Two photon exchange: theoretical issues

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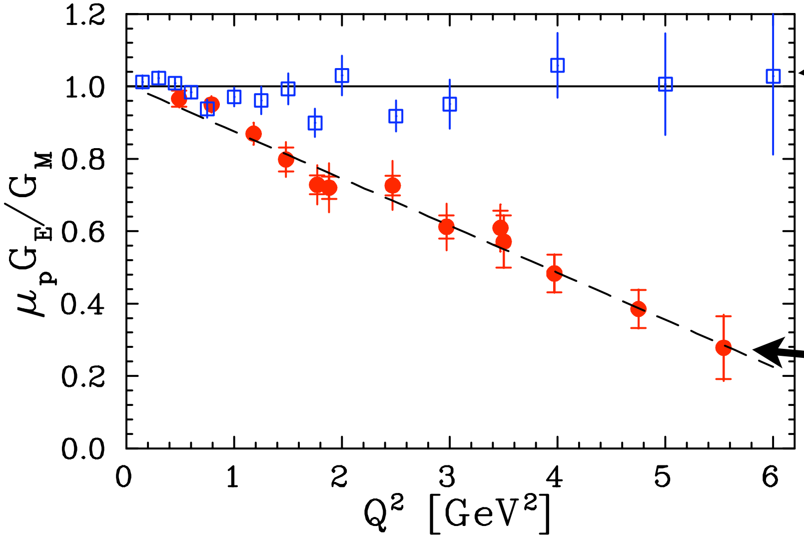
Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \rightarrow e\, \vec{p}$

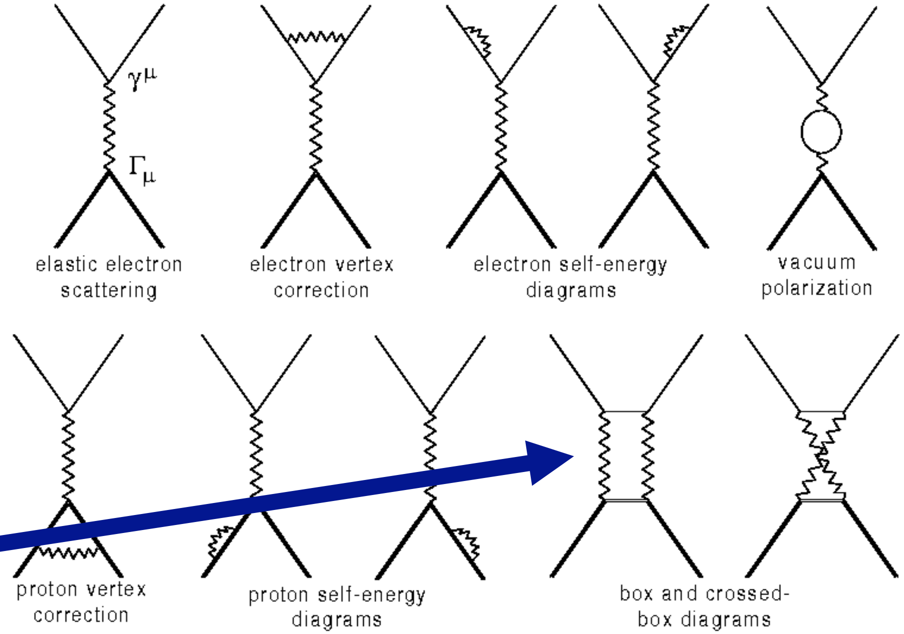
Radiative corrections

$d\sigma_0 \rightarrow d\sigma = d\sigma_0\,(1+\delta_{RC})$

Missing effect is

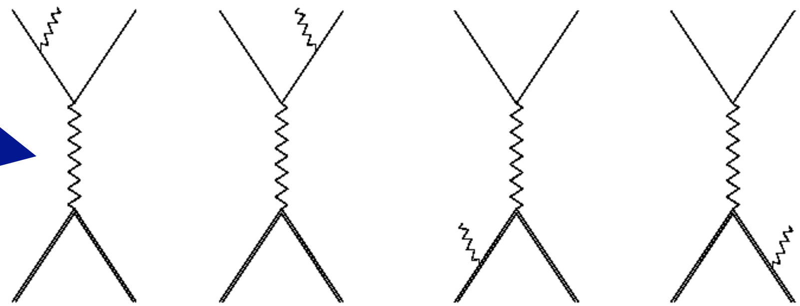
- approximately linear in ε
- not strongly Q^2 dependent

Two-photon exchange



Bremsstrahlung

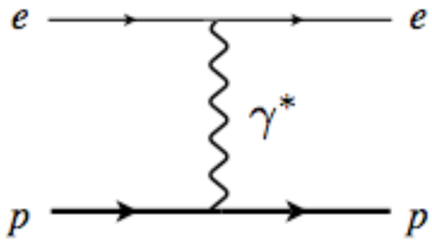
- SuperRosenbluth (detect proton)



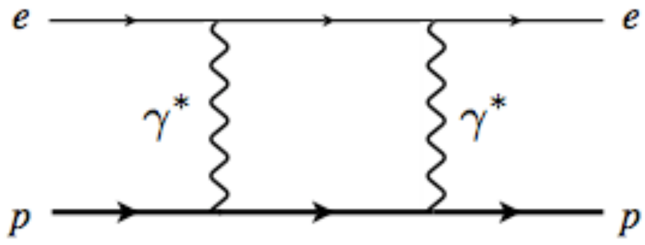
inelastic amplitudes

Two-photon exchange

- interference between Born and two-photon exchange amplitudes



x



$\mathcal{M}_0$ $\mathcal{M}_{\gamma\gamma}$

- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\,\mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

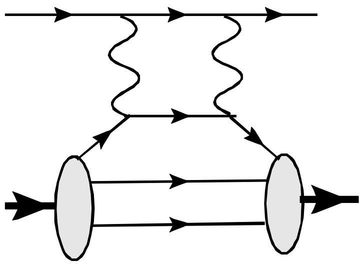
- standard "soft photon approximation" (used in most data analyses)

⟶ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

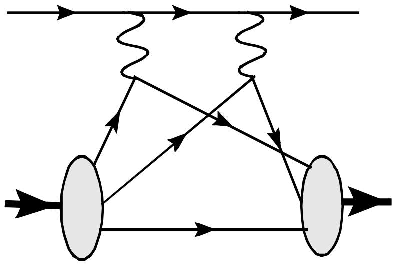
⟶ neglect nucleon structure (no form factors) *Mo, Tsai (1969)*

Partonic (GPD) calculation of two-photon exchange contribution

(Chen et al.)



"handbag"



"cat's ears"

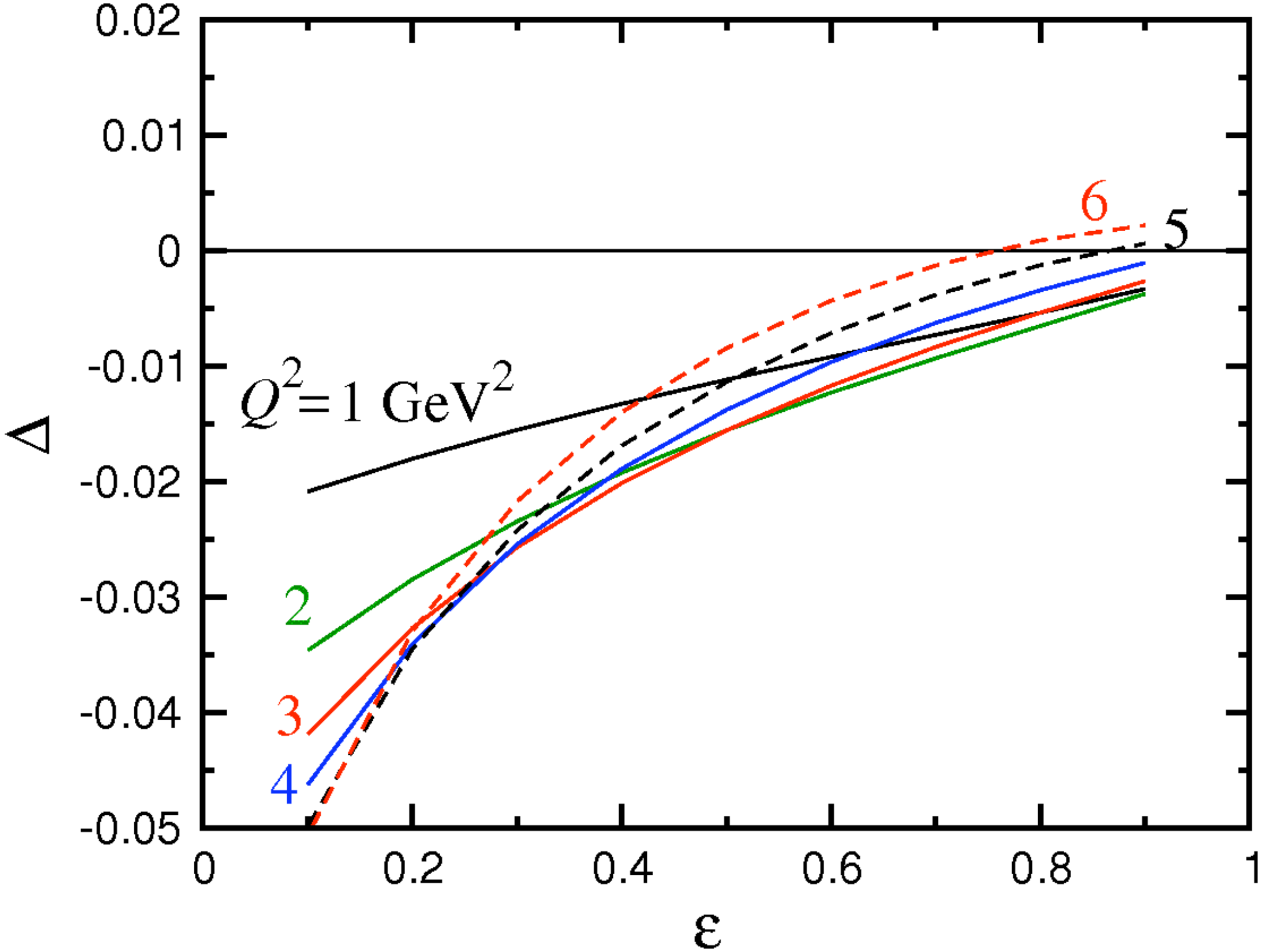
valid at large Q^2 : δ^{hard}

handbag diagrams (one active quark)

to reproduce the IR divergent contribution at nucleon correctly (Low Energy Theorem): δ^{soft}

need cat's ears diagrams (two active quarks)

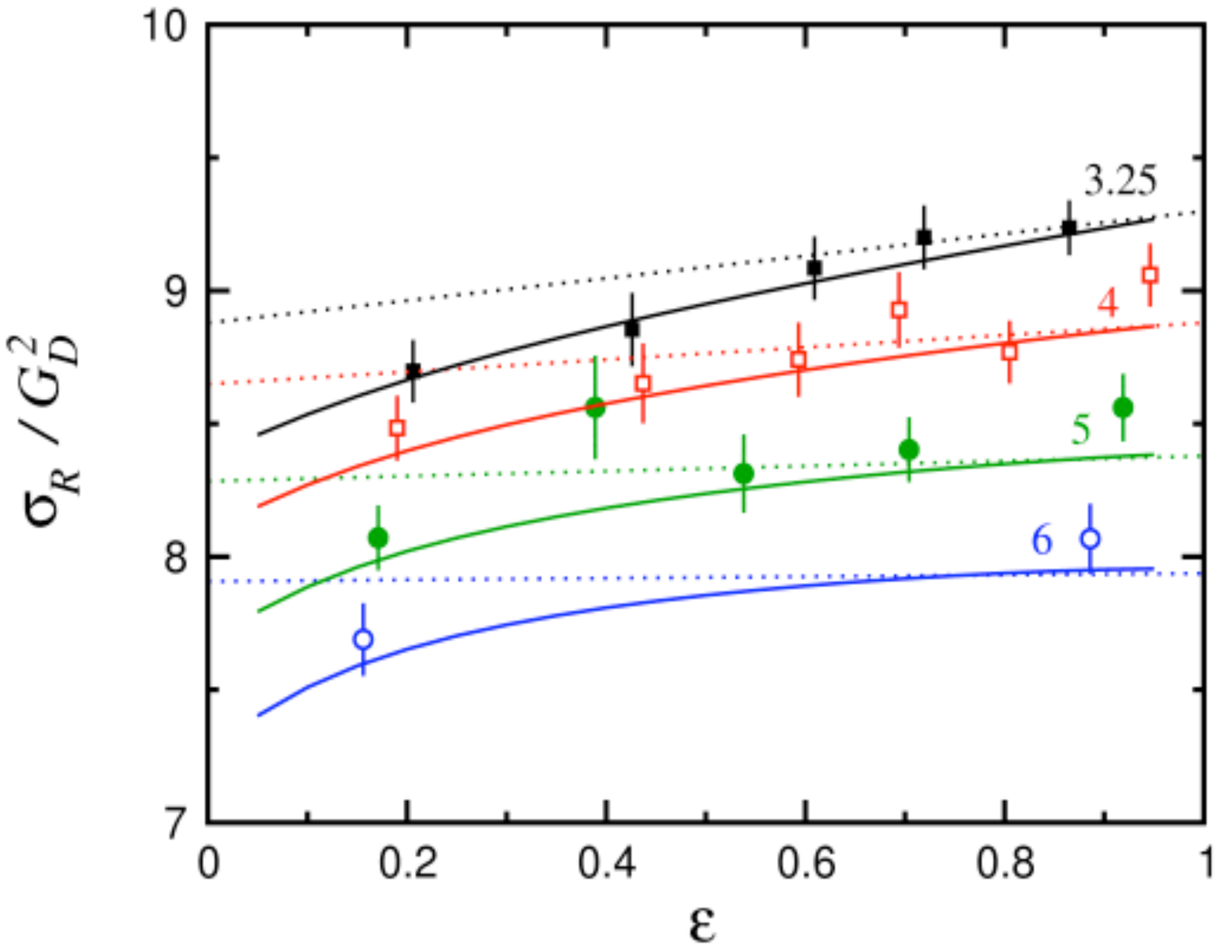
Corrections to unpolarized cross sections for Q^2=1 to 6 GeV^2



- Effect largest at small ε (backward angles)
- Vanishes as ε ->1
- Nonlinearity grows with Q^2
- JLAB E05-017 (Arrington) will set limits on nonlinearity

Effect on SLAC reduced cross sections at different Q^2

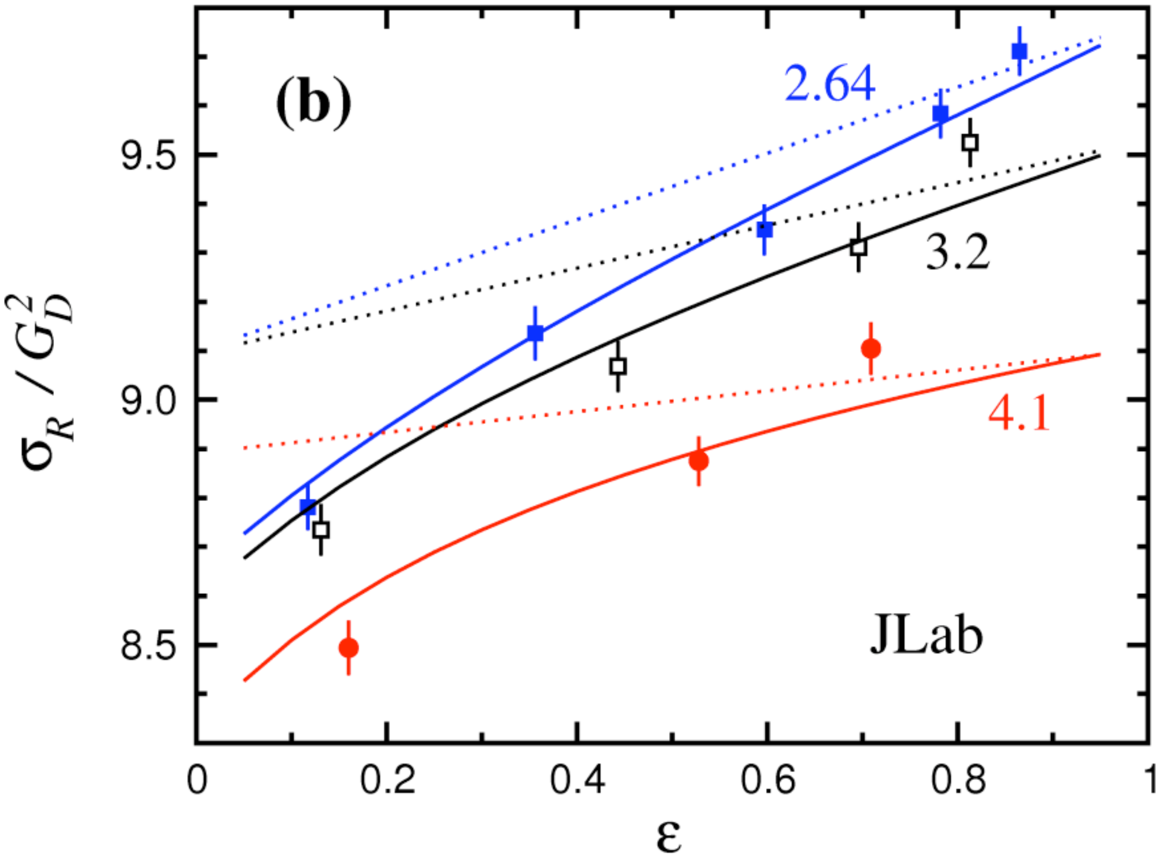
(normalized to dipole $G_D{}^2$)



Nonlinearity in ε is displayed here

JLAB proposals to measure nonlinearity

SuperRosenbluth (JLAB) data



Curves shifted by

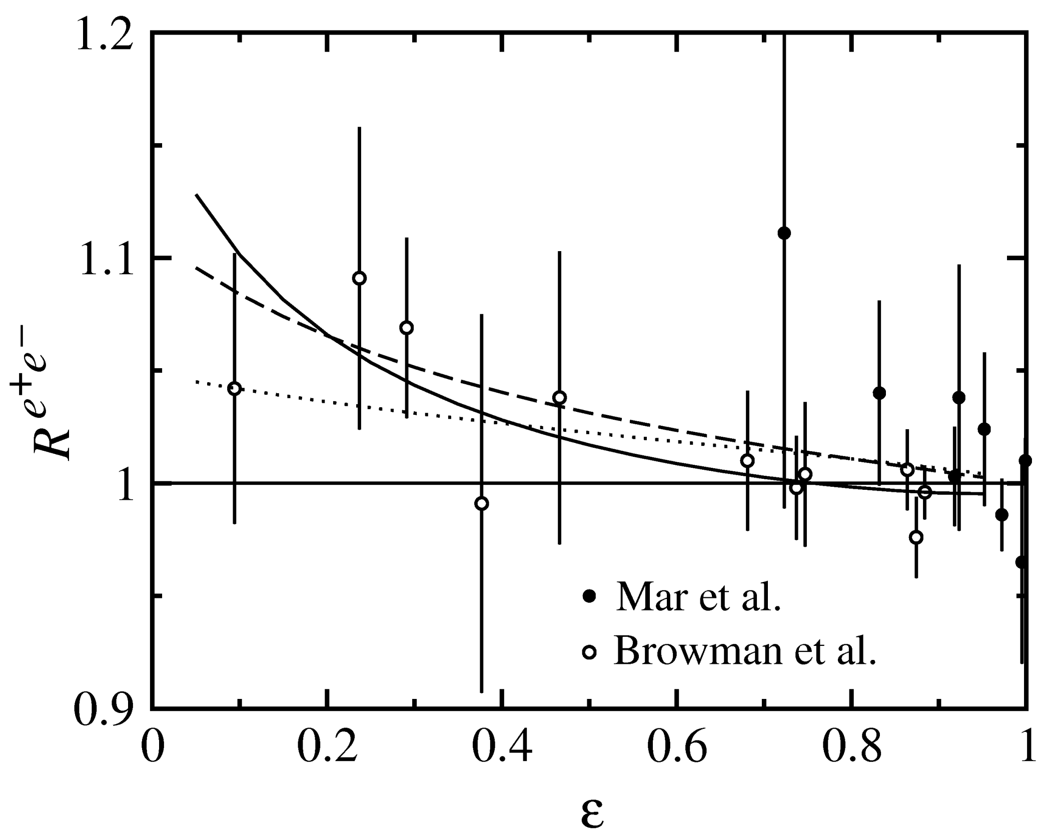
+1.0% 2.64

+2.1% 3.20

+3.0% 4.10

(Effect on determination of G_M)

Effect on ratio of e^+p to e^-p cross sections (SLAC, Q^2 from 0.01 to 5 GeV^2)



M_{Born} opposite sign for e^+p vs. e^-p, so enhancement instead of suppression as $\varepsilon \to 0$

$R(e^+p/e^-p) = (1-\Delta)/(1+\Delta)$

$= 1-2\Delta$

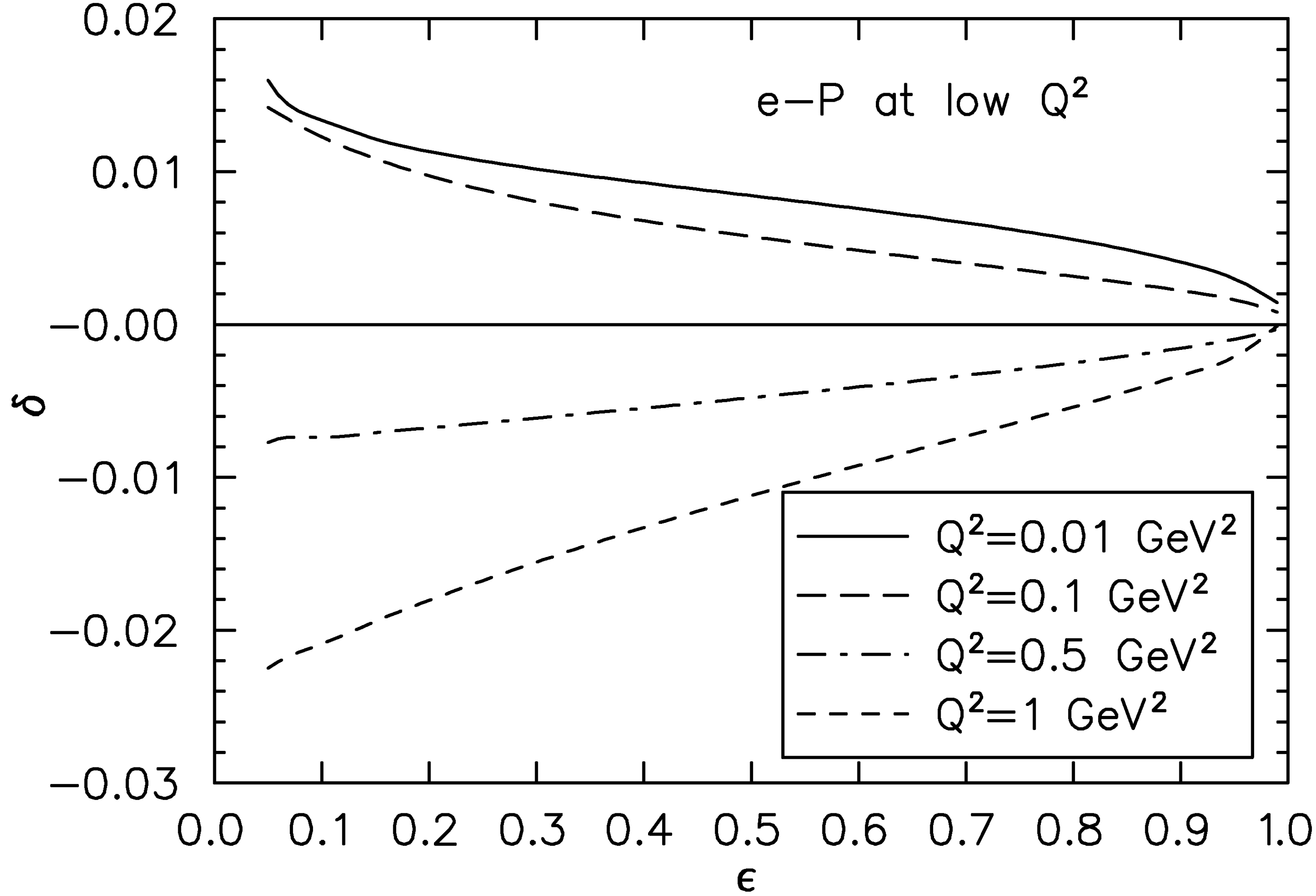
Curves are elastic results for Q^2=1, 3, 6 GeV^2

Expts.

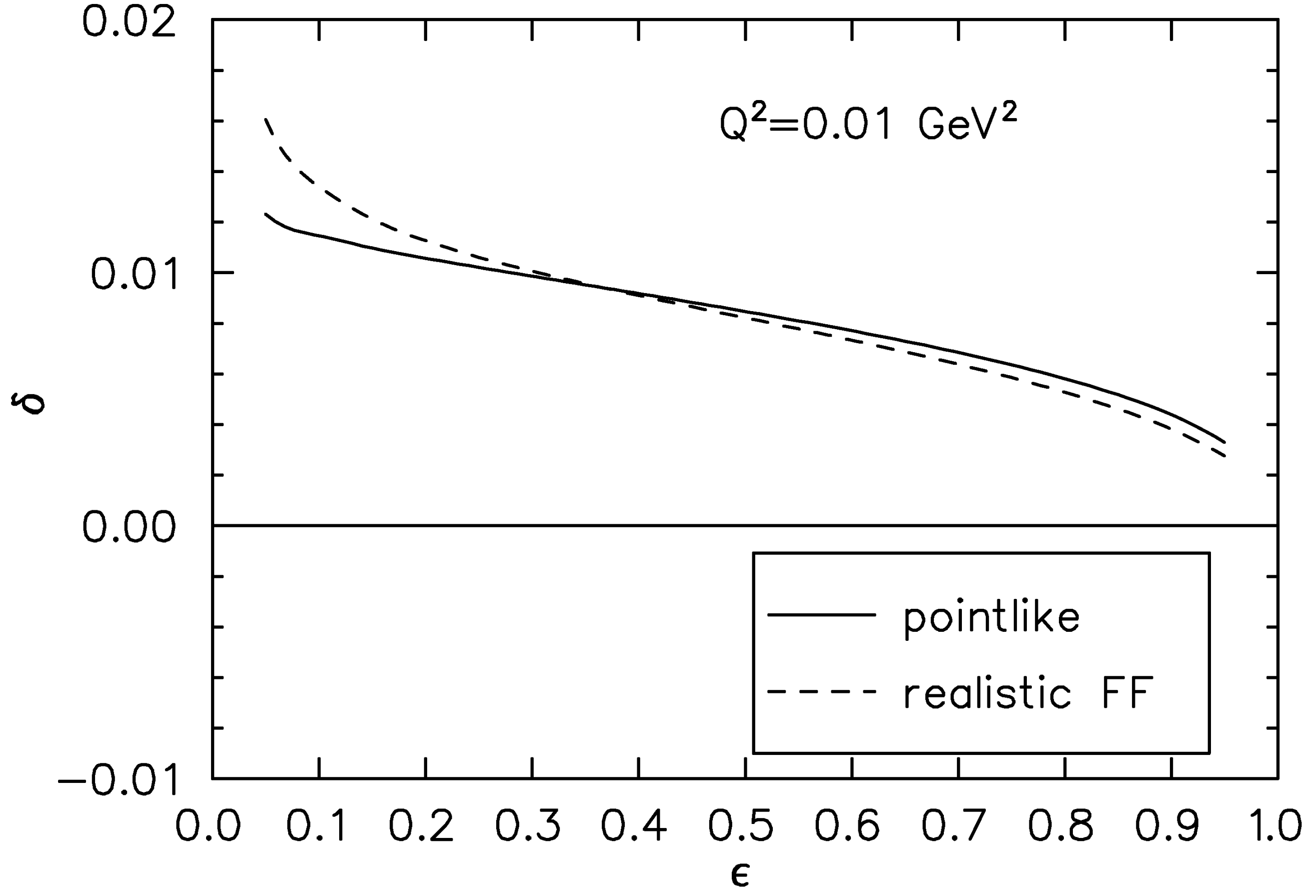
E04-116 $Q^2 < 2$ GeV^2

VEPP-3 Q^2=1.6 GeV^2, $\varepsilon = 0.4$

proton correction at low Q^2



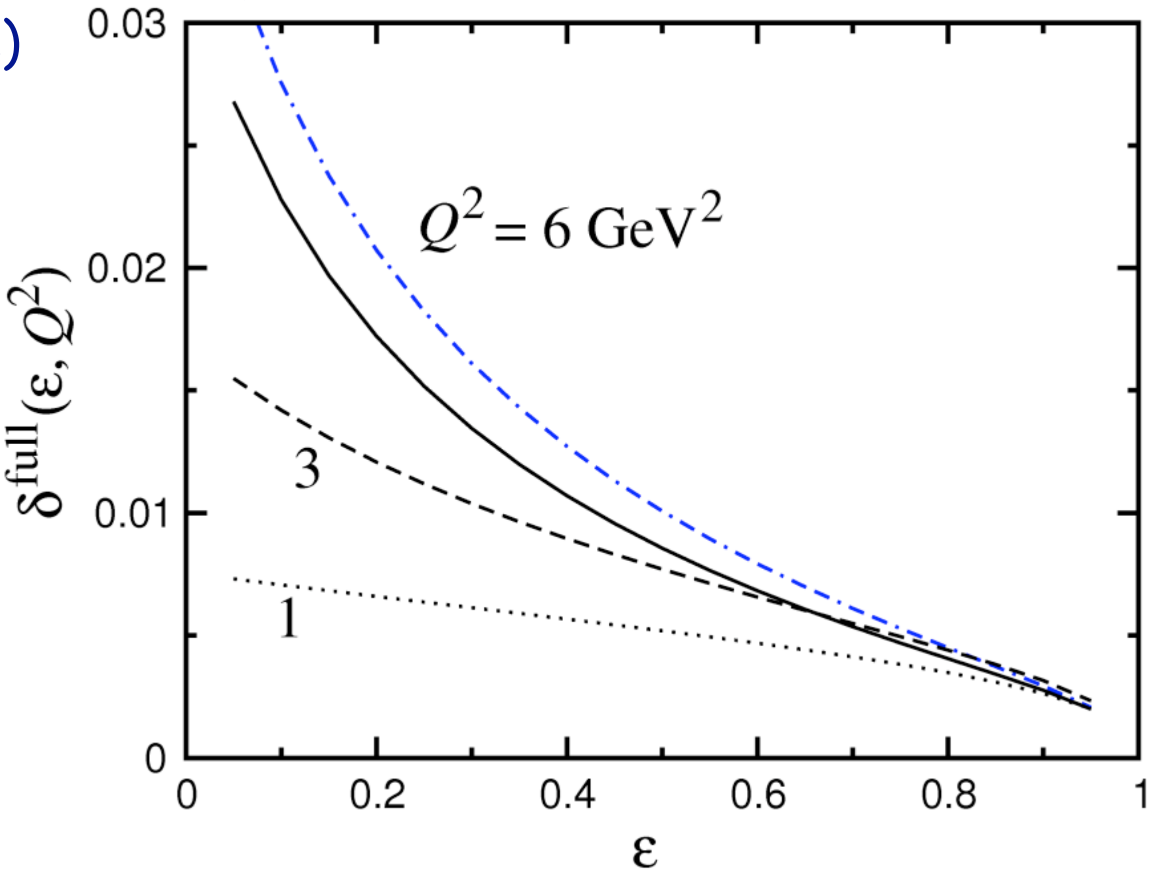
proton correction at Q^2=0.01 GeV^2



- Essentially independent of mass (same for muon, free quarks)
- At high Q^2, G_M dominates the loop integral
- At low Q^2, G_E dominates
- neutron correction vanishes at low Q^2 (pointlike neutron)

Neutron

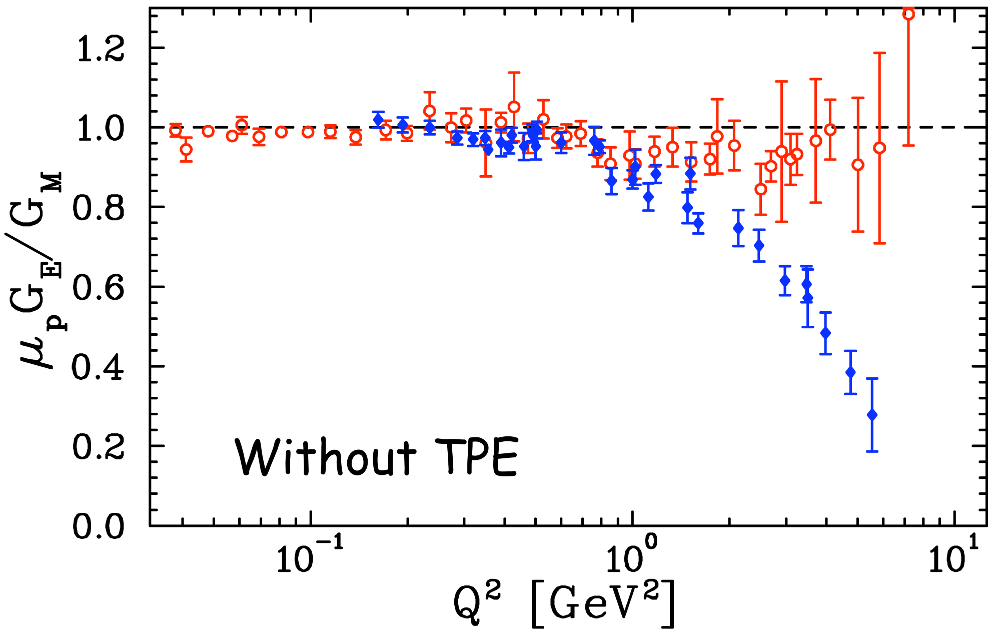
- No infrared divergences
- Positive and about 2-3 times smaller than proton (dominance of magnetic form factor?)
- Some model dependence due to choice of form factors (blue curve)

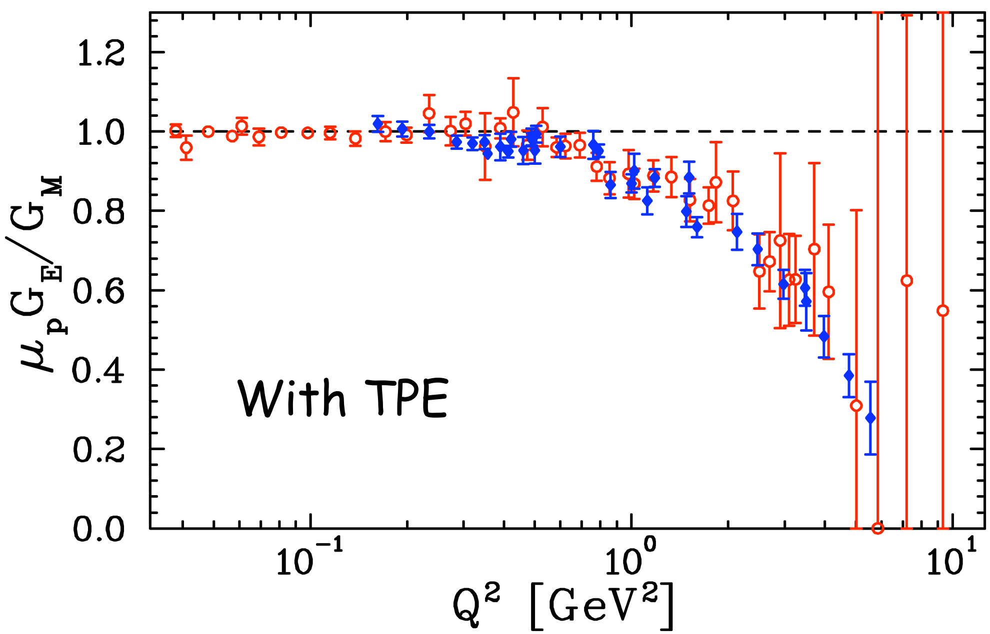


Effect on ratio R

Global Analysis:

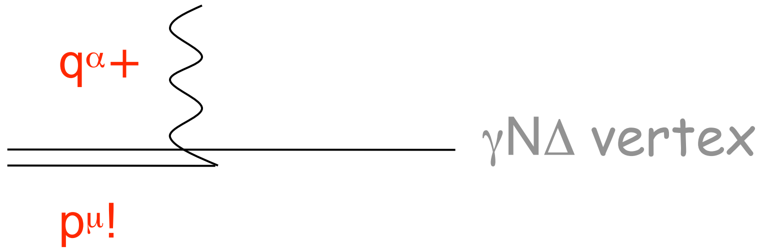
(Arrington, Melnitchouk & Tjon, PRC, 2007)





Resonance (Δ) contribution:

$\gamma(q^{\alpha}) + \Delta(p^{\mu}) \rightarrow N$



- Lorentz covariant form
- Spin $\frac{1}{2}$ decoupled
- Obeys gauge symmetries

$$p_{\mu}\Gamma^{\alpha\mu}(p,q) = 0$$
$$q_{\alpha}\Gamma^{\alpha\mu}(p,q) = 0$$

$$\Gamma^{\alpha\mu}_{\gamma\Delta\rightarrow N}(p,q) = \frac{ieF_{\Delta}(q^2)}{2M_{\Delta}^2}\{g_1(g^{\alpha\mu}\not{p}\not{q} - p^{\alpha}\gamma^{\mu}\not{q} - \gamma^{\alpha}\gamma^{\mu}p\cdot q + \gamma^{\alpha}\not{p}q^{\mu}) + g_2\,(p^{\alpha}q^{\mu} - g^{\alpha\mu}p\cdot q) + (g_3/M_{\Delta})\left(q^2(p^{\alpha}\gamma^{\mu} - g^{\alpha\mu}\not{p}\right) + q^{\alpha}\,(q^{\mu}\not{p} - \gamma^{\mu}p\cdot q)\}\gamma_5 T_3$$

3 coupling constants g_1, g_2, and g_3

At Δ pole:

g_1	magnetic
$(g_2\text{-}g_1)$	electric
g_3	Coulomb

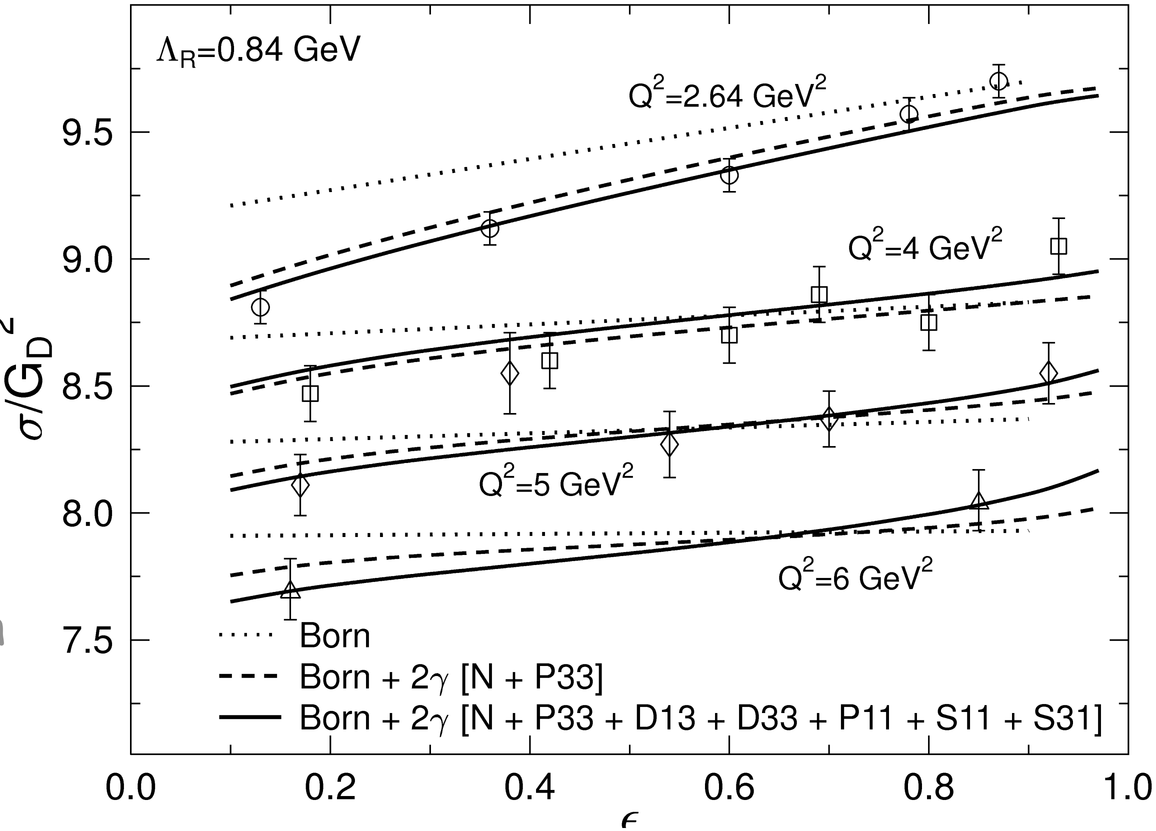
Take dipole FF $F_{\Delta}(q^2) = 1/(1-q^2/\Lambda_{\Delta}^2)^2$ with $\Lambda_{\Delta} = 0.84$ GeV

Other resonances

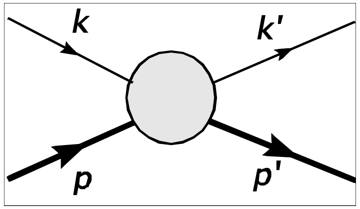
- N (P11), Δ (P33) + D13, D33, P11, S11, S31
- Parameters from dressed K-matrix model

Results

• contribution of heavier resonances much smaller than N and Δ
• D13 next most important (consistent with second resonance shape of Compton scattering cross section)
• partial cancellation between spin 1/2 and spin 3/2
• leads to better agreement, especially at high Q^2



Phenomenology: Generalized form factors



$$P \equiv \frac{p+p'}{2}, \qquad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$q^2 = (p'-p)^2 \equiv -Q^2$$
$$\nu = K \cdot P = p \cdot k + q^2/4$$

In limit $m_e \to 0$ (helicity conservation) general amplitude can be put in form

$$T = (\gamma_\mu)^{(e)} \otimes \left(\tilde{F}_1 \gamma^\mu + i\frac{\tilde{F}_2}{2M}\sigma^{\mu\nu}q_\nu + \frac{F_3}{M^2}\gamma \cdot K P^\mu \right)^{(p)}$$

In general, 16 independent amplitudes:

parity 16 → 8; time reversal 8 → 6; helicity conservation (m_e=0) 6 → 3

Generalized (complex) form factors

$$\tilde{F}_1(\nu, Q^2) = F_1(Q^2) + \delta F_1$$
$$\tilde{F}_2(\nu, Q^2) = F_2(Q^2) + \delta F_2$$

$$\tilde{G}_M = \tilde{F}_1 + \tilde{F}_2$$
$$\tilde{G}_E = \tilde{F}_1 - \tau \tilde{F}_2$$
$$Y_2 = \frac{\nu}{M^2}\frac{F_3}{G_M}$$

Observables including two-photon exchange

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\left\{\epsilon\left(\frac{\delta G_E}{G_E}\right)G_E^2 + \tau\left(\frac{\delta G_M}{G_M}\right)G_M^2 + \epsilon Y_2(\tau G_M^2 + G_M G_E)\right\}}{\epsilon G_E^2 + \tau G_M^2}$$

$$\frac{\delta P_L}{P_L} = 2\left(\frac{\delta G_M}{G_M}\right) + 2\frac{\epsilon}{1+\epsilon}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

$$\frac{\delta P_T}{P_T} = \left(\frac{\delta G_M}{G_M}\right) + \left(\frac{\delta G_E}{G_E}\right) + \frac{G_M}{G_E}Y_2 - \frac{\delta\sigma}{\sigma_0}$$

Caution needed about assumptions (generalized FF's are not observables)

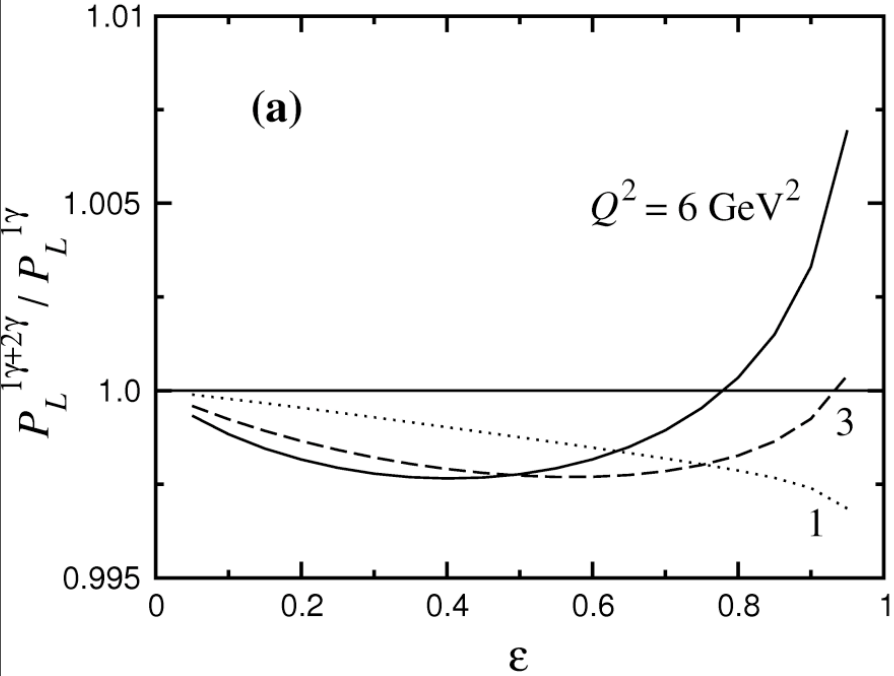
- Parametrization of amplitude NOT unique

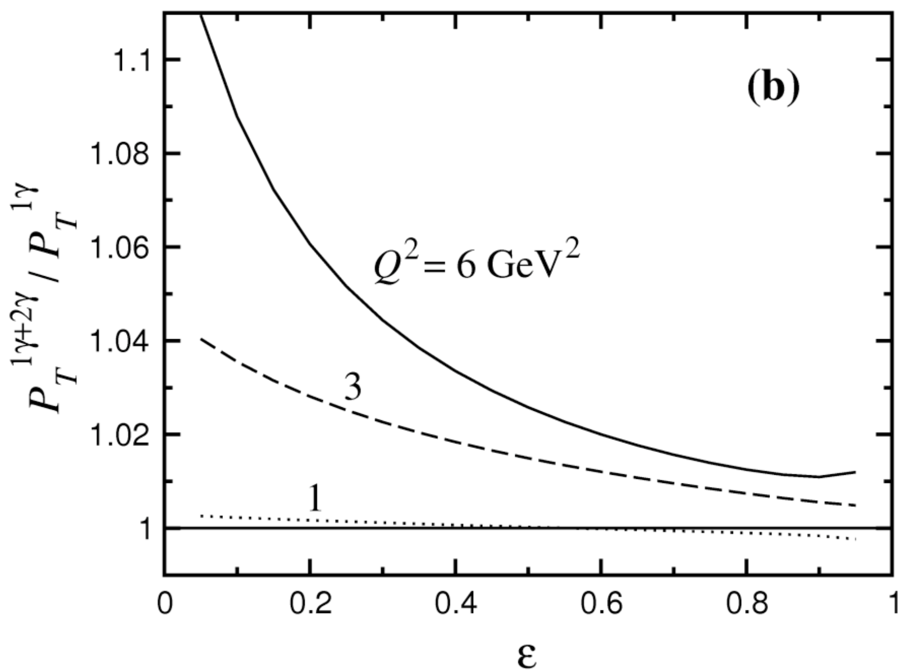
  Axial parametrization: G_A' $(\gamma_\mu\gamma_5)^{(e)}$ $(\gamma^\mu\gamma_5)^{(p)}$ instead of F_3 (or Y_2) term

  shifts some F_3 into δF_1 (and hence into δG_E and δG_M)

$$\vec{e} + p \rightarrow e + \vec{p}$$

Corrections to P_L and P_T at Q^2=1, 3, and 6 GeV^2





P_T/P_L will show some variation with ε, esp. at low ε

JLab data taken at ε~0.7

JLAB expt (Gilman) measures P_T/P_L at low ε

GPD calculation predicts suppression of P_T/P_L

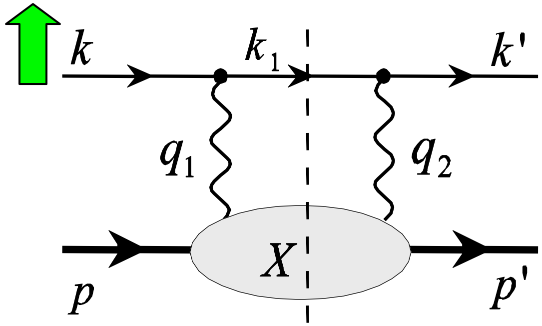
SSA in elastic eN scattering

spin of beam OR target

OR recoil proton

NORMAL to scattering

plane



$$s = (k+p)^2$$

on-shell intermediate state (M_X = W)

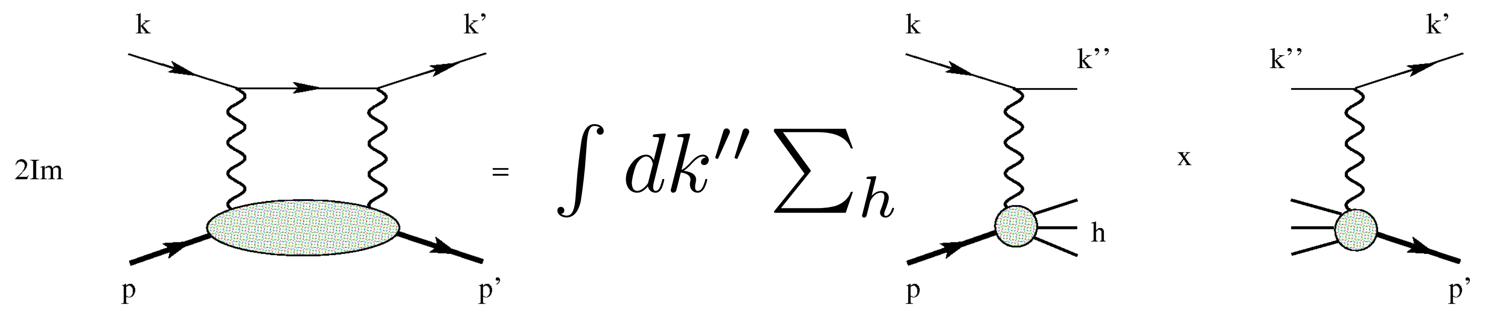
→ involves the imaginary part of two-photon exchange amplitudes

Target: general formula of order e^2
- GPD model allows connection of real and imaginary amplitudes
- Hadronic models sensitive to intermediate state contributions, no reliable theoretical calculations at present

Beam: general formula of order $m_e\, e^2$ (few ppm)
- Measured in PV experiments (longitudinally polarized electrons) at SAMPLE and A4 (Mainz)
- Only non-zero result so far for TPEX

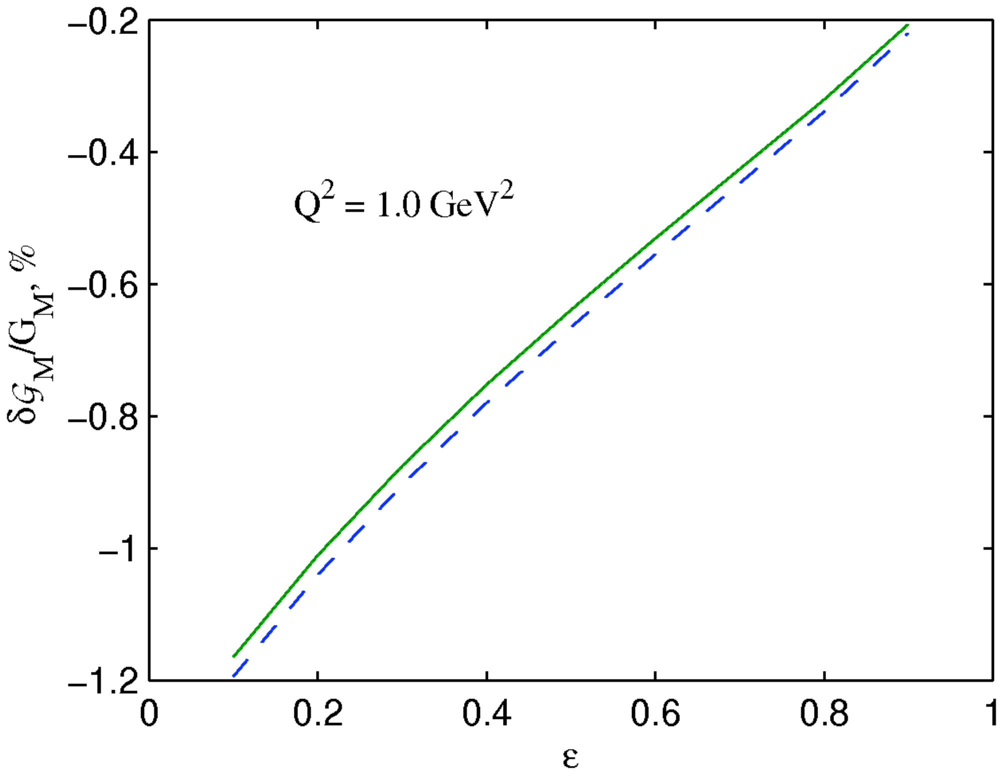
TPEX using dispersion relations

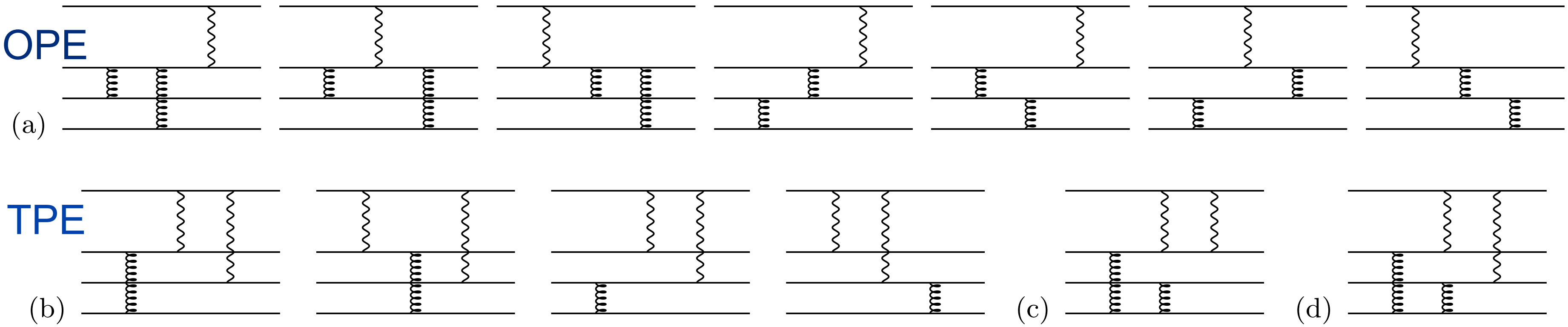
(Borisyuk & Kobushkin, PRC **78**, 2008)



- Imaginary part determined by unitarity
- Only on-shell form factors
- Real part determined from dispersion relations
- Numerical differences between naive (solid) and dispersion (dashed) analyses are small

- Similar insensitivity seen for Δ (Tjon, Blunden, Melnitchouk)





Recent pQCD calculation: Borisyuk & Kobushkin, PRD **79**, 2009

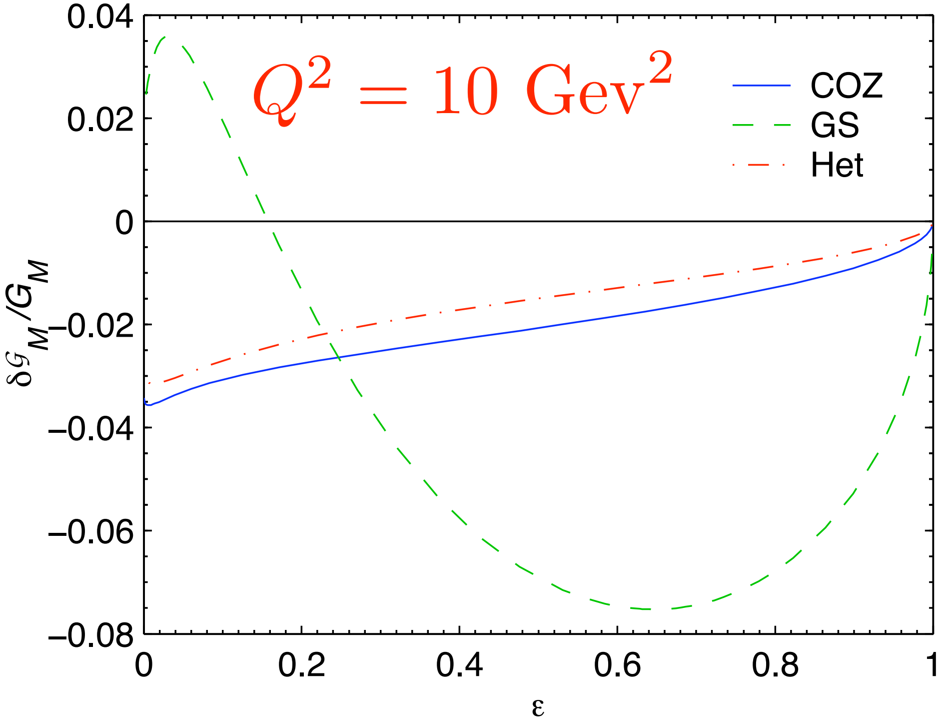
(a) one-photon exchange: need 2 hard gluons to turn momentum of all 3 quarks

$$\alpha\alpha_s^2/Q^6$$

(b) two-photon exchange:
leading order needs 1 hard gluon

$$\alpha^2\alpha_s/Q^6 \quad \text{TPE/OPE} \sim \alpha/\alpha^s$$

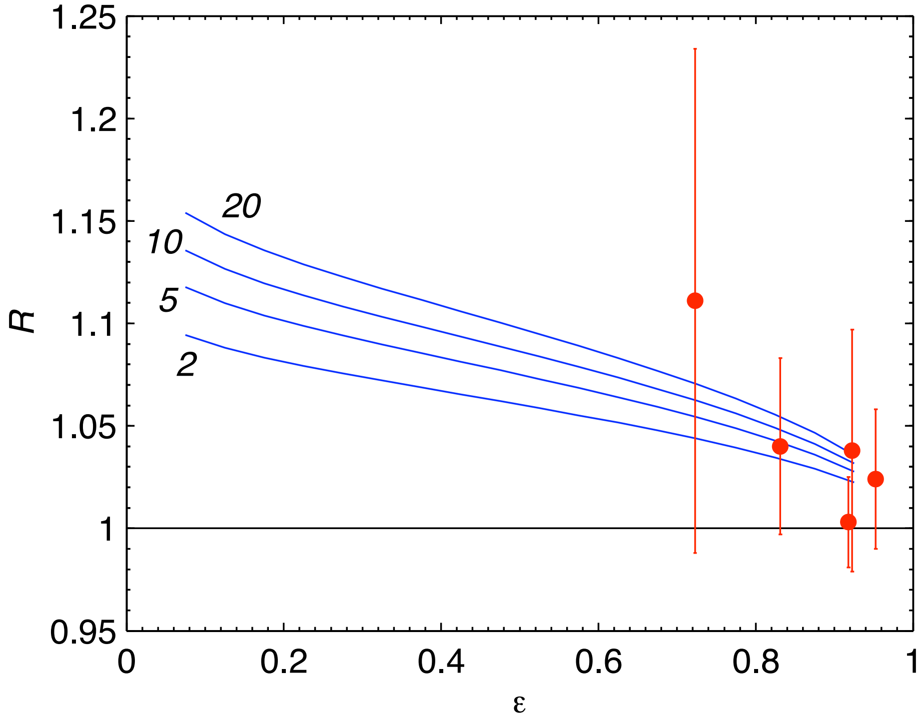
subleading order (both photons on one quark) requires 2 hard gluons

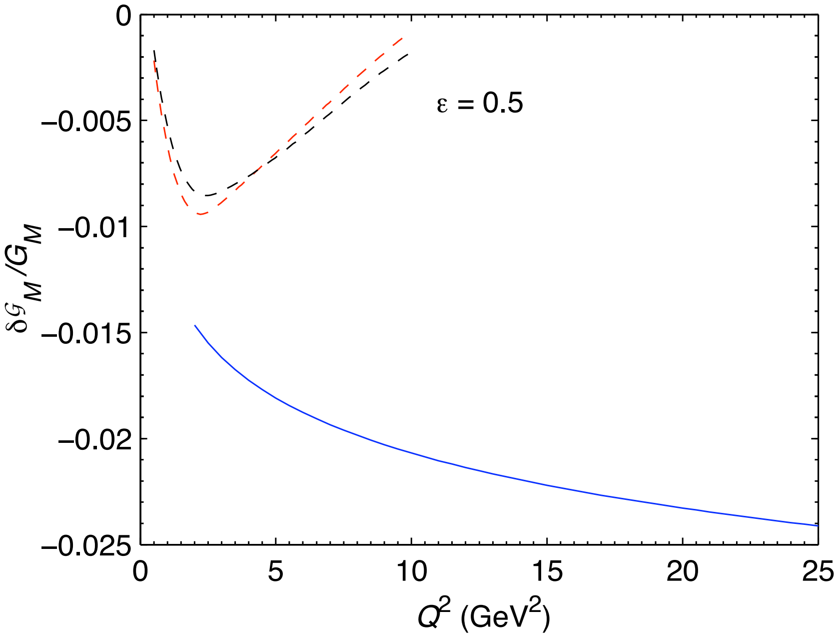


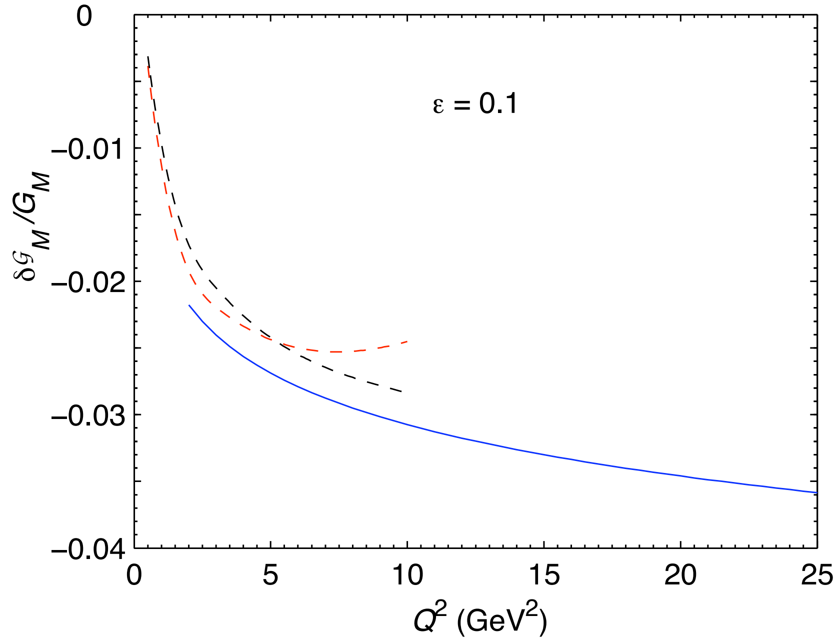
Contribution to *ep* for different quark wavefunctions

Approx linear in ε

e^+e^-ratio







Comparison of hadronic and pQCD results

Connect smoothly around $Q^2 = 3$ GeV^2

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

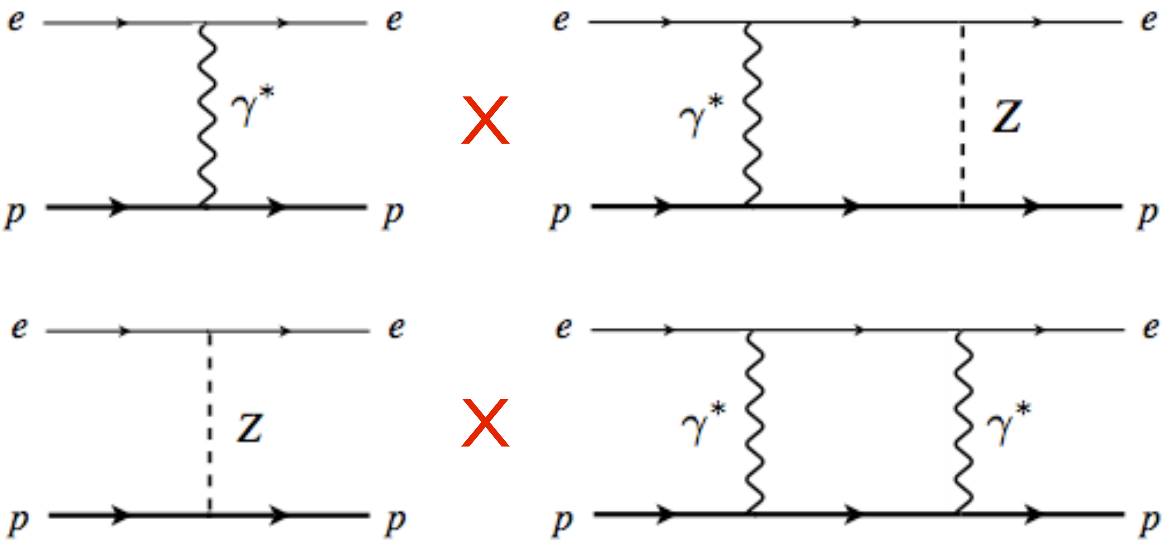
$$\mathcal{A}_{PV} = \frac{2\Re\left\{M_\gamma^\dagger M_Z\right\}}{|M_\gamma|^2}$$

Electromagnetic radiative corrections interfere with M_Z ($M_\gamma \to M_\gamma + M_{\gamma\gamma}$)

plus weak radiative corrections interfere with M_γ ($M_Z \to M_Z + M_{\gamma Z}$)

Afanasev and Carlson (PRL 2005) used generalized form factors to analyze effect of $\gamma\gamma$ on A (GPD model)

Two-boson exchange corrections



“$\gamma(Z\gamma)$”

“$Z(\gamma\gamma)$”

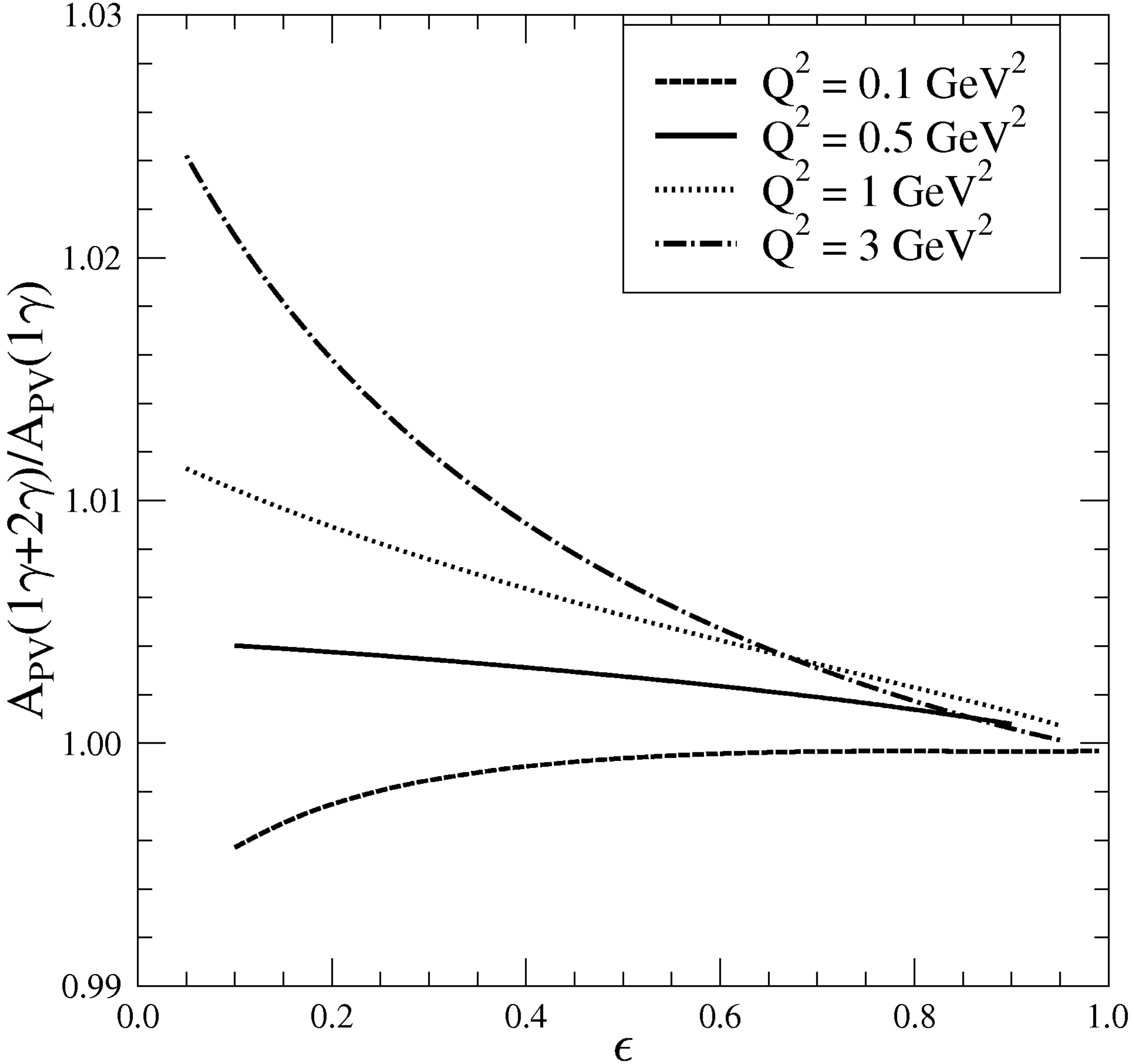
- current PDG estimates (of “$\gamma(Z\gamma)$”) computed at $Q^2 = 0$

Marciano, Sirlin (1980)

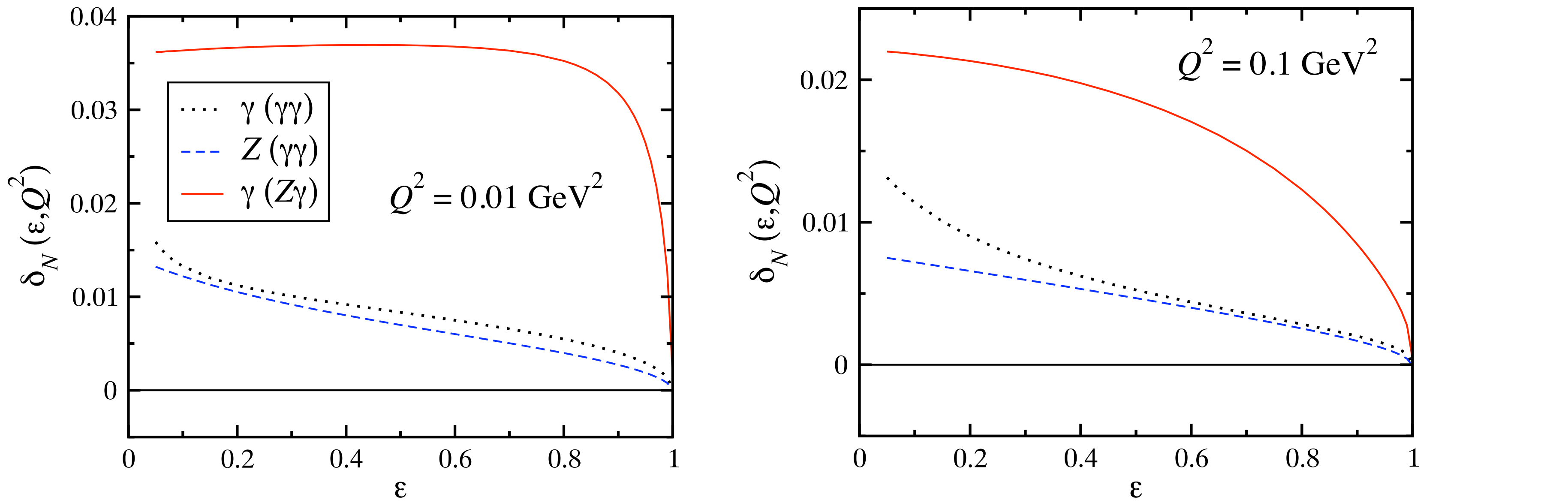
Erler, Ramsey-Musolf (2003)

Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008

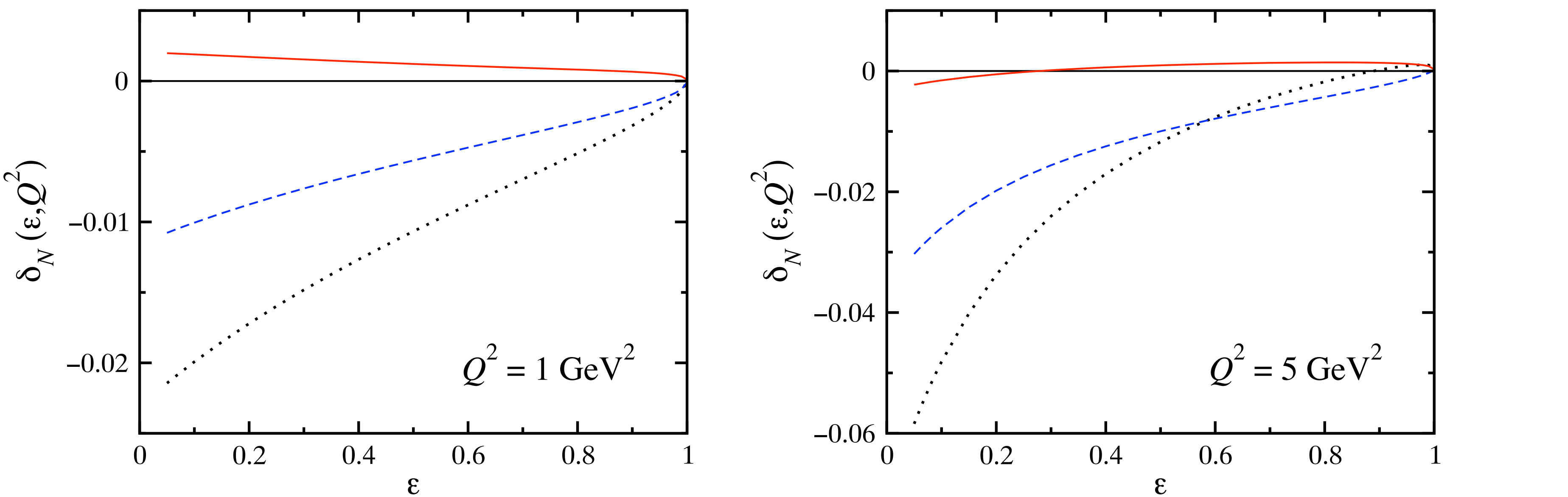
A_{PV} vs. ε for Q^2 = 0.1, 0.5, 1.0, 3.0 GeV^2 (TPE only)



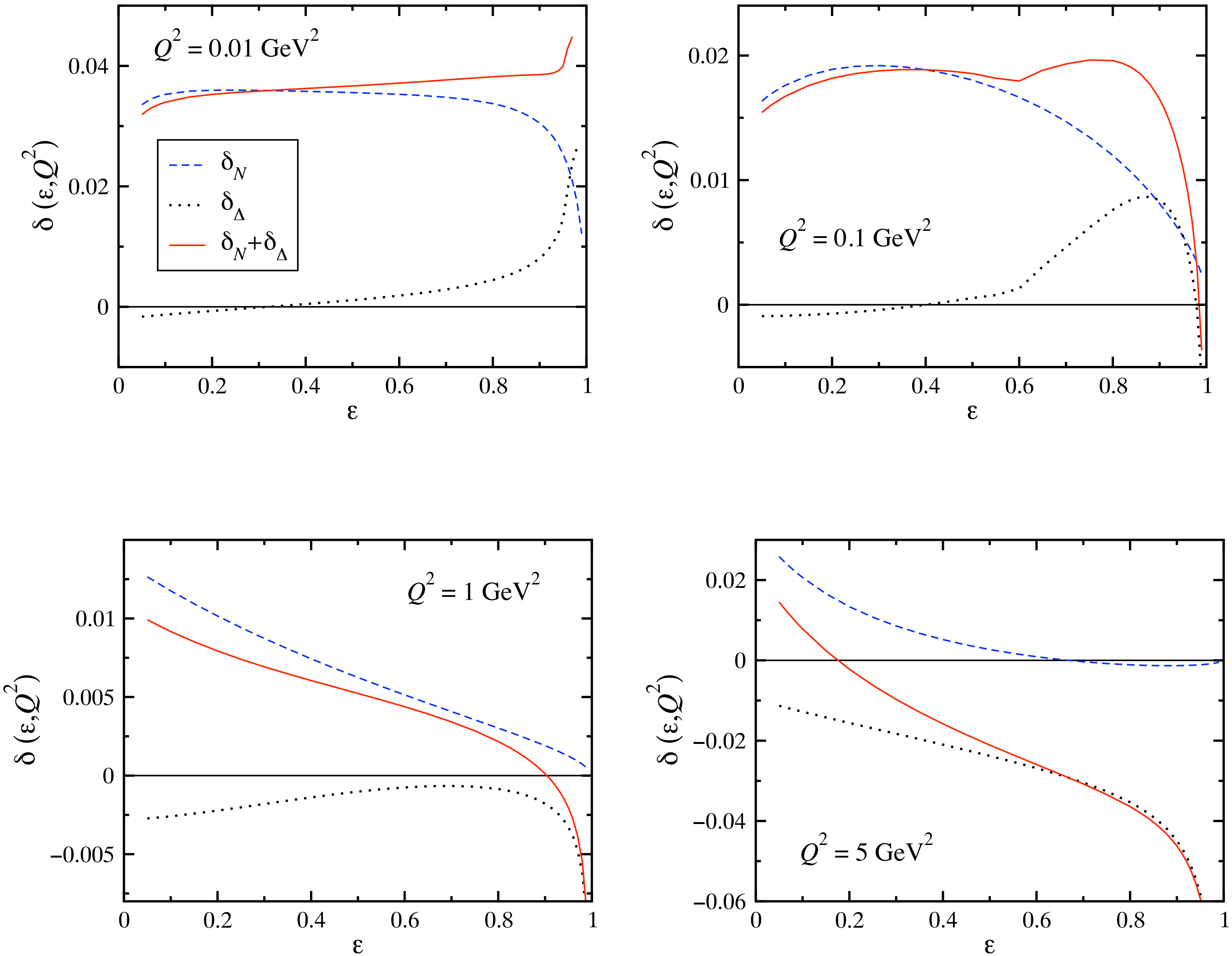
Tjon, Blunden & Melnitchouk, nucl-th/0903.2759



$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

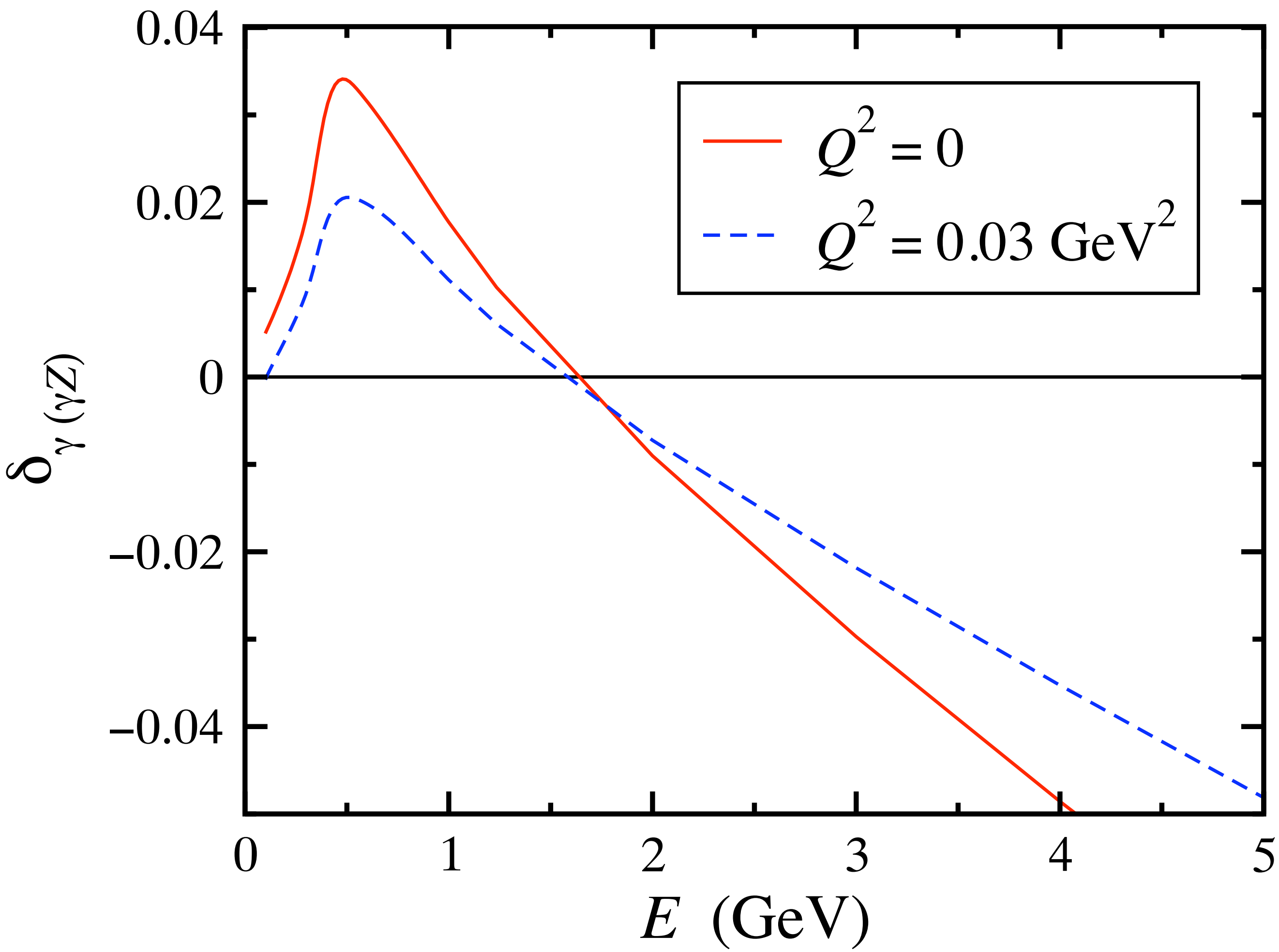


Nucleon and Delta contribution



$\delta_{\gamma(\gamma Z)}$ Δ contribution enhanced at forward angles and low Q^2

enhancement $(1 + Q_{\rm weak})/Q_{\rm weak} \approx 14$



γZ contribution to Q_{weak} using dispersion relations

(Gorchtein & Horowitz, PRL 2009)

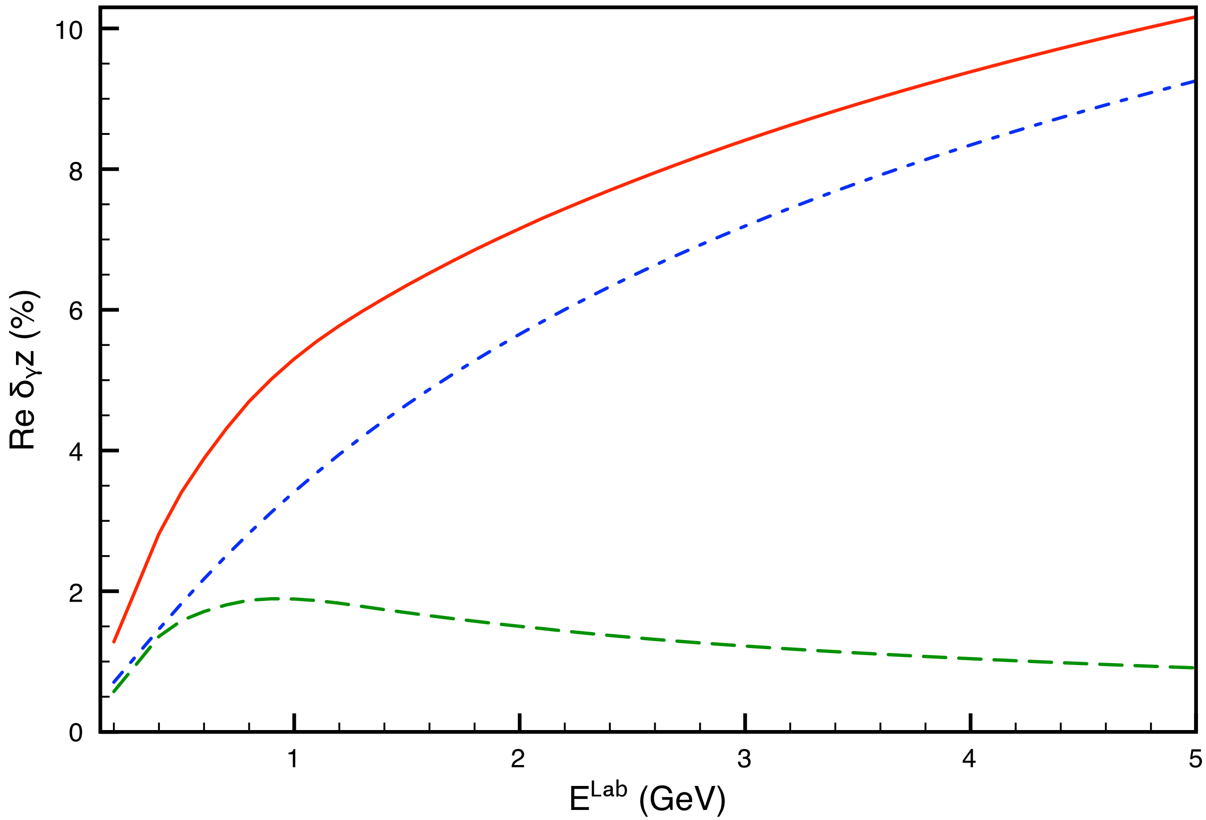
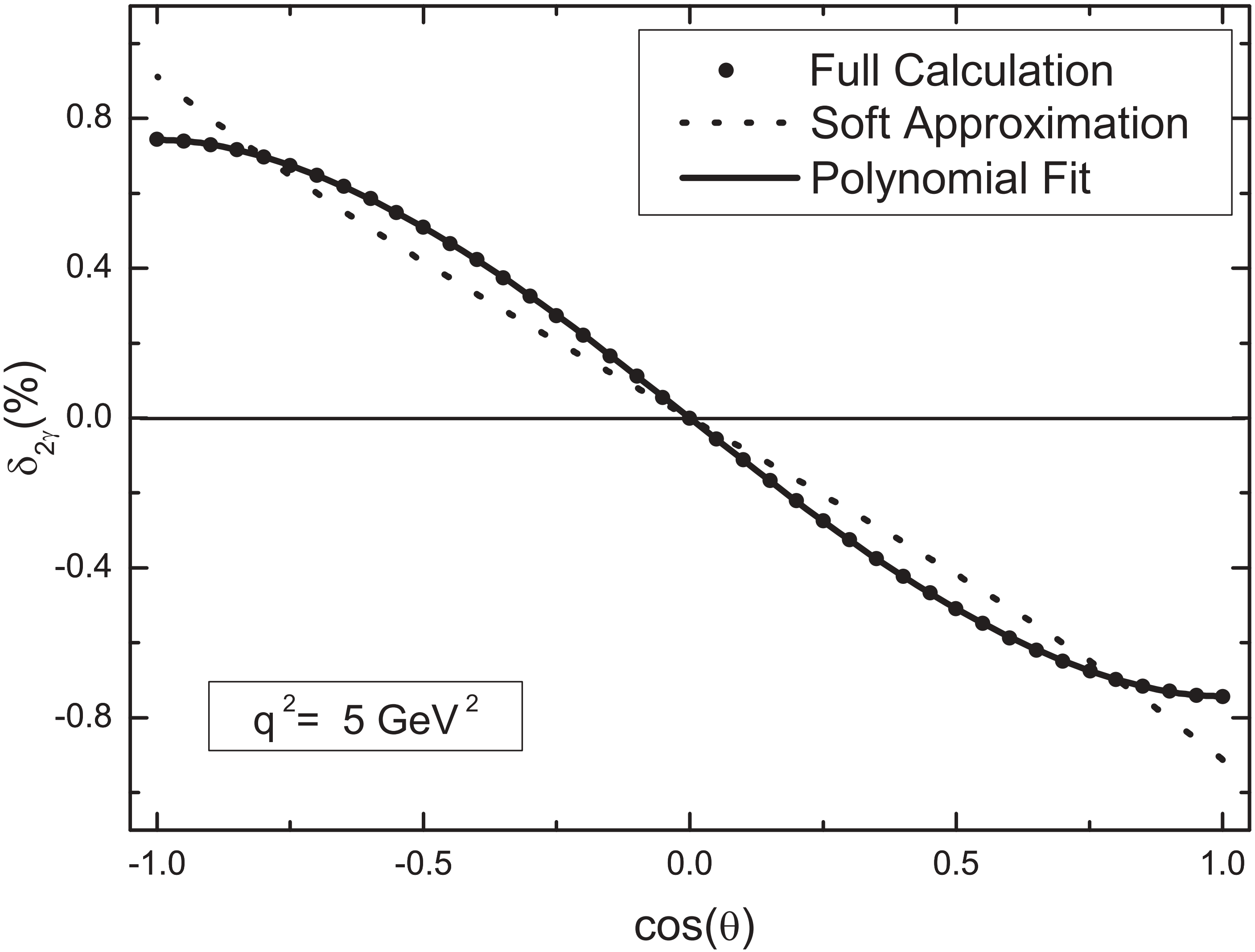


FIG. 3: Results for $\text{Re}\delta_{\gamma Z_A}$ as function of energy. The contributions of nucleon resonances (dashed line), Regge (dash-dotted line) and the sum of the two (solid line) are shown.

TPE contribution to proton FF's in time-like region:

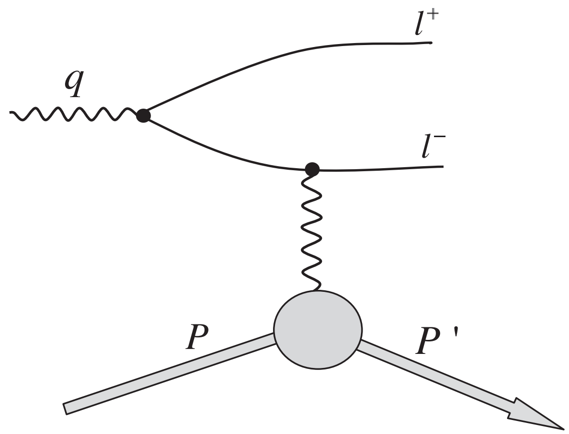
$$e^+ + e^- \to p + \bar{p}$$

Chen, Zhou & Dong, PRC **78** (2008)



Lepton-antilepton photoproduction using real photons

(Pervez Hoodbhoy, PRD 2006)



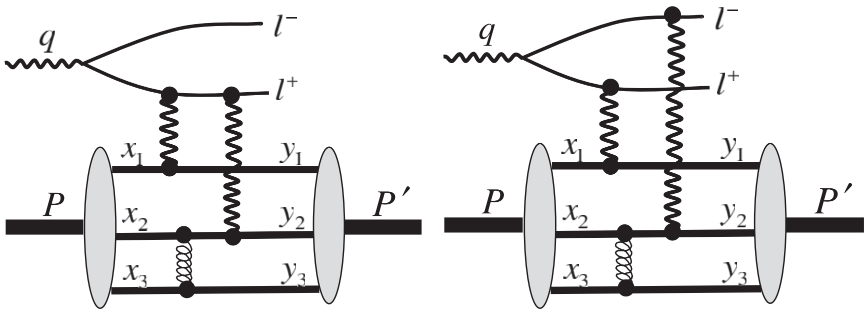


FIG. 6. Typical diagrams for lepton pair production from a 3-quark proton.

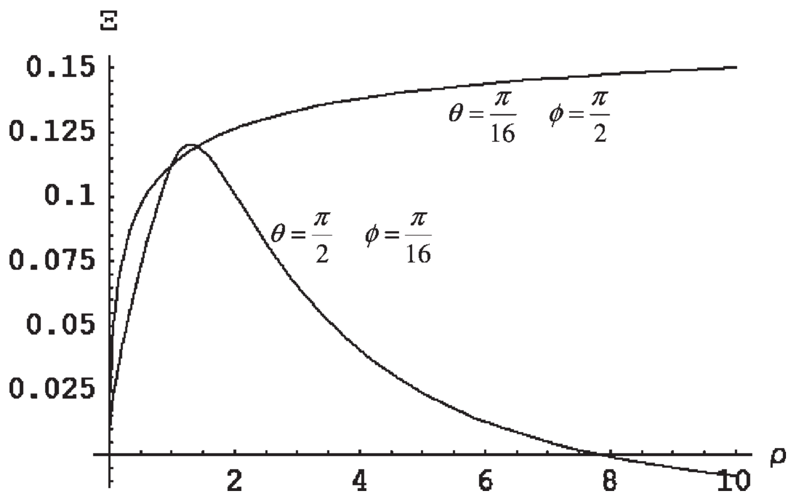


FIG. 7. Lepton pair asymmetry from a proton target.

Outlook

- Use phenomenological form factors in analyzing data, extracting strange form factors, etc.
- Merge hadronic models with GPD or pQCD calculations for $\gamma\gamma$ and γZ?
- Recent work on TPE seems to indicate insensitivity to off-shell form factors

Collaborators: Melnitchouk, Tjon + Kondratyuk