GPDs and DVCS with Positrons

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Outline

- probing GPDs in DVCS
- Ji-relation
- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
- more on DVCS $\longrightarrow$ GPDs
- Summary

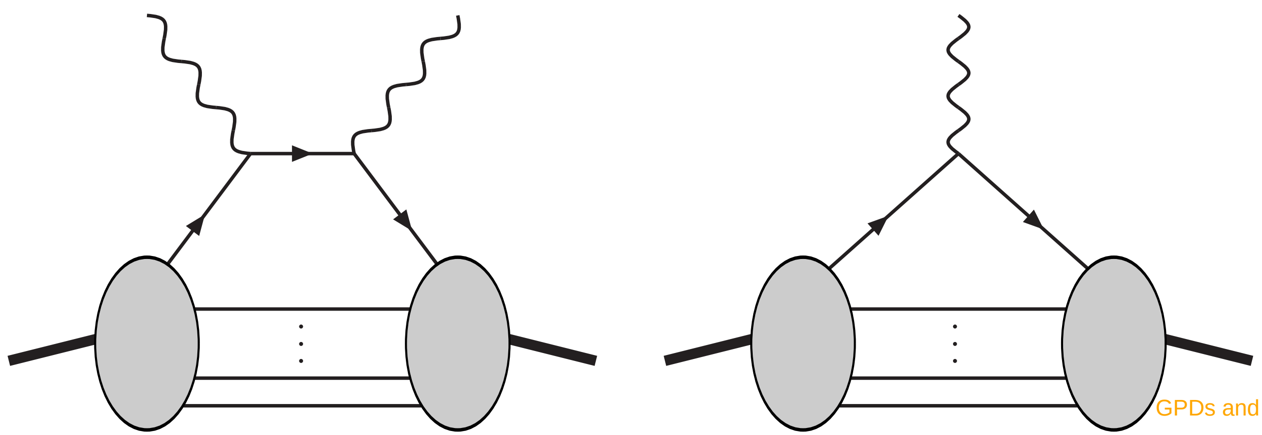
Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

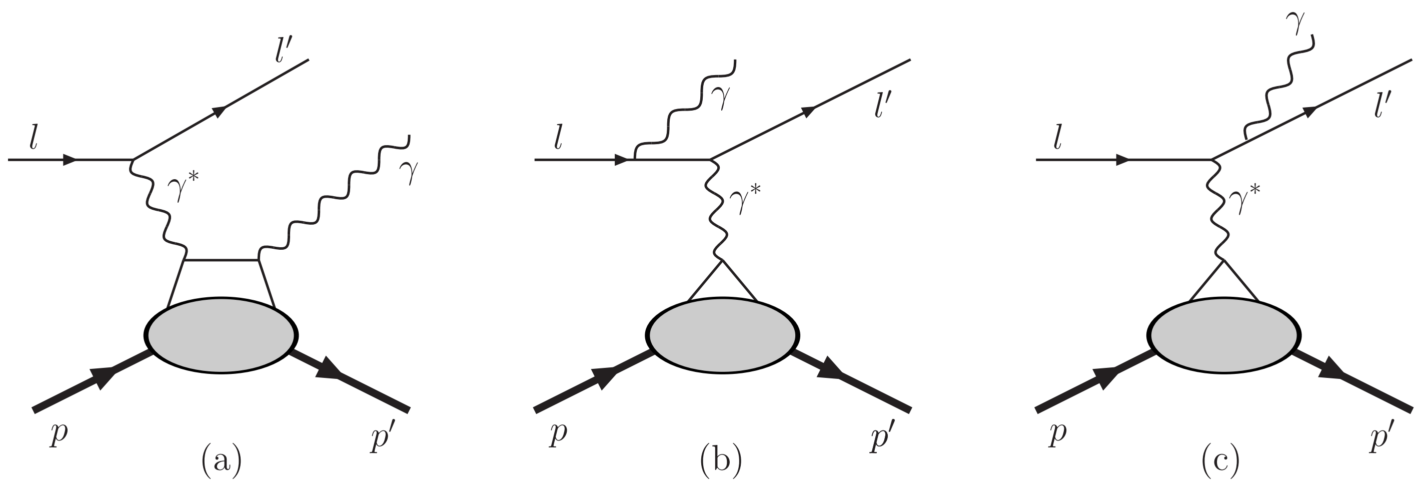
$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$

$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Phase of DVCS Amplitude



- $\sigma = |\mathcal{A}_{BH} + \mathcal{A}_{DVCS}|^2 = |\mathcal{A}_{BH}|^2 + |\mathcal{A}_{DVCS}|^2 + 2\Re\{\mathcal{A}_{BH}\mathcal{A}^*_{DVCS}\}$

$\hookrightarrow$ clean separation of real part with beam charge asymmetry (e^+ v. e^-)

$$\Re\mathcal{A}_{DVCS}(\xi, t) \sim \int_{-1}^{1} dx \frac{GPD(x, \xi, t)}{x - \xi}$$

- $\Im\mathcal{A}_{DVCS}(\xi, t) \sim GPD(\xi, \xi, t)$ from beam spin asymmetry

Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$

$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- DVCS amplitude

$$\mathcal{A}(\xi,t) \sim \int_{-1}^{1} \frac{dx}{x-\xi+i\varepsilon} GPD(x,\xi,t)$$

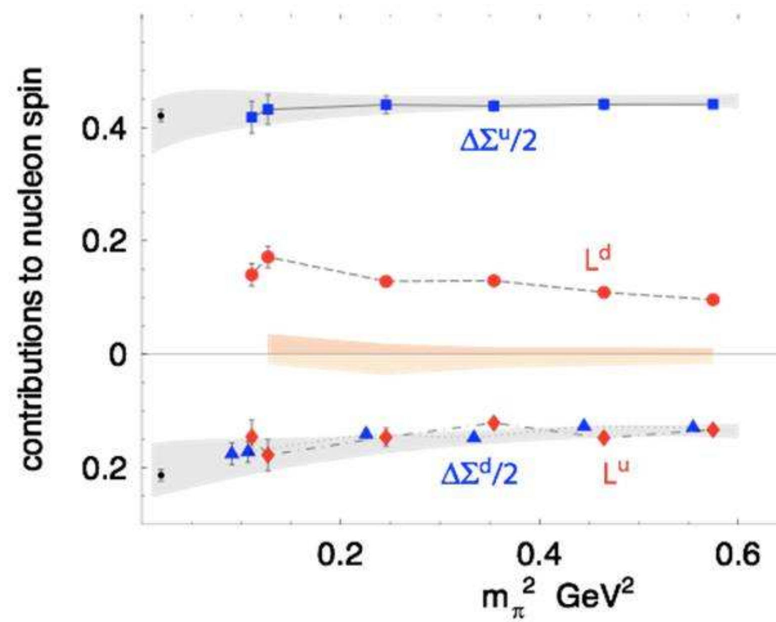
Ji Relation

- Interesting observation: X.Ji, PRL**78**,610(1997)

$$\langle J_q \rangle = \frac{1}{2}\int_0^1 dx\, x\,[H_q(x,0,0) + E_q(x,0,0)]$$

$$\boxed{\text{DVCS}} \quad \Leftrightarrow \quad \boxed{\text{GPDs}} \quad \Leftrightarrow \quad \boxed{\vec{J}_q}$$

- lattice QCD (LHPC,QCDSF)
  - $L^u + L^d \approx 0$ (disconnected diagrams?)
  - $L^u - L^d < 0$!



- But: what other “physical information” about the nucleon can we obtain by measuring/ calculating GPDs?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

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$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
$\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \quad \begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2), \end{aligned}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)
- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

$\hookrightarrow$ for $x \rightarrow 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \rightarrow 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

- Note that this does not necessarily imply that 'hadron size' goes to zero as $x \rightarrow 1$, as separation $\mathbf{r}_\perp$ between active quark and COM of spectators is related to impact parameter $\mathbf{b}_\perp$ via $\mathbf{r}_\perp = \frac{1}{1-x} \mathbf{b}_\perp$.

Transversely Deformed Distributions and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\uparrow\rangle = H(x,0,-\boldsymbol{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\boldsymbol{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} E(x,0,-\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}$$

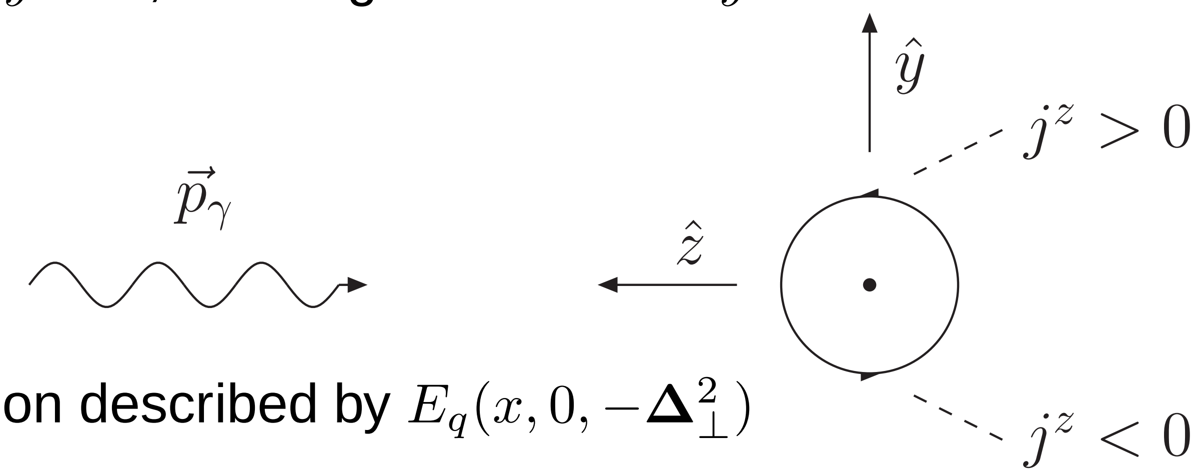
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\hookrightarrow$ j^+ larger than j^0 when quark current towards the γ^*; suppressed when away from γ^*

$\hookrightarrow$ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x .$$

Transversely Deformed PDFs and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

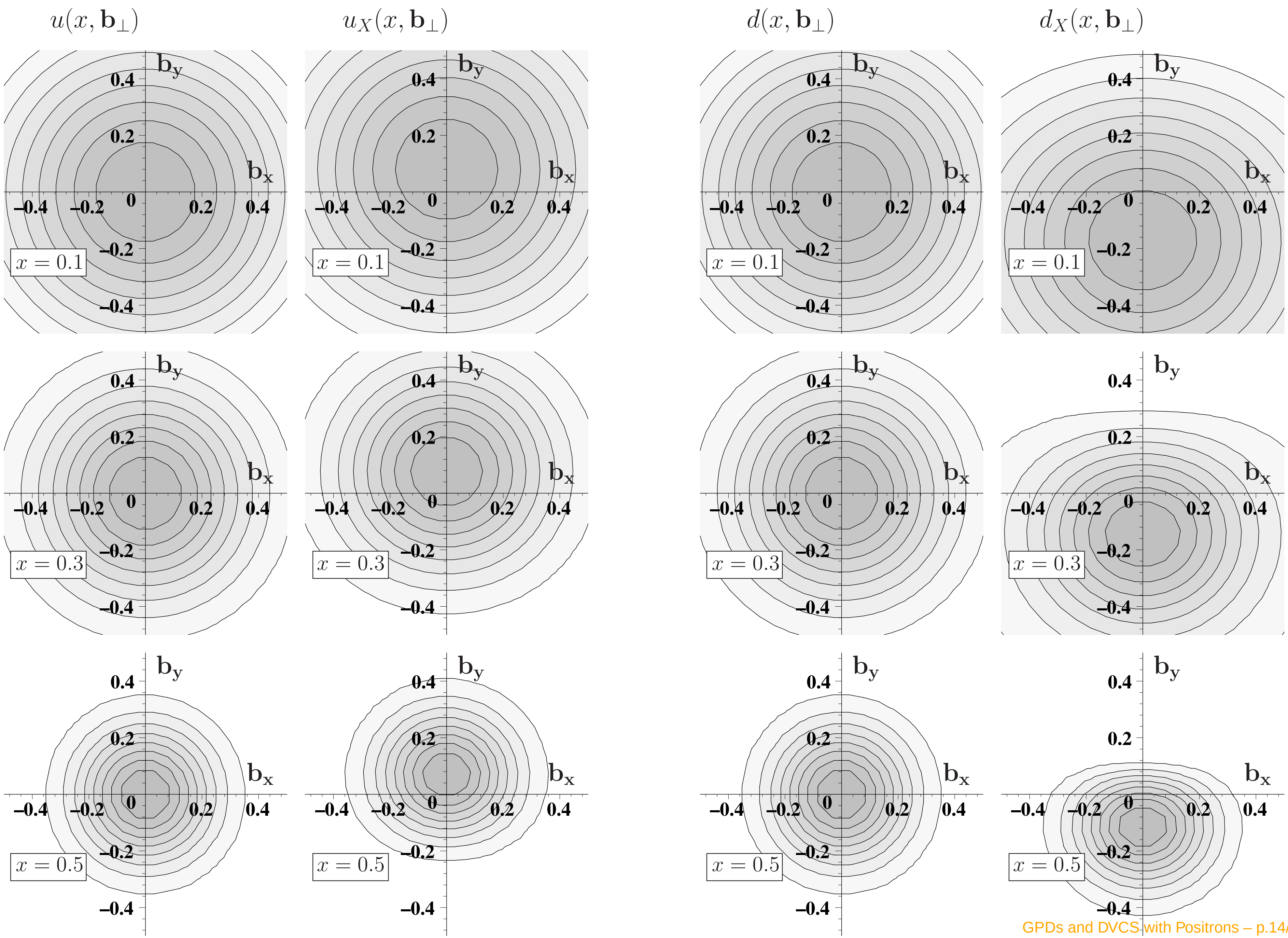
with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\ E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2) \end{aligned}$$

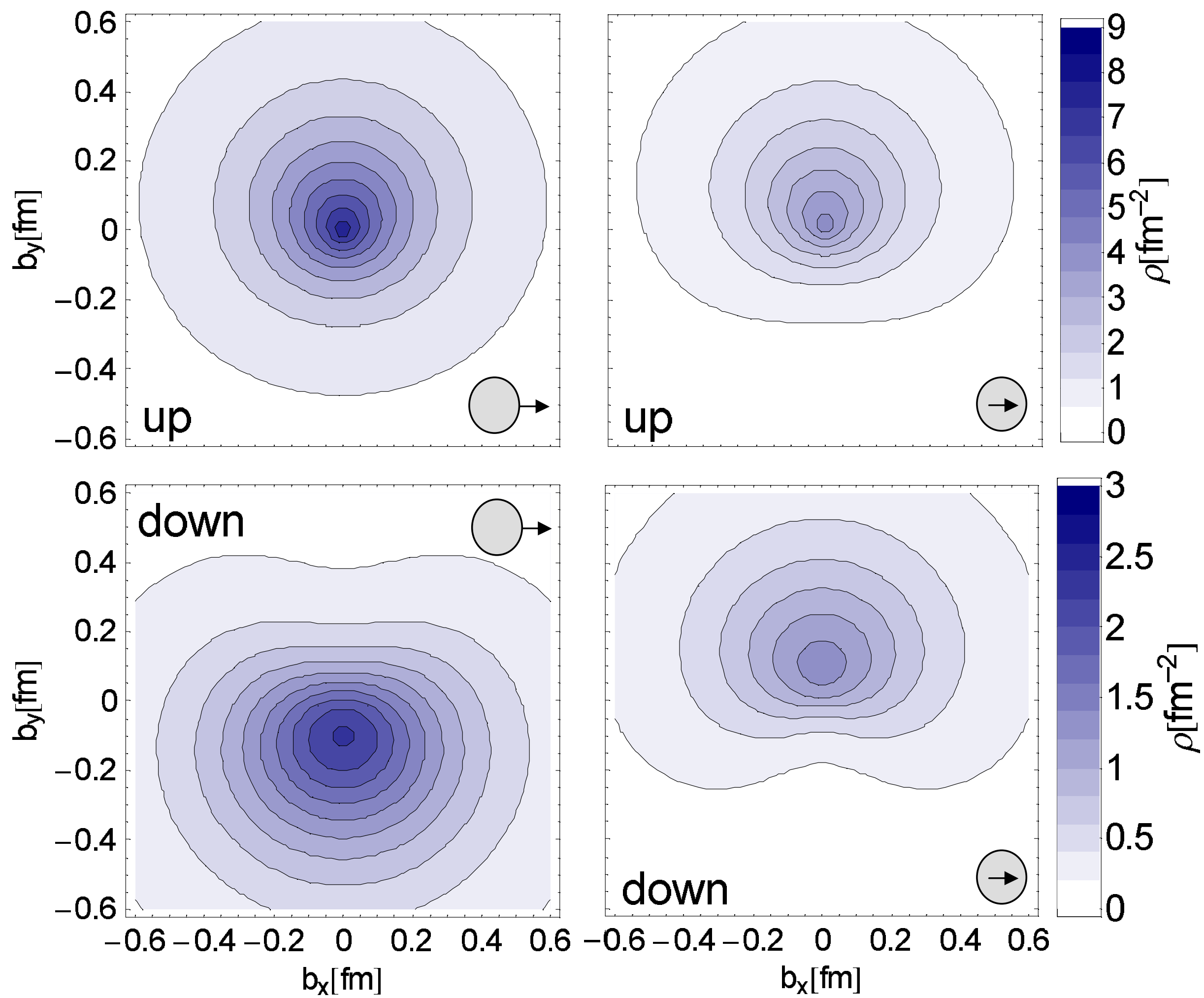
with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_y}$
$\mathbf{b_x}$
0.4
0.2
0
−0.2
−0.4

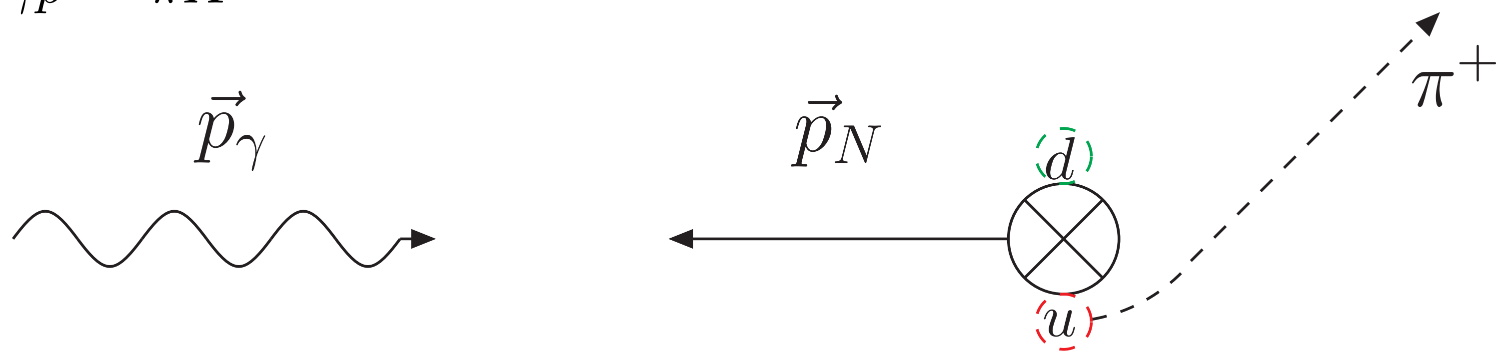
IPDs on the lattice (Hägler et al.)

- lowest moment of distribution of unpol. quarks in ⊥ pol. proton (left) and of ⊥ pol. quarks in unpol. proton (right):



GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

GPD $\longleftrightarrow \bar{g}_2$ (twist-3 polarized DIS)

- σ_{LT} in polarized DIS $\longrightarrow g_1 + g_2$
- $g_1 \longrightarrow$ spin fraction carried by quark spin
- g_2 (after subtracting 'Wandzura-Wilzcek piece') sensitive to quark gluon correlations $d_2 \sim \int dx x^2 \bar{g}_2(x)$

$\hookrightarrow$ (MB, arXiv 0810.3589) $\perp$ color Lorentz force acting on quark in DIS from $\perp$ polarized target

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$ provides intuitive explanation for sign of d_2

DVCS $\leadsto GPD(x,\xi,t)$

- $\Im \mathcal{A}_{DVCS}(\xi,t) \sim GPD(\xi,\xi,t)$
- $\Re \mathcal{A}_{DVCS} \sim \int dx \frac{GPD(x,\xi,t)}{x-\xi}$
- dispersion relation $\Rightarrow \Re \mathcal{A}_{DVCS} \sim \int dx \frac{GPD(x,x,t)}{x-\xi} + \Delta(t)$

$\hookrightarrow$ In addition to information along diagonal $x=\xi$ that is also available from $\Im \mathcal{A}_{DVCS}(\xi,t)$ $\Re \mathcal{A}_{DVCS}$ provides access to

  - GPDs along diagonal that is not kinematically accessible through $\Im \mathcal{A}_{DVCS}(\xi,t)$
  - ‘D-form factor’ Δ (Polyakov Weiss)

- Ji relation requires $GPDs(x,\xi,t)$ for $-1<x<1$ at **fixed** ξ
- $q(x,\mathbf{b}_\perp)$ requires $GPDs(x,0,t)$

DVCS $\rightsquigarrow GPD(x, \xi, t)$

- Information away from diagonal ($x = \xi$):
  - $\Re \mathcal{A}_{DVCS}$ (positrons!) $\quad \Rightarrow \quad$ $D-$form factor
  - polynomiality condition: n-th Mellin moment of $GPD(x, \xi)$ must be even polynomial in ξ of order n

  $\hookrightarrow$ $GPD(x, \xi)$ cannot depend on variables x and ξ completely independently
  - Q^2 evolution: changes x distribution in a known way for fixed ξ
  - Double Deeply Virtual Compton Scattering D^2VCS (lepton pair instead of real photon in final state)

DVCS $\rightsquigarrow GPD(x, \xi, t)$

- example: dispersion relations/polynomiality $\Rightarrow$

$$\int_{-1}^{1} dx \frac{H^{(+)}(x,0,t)}{x} = \int_{-1}^{1} dx \frac{H^{(+)}(x,x,t)}{x} + \Delta(t)$$

$\hookrightarrow$ DVCS allows access to same generalized form factor $\int_{-1}^{1} dx \frac{H^{(+)}(x,0,t)}{x}$ also available in WACS (wide angle Compton scattering), but t does not have to be of order Q^2

$\hookrightarrow$ after flavor separation, comparing $\frac{1}{F_1(t)} \int_{-1}^{1} dx \frac{H^{(+)}(x,0,t)}{x}$ at large t provides information about the ‘typical x’ that dominates large t form factor

Summary

- Deeply Virtual Compton Scattering $\Rightarrow$ GPDs
- beam charge asymmetry $\Rightarrow$ clean separation of $\Re\mathcal{A}_{DVCS}$
- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $H(x,0,-\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longrightarrow} q(x,\mathbf{b}_\perp)$ for unpolarized target
- $\mathbf{\Delta}_\perp E(x,0,-\mathbf{\Delta}_\perp^2) \stackrel{FT}{\longrightarrow} \perp$ deformation of PDFs for $\perp$ polarized target
- $\hookrightarrow$ $\kappa^{q/p} \Rightarrow$ sign of deformation
- $\hookrightarrow$ attractive FSI $\Rightarrow f_{1T}^{\perp u} < 0$ & $f_{1T}^{\perp d} > 0$
- $\hookrightarrow$ Interpretation of sign of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as sign of $\perp$ force on active quark in DIS on $\perp$ polarized target
- $\Im\mathcal{A}_{DVCS}$ only sensitive to $GPDs(\xi,\xi,t)$
- use $\Re\mathcal{A}_{DVCS}$/polynomiality/dispersion relations/ Q^2-evolution/DDVCS to get information on $GPDs(x,\xi,t)$ for $x \neq \xi$