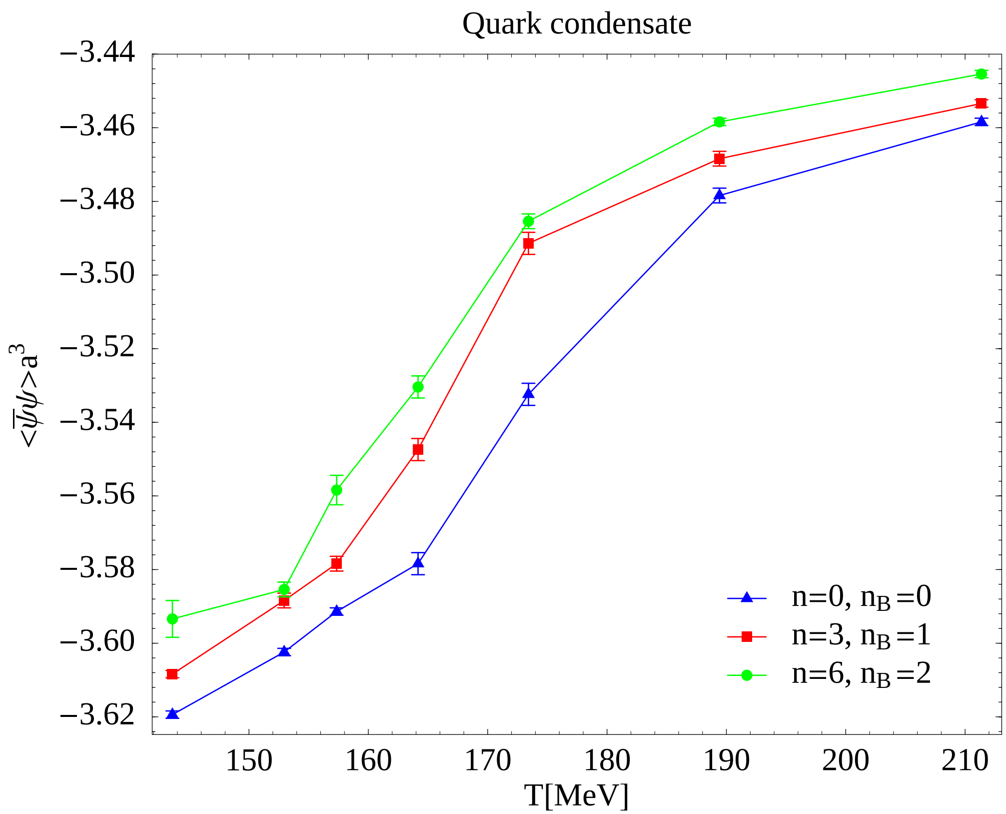
At T~170MeV we observe a sharp increase in |P|.

This signals deconfinement.

To see a shift in the transition temperature we need more data.

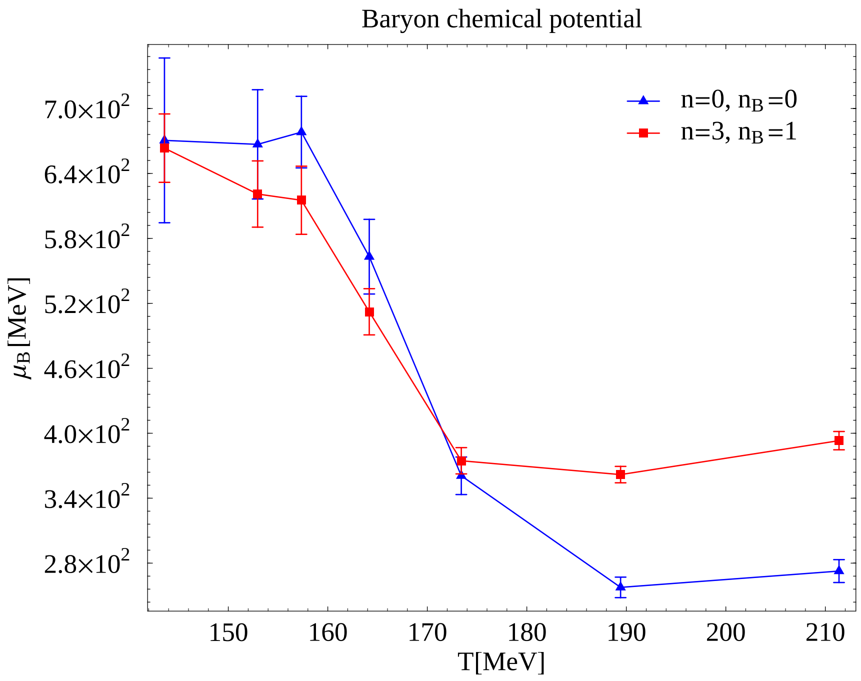
Chiral condensate

$$\left\langle \bar{\psi}\Gamma\psi \right\rangle_n = \left\langle \sum_{n'=0}^{N-1} \frac{\det_{n'} M^2}{\det_n M^2} \left(-2Tr_{n-n'}\Gamma M^{-1}\right) \right\rangle_n \qquad Tr_n \Gamma M^{-1} \equiv \frac{1}{N} \sum_{j=0}^{N-1} e^{-in\varphi_j} Tr\Gamma M(U_{\varphi_j})^{-1}$$



Chemical potential

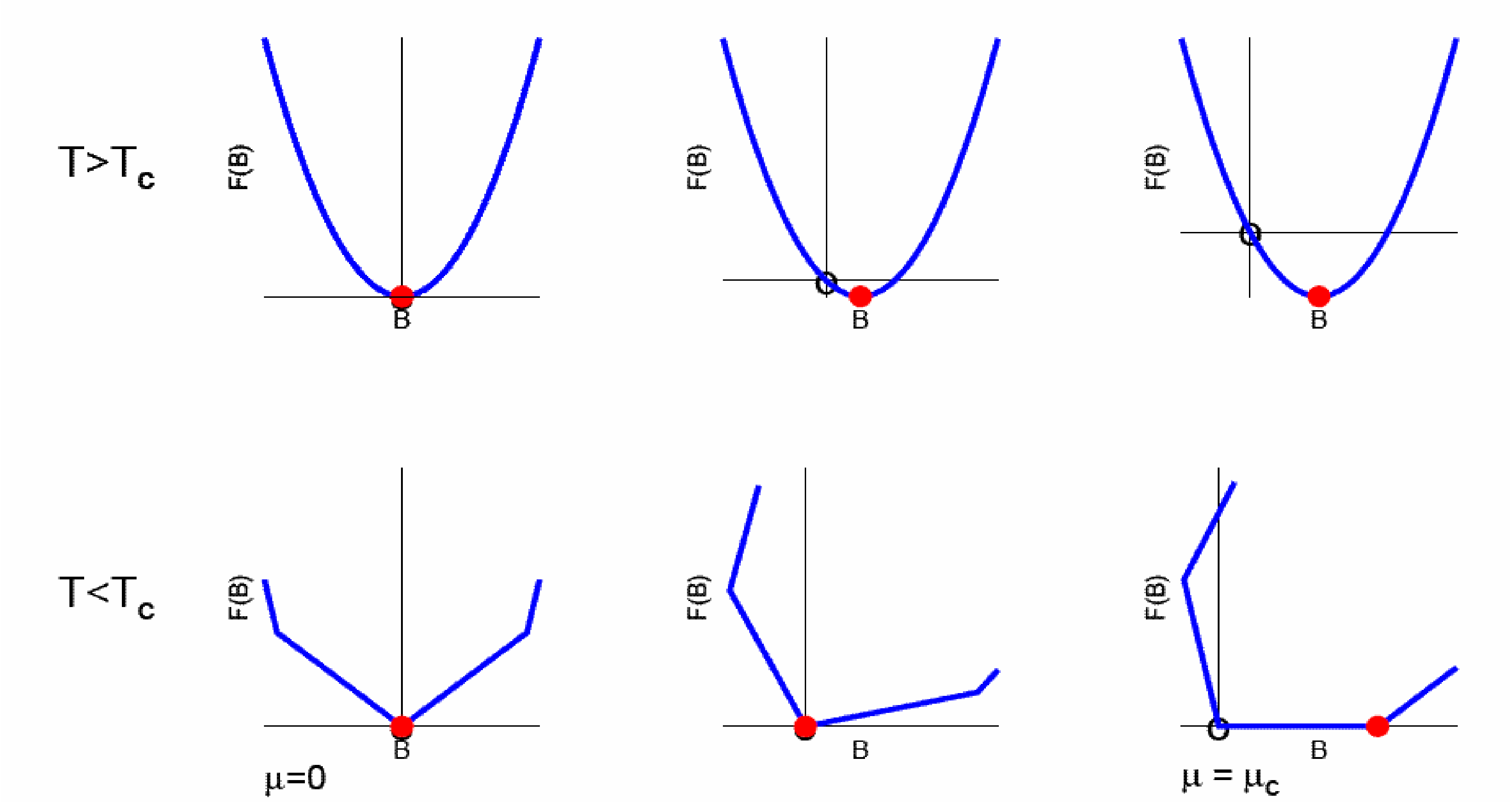
$$\mu_B = F(B+1) - F(B) = -\frac{1}{\beta}\ln\frac{Z_{B+1}}{Z_B} = -\frac{1}{\beta}\ln\left\langle e^{-i3\theta}\right\rangle_{3B}$$



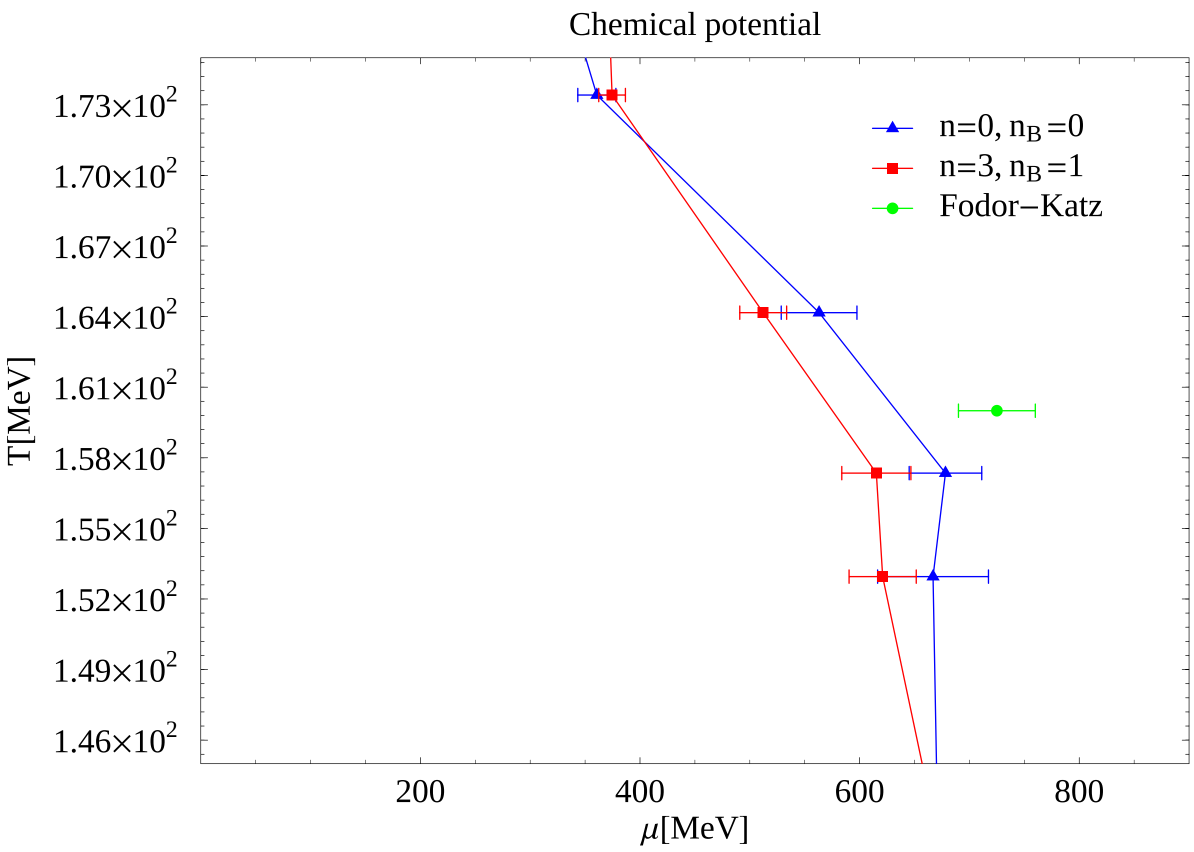
Below T~170MeV we see that the chemical potential is the same for both densities.

Above T~170MeV we have a repulsive interaction between quarks.

Simple model



Transition line



Conclusions and Outlook

- This is an exploratory study. We are mainly interested in the feasibility of the algorithm and to find ways to improve it.
- We are able to simulate at large densities at least in the vicinity of the critical temperature.
- This method can be used to determine the reliability of the reweighting techniques.

- Implement an estimator to explore larger lattices, smaller densities.